Prospects for resonant photoexcitation as a pumping mechanism for x-ray lasers

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Using analytic estimates we compare the relative merits of resonant photoexcitation pumping with other excitation methods of x-ray lasers, specifically electron-collision and -recombination pumping. It is shown that the potential of line pumping to achieve high gain in the x-ray region (including the range $2.3 < \lambda < 4.4$ nm) is higher than that of the other methods using high-power lasers or Z-pinch plasmas as pump sources. In the second part of the paper, we investigate the conditions required to obtain an inversion if the pumping ion and the lasing ion are of the same kind. The analysis is performed for laser transitions of the type $\Delta n \neq 0$ as well as for $\Delta n = 0$.

I. INTRODUCTION

Despite the recent success of electron-collision and -recombination pumped x-ray laser schemes[1–6] there is strong motivation to investigate other excitation methods as well. The main shortcomings of the present achievements are the difficulties in the creation of sufficient gain to saturate the lasing transition and the need for excessively high pump power, which can be provided only by large laser systems. It is desirable to obtain strong sources of coherent x rays (possibly in the so-called water window $2.3 < \lambda < 4.4$ nm, in which carbon is absorbing but water is still transmitting, making it a useful range for biological applications) from relatively small laboratory installations such as medium size lasers or Z pinches.

Since the demonstration of gain by electron-collision and -recombination pumping, photoresonant pumping has been given only modest attention but investigations continue.[7–9] Line-pumped x-ray lasing, however, has yet to be demonstrated, the shortest lasing wavelength obtained so far being 216.3 nm,[10] in spite of a large number of experimental and theoretical investigations.[11–19]

In this paper we analyze the potential of resonant photopumping in comparison with collisionally pumped and recombination pumped schemes using analytic expressions for the basic excitation and deexcitation rates. We restrict the number of levels considered to the minimum required. These approximations render the analysis not highly accurate but quite general. We show further how the basic deficiency of line pumping, i.e., the need for a matched pair of lines in the pumping and lasing plasmas, can be overcome by using the same ions for pumping and lasing.

We analyze only the case when a "lamp"—a source of x rays—is spatially separated from an active medium. As an example of such a configuration we can consider two laser-produced plasmas at a small distance from each other (Fig. 1). (We will use the symbols s and a to characterize the parameters of the source plasma and the active medium). The lamp is a plasma created by focusing of high-power laser radiation into a strip about a hundred micrometers wide and a few centimeters long. An active medium in the form of a plasma jet is separated by several hundred micrometers from the lamp so that for nanoseconds the expanding plasmas do not collide.

Since only part of the radiation goes into the active medium, one must introduce a geometric factor $\delta < 1$ which takes this into account. The mathematical definition of δ will be given later. For a distance between the two plasmas of $R \simeq 200$ μm one has $\delta \simeq 0.1$, for $R \simeq 1$ mm, $\delta \simeq 0.02$.

The other example involves two spatially separated Z-pinch discharges[9] or a gas-puff Z-pinch discharge with a transverse geometry as shown in Fig. 2. A detailed description of an experimental realization of such a scheme is given in Ref. 20. A pulse of gas in the form of a cylindrical shell is injected into the discharge gap, along the axis of an evacuated chamber. The discharge in this gas shell produces a current shell, which is accelerated inward by its own magnetic field and stagnates on the surface of a cylindrical target. Subsequently a dense high-

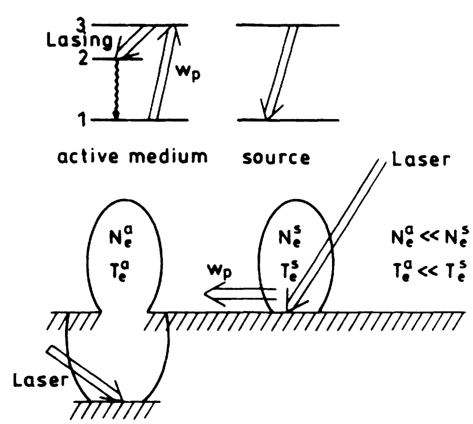

FIG. 1. Possible geometry of photoresonant pumping using two laser-produced plasmas.

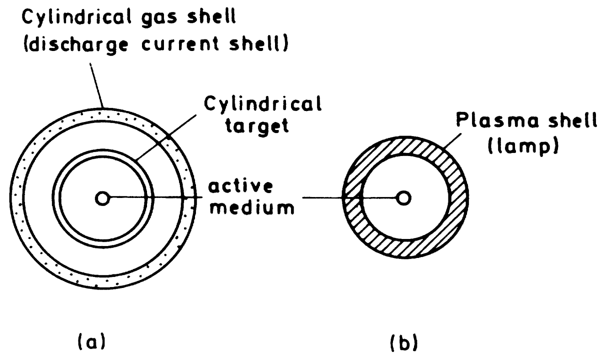

FIG. 2. Geometry of a gas-puff Z pinch. The figure shows a section through the cylindrical arrangement (a) before and (b) after the stagnation.

temperature plasma cylinder is formed, which serves as a source of x rays. This radiation can pump an active medium located on the axis of the cylinder. The active medium may be specially prepared previously (for example, it can be a laser plasma produced in a gas jet directed along the discharge axis or a plasma generated in a discharge or wire explosion) or it may be created by the radiation itself. It is clear that by its very nature this Z-pinch geometry is ideal for photopumping schemes, the geometric factor δ being $\simeq 0.5$ in this case, but its realization may require considerable experimental effort.

In Sec. II of this paper we compare the potential of photoresonant, electron-collision and -recombination pumping to obtain higher gain on the Balmer α line ($n=3\rightarrow 2$) in case of one- and multicomponent plasmas. In Sec. III we discuss the parameters of the pump plasma necessary for photopumping. We then turn to ions of the configuration $2s$ or $2s^2 2p^k$, with $k\leq 5$, and show that with these ions it is possible to use the same kind of ion for pumping and lasing (Sec. IV). Finally, in Sec. V we treat $\Delta n=0$ lasing transitions in neonlike ions. We present a scheme in which photopumping by the same neonlike ion leads to high gain on the $3p\rightarrow 3s$ transition.

II. LASER TRANSITIONS WITH $\Delta n\neq 0$

First we analyze the case of laser transitions of the type $\Delta n\neq 0$ in an active medium consisting of ions with "spectroscopic symbol" Z (equal to charge state $+1$). As an example we consider hydrogenic ions but our estimates and conclusions will remain approximately true for other ions as well, such as He-like and Li-like ions, etc.

We write the gain coefficient for a lasing transition between the levels $n=3$ and 2 (Fig. 1) as

$$\alpha=\frac{\lambda_{32}^2 A_{32}}{4\Delta\omega_{32}}N_1\frac{N_3}{N_1}\ , \qquad (1)$$

where A_{32} is the Einstein-A coefficient of the transition, $\Delta\omega_{32}$ is the linewidth, and N_1 and N_3 are the populations of the levels with $n=1$ and 3. The use of main quantum numbers n means that we assume that different fine-structure components belonging to the same n are populated proportionally to their statistical weights. Such mixing is produced by electric microfields created by ions and collisions with electrons (see Ref. 15). We assume here also $N_3 \gg N_2 g_3/g_2$, where g_i is the statistical weight.

