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Three-body resonances in $t\alpha\mu$ and $d\alpha\mu$

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Resonances in muonic systems $t\alpha\mu$ and $d\alpha\mu$ have been predicted for $J=0$ and 1 using the complex-rotation method. This method has the advantage that the resonance position and total width are calculated at the same time. These resonances are found to be below the $n=4$ threshold of $\alpha\mu$ and are of Feshbach type. They decay to $t\mu+\alpha$ and $d\mu+\alpha$ and therefore could be of importance in the reactivation of μ from $\alpha\mu$ in the fusion of t and d in the presence of a muon (muon-catalyzed fusion). The channel $(\alpha\mu)_{n=3}+t$ or d could be followed by multiple scattering resulting in stripping of the muon. This process is highly density dependent.

Feshbach-type resonances or autoionization states in the muonic system $td\mu$ have been calculated recently.[1-3] Such resonances could also occur in other muonic systems. Resonances in $t\alpha\mu$ could be of particular importance because of the formation of $\alpha\mu$ in the reaction (muon-catalyzed fusion[4])

$$t\mu+d\rightarrow td\mu\rightarrow\alpha+\mu+n+17.6\text{ MeV}$$
$$\rightarrow(\alpha\mu)+n\,. \tag{1}$$

After about 150 reactions μ gets attached to the α particle and is lost for catalyzing further reactions. It is conceivable that $\alpha\mu$ forms resonance states of $t\alpha\mu$ that lie above the $n=1$ threshold of $t\mu$ and decay by releasing α,

$$(t\alpha\mu)^{**}\rightarrow t\mu+\alpha\ . \tag{2}$$

This reactivation would lower the sticking probability thereby increasing the number of reactions where μ is free to continue catalyzing fusion reactions. Resonances in $p\alpha\mu$, $d\alpha\mu$, and $t\alpha\mu$ have been investigated by Kravtsov *et al.*[5-7] for their role in charge exchange processes such as

$$p\mu+\alpha\rightarrow(p\alpha\mu)^{2+}\rightarrow(\alpha\mu)^{+}+p\ . \tag{3}$$

The resonances in $t\alpha\mu$ and $d\alpha\mu$ investigated by them lie below the $n=1$ thresholds of $t\mu$ and $d\mu$, have very small widths (generally less than a meV), and are therefore not very useful as far as muon-catalyzed fusion is concerned. Radiative transitions from the decay of these lower resonances in $(d\alpha\mu)^{2+}$ have been observed by Matsuzaki, Ishida, and Nagamine.[8]

It is possible that $\alpha\mu$ from the fusion of t and d in $td\mu$, could form a resonant state that subsequently decays to $t\mu$

$$\alpha\mu+t\rightarrow(t\alpha\mu)\rightarrow\begin{cases}(\alpha\mu)_{n=3}+t\,,\\ t\mu+\alpha\,,\\ (\alpha\mu)_{n=1,2}+t\,.\end{cases} \tag{4}$$

The decay channel $(\alpha\mu)_{n=3}+t$ is of particular importance because of multiple scattering resulting in stripping of muons. This multiple scattering is highly density dependent. The decay channel $t\mu+\alpha$ is also important because $t\mu$ could start the fusion reactions again. We have investigated such resonances by using the complex-rotation method.[9] This method has the advantage of giving the resonance position and width at the same time. It would be interesting to know the branching ratios of the above-mentioned processes but partial widths cannot be calculated easily by this method. Similar reactions with t replaced by d are also possible.

The complex-rotation method has been used successfully in numerous calculations[9] to find resonance parameters in three-body systems, e^-H, e^-He$^+$, e^+H, etc., and also in molecular systems.

The Hamiltonian given by

$$H=T+V \tag{5}$$

is analytically continued in the complex energy plane by using the transformation $r\rightarrow r\exp(i\theta)$. For Coulomb potentials, it can be seen that

$$H(\theta)=T\exp(-2i\theta)+V\exp(-i\theta)\ , \tag{6}$$

where θ is chosen between 0° and 45°. The eigenvalues are obtained by diagonalizing the expression

$$E=\langle\Psi H(\theta)\Psi\rangle/\langle\Psi\Psi\rangle\ , \tag{7}$$

wherein eigenvalues and eigenvectors are complex.

$$E=E_r-i\Gamma/2\ , \tag{8}$$

E_r gives the resonance position and Γ gives the width.

In this method a resonance, if it exists, is "uncovered" for θ greater than the absolute value of $\arg(E)/2$ and its position remains constant, while the other roots follow the branch cut, as the cut associated with that threshold is rotated through various angles.

