PHYSICAL REVIEW A VOLUME 42, NUMBER 1 1 JULY 1990

Dressed-state lasers and masers

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(Received 21 February 1990)

We show that laser and maser action is possible using atoms that have been prepared in dressed states. We develop the microscopic theory and derive the Fokker-Planck equation for the field distribution for such a laser (maser) system.

In the normal operation of lasers and masers one assumes that the atoms are continuously pumped into an excited state to create population inversion which in turn amplifies an input radiation. Recently, however, it has been demonstrated using various models that the population inversion is not essential for achieving laser action.[1-5] The basic idea behind all these models is to modify the absorption line shapes. This can be done either by using external fields[1] or by using level schemes in which several pathways for absorption exist such that there is a possibility of destructive interference[2,3] between these path ways. In this Rapid Communication we demonstrate how *atomic coherence effects* can be used to achieve laser action without population inversion. We consider atoms injected in the cavity such that initially they are in a *dressed state.* The dressed-state preparation modifies the absorption and emission profiles.

Consider a two-level atom with states $|1\rangle$ and $|2\rangle$ separated by $\hbar\omega_0$. Consider its interaction with a *resonant* electromagnetic field such that the interaction Hamiltonian in the interaction picture is

$$H_e = -\hbar(gs^+ + g^*s^-), \quad g = \frac{\mathbf{d}\cdot\epsilon}{\hbar}, \quad s^+ = |1\rangle\langle 2| . \qquad (1)$$

We assume that the atom is initially prepared in the dressed-state $|\psi_+\rangle$ where dressed states are defined by

$$|\psi_\pm\rangle = 1/\sqrt{2}(|1\rangle \mp |g|/g|2\rangle), \quad H_e|\psi_\pm\rangle = \pm\hbar|g|\,|\psi_\pm\rangle . \qquad (2)$$

Mossberg and co-workers[6] have shown how the dressed states can be prepared experimentally by using $\pi/2$-resonant pulses and by switching the phase of the field. We assume that the pumping field is large compared to the decay rate of the excited state and the inverse of the time of flight. Note that (2) is such that the populations in the bare states $|1\rangle$ and $|2\rangle$ are equal and the atomic coherence ρ_{12} is maximum

$$\rho_{11}-\rho_{22}=0, \quad \rho_{12}=-\frac{|g|}{2g^*} . \qquad (3)$$

The radiative properties of the system in the state (2) can be calculated from the dipole-dipole correlation function.[7] It can be shown that

$$\langle s^+(t+\tau)s^-(t)\rangle = \tfrac{1}{4}(1+e^{2i|g|\tau})e^{i\omega_0\tau} . \qquad (4)$$

Thus the spontaneous emission spectrum will consist of lines at

$$\omega=\omega_0, \quad \omega_0+2|g| . \qquad (5)$$

The amplification of an input field can be studied in terms of a different correlation function,[8] viz., $\langle[s^+(t+\tau), s^-(t)]\rangle$, which on using Eqs. (1) and (2) is found to be

$$\langle[s^+(t+\tau), s^-(t)]\rangle = \tfrac{1}{4}[e^{2i|g|\tau}-e^{-2i|g|\tau}]e^{i\omega_0\tau} . \qquad (6)$$

Thus the input field in the *frequency range* $\omega_0+2|g|$ *will be amplified.* However the field in the frequency region $\omega_0-2|g|$ will experience absorption. Therefore it is clear that we have the possibility of *laser or maser action* in a single-mode cavity if the *cavity frequency* is tuned in the region $\omega_0+2|g|$.

Having discussed the mechanism, we now consider the derivation of the laser-maser equations[9] for the model shown schematically in Fig. 1. Note that the geometry is such that the macroscopic polarization associated with the dressed states does not drive the cavity mode. The interaction Hamiltonian of the atoms and the cavity mode can be written as

$$H_c = -\hbar(g_c s^+ a e^{-i(\omega_c-\omega_0)t} + g_c^* s^- a^+ e^{i(\omega_c-\omega_0)t}) , \qquad (7)$$

where ω_c is the frequency of the cavity mode. The total Hamiltonian of the atom-cavity mode system is H_e+H_c.

