Nonlinear kinetic theory of the free-electron laser

R. Pratap* and A. Sen
Institute for Plasma Research, Bhat, Gandhinagar 382 424, India
(Received 6 July 1990)

A kinetic analysis of the nonlinear evolution of the free-electron-laser (FEL) instability is presented. The governing equations are the coupled Vlasov-Maxwell equations, which are investigated for a system consisting of a relativistic electron beam propagating through a helical wiggler magnetic field. Assuming that a single cavity mode of the electromagnetic field takes part in the lasing, a general nonlinear solution of the Vlasov equation is obtained in the resolvent formalism. Use of this solution in the wave equation provides a nonlinear description of the FEL. The saturation properties of the FEL are discussed by numerical and analytical solutions of this equation.

I. INTRODUCTION

The nonlinear evolution of the free-electron-laser (FEL) instability is a subject of considerable interest, particularly the study of saturation properties.[1–16] While linear theory is adequate to describe the exponential gain regime, the saturation regime is difficult to model analytically, and it is customary to rely on the help of computer simulations. In this paper we develop an analytical model for the nonlinear regime, which is based on the resolvent formalism developed by Prigogine and his coworkers[17] within the framework of general statistical mechanics of nonequilibrium processes. The principal advantage of the method lies in its being a nonperturbative approach and in that it permits a general formulation of the nonlinear evolution problem in the strong-signal regime. For the example studied by us, that of a low-density relativistic electron beam propagating through a helical wiggler magnetic field to amplify a single cavity mode, the nonlinear evolution equation is quite compact and can be solved either numerically or by analytical techniques in some limiting situations. Another unique feature of the method, for the problem at hand, is that it allows an exact solution of the Vlasov equation for an arbitrary amplitude of the signal strength. Basically this is achieved by formally writing the solution of the Vlasov equation as an inhomogeneous Volterra equation and employing an iterative method to obtain an infinite-series solution. This series can be closely approximated by a geometric series and summed exactly. This solution is used in the wave equation to obtain a nonlinear evolution equation for the FEL. This equation can be further generalized to include dielectric effects (for Cerenkov radiation problems), self-fields, and other wiggler geometries. For the small-signal limit it easily reduces to the standard evolution equation discussed in the literature.[1] We discuss the saturation properties of the FEL by analyzing the evolution equation for a simple Gaussian form of the initial beam distribution function.

The paper is organized as follows. Section II describes the basic equations and the physical model. The coupled Vlasov-Maxwell equations are reduced to one-dimensional forms under some standard approximations. In Sec. III we obtain a nonlinear solution to the Vlasov equation in the resolvent formulation. This solution is used in Sec. IV to obtain the final nonlinear evolution equation for the laser wave amplitude, and this equation is solved both numerically and analytically. Saturation properties are discussed. Conclusions and summary discussions are given in Sec. V.

II. BASIC EQUATIONS

The dynamical system consists of a beam of relativistic electrons traveling through a spatially periodic static magnetic field. We assume that the electron density is sufficiently low that self-fields as well electrostatic effects (representative of collective effects) can be neglected. The relativistic Vlasov equation governing the dynamics of the system is given by

$$\frac{\partial f}{\partial t}+\frac{\partial \mathcal{H}}{\partial \mathbf{P}}\frac{\partial f}{\partial \mathbf{X}}-\frac{\partial \mathcal{H}}{\partial \mathbf{X}}\frac{\partial f}{\partial \mathbf{P}}=0\ , \tag{1}$$

where the Hamiltonian for the single electron is given by

$$\mathcal{H}=\left[m^2c^4+c^2\left[\mathbf{P}-\frac{e\,\mathbf{A}}{c}\right]^2\right]^{1/2}, \tag{2}$$

m and e being the mass and charge of the electron, $\mathbf{P}$ the canonical momentum, and $\mathbf{A}$ the vector potential. $\mathbf{A}$ can be written in terms of a sum of plane waves representing the laser field and the static wiggler magnetic field. We shall assume that only one mode takes part in the lasing. Further, choosing a helical wiggler field (right circularly polarized), the total field in the cavity can be written as

