Screening effects in pair production by 10 and 15 $m_e c^2$ photons

H. K. Tseng
Department of Physics, National Central University, Chung-Li, Taiwan 32054, Republic of China
(Received 23 April 1990; revised manuscript received 19 July 1990)

With a partial-wave interpolation method we calculated the positron energy spectra of pair production in the field of atoms with atomic number $Z=82$ for photons of energies $k=10$ and 15 $m_e c^2$. Our results show that the screening effect is less than about 2%. Our results also indicate that the approximate treatment of screening through energy-shift screening theory becomes inadequate for photons of energy $k \gtrsim 10 m_e c^2$.

Recently[1,2] there has been interest in the calculation of accurate cross sections of electron-positron pair production by intermediate-energy photons in the field of atomic nucleus. There is a need for an accurate calculation of pair-production cross sections for intermediate-energy photons in the field of atoms. We report here a preliminary study of this problem. We wish to present results on the positron energy spectra of pair production by photons of energies $k=10$ and 15 $m_e c^2$, obtained with an extension of our previous numerical partial-wave calculation techniques that utilizes interpolation in partial-wave cross sections.[3]

We use the same formalism we used for the atomic-field bremsstrahlung calculation.[4] Following our previous bremsstrahlung work,[4] we write the pair-production cross section, differential with respect to positron energy E_+ and to positron and electron angles, as

$$\frac{d\sigma}{dE_+ d\Omega_+ d\Omega_-}=(2\pi)^{-5} p_- E_- p_+ E_+ |M_{fi}|^2 . \tag{1}$$

Here the pair-production matrix element is

$$M_{fi}=(2\pi\alpha/k)^{1/2} \times \int d^3x \, \psi_2^\dagger(\mathbf{p}_-,\mathbf{r},\zeta_-)\boldsymbol{\alpha}\cdot\boldsymbol{\epsilon}\psi_1(-\mathbf{p}_+,\mathbf{r},\zeta')e^{i\mathbf{k}\cdot\mathbf{r}} . \tag{2}$$

The photons are specified by momentum $\mathbf{k}$, energy k, and photon polarization vector $\boldsymbol{\epsilon}$ such that

$$\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}=1, \quad \boldsymbol{\epsilon}\cdot\mathbf{k}=0 . \tag{3}$$

Here, $\psi_2(\mathbf{p}_-,\mathbf{r},\zeta_-)$ is the electron wave function asymptotically normalized to a unit-amplitude plane wave (or distorted plane wave in the point Coulomb case) of four-momentum $(E_-,\mathbf{p}_-)$ and four-polarization $(0,\zeta_-)$ in its rest frame plus an incoming spherical wave; and the positron wave function contains asymptotically spherical incoming waves, as the substitutions $E_1\rightarrow -E_+$, $\mathbf{p}_1\rightarrow -\mathbf{p}_+$ (but $|\mathbf{p}_1|\rightarrow|\mathbf{p}_+|$), and $\zeta_1\rightarrow\zeta'=\zeta_+ -2\hat{\mathbf{p}}_+(\hat{\mathbf{p}}_+\cdot\zeta_+)$ change outgoing into incoming spherical waves, namely

$$i\gamma^2[\psi_+^{\rm in}(\mathbf{p}_+,\mathbf{r},\zeta_+)]^*=\psi_1^{\rm out}(-\mathbf{p}_+,\mathbf{r},\zeta') , \tag{4}$$

where ψ_+ is the positron wave function. That is,[5]

$$\left.\begin{matrix}\psi_1(\mathbf{p},\mathbf{r},\zeta)\\ \psi_2(\mathbf{p},\mathbf{r},\zeta)\end{matrix}\right\}=4\pi\sum_{\kappa,m}[\Phi_{\kappa m}^\dagger(\hat{\mathbf{p}})\chi(\zeta)]i^l e^{(\pm)i\delta_\kappa}\psi_{\kappa m}(\mathbf{r}) , \tag{5}$$