The basis functions used are similar to the ones used in the calculation of binding energies and resonances in $td\mu$, Refs. 3, 10, and 11. The wave function for $J=0$ is

$$\Psi=f_1\,, \tag{9}$$

where

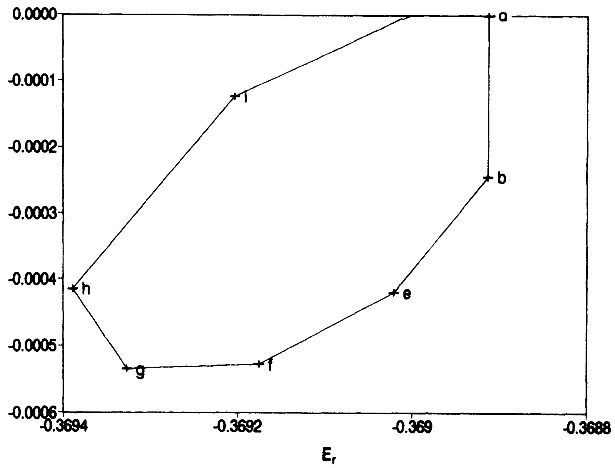

FIG. 1. $t\alpha\mu$ $J=0$ rotation paths through various angles (indicated by the labels, see Table I).

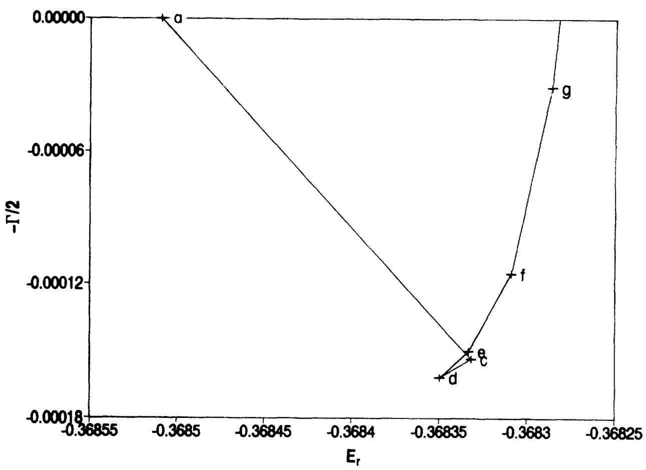

FIG. 2. $t\alpha\mu$ $J=1$ rotation paths through various angles (indicated by the labels, see Table I).

$$f_1=\sum_{l,m,n} C_{lmn}r_{12}^l(r_{13}^m r_{23}^n+r_{13}^n r_{23}^m)[\exp(-a_1r_{12}-a_2r_{13}-a_3r_{23})+\exp(-b_1r_{12}-b_2r_{13}-b_3r_{23})]$$
$$+\sum_{\substack{l,m,n\\ m(\neq n)}} D_{lmn}r_{12}^l(r_{13}^m r_{23}^n-r_{13}^n r_{23}^m)[\exp(-e_1r_{12}-e_2r_{13}-e_3r_{23})+\exp(-g_1r_{12}-g_2r_{13}-g_3r_{23})] \ , \quad (10)$$

r_{ij} are interparticle distances. The subscripts 1, 2, and 3 refer to alpha, triton (or deuteron), and muon, respectively. The a's, b's, e's, and g's, are the nonlinear parameters. Some of the b's and g's are subjected to cusp constraints described in Ref. 10. There are 1101 linear parameters. Three to four sets of nonlinear parameters have been used in the calculation for $J=0$.

The wave function for $J=1$ is

$$\Psi=f_1\mathbf{r}_{12}+f_2\mathbf{r} \ . \quad (11)$$

Both f_1 and f_2 have the same structure, Eq. (10), with different nonlinear parameters, and $\mathbf{r}$ is the distance of μ from the center of mass of α and t (or d). The wave function for $J=1$ has 1070 terms. Again, three sets of nonlinear parameters have been used. The masses used are $M_\alpha=7294.295$, $M_t=5496.918$, $M_d=3670.481$, and $M_\mu=206.769$, all in the units of electron mass. The unit of energy is $m_\mu R_\infty/m_e=2813.2584$ eV.