From the equation of balance for the level with $n=3$ one obtains

$$N_3=N_1 W_p/(A_{32}+A_{31})\ , \qquad (2)$$

and therefore

$$\alpha=\frac{\lambda_{32}^2 A_{32}}{4\Delta\omega_{32}}N_1\frac{W_p}{A_{32}+A_{31}}\ , \qquad (3)$$

where W_p is the rate of pumping of the upper laser level.

It is well known that population inversion can be achieved only if the active region is optically thin for the radiation $2\rightarrow 1$, i.e., the optical thickness of this transition

$$\tau=N_1\sigma_{12}d\leq\tau^{\max}\ , \qquad (4)$$

where

$$\sigma_{12}\frac{\lambda_{12}^2}{4}\frac{A_{12}}{\Delta\omega_{12}} \qquad (5)$$

is the absorption cross section for the transition $1\rightarrow 2$ and d is the transverse scale length of the active medium. We define generally $A_{ki}=(g_i/g_k)A_{ik}$ for $i>k$.

Inserting the value for the maximum ion density from (4) into the expression (3) for the gain coefficient one obtains

$$\alpha=\left[\frac{\lambda_{32}}{\lambda_{12}}\right]^2\frac{A_{32}}{A_{12}}\frac{\Delta\omega_{12}}{\Delta\omega_{32}}\frac{\tau^{\max}}{d}\frac{W_p}{A_{32}+A_{31}}\ . \qquad (6)$$

Assuming both lines to be Doppler broadened one can write the gain coefficient

$$\alpha=CN_3/N_1=C\frac{W_p}{A_{32}+A_{31}}\ , \qquad (7)$$

with the Z-independent constant

$$C=\left[\frac{\lambda_{32}}{\lambda_{12}}\right]^3\frac{A_{32}}{A_{12}}\frac{\tau^{\max}}{d}\ . \qquad (8)$$

Taking $\tau^{\max}=1$ for hydrogenic ions and $d=100$ μm gives $C\simeq 370$ cm^{-1}.

Inserting the expression for the Doppler width and using the above values for $\tau^{\max}$ and d one obtains from (4) the maximum number density of ions in the hydrogenic ground state in cm^{-3}

$$N_1^{\max}=Z^{2.5}(T_i^a/Z^2\mathcal{R})^{1/2}(4.46\times 10^{15})\ , \qquad (9)$$

where $\mathcal{R}=13.6$ eV is the Rydberg constant and T_i^a is the kinetic ion temperature in the active medium. We assume $T_i^a\simeq T_e^a$ in the following analysis, where T_e^a is the electron temperature of the active medium.

It should be mentioned that the assumption of a Doppler-broadened laser line may not be correct in a dense plasma. Stark broadening (mainly Holtsmark broadening) will reduce the gain coefficient in this case. The reduction applies only to electron-collision and -recombination pumping since these schemes operate at higher density than photoresonant pumping.

A. Comparison of pumping mechanisms for a one-component plasma

If the plasma consists only of one species of material, the limit on electron density follows from the limit of ion density and one can write

$$N_e^{\max}=\gamma N_1^{\max} Z \simeq Z^{3.5} , \tag{10}$$

where γ is a factor of proportionality which can be found from the equation

$$N_e^{\max}=N_+ Z+N_1^{\max}(Z-1)+N_-(Z-2) , \tag{11}$$

where N_+ is the density of the bare nuclei and N_- is the density of He-like ions.

Usually the plasma of the active medium is prepared such that the maximum of the ion charge distribution curve coincides with laser ions (i.e., hydrogenic ions). In this case one can assume $\gamma \simeq 1$. However, if it is necessary to have a higher electron density, the active medium can be either overionized or underionized, so that the density of hydrogenic ions is still $N_1^{\max}$, but $N_+ \gg N_1^{\max}$ or $N_- \gg N_1^{\max}$. In this case the factor γ can be greater than 1. We think that for a one-component plasma a factor $\gamma > 10$ (as, for example, in a plasma consisting almost entirely of bare nuclei) is not feasible for x-ray laser schemes. In our further analysis we will assume $\gamma \le 10$.

For *photoresonant pumping* one has

$$W_p^p=\delta A_{13}\theta \sim Z^4 , \tag{12}$$

where θ is the number of photons per mode, averaged over the linewidth, and δ is a geometry factor.

In general δ would have to be calculated by an integration over the volumes of the source and the active medium.[15] However, in practical applications one can assume the source plasma to be optically thick in the pumping lines and for sufficiently small δ a good approximation is $\delta=\bar{\Omega}_s/4\pi$, where $\bar{\Omega}_s$ is the average solid angle of the source as seen from the active medium.

Note that the maximum number of photons per mode from a local-thermodynamic-equilibrium radiation source with electron temperature T_e^s is given by the Planck formula

$$\theta_p=\frac{1}{\exp(\hbar\omega/kT_e^s)-1} . \tag{13}$$

A more general expression for θ will be given in the Appendix.

Inserting (12) into the gain equation (7) one obtains a simple expression for the gain coefficient in case of photoresonant pumping,

$$\alpha^p \simeq 1850\theta\delta \tag{14}$$

(in cm^{-1}), independent of Z. This equation is valid provided the number of photons per mode arriving at the active medium (i.e., $\theta\delta$) is much greater than the number of photons per mode generated in the active medium itself in the same frequency region. Furthermore, the number of photons per mode supplied by the pump plasma in the "spoil" region (corresponding to the $1\to2$ transition in hydrogenic ions) must be smaller than the relevant number generated in the active medium.

When applying Eq. (14) it must be kept in mind that in its derivation only spontaneous decay of the upper laser level has been taken into account. For $\theta\delta \simeq 1$ decay by stimulated emission of the pump radiation becomes comparable and leads to a saturation of the excitation. The equation is therefore only applicable for $\theta\delta < 1$.

With *electron-collision pumping* it is not possible to get an inversion on the $3\to2$ transition of hydrogenic ions. In case of heliumlike ions the transition $1s3s\,[^1S_0]\to 1s2p\,[^1P_1]$ may be pumped to inversion by collisional excitation from the ground state[21] but the gain is found to be insignificant. Here we do not imply any specific electron-collision laser scheme, we only wish to compare the basic excitation mechanisms.