where

$$\psi_{\kappa m}(\mathbf{r})=r^{-1}\begin{bmatrix} g_\kappa(r)\Phi_{\kappa m}(\hat{\mathbf{r}}) \\ if_\kappa(r)\Phi_{-\kappa m}(\hat{\mathbf{r}})\end{bmatrix} \tag{6}$$

and

$$\Phi_{\kappa m}(\hat{\mathbf{r}})=\sum_{s=\pm 1/2} C(l\tfrac{1}{2}j;m-s,s)Y_{l,m-s}(\hat{\mathbf{r}})\chi^s \tag{7}$$

is an eigenstate of J^2, J_z, and L^2. The angular-momentum operator is $\mathbf{J}=\mathbf{L}+\mathbf{S}$, the quantity $C(l\frac{1}{2}j;m-s,s)$ is the Clebsch-Gordan coefficient, and the quantum number $\kappa=\mp(j+\frac{1}{2})$ as $j=l\pm\frac{1}{2}$. The radial wave functions g_κ and f_κ satisfy the radial Dirac equations

$$\begin{aligned}\frac{dg_\kappa(r)}{dr}&=(p_0+1-V)f_\kappa(r)-\kappa g_\kappa(r)/r ,\\ \frac{df_\kappa(r)}{dr}&=-(p_0-1-V)g_\kappa(r)+\kappa f_\kappa(r)/r ,\end{aligned} \tag{8}$$

with $p_0=-E_+$ for ψ_1 and $p_0=E_-$ for ψ_2, where V is the central potential described by the target atom. The target atom is assumed to be unpolarized. We use the spinor representation for Dirac electron and positron wave functions. In this representation any matrix element between four-component states may be reduced to matrix elements between two-component spinors:

$$\chi=c_{1/2}\chi^{1/2}+d_{1/2}\chi^{-1/2} \tag{9}$$

with

$$\chi^{1/2}=\begin{bmatrix}1\\0\end{bmatrix}, \quad \chi^{-1/2}=\begin{bmatrix}0\\1\end{bmatrix} , \tag{10}$$

and

$$\chi^+\chi=1 . \tag{11}$$

Choosing a coordinate system centered at the atomic nucleus with the z axis along $\mathbf{k}$, $\hat{\mathbf{y}}$ along $\mathbf{k}\times\mathbf{p}_+$, and $\hat{\mathbf{x}}$ in the $(\mathbf{k},\mathbf{p}_+)$ plane, and inserting Eqs. (5) into Eq. (2), we obtain

$$M_{fi}=16\pi^2(2\pi\alpha/k)^{1/2}\sum_{\kappa_1 m_1 \kappa_2 m_2}[\Phi_2^\dagger(\hat{\mathbf{p}}_-)\chi_2]^\dagger[\Phi_1^\dagger(-\hat{\mathbf{p}}_+)\chi_1](-1)^{l_1}e^{i(\delta_{\kappa_1}+\delta_{\kappa_2})}[\epsilon_- R_+(m_2)+\epsilon_+ R_-(m_2)]\ , \tag{12}$$

where

$$R_\pm(m_2)=R^\pm_{\kappa_2\kappa_1}(m_2)\delta_{m_1,m_2\mp1}\ ,$$
$$R^\pm_{\kappa_2\kappa_1}(m)=\sum_{n=1}^{2}Q_n^\pm(m)\sum_l{}'P_n^\pm(m)s_n\ . \tag{13}$$

The index l runs from $|l_2'-l_1|$ to $l_2'+l_1$ in steps of 2 for $n=1$, and from $|l_2-l_1'|$ to l_2+l_1' in steps of 2 for $n=2$. Here,

$$\epsilon_\pm=\epsilon_x\pm i\epsilon_y\ , \tag{14}$$

$$l'=l+\eta_\kappa,\quad \eta_\kappa=-\kappa/|\kappa|\ , \tag{15}$$

$$Q_1^\pm(m)=\eta_{\kappa_2}(-1)^{m\mp1/2}[(2l_2'+1)(2l_1+1)]^{1/2}C^\pm_{-\kappa_2,m}C^\mp_{\kappa_1,m\mp1}\ ,$$
$$Q_2^\pm(m)=-\eta_{\kappa_1}(-1)^{m\mp1/2}[(2l_2+1)(2l_1'+1)]^{1/2}C^\pm_{\kappa_2,m}C^\mp_{-\kappa_1,m\mp1}\ , \tag{16}$$