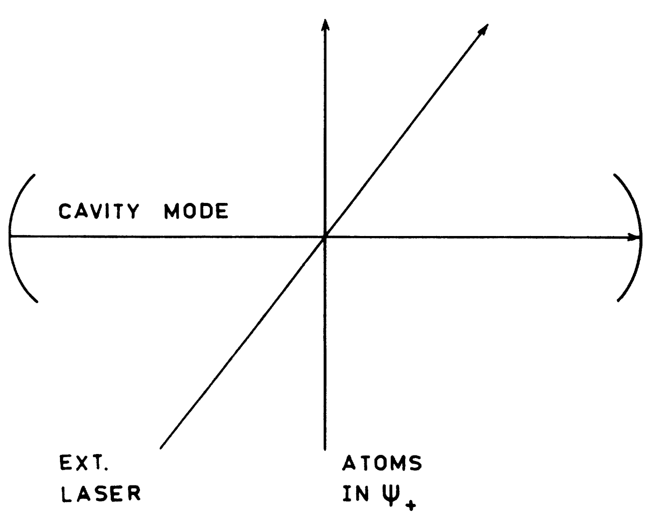

FIG. 1. Schematic diagram of the model.

We now make canonical transformation to a new interaction picture so that the interaction Hamiltonian (7) becomes

$$\tilde{H}_c = e^{iH_e t/\hbar} H_c e^{-iH_e t/\hbar} . \quad (8)$$

On introducing new angular momentum operators $R^{\pm}$, R^z in terms of the dressed-state basis

$$R^+ = |\psi_+\rangle\langle\psi_-| , \quad R^- = |\psi_-\rangle\langle\psi_+| ,$$
$$R^z = \tfrac{1}{2}(|\psi_+\rangle\langle\psi_+| - |\psi_-\rangle\langle\psi_-|) , \quad (9)$$

we find that

$$\tilde{H}_c = -\frac{\hbar g g_c^*}{|g|} a^+ e^{i(\omega_c-\omega_0)t}\{-R^z + \tfrac{1}{2}R^- e^{-2i|g|t} - \tfrac{1}{2}R^+ e^{2i|g|t}\} + \text{H.c.} \quad (10)$$

In what follows we absorb the phase factor $g/|g|$ in the definition g_c^*. If we now assume that $\omega_c \sim \omega_0 + 2|g|$ and that $|g|$ is large, then one can make secular approximation which reduces $\tilde{H}_c$ to

$$\tilde{H}_c = -\frac{\hbar g_c^*}{2} a^+ R^- e^{i(\omega_c-\omega_0-2|g|)t} + \text{H.c.} , \quad (11)$$

$$= -\frac{\hbar g_c^*}{2} a^+ R^- + \text{H.c., if } \omega_c = \omega_0 + 2|g| . \quad (12)$$

For simplicity we derive laser-maser equation[10] for the resonant case only. Note that Eq. (12) shows how the present problem is isomorphic to the usual laser-maser problem. Table I compares the two situations.

The case of dressed-state maser is the simplest as then all complications associated with spontaneous emission can be ignored. From (12) it is clear that

$$\langle \dot{a} \rangle = +\frac{i g_c^*}{2} \sum_{t_j} \langle R^-(t-t_j) \rangle , \quad (13)$$

where we have now summed over all the atoms assuming that the jth atom enters the cavity at time t_j. The mean dipole moment can be calculated assuming that the field a is prescribed by using the Hamiltonian (12) and the initial condition $\psi_+(0) = 1$, $\psi_-(0) = 0$. The result is

$$\langle R^-(t) \rangle = -\frac{i g_c}{2} a t \left[\frac{\sin|g_c t \langle a \rangle|}{|g_c t \langle a \rangle|} \right] . \quad (14)$$

TABLE I. Comparison between the usual laser and the dressed-state laser.