The electron-collision excitation rate of the upper laser level is

$$W_p^c=N_e\langle v\sigma_{13}\rangle , \tag{15}$$

with the rate coefficient for an optically allowed transition (in $\mathrm{cm}^3\,\mathrm{s}^{-1}$) given by the well-known formula[22]

$$\langle v\sigma_{13}\rangle=(6.4\times10^{-8})f_{13}\frac{1}{Z^3}\left[\frac{Z^2\mathcal{R}}{\Delta E_{13}}\right]\left[\frac{Z^2\mathcal{R}}{T_e^a}\right]^{1/2} \times\exp(-\Delta E_{13}/T_e^a) . \tag{16}$$

In this equation f_{13} is the oscillator strength of the transition, ΔE_{13} the energy difference of the levels, and T_e^a the electron temperature in the active medium. With $N_e^{\max}$ from Eq. (10) one gets the maximum value for the electron-collision pumping rate in the case of a one-component plasma,

$$W_p^c=(2.85\times10^8)f_{13}Z^{0.5}\gamma\left[\frac{Z^2\mathcal{R}}{\Delta E_{13}}\right]\exp(-\Delta E_{13}/T_e^a). \tag{17}$$

From (7) one sees that the gain coefficient in this case scales as

$$\alpha^c \sim Z^{-3/5} . \tag{18}$$

From (12) and (17) one derives as the condition on the number of photons per mode when photoresonant pumping results in higher excitation of the upper laser level than electron-collision pumping

$$\theta\delta > (3.5\times10^{-2})\left[\frac{Z^2\mathcal{R}}{\Delta E_{13}}\right]^3 \gamma Z^{-3/5}\exp(-\Delta E_{13}/T_e^a) . \tag{19}$$

Remembering that we assume here H-like ions this becomes

$$\theta\delta > 0.05Z^{-3.5}\gamma \exp(-\Delta E_{13}/T_e^a) . \quad (20)$$

Evaluating the number of photons from the black-body equation (13) we obtain for $\hbar\omega_{13} \gg T_e^s$

$$\theta = \exp(-\hbar\omega_{13}/T_e^s) \geq \exp(-\Delta E_{13}/T_e^a) \quad (21)$$

as $\Delta E_{13} = \hbar\omega_{13}$ and we assume $T_e^s > T_e^a$. Thus, using $\gamma = \gamma_{\max} = 10$ one obtains for the geometry factor

$$\delta > 0.5Z^{-3/5} , \quad (22)$$

which can be easily realized.

For *recombination pumping* we assume that the three-body recombination flow goes entirely into the upper laser level, thus deriving an upper limit for the achievable recombination gain. The formula used for the coefficient of three-body recombination[23,24] (in $cm^6 s^{-1}$) is

$$\chi_r = (4.3 \times 10^{-32}) Z^{-6} \left[\frac{Z^2 \mathcal{R}}{T_e^a} \right]^{9/2} , \quad (23)$$

whence the recombination pumping rate becomes

$$W_p^r = (4.3 \times 10^{-32})(N_+/N_1) N_e^2 Z^{-6} \left[\frac{Z^2 \mathcal{R}}{T_e^a} \right]^{9/2} . \quad (24)$$

The ratio N_+/N_1 appears here because in our previous formula W_p was defined with respect to N_1.

For $N_+ > N_1$ one can approximate

$$N_+/N_1 \simeq \gamma . \quad (25)$$

We assume an undercooled plasma with $Z^2\mathcal{R}/T_e^a = 50$. This means that the electron temperature is a factor of 10 lower than the equilibrium plasma electron temperature $T_e^{eq} = Z^2\mathcal{R}/5$. From various numerical simulations (e.g., for expansion cooled carbon[25] and for radiatively cooled neon[26]) $T_e^a = T_e^{eq}/10$ seems to be a rather optimistic value.[27]

Under this assumption one obtains for the pumping rate

$$W_p^r \simeq (2 \times 10^{-24}) Z^{-6} \gamma N_e^2 . \quad (26)$$

For a one-component plasma with $N_e^{\max}$ from Eq. (10) we get

$$W_p^r = 0.8 \times 10^6 (N_+/N_1) \gamma^2 Z \simeq (0.8 \times 10^6) \gamma^3 Z \quad (27)$$

and a gain coefficient (in cm^{-1})

$$\alpha^r = 3(N_+/N_1)\gamma^2 Z^{-3} \simeq 3\gamma^3 Z^{-3} . \quad (28)$$

Experimentally with a one-component C VI plasma a maximum gain coefficient of 6 cm^{-1} has been reported.[2]

Photoresonant pumping leads to a higher gain if

$$\theta\delta > 1.6(N_+/N_1)\gamma^2 10^{-3} Z^{-3} \simeq 1.6\gamma^3 10^{-3} Z^{-3} . \quad (29)$$

The conditions of Eqs. (20) and (29) can be fulfilled for photoresonant pumping, since the value of $\theta\delta$ can in principle be as high as 1. (This is the maximum possible value because above this limit rapid photoionization from the upper laser level will destroy the active medium.) We conclude that in the case of a one-component active medium and laser transitions of the type $\Delta n \neq 0$ photoresonant pumping can be much more effective than electron-collision and -recombination pumping, especially for high-Z ions and short laser wavelengths. The main reason for this is that, in photoresonant pumping, the electron density plays only a secondary role and has no influence on the pumping rate of the upper laser level, provided that it is still high enough for efficient collisional mixing of fine-structure sublevels (see Ref. 15).

B. Comparison of pumping mechanisms for a multicomponent plasma

Since the density of active ions is restricted by the radiation trapping condition [Eq. (4)] it has been suggested that the medium could be "seeded" with high-Z ions acting as electron donors to the plasma.[26] In this case the maximum electron density is not restricted by the maximum density of active ions but can be considerably higher.

In a multicomponent plasma the upper limit on the electron density is determined by the condition of equal probability of electron deexcitation and radiative decay of the upper laser level,

$$N_e^{\max} = (3.3 \times 10^{14}) Z^7 \left[\frac{T_e^a}{Z^2 \mathcal{R}} \right]^{1/2} \quad (30)$$

(in cm^{-3}) for H-like ions.

For such high electron and ion densities the linewidth of the laser transition will be determined by Holtsmark broadening and will be larger than the width due to Doppler broadening. This leads to a reduction of the gain coefficient. We do not take this into account and thus get upper bounds for the gain coefficients in case of electron-collision and recombination pumping.

Applying this limit one can still use the relation $N_e^{\max} = \gamma N_1^{\max} Z$ as previously but γ now becomes Z dependent and, with $N_1^{\max}$ from (9), is equal to

$$\gamma^{\max} = 0.07 Z^{3.5} . \quad (31)$$

For example, for $Z = 13$ (the first hydrogenic ion for which the laser transition lies in the water window) one obtains $\gamma^{\max} \simeq 550$. To get such a high value for γ the number density of donor ions must be much higher than that of the lasing ions.

Inserting (31) into (17) one obtains for the electron-collision excitation rate

$$W_p = (1.8 \times 10^6) Z^4 \exp(-\Delta E_{13}/T_e^a) \quad (32)$$

and the gain coefficient becomes independent of Z.

For photoresonant pumping to be more effective than electron-collision pumping a condition

$$\theta\delta > (3.5 \times 10^{-3}) \exp(-\Delta E_{13}/T_e^a) \quad (33)$$

is obtained. Thus even for a multicomponent plasma with its higher electron densities photopumping remains more effective than electron-collision pumping.

Turning now to *recombination pumping* we note that with increasing Z of the active medium it becomes exceedingly difficult to achieve the required overcooling of the plasma. The reason for this is that the temperature (normalized to Z^2) required to obtain a given fraction of bare nuclei increases with increasing Z in a nearly optically thin plasma. For example it was found by numerical simulations that the electron temperature required to reach 85% stripped nuclei is approximately $\sim Z^{3.4}$.[26] Also, since the density increases with Z [Eq. (30)] the recombination rate increases rapidly. Therefore less and less time for both radiative and hydrodynamic cooling is available as Z is increased and the inversion appears at successively higher temperatures.