$$P_1^\pm(m)=(-1)^{(l_2'+l_1-l)/2}T(l_2',l_1,l;m\mp\tfrac{1}{2})\ ,$$
$$P_2^\pm(m)=(-1)^{(l_2+l_1'-l)/2}T(l_2,l_1',l;m\mp\tfrac{1}{2})\ , \tag{17}$$

$$T(l_2,l_1,l;m)=(2l+1)\begin{bmatrix}l_2 & l_1 & l\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}l_2 & l_1 & l\\ -m & m & 0\end{bmatrix}\ , \tag{18}$$

$$C^\pm_{\kappa,m}=C(l\tfrac{1}{2}j;m\mp\tfrac{1}{2},\pm\tfrac{1}{2})\ , \tag{19}$$

and

$$s_1=\int_0^\infty dr\, j_l(kr)g_{\kappa_1}f_{\kappa_2}\ ,$$
$$s_2=\int_0^\infty dr\, j_l(kr)g_{\kappa_2}f_{\kappa_1}\ . \tag{20}$$

To calculate the pair-production cross section $d\sigma/dE_+d\Omega_+d\Omega_-$ from Eq. (1), we need $|M_{fi}|^2$, which can be obtained from Eq. (12). To obtain the unpolarized pair-production cross section $(d\sigma/dE_+d\Omega_+d\Omega_-)_{\text{unpol}}$ we average the $d\sigma/dE_+d\Omega_+d\Omega_-$ over the initial-photon polarizations and sum over the final electron and positron spins. Integrating the unpolarized pair-production cross section $(d\sigma/dE_+d\Omega_+d\Omega_-)_{\text{unpol}}$ over the positron and electron angles $d\Omega_+$ and $d\Omega_-$, we obtain the unpolarized pair-production positron energy spectrum

$$\sigma(E_+)\equiv\left[Z^{-2}\frac{d\sigma}{dE_+}\right]_{\text{unpol}}$$
$$=\lambda_0\sum_{\kappa_2,\kappa_1,m=|m_2|}\{[R^+_{\kappa_2\kappa_1}(m)]^2+[R^-_{\kappa_2\kappa_1}(m)]^2\}\ , \tag{21}$$

where

$$\lambda_0=\frac{32\alpha}{Z^2k}p_-E_-p_+E_+ \tag{22}$$

and $R^\pm_{\kappa_2\kappa_1}(m)$ are given by Eqs. (13).

The problem of calculating the unpolarized pair-production positron energy spectrum has been reduced to computing the $R^\pm_{\kappa_2\kappa_2}(m)$. We used the same numerical method that we used for our relativistic bremsstrahlung calculations.[4] The $Q_n^\pm(m)$ and $P_n^\pm(m)$ factors present no great problem. Electron and positron wave functions are obtained in partial-wave series by numerically solving the radial Dirac equation. The radial integrals s_n are calculated numerically to the point where the continuum wave functions of electrons and positrons can be approximately considered as the modified phase-shifted free-field wave functions and an integration by parts method can be used. Then the rest of the radial integrals were calculated by the integration by parts method analytically.

The unpolarized pair-production positron energy spectrum has the form

$$\sigma(E_+)=\sum_{l_1,l_2}\sigma_{l_1,l_2}(E_+)=\sum_{l_1}\sigma_1(E_+)\ , \tag{23}$$

where l_1 and l_2 are the orbital angular-momentum quantum numbers of the positron and the electron, respectively. The brute-force application of this partial-wave method is feasible for obtaining rather accurate theoretical predictions for the cross sections of pair production by low-energy photons.