$$\mathbf{A}=\sqrt{2}\hat{\mathbf{e}}_x[A_i\cos(k_iz+\omega_it)+A_s\cos(k_sz-\omega_st)] +\sqrt{2}\hat{\mathbf{e}}_y[-A_i\sin(k_iz+\omega_it)+A_s\sin(k_sz-\omega_st)]\ , \tag{3}$$

where $\hat{\mathbf{e}}_x,\hat{\mathbf{e}}_y$ are unit polarization vectors. This form is known in the literature as the Williams-Weizsacker approximation,[18] in which the static magnetic field is replaced by a traveling wave propagating in the opposite direction to the electron beam. In the above, A_i is the amplitude of the wiggler field and A_s is the amplitude of the laser mode which is assumed to be a slowly varying function of time. The Hamiltonian $\mathcal{H}$ leads to the equations of motion

$$\frac{d\mathbf{X}}{dt}=\frac{\partial\mathcal{H}}{\partial\mathbf{P}}=\mathbf{V}\ , \tag{4}$$

$$\frac{d\mathbf{P}}{dt}=-\frac{\partial\mathcal{H}}{\partial\mathbf{X}}\ . \tag{5}$$

Since there is no dependence on the transverse coordinates, Eq. (5) gives

$$\frac{d\mathbf{P}_T}{dt}=0\ .$$

We choose the transverse canonical momentum $\mathbf{P}_T=0$, so that the kinetic momentum $\mathbf{p}_T$ is given by

$$\mathbf{p}_T=-\frac{e\,\mathbf{A}}{c}\ . \tag{6}$$

Likewise $p_z=P_z$ (since $A_z=0$), and from (5) we have

$$\frac{dp_z}{dt}=-\frac{\partial\mathcal{H}}{\partial z}=-\frac{e^2}{2m\gamma c^2}\frac{\partial A^2}{\partial z}\ , \tag{7}$$

where

$$\gamma=\left[\frac{p_z^2}{m^2c^2}+\frac{e^2A^2}{m^2c^4}+1\right]^{1/2} \tag{8}$$

is the relativistic factor. From (4) we also obtain

$$V_z=\frac{p_z}{m\gamma},\quad \mathbf{V}_T=-\frac{e\,\mathbf{A}}{mc\gamma}\ .$$

Substituting these into the Vlasov equation we get

$$\frac{\partial f}{\partial t}-\frac{e\,\mathbf{A}}{mc\gamma}\cdot\nabla_T f+\frac{p_z}{m\gamma}\frac{\partial f}{\partial z}-\frac{e^2}{2m\gamma c^2}\frac{\partial A^2}{\partial z}\frac{\partial f}{\partial p_z}=0. \tag{9}$$

We derive a one-dimensional equation from (9) by integrating over the transverse positions and momenta and noting that f vanishes at infinity. Defining the one-dimensional distribution function h by

$$h(p_z,z,t)=\frac{1}{a_0}\int dx\,dy\,dP_x dP_y f\ , \tag{10}$$

where a_0 is the electron-beam area, we obtain from (9) the following one-dimensional form:

$$\frac{\partial h}{\partial t}+V_z\frac{\partial h}{\partial z}-\frac{e^2}{2m\gamma c^2}\frac{\partial A^2}{\partial z}\frac{\partial h}{\partial p_z}=0\ . \tag{11}$$

h is normalized as follows:

$$\int_0^L dz\int_{-\infty}^{\infty}dp_z h(z,p_z,t)=\frac{N}{a_0}\ , \tag{12}$$

where L is the cavity length. The longitudinal force acting on the electrons is seen to be proportional to the gradient of A^2 in the Vlasov equation. This is the so-called ponderomotive force that provides the basic mechanism for electron bunching and "stimulated" emission. The evolution of $\mathbf{A}$ is given by the wave equation

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{A}=-\frac{4\pi}{c}\mathbf{J}_T\ , \tag{13}$$

where $\mathbf{J}_T$, the transverse current density, is defined by

$$\mathbf{J}_T=e\int d^3P\,\mathbf{V}_T f(\mathbf{X},\mathbf{P},t)\ . \tag{14}$$

Integrating (13) over the transverse dimensions and using the relation for $\mathbf{V}_T$, we obtain