We take this fact into account by assuming $T_e^a/Z^2 \sim Z$ and write specifically

$$T_e^a = \frac{Z}{50} T_e^{\mathrm{eq}} , \tag{34}$$

where $T_e^{\mathrm{eq}} = Z^2 \mathcal{R}/5$ is the equilibrium temperature of the plasma. The choice of Eq. (34) results in our original value of $T_e^a = Z^2\mathcal{R}/50$ for $Z=5$ and is quite optimistic for higher Z under the assumption.[27]

Inserting the temperature given by (34) into Eqs. (24) and (30) one obtains the pumping rate for recombination pumping in a multicomponent plasma

$$W_p^r = 10^6 (N_+/N_1) Z^{4/5} \tag{35}$$

and, using (7) for the gain coefficient (in cm^{-1}),

$$\alpha^r = 4(N_+/N_1)\sqrt{Z} . \tag{36}$$

For resonant photopumping to be more effective than recombination pumping one gets the condition

$$\theta\delta > (2\times 10^{-3})(N_+/N_1)\sqrt{Z} . \tag{37}$$

Taking $N_+/N_1 = 10$, these estimates appear to give upper limits for recombination pumping.

C. Upper density limit

In case of photoresonant pumping the active medium will be a one-component plasma with an electron density $N_e^{\max} = Z N_1^{\max}$ and $N_1^{\max}$ given by Eq. (9), a value which remains below, say, 10^{23} cm^{-3} for all Z of interest.

In contrast, in the case of electron-collision and -recombination pumping the electron density should be as high as possible, limited to $N_e^{\max}$ of Eq. (30). This value is lower than 10^{23} cm^{-3} only for $Z \le 18$.

Although, in principle, a plasma with an electron density $> 10^{23}$ cm^{-3} can be generated by using *fs* pulses or by plasma compression, we believe that it will be very difficult to realize an x-ray laser working at such high electron densities. Assuming a maximum electron density of $N_e^{\max} = 10^{23}$ cm^{-3} the gain coefficient at high Z is not affected in the case of photoresonant pumping, because the optimum electron density remains lower than this value. Under this condition, however, the gain scales as

$$\alpha^c \sim Z^{-7} \tag{38}$$

for electron-collision pumping and as

$$\alpha^r \sim Z^{-10} \tag{39}$$

for recombination pumping.

This shows that, at laser wavelengths $\lambda \le 2$ nm electron-collision and -recombination pumping runs into severe problems, which are of no concern to photoresonant pumping. The conclusion of these considerations is that from its very principle, i.e., the physical separation of the active medium and pumping plasma, photoresonant excitation seems to be more promising for high-gain lasing and lasing at short wavelengths than the other two mechanisms. The main results of the previous analysis are summarized in Table I.

III. PUMP-PLASMA PARAMETERS FOR PHOTORESONANT PUMPING SCHEMES

The required number of photons per mode and the pump intensity needed to obtain gain coefficients of 2 and 10 cm^{-1} for various nuclear charges have been calculated in Ref. 15. In the following we give an estimate of the parameters of the pump plasma to realize an amplified spontaneous emission (ASE) laser by photoresonant pumping.

We require a minimum gain coefficient of 10 cm^{-1} for ASE lasing. Equation (14) leads to the condition

$$\theta \ge 0.005 \frac{1}{\delta} \tag{40}$$

for the number of photons per mode emitted by the pump source. This can be transferred into a pump intensity by multiplying by $\hbar\omega$ and by the density of modes in the pumping radiation. Assuming a Doppler-broadened line

TABLE I. Maximum gain coefficient α (cm^{-1}) for various pumping mechanisms and plasma conditions of active medium; $\Delta n \neq 0$.

Pumping mechanism	One-component active medium	Multicomponent active medium	$N_e = 10^{23}$ cm^{-3}
Photoresonant pumping	$1850\ \theta\delta$	Not necessary	Not reached
Recombination pumping	$3\times 10^3 Z^{-3}$	$40\sqrt{Z}$	$\sim Z^{-10}$
Electron-collisional pumping (no real scheme)	$\sim Z^{-3.5}$	$\sim Z^0$	$\sim Z^{-7}$

$\Delta\omega^a$ for the transition to be pumped in the active medium one derives

$$I_p = \hbar\omega \frac{\omega^2 \Delta\omega^s}{\pi^2 c^2} \theta \geq (1.4\times 10^3) \left[\frac{T_e^a}{Z^2 \mathcal{R}} \right]^{0.5} Z^{8.5} \frac{1}{\delta} \frac{\Delta\omega^s}{\Delta\omega^2} \quad (41)$$

in W/cm^2, where $\Delta\omega^2$ is the linewidth of the pump source. For $Z=13$ and $Z^2\mathcal{R}/T_e^a=5$ one obtains a pump intensity

$$I_p = (2\times 10^{12}) \frac{1}{\delta} \frac{\Delta\omega^s}{\Delta\omega^a} \ . \quad (42)$$

Assuming that radiation in a pumping line is close to the blackbody limit (see below) with the radiation temperature equal to the plasma electron temperature T_e^s one obtains from (40) the following condition on photon energy necessary to pump to a gain coefficient of 10 cm^{-1}:

$$\hbar\omega_p \leq T_e^s \ln(1+200\delta) \ . \quad (43)$$

Recalling the geometry factors for the Z pinch and for two laser plasmas with a distance of 200 μm and 1 mm one obtains

$$\delta = \begin{cases} 0.5, & \hbar\omega \leq 4.6 T_e^s \\ 0.1, & \hbar\omega \leq 3 T_e^s \\ 0.02, & \hbar\omega \leq 1.6 T_e^s \ . \end{cases} \quad (44)$$

These conditions are visualized in the qualitative diagram of Fig. 3. The figure shows the number of photons per mode and the intensity (in arbitrary units) obtained from a blackbody source versus pump photon energy normalized to the source temperature. In the figure the qualitative appearance of the spectrum of a laser-produced plasma with electron temperature T_e^s is also drawn.

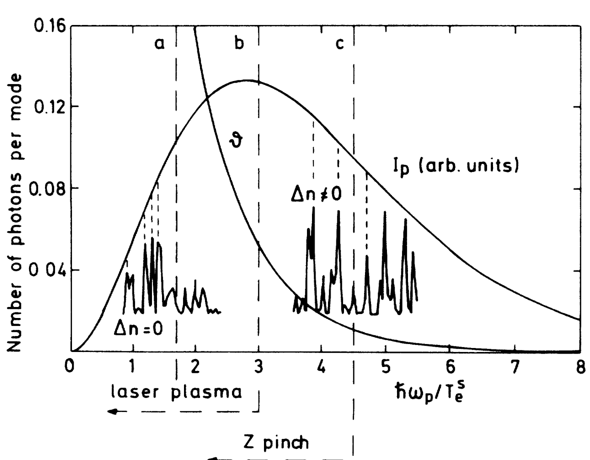

FIG. 3. Number of photons per mode and intensity vs pump photon energy normalized to source temperature. Regions defined by the limits of Eq. (44) are indicated by vertical dashed lines: (a) $\delta=0.02$; (b) $\delta=0.1$; (c) $\delta=0.5$. The schematic appearance of an x-ray spectrum showing lines with various optical thickness is inserted into the figure.