It is well known that many partial waves are needed for obtaining accurate positron energy spectra of pair production by intermediate-energy photons with the partial-wave method. In Fig. 1 we show the variation of the pair-production partial cross section $\sigma_{l_1}(E_+)$ as a function of l_1 for the cases $Z=82$, $k=10m_ec^2$, and $y=0.1$, 0.3, 0.5, 0.7, and 0.9, where $y=(E_+-1)/(k-2)$. Here the pair-production partial cross sections are calculated numerically both with the Hartree-Fock-

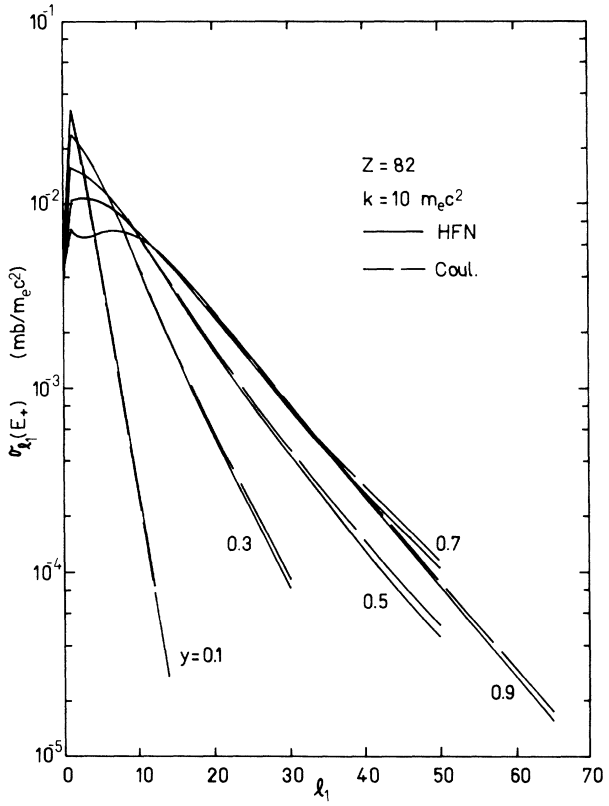

FIG. 1. The variation of the partial cross section $\sigma_{l_1}(E_+)$ of the positron energy spectrum as a function of the orbital angular-momentum quantum number l_1 of the positron for the cases $Z=82$, $k=10m_ec^2$, and $y=(E_+-1)/(k-2)=0.1$, 0.3, 0.5, 0.7, and 0.9. Coul. and HFN refer to the point Coulomb potential and the Hartree-Fock-Slater potential with the exchange term omitted, respectively.

Slater potential with the exchange term omitted[6] (HFN potential, solid line) and with the point Coulomb potential (dashed line). Our results show that the partial cross section $\sigma_{l_1}(E_+)$ is a smoothly varying function of l_1 as $l_1>10$. A modified partial-wave method is therefore possible, which directly calculated a finite set of l value on a grid in l whose spacing increases with l, and then interpolates the intermediate terms. Figure 1 also indicates that for a given photon energy k, the larger the value for y the higher the positron energy, and more partial waves are needed for positron wave functions, as expected.

With this partial-wave interpolation method we have obtained the positron energy spectra of pair production $\sigma(E_+)$ for incident photons of energy $k=10$ and 15 m_ec^2, for the elements of atomic number $Z=82$. In Table I, we show comparisons of unpolarized pair-production cross section $\sigma_C(E_+)$ by photons of energy $k=10m_ec^2$ in the field of atomic nucleus with $Z=82$ between the results[2] of Øverbø, Mork, and Olsen (ØMO) for the point Coulomb potential and our results calculated with the partial-wave interpolation method also for the point Coulomb potential. The agreement is very good. This provides a check of our calculation.

In Tables II and III, we give our results $\sigma(E_+)$ calculated with the partial-wave interpolation method for the cases $Z=82$ and $k=10$ and 15 m_ec^2, with the point Coulomb potential (σ_C) and with the HFN potential ($\sigma_{\rm HFN}$). Our results show that the screening effect is less than about 2%. In Tables II and III, we show comparisons of our results $\sigma_{\rm HFN}$ with the results calculated with the energy-shift screening theory[7] ($\sigma_{\rm EST}$). Our results indicate that the approximate treatment of screening through energy-shift screening theory becomes inadequate for photons of energy $k\gtrsim 10m_ec^2$.