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{A}=\frac{4\pi e^2}{mc^2}F\,\mathbf{A}\int_{-\infty}^{\infty}dp_z\frac{h}{\gamma}\ , \tag{15}$$

where F, the filling factor, is the ratio of the electron-beam area to the laser-mode area. Equation (15) can be further simplified by substituting the form of $\mathbf{A}$ from (3) and using the standard approximations of treating A_s to be slowly varying and projecting $\mathbf{A}$ onto the lasing mode. The evolution equation for the mode amplitude can then be reduced to

$$\frac{\partial A_s}{\partial t}=-\frac{2\pi e^2FA_i}{mc^2\omega_sL}\int_0^L dz\sin(Kz+\Omega t)\int dp_z\frac{h}{\gamma}\ , \tag{16}$$

where $K=k_s+k_i$ and $\Omega=\omega_i-\omega_s$. Equations (11) and (16) are the basic classical equations for the free-electron laser and the starting point of several investigations in the past. Further details of the derivation sketched in this section can be found in many past works— that of Hopf *et al.*[1] is one such good reference.

III. NONLINEAR SOLUTION OF THE VLASOV EQUATION

In this section we will obtain an analytic solution of the one-dimensional nonlinear Vlasov equation (11). Before proceeding to do so we note that the ponderomotive-force term is of a sinusoidal form. This can be seen by writing A^2 in detail, using expression (3) and noting that the z dependence only exists in the cross terms proportional to A_iA_s. Specifically we get

$$\frac{\partial A^2}{\partial z}=-KA_i^*A_s\sin(Kz+\Omega t)\ . \tag{17}$$

We substitute this in (11) and rewrite the equation in the following dimensionless form as

$$\frac{\partial h}{\partial t}+\xi\frac{\partial h}{\partial z}+\alpha\sin\Psi\frac{\partial h}{\partial\xi}=0\ , \tag{18}$$

where $t=\Omega t$, $z=kz$; $\xi=kV_z/\Omega$, $\alpha=e^2A_i^*A_sK^2/2m^2\gamma^2\Omega^2c^2$, and $\Psi=Kz+\Omega t$. In order to obtain a solution of (18) in the strong-signal regime, we adopt the slowly varying amplitude ansatz similar to the quasi-Bloch approach in high-energy-laser theory. Thus we represent h by a harmonic expansion,

$$h=h^{(0)}(\xi)+\alpha\sum_{n=0}^{\infty}[C_n(\xi,z)\cos n\Psi+S_n(\xi,z)\sin n\Psi]\ . \tag{19}$$

$h^{(0)}(\xi)$ is the equilibrium distribution function and C_n, S_n are assumed to be independent of t and slowly varying in z compared to K^{-1}. Our aim is to solve for S_1 (which drives the lasing mode) to all orders of α, and not arbitrarily truncate the series (19). We note that S_1 is coupled to S_2 and S_0 through the nonlinear term and likewise S_0 and S_2 are themselves further coupled to the other neighboring harmonics, and so on, *ad infinitum*.

Substituting (19) in (18) we get an infinite set of coupled equations, which can be expressed compactly in a matrix form as

$$\frac{\partial H_n}{\partial z}+\mathcal{L}H_n=-\alpha(\delta L)(H_{n+1}-H_{n-1})+2\delta_{n,1}(\delta L)\bar{h}^{(0)} . \tag{20}$$

Here

$$H_n=\begin{bmatrix} C_n \\ S_n \end{bmatrix},\quad \mathcal{L}=\bar{I}n\left[1+\frac{1}{\xi}\right],\quad \delta L=\bar{I}\frac{\partial}{\partial \xi^2} ,$$
$$\bar{I}=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\quad \bar{h}^{(0)}=\begin{bmatrix} 1 \\ 0 \end{bmatrix} h^{(0)} \tag{21}$$

and $\delta_{n,1}$ is the Kronecker delta function. A formal solution of Eq. (20) can be written in the form of an inhomogeneous Volterra equation,

$$H_n=e^{-\mathcal{L}z}H_n(0)+2\delta_{n,1}\int_0^z dz_1 e^{-\mathcal{L}(z-z_1)}(\delta L)\bar{h}^{(0)}$$
$$-\alpha\int_0^z dz_1 e^{-\mathcal{L}(z-z_1)}(\delta L)[H_{n+1}(z_1)-H_{n-1}(z_1)] . \tag{22}$$