It is useful to discuss the origin of the lines with respect to the average charge state $\bar{Z}$ of the plasma. We take only transitions to the ground state of an ion into account. For ions at the maximum of the charge-state distribution ($Z \simeq \bar{Z}$) transitions of the type $\Delta n \neq 0$ are located on the right-hand side of the Planck radiation curve, whereas $\Delta n=0$—transitions are on the left side. However, ions with lower charge state, $Z<\bar{Z}$, may have transitions of the type $\Delta n=1$ on the left-hand side of the radiation curve. The density of these ions is relatively low, however, since their charge state is below the maximum of the charge-state distribution.

The intensity of any spectral line reaches the Planck intensity curve only if the plasma is optically thick in this line and if the populations of the corresponding levels meet the Boltzmann distribution with $T=T_e^s$. In this case the number of photons per mode is given by the Planck distribution [see Appendix, Eq. (A10)].

It is important to note that lines on the left side of the Planck radiation curve are preferable for photoresonant pumping. Despite their lower brightness the number of photons per mode can be quite high in these lines, reaching values ≥ 0.1. Correspondingly, one can have a gain coefficient $\alpha \geq 180\delta$ cm^{-1}.

IV. PUMPING AND LASING IONS OF THE SAME KIND

The use of a blackbody source without any kind of filter for the excitation of an x-ray laser is not possible (except for the highly transient case) since such a radiator would pump the lower level of the laser transition as well. Furthermore, it is very difficult to realize a source which is blackbody over an extended region of the spectrum. Therefore photoexcitation of the upper level of an x-ray laser has to rely on resonant line pumping, requiring a match between the pumping and pumped transition within the linewidth. This is in general considered as the main deficiency of the photoresonant pumping scheme despite the fact that there exist quite a number of different resonant pairs (see, for example, Refs. 14, 15, 16, and 18). However, if resonant photopumping is to be applied as a universal method, a systematic way of generating matched line pairs has to be found.

In hydrogenic ions there is an automatic coincidence of the Lyman α and Balmer β transitions if $Z_l=2Z_s$, where Z_l is the nuclear charge of the lasing ion (pumped on its Balmer β transition) and Z_s is the nuclear charge of the source ion, emitting its Lyman α line.[8,13] In general, however, this coincidence does not seem to be useful since the electron temperature required to obtain enough source ions is the hydrogenic state is much lower than the electron temperature needed for the lasing ions. Therefore it is virtually impossible to provide the required number of photons per mode.

A more promising way to use "automatic" coincidences is to take the same kind of ion as the source and the lasing ion. This approach has been suggested with respect to hydrogenic ions.[13] In this case, however, it is necessary to filter the Lyman α radiation. Since the wavelength of the filtered radiation is quite close to that of the pumping line (Lyman β) such filtering will be hard to achieve. Thus one can exclude hydrogenic as well as heliumlike ions as possible candidates for using the same kind of ion for lasing and pumping.

It is likely that some filtering will be necessary if the pumping and lasing ions are the same. Favorable conditions exist if the filtered radiation has a much longer wavelength than the pumping radiation. This case applies to Li-like, Be-like, and other ions of the electron configuration $2s^2p^k$ with $k \le 5$. In Fig. 4 the situation is illustrated. The figure displays a qualitative level diagram, with pumping transitions and excitation paths indicated. The wavelengths in brackets refer to Be-like Mg. Two lasing lines, $4p \to 3d$ and $4f \to 3d$, are feasible.

It is worth mentioning that photoresonant pumping of Be-like ions by the radiation of ions of another kind was proposed in Ref. 16 and indeed a pumping effect was demonstrated,[10] the first promising experimental result in this direction.

It is evident from Fig. 4 that the radiation corresponding to the transition $2p \to 2s$ has to be filtered out to prevent population of the level $2p$ by the pumping radiation and subsequent photoresonant pumping of the lower laser level $3d$. Thus, in the active medium the condition

$$N_{2s} \gg N_{2p} \tag{45}$$

should be fulfilled. The wavelength of the radiation to be filtered, however, is at least a factor of 5 greater than the wavelength of the pumping radiation and therefore the realization of such a filter is quite feasible.

We require, further, that the electron temperature and also the electron density in the source plasma are higher than in the active medium. If $T_e^{\rm opt}$ is the equilibrium plasma temperature which optimizes the relative concentration of the required ion species, then the best conditions for pumping and lasing will exist if $T_e^s > T_e^{\rm opt} \ge T_e^a$. We also assume that the transverse size of the source plasma is large enough to render the $4p \to 2s$ line optically thick providing an emission at this line close to the Planck limit with temperature T_e^s.

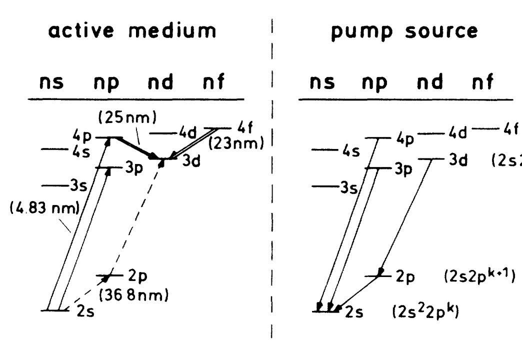

FIG. 4. Schematic level scheme of Li-like ions and ions with ground-state configuration $2s^2 2p^k$; $0 \le k \le 5$. Arrows indicate photoresonant excitation transitions and possible laser lines in the active medium and pumping transitions in the source ion. The wavelengths in brackets are for Be-like Mg.

In contrast, the transverse size and electron density of the active plasma should be so low that its own radiation is negligible in comparison with radiation from the source plasma.

The condition (45) on densities in the active medium can be written as

$$N_{2p} \simeq N_{2s} \frac{N_e^a \langle v\sigma_{2s,2p} \rangle}{A_{2p,2s}} \ll N_{2s} \ , \tag{46}$$

where $\langle v\sigma_{2s,2p} \rangle$ is the electron excitation rate coefficient, for which we use[24]

$$\langle v\sigma_{2s,2p} \rangle = (1.74\times 10^{-7}) f_{2s,2p} Z^{-3} \left[\frac{Z^2 \mathcal{R}}{\Delta E_{2s,2p}} \right] \times \beta^{1/2} \ln \left[\frac{E_{2s}}{\Delta E_{2s,2p} \sqrt{\beta}} \right] \tag{47}$$

(in $\mathrm{cm^3\,s^{-1}}$), where $f_{2s,2p}$ and $\Delta E_{2s,2p}$ are the oscillator strength and energy of the transition, respectively, $\beta = Z^2 \mathcal{R} / T_e^a$, and E_{2s} is the ionization energy.