	Usual laser	Dressed-state laser
Excited state	$\|1\rangle$	$\|\psi_+\rangle$
States are eigenstates of −	$s^z\|1\rangle = \frac{\omega_0}{2}\|1\rangle$	$R^z\|\psi_+\rangle = \frac{2\|g\|}{2}\|\psi_+\rangle$
Emission operator	$s^+ = \|1\rangle\langle 2\|$	$R^+ = \|\psi_+\rangle\langle\psi_-\|$
Atoms injected in the state	$\|1\rangle$	$\|\psi_+\rangle$
Frequency of emission, i.e., frequency at which amplification occurs	ω_0	$\omega_0 + 2\|g\|$

On substituting (14) in (13) and assuming that the atoms are injected at the rate Λ, we get

$$\langle \dot{a} \rangle = \frac{|g_c|^2 \langle a \rangle \Lambda}{4} \frac{1-\cos(g_c T|\langle a \rangle|)}{|g_c \langle a \rangle|^2} , \quad (15)$$

where T is the transit time through the cavity. For small photon numbers (15) can be approximated by

$$\langle \dot{a} \rangle \cong T^2 \frac{|g_c|^2 \langle a \rangle \Lambda}{8} \left[1 - \frac{|g_c|^2 |\langle a \rangle|^2 T^2}{12} \right] . \quad (16)$$

The two terms on the right-hand side of (16) have a simple interpretation—the first (second) term gives the linear gain (nonlinear loss). In order to obtain the full quantum-mechanical description[11] of the laser, we need to add the terms corresponding to losses from the cavity mirrors and the quantum noise terms. The quantum noise terms are related to the dipole-dipole correlation function (4). More precisely, the quantum noise term is found to be

$$q = \frac{\Lambda |g_c|^2}{4} \int_0^T dt_1 \int_0^{t_1} dt_2 [\langle R^+(t_1-t_2) R^-(t_1) \rangle - \langle R^+(t_1-t_2) \rangle \langle R^-(t_1) \rangle] , \quad (17)$$

which for the initial dressed-state preparation $|\psi_+\rangle$ reduces to

$$q = \frac{\Lambda |g_c|^2 T^2}{8} , \quad (18)$$

which is identical to the linear gain parameter as one would expect on physical grounds. On using (16) and (17) and on introducing P distribution for the radiation field, we find that the dynamics of the dressed-state maser is governed by the Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \alpha} \left[\alpha \left(\kappa - \frac{T^2 |g_c|^2 \Lambda}{8} + \frac{|g_c|^4 T^4 \Lambda}{96} |\alpha|^2 \right) P(\alpha) \right] + q \frac{\partial^2 P}{\partial \alpha \partial \alpha^*} + \text{c.c.} \quad (19)$$

Here $\kappa = \omega/2Q$, where Q is the quality factor of the cavity. The solution of the Fokker-Planck equation (19) can be written down from the earlier work of Risken and coworkers.[12] The only difference is that the coefficient in the Fokker-Planck equation (19) are those relevant for the dressed-state maser. A more exact dynamical equation can be obtained by using (14) in the same way as for the micromaser.[13] Thus the dressed state maser, like the micromaser, would show subpoissonian photon statistics in a certain parameter range.

Let us next discuss how (19) is modified for the case of dressed-state laser. We assume that the excited state $|1\rangle$ decays at the rate of 2γ to some other state $|3\rangle$. A calculation shows that the elements of the density matrix in the dressed-state basis decay at the rate γ, i.e.,

$$\dot{\rho}_{\alpha\beta} = -\gamma \rho_{\alpha\beta}, \quad \alpha = \pm, \quad \beta = \pm . \quad (20)$$