Equation (22) can be iterated by expanding H_n formally as

$$H_n(z)=H_n^{(0)}+\alpha H_n^{(1)}+\alpha^2 H_n^{(2)}+\cdots \tag{23}$$

and substituting in Eq. (22). Comparing coefficients of α gives us an infinite sequence of relations between the $H_n^{(1)}$. Choosing $H_n(0)\equiv 0$ (i.e., the beam is smooth at the entrance of the wiggler), we get

$$H_n^{(0)}=+2\delta_{n,1}\int_0^z dz_1 e^{-\mathcal{L}(z-z_1)}(\delta L)\bar{h}^{(0)} ,$$
$$H_n^{(1)}=-2(\delta_{n+1,1}-\delta_{n-1,1})\int_0^z dz_1 e^{-\mathcal{L}(z-z_1)}(\delta L)\int_0^{z_1} dz_2 e^{-\mathcal{L}(z_1-z_2)}(\delta L)\bar{h}^{(0)} . \tag{24}$$

The integrals in the above relation are in convolution form and hence can be conveniently carried out in terms of the Laplace transform, which for a function $f(z)$ is defined as

$$\tilde{f}(s)=\int_0^\infty dz\, e^{isz}f(z) . \tag{25}$$

For $n=1$ only the even terms $(H_n^{(0)},H_n^{(2)},H_n^{(4)}\cdots)$ contribute to the series (23). Taking the Laplace transform for this case, we get

$$\alpha\tilde{H}_1(s)=\left[-2a+\sum_{m=0}^{\infty}(-1)^m C_{m+1}^{2m+2}a^{2m+3}\right]\bar{h}^{(0)} , \tag{26}$$

where a is the operator

$$a=\alpha\frac{1}{\mathcal{L}-is}(\delta L) \tag{27}$$

and C_n^m is the combinatorial factor. This infinite series is absolutely convergent for $|a^2|<\frac{1}{4}$. The series can be closely approximated by

$$\alpha\tilde{H}_1=2a\frac{1}{1+2a^2}\bar{h}^{(0)} . \tag{28}$$

Taking the inverse Laplace transform of (28) we get

$$H_1(z)=\int_0^z dz_1 e^{-\mathcal{L}(z-z_1)}(e^{\sqrt{2}\alpha\delta L(z-z_1)}+e^{-\sqrt{2}\alpha\delta L(z-z_1)})\delta L\bar{h}^{(0)} . \tag{29}$$

In carrying out the inverse Laplace transform we have made use of the commutation relation $\delta L.(\mathcal{L}-is)=(\mathcal{L}-is)\delta L+\bar{I}(1/2\xi^3)$ and neglected the $\bar{I}(1/2\xi^3)$ term. This is valid for $\alpha \gg \zeta_{\rm th}^2/\xi^2$, where ζth is the typical thermal spread in the equilibrium distribution function. The above integration can be carried out for a simple beam equilibrium distribution function $h^{(0)}$,

$$h^{(0)}=(\beta/\pi)^{1/2}e^{-\beta(\xi-\xi_0)^2} , \tag{30}$$

where ξ_0 is the mean axial momentum and $\beta^{-1/2}$ is the thermal spread.

Using (30) in Eq. (2) we obtain an analytic expression for S_1 as

$$S_1(z)=-\left[\frac{\sin(l+\sqrt{2}\alpha\beta)z}{(l+\sqrt{2}\alpha\beta)}+\frac{\sin(l-\sqrt{2}\alpha\beta)}{(l-\sqrt{2}\alpha\beta)}\right]\frac{dh^0}{d\xi^2}\ , \qquad (31)$$

where $l=1+1/\xi$. It may be realized that S_1 is the coefficient of $\sin\Psi$ in the expansion (19), and we shall now substitute $S_1\sin\Psi$ in Eq. (16) and reduce this equation to a phase-independent one defining the nonlinear evolution equation for A_s. For $\alpha\to 0$, $S_1(z)$ reduces to the linear distribution function used in calculating gain functions in the small-signal regime.