To fulfill (46) we require $N_e^a \le \frac{1}{10} A_{2p,2s} / \langle v\sigma_{2s,2p} \rangle$ and obtain

$$N_e^a(\max) = (4.6\times 10^{15}) Z^4 \frac{g_{2s}}{g_{2p}} \left[\frac{\Delta E_{2s,2p}}{Z \mathcal{R}} \right]^3 \times \beta^{-1/2} \left[\ln \frac{E_{2s}}{\Delta E_{2s,2p} \sqrt{\beta}} \right]^{-1} \tag{48}$$

(in $\mathrm{cm^{-3}}$), where we have used

$$A_{2p,2s} = 8.05\times 10^9 (\Delta E_{2s,2p} / \mathcal{R})^2 f_{2s,2p,g2s} / g_{2p} \ .$$

From Eq. (48) it follows that $N_e^a(\max)$ scales as Z^4, since $\Delta E_{2s,2p} \sim Z$ and therefore $N_e^a(\max)$ has approximately the same Z dependence as derived from the optical thickness criterion (10).

The maximum density of active ions in the ground state is given approximately by $N_{2s}^{\max} = N_e^a(\max)/Z$ and one can estimate the gain coefficient from Eq. (3). Taking as an example the case of Mg IX (Be-like magnesium) and using the atomic parameters given in Refs. 16 and 24 we obtain for the gain coefficients (in $\mathrm{cm^{-1}}$) of two possible lasing lines

$$\alpha_{4p,3d} = 16 \left[\frac{N_{4p}}{N_{2s}} \right] \simeq 5.3\sqrt{Z} \left[\frac{N_{4p}}{N_{2s}} \right] , \tag{49a}$$

$$\alpha_{4f,3d} = 190 \left[\frac{N_{4f}}{N_{2s}} \right] \simeq 63\sqrt{Z} \left[\frac{N_{4f}}{N_{2s}} \right] . \tag{49b}$$

As was pointed out in Ref. 16 electron-collision mixing will transfer a great deal of the population of level $4p$ to the level $4f$ making the $4f \to 3d$ transition the more favorable for lasing. Since a slight Z dependence is included in Eqs. (49) and expressions can be used for other kinds of ions as well.

It should be pointed out that for Be-like ions in a quasi-cw scheme there appears to exist a limit on the achievable ratio N_{4f}/N_{2s} due to the specific values of the relevant radiative decay rates. Since

$$N_{4f}A_{4f,3d}=N_{3d}A_{3d,2p} , \quad (50a)$$

$$N_{3d}A_{3d,2p}=N_{2p}A_{2p,2s} , \quad (50b)$$

we obtain

$$N_{2p}=N_{4f}\frac{A_{4f,3d}}{A_{2p,2s}}\leq\frac{1}{10}N_{2s} \quad (51)$$

to fulfill the condition (45). The values of the Einstein-A coefficients for Mg IX in Eq. (51) lead to a limit

$$N_{4f}/N_{2s}\leq 1.5\times 10^{-2} \quad (52)$$

and to a maximum achievable gain in continuous operation (in cm^{-1})

$$\alpha_{4f,3d}^{cont}\leq 3 . \quad (53)$$

Note that in case of an optically thick pumping line (close to the blackbody limit) the required number of photons in the $2s\rightarrow 4p$ line is generated by a plasma with an electron temperature of 60 eV, the equilibrium temperature of Be-like Mg.

The above simple estimates show that it is possible to use the same kind of ion for pumping as for lasing. Of course detailed calculations are necessary to give more exact values for the gain coefficients.

We close this section by making the following remarks.

(1) More favorable conditions for lasing than with Be-like ions may be obtained in ions of the electron configuration $2s^22p^k$ with $k\geq 1$ since for these ions the maximum electron density [Eq. (48)] at the same spectroscopic symbol Z is usually higher than for Be-like ions.

(2) The analogous idea can be applied to the configuration $3s^23p^k$ possibly with even greater success.

(3) It seems promising to analyze the following scheme for ions with configuration $3s^23p^k$ $(k\geq 1)$ or $3s^23p^63d^k$ $(k\geq 0)$: apply photoresonant pumping to the $3p\rightarrow 5d$ transition with subsequent lasing on the transitions $5d\rightarrow 4f$ and $5g\rightarrow 4f$. The radiation corresponding to the transition $3d\rightarrow 3p$ of the source has to be filtered out in this case.

A more detailed knowledge of the atomic parameters of ions of the kind mentioned above would be necessary to be able to do the required analysis.

V. RESONANT PHOTOPUMPING OF $\Delta n=0$: LASING TRANSITIONS

For this kind of transition successful experiments with electron-collision pumping have been performed using neonlike and nickel-like ions with corresponding laser transitions $3p\rightarrow 3s$ and $4d\rightarrow 4p$, respectively.[1,6] In this section we want to prove that the potential of photopumping is quite favorable for obtaining the same, or higher, gain as with electron-collision pumping but with considerably lower electron density in the active medium.

In Fig. 5 a schematic energy diagram for neonlike ions is shown. For convenient notation in the following analysis the relevant levels are numbered from (1) to (5),

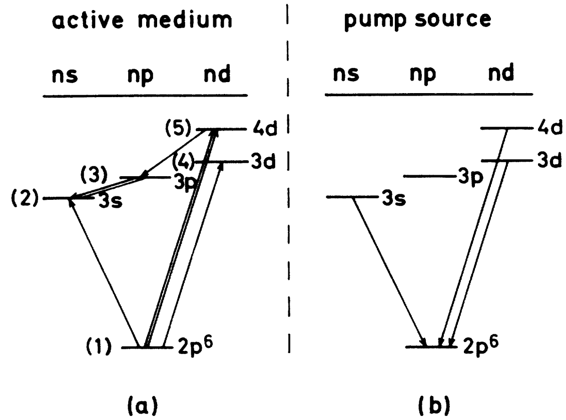

FIG. 5. Schematic level scheme of neonlike ions. Only the levels relevant to the present proposal are shown. The laser transition is $3p\rightarrow 3s$.

as shown in the figure, with the laser transition between (3) and (2).

As for hydrogenic ions, the condition of sufficiently fast radiative decay of the lower laser level determines a maximum density of active ions given by Eq. (4). Using the above notation Eq. (7) remains valid and, assuming Doppler broadening for both transitions, the gain becomes

$$\alpha=\left[\left[\frac{\lambda_{32}}{\lambda_{12}}\right]^3\frac{A_{32}}{A_{12}}\right]\frac{\tau^{max}}{d}\frac{N_3}{N_1} . \quad (54)$$

The bracketed part of this expression is Z independent, since $\lambda_{32}\sim Z^{-1}$, $\lambda_{12}\sim Z^{-2}$, $A_{32}\sim Z$, and $A_{12}\sim Z^4$.

The maximum optical thickness τ^{max} of the active medium can be determined from the condition

$$A_{21}^{eff}>\frac{g_3}{g_2}A_{32} , \quad (55)$$

where A_{21}^{eff} is the effective radiative decay rate, subject to radiative trapping. Using $A_{21}^{eff}=A_{21}/(\tau\sqrt{\ln\tau})\simeq A_{21}/\tau$ for $\tau>1$ one obtains

$$\tau^{max}<\frac{g_2}{g_3}\frac{A_{21}}{A_{32}}\sim Z^3 . \quad (56)$$

Turning now to the ratio N_3/N_1 in Eq. (54) we note that electron-collision pumping operates at an electron density high enough that $A_{32}<N_e\langle v\sigma_{32}\rangle$, and therefore N_3/N_1 is given approximately by the ratio of the electron-collision excitation and deexcitation rates,

$$N_3/N_1\simeq\langle v\sigma_{13}\rangle/\langle v\sigma_{32}\rangle , \quad (57)$$

independent of Z.