The diffusion coefficient in (17) is now obtained by calculating the correlation function including the decay (20) and by letting $T \rightarrow \infty$. We find the result

$$q = \frac{\Lambda |g_c|^2}{4\gamma^2} . \tag{21}$$

The linear gain and the nonlinear loss terms are obtained by integrating the right-hand side of (14) over the interval $(0,\infty)$ with weight factor $e^{-\gamma t}$. Using the result so obtained and (21) we find the Fokker-Planck equation for the dressed-state laser model[14]

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \alpha}\left[\alpha\left(\kappa - q + \frac{|g_c|^4 \Lambda |\alpha|^2}{4\gamma^4}\right)P(\alpha)\right] + q\frac{\partial^2 P(\alpha)}{\partial\alpha\partial\alpha^*} + \text{c.c.} \tag{22}$$

From the Fokker-Planck equations (19) and (22) it is clear that the physical properties of the dressed-state laser and maser will be identical to the properties of the conventional single-mode lasers and masers. It is, however, important to remember that the atoms are injected in the dressed-state $|\psi_+\rangle$. Thus initially the *population inversion in terms of the bare states of the atom is zero,* i.e., $\rho_{11} = \rho_{22}$, though the population inversion in terms of the dressed states is nonzero and which is responsible for laser action.

The author is grateful to the Department of Science and Technology, Government of India for partially supporting this work.

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[1]A. Lezema, Y. Zhu, M. Kanskar, and T. W. Mossberg, Phys. Rev. A **41**, 1576 (1990) use the amplification on three-photon Mollow sideband to produce laser action. G. S. Agarwal, *ibid.* **41**, 2886 (1990) has developed a quantum statistical modeling of such a laser.

[2]S. Harris, Phys. Rev. Lett. **62**, 1033 (1989); G. S. Agarwal, S. Ravi, and J. Cooper, Phys. Rev. A **41**, 4727 (1990).

[3]M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).

[4]D. Grandclément, G. Grynberg, and M. Pinard [Phys. Rev. Lett. **59**, 40 (1987); **59**, 44 (1987)] used pressure-induced resonances to produce laser oscillation.

[5]R. Ghosh and G. S. Agarwal [Phys. Rev. A **39**, 1582 (1989)] use parametric gain in two-photon media to produce laser oscillation.

[6]Y. S. Bai, A. G. Yodh, and T. W. Mossberg, Phys. Rev. Lett. **55**, 1277 (1985).

[7]Cf. B. R. Mollow, Phys. Rev. **188**, 1969 (1969).

[8]Cf. B. R. Mollow, Phys. Rev. A **5**, 2217 (1972).

[9]Note that our model is different from that of C. Benkert, M. O. Scully, J. Bergou, and N. Lu (unpublished), who also use the idea of injecting the atoms with initial coherence. We assume that the external field continues to drive the atoms even in the interaction region. We consider the cavity to be tuned to the right Rabi sideband, whereas in the work of Benkert *et al.*, the cavity is tuned to the atomic resonance frequency. It should be further noted that N. Lu [Opt. Commun. **73**, 479 (1989)] has demonstrated that atoms with injected coherence can be used for laser action even in the absence of population inversion.

[10]The derivation of gain, loss, and diffusion parameters for the case of laser is now well documented. We follow the procedure used by Scully and co-workers, see for example, J. Bergou, M. Orszag, and M. O. Scully, Phys. Rev. A **38**, 754 (1988).

[11]H. Haken, in *Laser Theory, Handbuch der Physik,* edited by S. Fluge (Springer-Verlag, Berlin, 1970), Vol. 25C.

[12]H. Risken and H. D. Vollmer, Z. Phys. **204**, 240 (1967); H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1984), p. 385.

[13]L. A. Lugiato, M. O. Scully, and H. Walther, Phys. Rev. A **36**, 740 (1987).

[14]The system treated here is to be contrasted from the one treated in Ref. 1. In Ref. 1 the atoms are pumped coherently so that the steady state is a superposition of the two dressed states. The pumping field is off resonance. In the present problem the atoms are prepared in one of the dressed states by using a resonant field.