IV. NONLINEAR EVOLUTION EQUATION

Substituting $h=\alpha S_1(z)\sin\Psi$ in Eq. (16), averaging over the fast phase Ψ, and carrying out the z integration we get

$$\frac{dA_s}{dt}=\frac{2\pi e^4F|A_i|^2K^2L}{m^2\gamma^2\Omega c^2}A_s\int d\xi\frac{\partial h^{(0)}}{\delta\xi^2}\left[\frac{2(1+2\alpha^2\beta^2/l^2)}{(1-2\alpha^2\beta^2/l^2)^2}\left[\frac{1-\cos lKL\cos\sqrt{2}\alpha\beta KL}{l^2K^2L^2}\right]-\frac{4\sqrt{2}\alpha\beta KL}{(1-2\alpha^2\beta^2/l^2)^2}\frac{\sin lKL\sin(\sqrt{2}\alpha\beta KL)}{K^3l^3L^3}\right]. \qquad (32)$$

The above equation can be cast in a more convenient form by converting to the detuning variable $\mu=-\Omega(K/\Omega-1/V_z)=-Kl$, and integrating the right-hand side (rhs) of (32) by parts. This yields

$$\frac{1}{A_s}\frac{dA_s}{dt}=\frac{2\pi e^4F|A_i|^2K^2L^2}{m^2\gamma^2\Omega^2c^2}\int d\mu h(0)\frac{d}{d(\mu L)}\left[\frac{(1+2\alpha^2\beta^2K^2L^2/\mu^2L^2)}{(1-2\alpha^2\beta^2K^2L^2/\mu^2L^2)^2}\left[\frac{1-\cos\mu L\cos(\sqrt{2}\alpha\beta KL)}{\mu^2L^2}\right]-\frac{2\sqrt{2}\alpha\beta KL}{(1-2\alpha^2\beta^2K^2L^2/\mu^2L^2)^2}\frac{\sin\mu L\sin(\sqrt{2}\alpha\beta KL)}{\mu^2L^2}\right]. \qquad (33)$$

(Note that $t=\Omega t$ in the above equation.)

The rhs of (33) can be viewed as the nonlinear gain function for the FEL, valid in the strong-signal regime. It involves a product of two terms within the integrand $-h^{(0)}(\mu)$ the initial distribution and a nonlinear function of the wave amplitude. For the "small cavity" or "cold beam" limit (i.e., for the $\beta\gg$width of the nonlinear function) Eq. (33) can be simplified to

$$\frac{1}{A_s}\frac{dA_s}{dt}=\frac{2\pi e^4F|A_i|^2K^2L^2}{m^2\gamma^2\Omega^2c^2}\frac{d}{d(\mu_0L)}\left[\frac{(1-2\alpha^2\beta^2K^2L^2/\mu_0^2L^2)}{(1+2\alpha^2\beta^2K^2L^2/\mu_0^2L^2)^2}\left[\frac{1-\cos\mu_0L\cos(\sqrt{2}\alpha\beta KL)}{\mu_0^2L^2}\right]-\frac{2\sqrt{2}\alpha\beta KL}{(1-2\alpha^2\beta^2K^2L^2/\mu_0^2L^2)^2}\frac{\sin\mu_0L\sin(\sqrt{2}\alpha\beta KL)}{\mu_0^3L^3}\right], \qquad (34)$$

where μ_0 is the value of μ where $h^{(0)}$ is centered. The familiar small-signal result can be easily recovered from (34) in the limit of $\alpha\to 0$ (i.e., neglecting α^2 terms). We get

$$\frac{1}{A_s}\frac{dA_s}{dt}=\frac{2\pi e^4F|A_i|^2K^2L^2}{m^2\gamma^2\Omega^2c^2}\frac{d}{d(\mu_0L)}\left[\frac{\sin\mu_0L/2}{\mu_0L}\right]^2. \qquad (35)$$

This has the characteristic antisymmetric form of stimulated scattering and shows maximum gain to occur at $\mu_0L=-\pi$. Retaining the nonlinear terms modifies the linear gain function and leads to a shift in the phase for the maximum gain as well as saturation effects. To discuss these properties we rewrite (34) in a more useful form by introducing the variables

$$x=\mu_0L\ ,$$

$$y=\sqrt{2}\alpha\beta KL\ ,$$

and the constant

$$R=\frac{2\pi e^4F|A_i|^2K^2L^2}{m^2\gamma^2\Omega^2c^2}\ .$$

We then get

$$\frac{1}{y}\frac{dy}{dt}=G(x,y)=R\frac{d}{dx}\left[\frac{(x^2+y^2)}{(x^2-y^2)^2}(1-\cos x\cos y)-\frac{2xy}{(x^2-y^2)^2}\sin x\sin y\right]. \qquad (36)$$