Thus, for electron-collision pumping the gain increases with Z (a well-known fact[28]) and scales approximately as τ_{max}. However, for a given ion or electron density there exists an optimum Z which results in the highest gain.[28]

Using the atomic data given in Refs. 29 and 30 we obtain for the lasing transition ${}^3P_2\rightarrow{}^1P_1$ in Kr VVXII a gain coefficient (in cm^{-1})

$$\alpha = 40 \frac{\tau^{max}}{d} \frac{N_3}{N_1} . \tag{58}$$

According to calculations[28,31] in the electron-collision scheme a maximum value for N_3/N_1 of approximately 10^{-2} can be obtained (corresponding to $\alpha \simeq 30$ cm^{-1}) and, furthermore, the optimum electron density N_e^c lies in the interval $N_{e_1} = A_{32}/\langle v\sigma_{32}\rangle$ to $N_{e_2} = A_{21}/\langle v\sigma_{32}\rangle$. The ratio $N_{e_2}/N_{e_1} \geq 10^2$ for $Z \geq 20$ and increases as Z^3.

In the following we will analyze two resonant photopumping schemes for neonlike ions and compare them with electron-collision pumping. In case of photopumping a high electron density is not required for the excitation and therefore it is preferable to work at an electron density N_e^{ph} which is much lower than in the electron-collision scheme. Introducing the ratio N_e^{ph}/N_e^c to take the correspondingly lower ion density into account we get the condition that photoresonant pumping leads to a higher gain than electron-collision pumping if

$$\left[\frac{N_3}{N_1} \right]^{ph} \frac{N_e^{ph}}{N_e^c} \geq 10^{-2} . \tag{59}$$

We discuss two possible excitation paths to obtain an inversion between the levels $3p$ and $3s$ in neonlike ions.

a. Photopumping of $2p \rightarrow 3d$ transitions ($1 \rightarrow 4$ in our notation) and subsequent electron-collision transfer of the excitation to the upper laser level 3p. This scheme has been explored experimentally using a two-component plasma.[7] The $1s \rightarrow 3p$ resonance line of Al XIII was used to pump the $2p^6(^1S_0) \rightarrow 2p^5 3d(^3D_1)$ resonance line of Sr XXIX. A factor of 2 increase in the population of the photopumped level $3d$ in comparison with electron-collision pumping was measured. The conclusions with respect to gain on the $3p \rightarrow 3s$ transition were not very optimistic, however, since one has to rely on efficient collisional coupling of the pumped $3d$ level and the upper laser level $3p$, requiring high electron densities. In fact the electron density probability must be as high as in the collisionally pumped scheme and photopumping does not seem to offer any specific advantage. We agree with the conclusion[7] that $2p \rightarrow 3d$ photopumping does not appear to be a promising method of excitation.

b. Photopumping of $2p \rightarrow 4d$ transitions in neonlike ions. Here we propose a new idea for getting inversion on the $3p \rightarrow 3s$ transition in neonlike ions which seems to be much more promising than $2p \rightarrow 3d$ photopumping.

We assume that the electron density in the active medium is quite low, $N_e^{ph} \leq N_{e_1}$, and thus the radiative decay probability for the $3p$ level is higher than the collisional deexcitation rate. We also neglect the effect of stimulated emission (we assume $\theta\delta < 1$). From the balance equations

$$N_1 A_{15} \delta\theta = N_5 (A_{51} + A_{53}) , \tag{60a}$$

$$N_3 A_{32} = N_5 A_{53} , \tag{60b}$$

one obtains, using $A_{51} \gg A_{53}$ and $A_{15} = g_5 A_{51}$ (since $g_1 = 1$),

$$N_3/N_1 = g_5 \theta\delta \frac{A_{53}}{A_{32}} . \tag{61}$$

In comparison with $2p \rightarrow 3d$ pumping an "enhancement factor" $A_{53}/A_{32} \sim Z^3$ appears due to the quasimetastability of the $3p$ level. For Mn XVI one has $A_{53}/A_{32} \simeq 30$.[32]

For photoresonant $2p \rightarrow 4d$ pumping the condition (59) reads

$$g_5 \theta\delta \frac{A_{53}}{A_{32}} \frac{N_e^{ph}}{N_e^c} \geq 10^{-2} . \tag{62}$$

Assuming maximum values for the electron densities, viz., $N_e^{ph} = N_{e_1}$ and $N_e^c = N_{e_2}$ (optimistic for electron-collision pumping), one obtains

$$g_5 \theta\delta \frac{A_{53}}{A_{21}} \geq 10^{-2} \tag{63}$$

for higher photoresonant than collisional gain. Realizing that $A_{53}/A_{21} \simeq 1$ one concludes that the gain of the photoresonant scheme can be higher than in the case of electron-collision pumping in spite of the lower density.

c. Pumping and lasing ions of the same kind. When considering the idea of using the same kind of ions for pumping and lasing one must keep in mind that it is desirable to have higher electron temperature in the source plasma than in the active medium but still a significant fraction of ions in both plasmas should be in the proper ionization stage. Neonlike ions seem to be ideal in this respect since they are a quite stable species, existing under a wide range of plasma temperatures. Calculations show[7] that the fraction of neonlike Sr, for example, is higher than 0.1 in the temperature interval 0.4–2 keV with the maximum at $T_{eq} \simeq 0.8$ keV. So it is possible to have $T_e^s > T_e^{eq}$ in the source plasma and $T_e^a < T_e^{eq}$ in the active medium.

If $4d \rightarrow 2p$ radiation from the pump source excites the $2p \rightarrow 4d$ transition in the active medium an inversion will be obtained automatically. Let the electron density in the active medium $N_e^a < N_{e_1}$. Recalling that $N_{e_1} = A_{32}/\langle v\sigma_{32}\rangle$ this means that the upper laser level decays only radiatively. Assuming further that the number of photons per mode emitted by the source plasma on the transition $4d \rightarrow 3p$ and $3p \rightarrow 3s$ satisfies the conditions $\theta_{53} \ll 1$ and $\theta_{32} < 1$ we obtain for the populations of the lasing transition [see Eq. (61)]

$$N_2 = g_2 \theta_{12} \delta N_1 , \quad N_3 = g_5 \theta_{15} \delta \frac{A_{53}}{A_{32}} N_1 . \tag{64}$$

For Z not too low the condition for inversion

$$\theta_{15} \gg \frac{g_3}{g_5} \frac{A_{32}}{A_{53}} \theta_{12} , \tag{65}$$

is easily fulfilled, since $A_{53}/A_{32} \gg 1$ and increases $\sim Z^3$.

We assume that the transverse size of the active medium is small and its electron density low enough so that its own radiation is negligible compared to the radiation from the source plasma. It should be pointed out that it is not necessary to filter the radiation from the transitions $3s \rightarrow 2p$ of the source plasma (which populates the lowest laser level). This is due to the quasimetastability of the upper laser level which makes (65) valid even if $\theta_{12} > \theta_{15}$.