TABLE I. Comparisons of unpolarized pair production cross section $\sigma_C(E_+)$ by photons of energy $k=10m_ec^2$ in the field of atomic nucleus with atomic number $Z=82$ between the results of Øverbø, Mork, and Olsen (ØMO) for the point Coulomb potential, and our results calculated with the partial-wave interpolation method also for the point Coulomb potential. Here, $y=(E_+-1)/(k-2)$.

y	$\sigma_C(E_+)$ ØMO	(μb/m_ec^2) This work
0.1	91.0	90.16
0.3	159.0	159.4
0.5	171.0	170.7
0.7	169.0	169.1
0.9	142.0	142.3

TABLE II. Comparisons of unpolarized pair production cross section $\sigma_C(E_+)$ by photons of energy $k=10m_ec^2$ in the field of atoms with $Z=82$ between our results calculated with the partial-wave interpolation method for the point Coulomb potential (σ_C), and for Hartree-Fock-Slater potential with the exchange term omitted ($\sigma_{\rm HFN}$), and the results calculated with the energy-shift screening theory ($\sigma_{\rm EST}$). Here, $y=(E_+-1)/(k-2)$; σ_C, $\sigma_{\rm HFN}$, and $\sigma_{\rm EST}$ are in units of μb/m_ec^2.

y	σ_C	$\sigma_{\rm HFN}$	$\sigma_{\rm HFN}/\sigma_C$	$\sigma_{\rm EST}/\sigma_C$
0.1	90.16	91.31	1.103	1.030
0.3	159.4	156.4	0.981	1.004
0.5	170.7	166.9	0.978	1.000
0.7	169.1	165.5	0.979	0.999
0.9	142.3	139.1	0.978	0.992

This work was supported, in part, by National Science Council, Republic of China, under Grant No. NSC 78-0208-M008-13.

TABLE III. Same as Table II, except for $Z=82$ and $k=15m_ec^2$.

y	σ_C	$\sigma_{\rm HFN}$	$\sigma_{\rm HFN}/\sigma_C$	$\sigma_{\rm EST}/\sigma_C$
0.1	91.71	91.02	0.992	1.012
0.3	136.4	132.4	0.971	1.002
0.5	142.8	138.2	0.968	1.000
0.7	143.1	139.0	0.971	1.000
0.9	122.7	119.9	0.977	0.995

[1]L. E. Wright, K. K. Sud, and D. W. Kosik, Phys. Rev. C **36**, 562 (1987); E. Borie, Z. Phys. A **302**, 219 (1981); I. Øverbø, Phys. Lett. **71B**, 412 (1977); J. H. Hubbell, H. A. Gimm, and I. Øverbø, J. Phys. Chem. Ref. Data **9**, 1023 (1980).

[2]I. Øverbø, K. J. Mork, and H. A. Olsen, Phys. Rev. **175**, 1978 (1968); Phys. Rev. A **8**, 668 (1973).

[3]H. K. Tseng and R. H. Pratt, Phys. Rev. A **4**, 1835 (1971); **19**, 1525 (1979). Unrationalized units are used throughout, i.e., $\hbar = m_e = c = 1$, unless otherwise specified.

[4]H. K. Tseng and R. H. Pratt, Phys. Rev. A **3**, 100 (1971).

[5]See, for example, H. A. Olsen, *Applications of Quantum Electrodynamics,* Vol. 44 of *Springer Tracts in Modern Physics* (Springer, Berlin, 1968).

[6]D. A. Liberman, J. B. Mann, and J. T. Waber, Comput. Phys. Commun. **2**, 107 (1971).

[7]H. K. Tseng and R. H. Pratt, Phys. Rev. A **6**, 2049 (1972); **21**, 454 (1980).