Equation (36) offers a simple model nonlinear evolution equation for the single-cavity mode FEL. The nonlinear gain function $G(x,y)$ is expressed in a simple analytic form in terms of the detuning parameter and the laser-mode amplitude. In the low-gain regime (away from resonance, i.e., for $x \ll -\pi$) the nonlinear gain function decreases as a function of y, leading to saturation of the mode. An approximate expression for the saturation amplitude can be obtained by expanding (36) for small y (retaining y^2 terms). This yields

$$\frac{1}{y}\frac{dy}{dt}=R[\lambda_1(x)+\lambda_2(x)y^2]\ , \tag{37}$$

where

$$\lambda_1(x)=-\frac{2}{x^3}(1-\cos x)+\frac{\sin x}{x^2}\ , \tag{38}$$

$$\lambda_2(x)=-\frac{12}{x^5}(1-\cos x)+\frac{9\sin x}{x^4}-\frac{3\cos x}{x^3}-\frac{\sin x}{2x^2}\ . \tag{39}$$

Equation (37) can be integrated to give

$$y^2=\frac{\lambda_1 y_0^2}{\lambda_1+\lambda_2 y_0^2}\,\frac{e^{2\lambda_1 Rt}}{1-[\lambda_2 y_0^2/(\lambda_1+\lambda_2 y_0^2)]e^{2\lambda_1 Rt}}\ . \tag{40}$$

The saturation amplitude is then obtained as

$$y_{\rm sat}^2 \rightarrow \left|\frac{\lambda_1}{\lambda_2}\right|\ . \tag{41}$$

For x closer to the resonance region, the gain function can be evaluated numerically and we show two typical plots of $G(x,y)$ versus y for $x=-4.0$, -3.0 in Fig. 1. The corresponding numerical solution of Eq. (36) for the mode amplitude y is displayed in Fig. 2 (R has been taken to be unity).

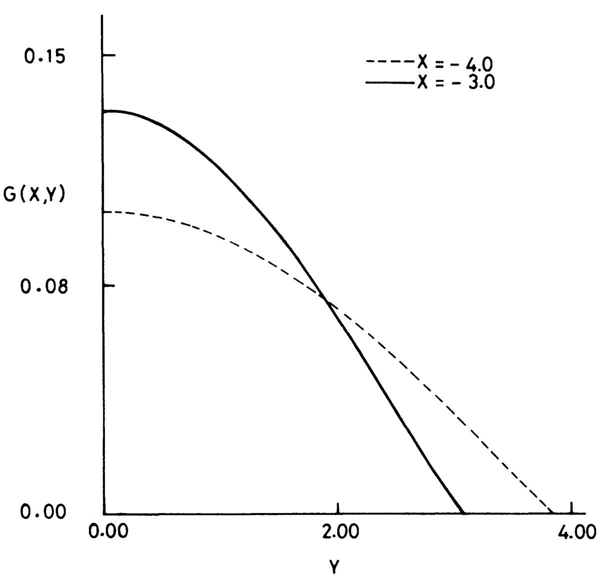

FIG. 1. Behavior of gain function at $x=-4.0$ and -3.0 ($R=1$).

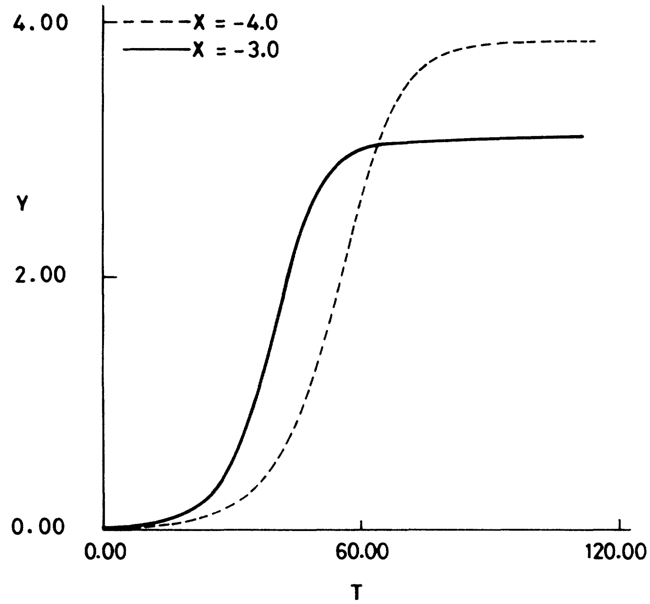

FIG. 2. Time evolution of lasing mode for $x=-4.0$ and -3.0 ($R=1$).