It is possible that this scheme can be realized without a filter between source and active medium, provided that the conditions $\theta_{53} \ll 1$ and $\theta_{32} < 1$ are fulfilled. If this is not possible, this radiation will have to be filtered out. Tabulated opacity values[33] together with the concept of using for the ionized material the "cold" cross sections with a shifted threshold[34,35] indicate that such filtering is not too difficult, since the wavelength of the radiation to be filtered differs by a large factor from the pump radiation.

Since the fraction of neonlike ions in the source plasma remains high (>0.1) for electron temperatures as high as the ionization potential of the ion[31] the number of photons per mode in $4d \rightarrow 2p$ radiation as given by expression (13) can in principle be higher than 0.5. In this case condition (63) can be fulfilled with two spatially separated laser plasmas (Fig. 1) or Z pinches and $\delta \geq 2\times 10^{-2}$.

We remark that the idea outlined above can be applied in an analogous way to Ni-like ions with resonant photopumping of $3d \rightarrow 5f$ and lasing on $4d \rightarrow 4p$.

VI. SUMMARY AND CONCLUSION

Using analytical estimates we have compared the merits of photoresonant pumping with electron-collision and -recombination pumping. The analysis was first performed for $\Delta n \neq 0$ transitions and hydrogenic ions. Using the criterion of low optical density at the resonance line to the lower laser level maximum gain formulas were derived for the three pumping methods. From these expressions it appears that photoresonant pumping may be favorable with respect to scaling to smaller wavelengths (higher Z). If a multicomponent plasma is used to supply more electrons all three excitation methods give similar Z dependences but photoresonant pumping again has the potential of achieving a higher gain.

The analysis was extended to $\Delta n \neq 0$ transitions in other than hydrogenic ions, specifically ions of the electron configuration $2s^2 2p^k$ ($0 \leq k \leq 5$). It was shown that in this case the basic problem of photoresonant pumping, i.e., the requirement of a matched pair of lines can be overcome by using the same kind of ion for pumping and lasing. A filter between source plasma and lasing plasma may have to be used. However, since the filtered radiation is of much lower frequency than the pumping radiation such a filter can readily be realized.

In the last part of the paper photoresonant pumping of $\Delta n = 0$ lasing transitions in neonlike ions was investigated. A promising scheme in which neonlike lasing ions can be pumped by neonlike source ions was suggested. It was shown that conditions can be found under which higher gain than in the collisionally pumped scheme can be achieved.

In conclusion it has been shown that photoresonant pumping is a promising way to obtain gain in the x-ray region. The favorable Z scaling of the achievable gain coefficient gives it a high potential for the realization of an x-ray laser in the water window. The main advantages over electron-collision and -recombination pumping result from the separation of the pump plasma and the active medium. For this reason the parameters in the two media can be optimized independently.

The following specific advantages of photoresonant pumping should be pointed out.

(i) A low electron density in the active medium eliminates problems due to refractive index disturbances.

(ii) The achievable gain coefficient is limited only by the available number of photons per mode in the pump radiation (and at hypothetically high values by saturation of the excitation transition).

(iii) Photoresonant pumping seems to be especially well suited for application in the gas puff Z pinch because of its favorable geometry factor.

A specific advantage of photoresonant pumping with ions of the same kind as the lasing ions applies to the situation in which the levels exhibit a fine-structure splitting (as, for example, in neonlike or berylliumlike ions): Since the level splitting in the pumping and lasing ions is the same the full manifold of levels in the lasing ion is excited by the pumping radiation.

All this seems to indicate that a photopumped x-ray laser may be realized using medium size laboratory installations, thus raising the hope for a successful verification of the many potential applications of coherent x rays. It should be also pointed out that not enough experimental work has been performed on photopumping schemes and that the pump-line intensities which ultimately determine the potential for resonantly photopumped x-ray lasers are essentially unknown and are difficult to accurately calculate.

ACKNOWLEDGMENTS

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APPENDIX: NUMBER OF PHOTONS PER MODE FOR VARIOUS PLASMA CONDITIONS

The number of photons per mode can be derived from the transport equation

$$\frac{d\theta}{dz} = -\kappa\theta + N_2 \sigma_{21}^{\mathrm{ph}} . \tag{A1}$$

In this equation $\kappa = [N_1 - (g_1/g_2)N_2]\sigma_{12}^{\mathrm{ph}}$ is the photoabsorption coefficient, N_1 the number density of ions in the lower level, N_2 the number density of ions in the upper level, g_i the degeneracy factor of level i, $\sigma_{12}^{\mathrm{ph}}$ the photoabsorption cross section, and $\sigma_{21}^{\mathrm{ph}} = (g_1/g_2)\sigma_{12}^{\mathrm{ph}}$ the cross section for stimulated emission.

With the boundary condition $\theta = 0$ at $z = 0$ one obtains after a propagation distance d

$$\theta = \frac{N_2/g_2}{N_1/g_1 - N_2/g_2}[1 - \exp(-\kappa d)] . \tag{A2}$$

Equation (A2) is valid in case of absorption or gain ($\kappa \gtrless 0$). Laser saturation is of course not included.

In an x-ray flashlamp the transition will be absorbing and we can distinguish the following cases.

(1) In case of low optical thickness τ ($\tau = N_1 \sigma_{12}^{\text{ph}} \simeq \kappa d \ll 1$)

$$\theta = N_2 \sigma_{21}^{\text{ph}} d \ . \quad \text{(A3)}$$

From the balance condition

$$N_1 A_{21} = N_1 N_e \langle v\sigma_{12} \rangle \quad \text{(A4)}$$

one obtains

$$\theta = \frac{g_1}{g_2} \frac{N_e \langle v\sigma_{12} \rangle}{A_{21}} \tau \ , \quad \text{(A5)}$$

where $\langle v\sigma_{12} \rangle$ is the coefficient for electron-collisional excitation of the upper from the lower level.

(2) For high optical thickness ($\tau \gg 1$, $\kappa d \gg 1$)

$$\theta = \frac{N_2}{(g_2/g_1)N_1 - N_2} \quad \text{(A6)}$$

(i) *Corona equilibrium.* As in the previous case, radiative decay, in spite of radiative trapping, is faster than electron collisional quenching,

$$A_{21}/(\tau\sqrt{\ln\tau}) \gg N_e \langle v\sigma_{21} \rangle \ . \quad \text{(A7)}$$

The factor $1/(\tau\sqrt{\ln\tau})$ takes the lower radiative decay rate due to radiative trapping into account. Using $N_2 \ll (g_2/g_1)N_1$, the number of photons per mode is given by

$$\theta = \frac{g_1}{g_2} \frac{N_e \langle v\sigma_{12} \rangle}{A_{21}} \tau\sqrt{\ln\tau} \ . \quad \text{(A8)}$$

(ii) *Boltzmann equilibrium.* The radiative decay rates are comparable or smaller than the electron collisional rates,

$$A_{21}/(\tau\sqrt{\ln\tau}) \le N_e \langle v\sigma_{12} \rangle \ , \quad \text{(A9)}$$

and the number of photons per mode is given by the Planck distribution

$$\theta = \frac{1}{\exp(\Delta E/kT_e) - 1} \ , \quad \text{(A10)}$$

where ΔE is the energy difference between levels 1 and 2 and T_e is the electron temperature.

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