V. CONCLUSIONS AND SUMMARY

We have carried out a nonlinear analysis of the free-electron-laser instability based on a nonperturbative solution of the coupled Vlasov-Maxwell equations. The approach is based on the resolvent formalism developed by Prigogine and co-workers and yields an infinite-series solution to an integral equation formulation of the Vlasov equation. Using this solution, we obtain a model nonlinear evolution equation for the amplification of a single-cavity mode driven by a low-density electron beam propagating through a helical wiggler magnetic field. This equation, Eq. (36), is the principal result of our calculation. It offers a simple analytic model for the saturation regime properties of the FEL and can be solved quite easily either numerically or analytically in limiting situations. The nonlinear saturation is due to the self-interaction of the mode and this effect has been incorporated by a nonperturbative solution of the Vlasov equation. Equation (36) also correctly describes the exponential gain behavior in the small-amplitude regime. Our approach has similarities with the quasi-Bloch approach, adopted in high-energy laser physics, but our iterative solution goes beyond the usual truncation of the harmonic expansion. The model can be further refined to include self-fields, different wiggler geometries, electrostatic effects, etc., and we are currently investigating these aspects.

ACKNOWLEDGMENTS

We would like to thank Professor P. K. Kaw for his keen interest in the present problem and for many useful discussions. One of us (R.P.) would like to thank him for extending hospitality at the Institute for Plasma Research during the period of this investigation.

*Department of Physics, Cochin University of Science and Technology, Cochin 682 022, India.

[1] F. A. Hopf, P. Meystre, G. T. Moore, and M. O. Scully, *Novel Sources of Coherent Radiation*, Vol. 5 of *Physics of Quantum Techniques* (Addison-Wesley, Reading, MA, 1978).
[2] N. M. Kroll, P. L. Morton, and M. N. Rosenbluth, IEEE J. Quantum Electron. **QE-17**, 1436 (1981).
[3] T. Taguchi, K. Mima, and T. Mochizuki, Phys. Rev. Lett. **46**, 824 (1981).
[4] R. C. Davidson and W. A. McMullin, Phys. Rev. A **26**, 410 (1982).
[5] N. S. Ginzburg and M. A. Shapiro, Opt. Commun. **40**, 215 (1982).
[6] W. B. Colson, IEEE J. Quantum Electron. **QE-17**, 1417 (1981).
[7] P. Sprangle, C. M. Tang, and W. M. Manheimer, Phys. Rev. A **21**, 302 (1980).
[8] B. Lane and R. C. Davidson, Phys. Rev. A **27**, 2008 (1983).
[9] A. M. Dimos and R. C. Davidson, Phys. Fluids **28**, 677 (1985).
[10] R. C. Davidson, Phys. Fluids **29**, 2689 (1986).
[11] R. C. Davidson, J. S. Wurtele, and R. E. Aamodt, Phys. Rev. A **34**, 3063 (1986).
[12] N. S. Ginzburg and M. I. Petelin, Int. J. Electron. **59**, 291 (1985).
[13] W. H. Louisell, J. F. Lam, D. A. Copeland, and W. B. Colson, Phys. Rev. A **19**, 288 (1979).
[14] R. C. Davidson, G. L. Johnston, and A. Sen., Phys. Rev. A **34**, 392 (1986).
[15] R. C. Davidson and J. S. Wurtele, Phys. Fluids **30**, 557 (1987).
[16] I. Kimel and L. R. Elias, Nucl. Instrum. Methods Phys. Res. A **285**, 132 (1989).
[17] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics* (Addison and Wesley, New York, 1975).
[18] R. H. Pantell, G. Soncini, and H. E. Puthoff, IEEE J. Quantum Electron. **4**, 905 (1971).