The resonances are uncovered using wave functions with different nonlinear parameters. Each of them traces out a curve in the complex energy plane as θ is increased. If a resonance exists, these curves come together in the vicinity of the resonance location and then diverge away from it as θ increases further. But for clarity we have shown only one curve for each case. Furthermore, we have used the criterion[12]

$$\Delta E/\Delta\theta \text{ is a minimum} \quad (12)$$

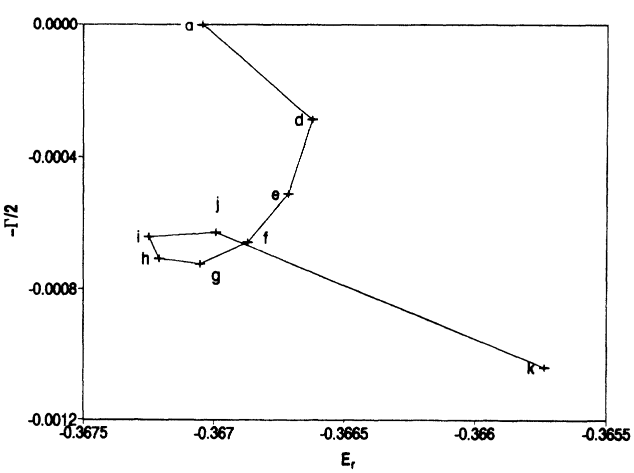

FIG. 3. $d\alpha\mu$ $J=0$ rotation paths through various angles (indicated by labels, see Table I).

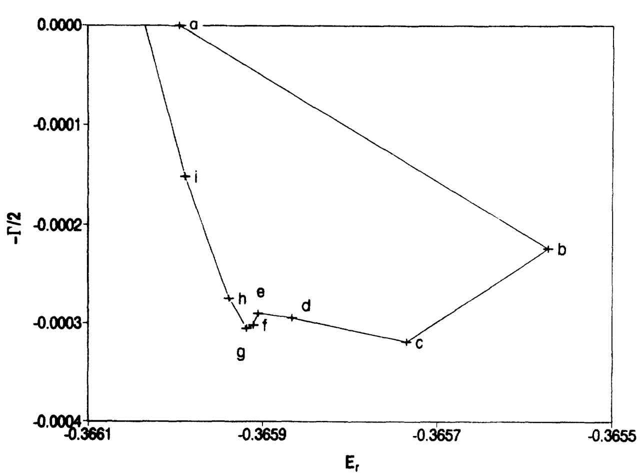

FIG. 4. $d\alpha\mu$ $J=1$ rotation paths through various angles (indicated by labels, see Table I).

TABLE I. Angles (deg) used in the calculation $t\alpha\mu$ and $d\alpha\mu$ in $J=0$ and 1 states.

Angle	Label	Angle	Label
0.0	*a*	7.5	*g*
0.46875	*b*	9.375	*h*
0.9375	*c*	11.25	*i*
1.875	*d*	13.125	*j*
3.75	*e*	15.0	*k*
5.625	*f*		

as θ increases. Here, $\Delta E=[|\Delta E_r|^2+|\Delta(\Gamma/2)|^2]^{1/2}$ and $\Delta\theta$ is the change in the angle. We have expressed energies in the units of $m_\mu R_\infty/m_e$. Figures 1–4 represent complex energies for various angles listed in Table I. Figures 1 and 2 are for $t\alpha\mu$ in $J=0$ and 1 states, and Figs. 3 and 4 are for $d\alpha\mu$ in $J=0$ and 1 states. These curves are for different sets of nonlinear parameters in each case. Clearly, there is an indication of a resonance in each case. Our final results are given in Table II. It should be noted that the angle required to uncover the resonance increases as the width increases. We have not investigated the resonance-formation mechanism. These resonances are very similar[13] to the resonances in $e^+\mathrm{He}^+=(\mathrm{PsHe}^{2+})$ which are believed to be due to the polarization potential $-\alpha/r^4$ of Ps in the presence of a He^{2+}, which has a charge $Z=2$, where α is the polarizability.

In Fig. 5, we give the energy-level diagram using the real part only and indicate the possible decay modes. The channels where the final state is $t\mu$, $d\mu$, or $(\alpha\mu)_{n=3}$ are of interest as far as muon-catalyzed fusion is concerned.

In order to obtain a qualitative picture of these resonances, we have calculated interparticle distances and they are given in Table III. We see that r_{12} and r_{23} are much larger in $J=1$ states than $J=0$ states, indicating that the angular momentum is carried by the heavy particles; it is also clear that the muon is in excited configurations.

Due to their large widths, it is conceivable that these resonances could be formed as a result of the collision between a fast moving $\alpha\mu$ with t or d. The laboratory energies required to form $J=0$ resonances in $t\alpha\mu$ and $d\alpha\mu$ are given in Table IV. We see that it is possible to form these resonances if the $\alpha\mu$ atoms from the fusion reaction, Eq. (1), are in the energy range indicated in Table IV. These resonances are broadened in the laboratory frame. The

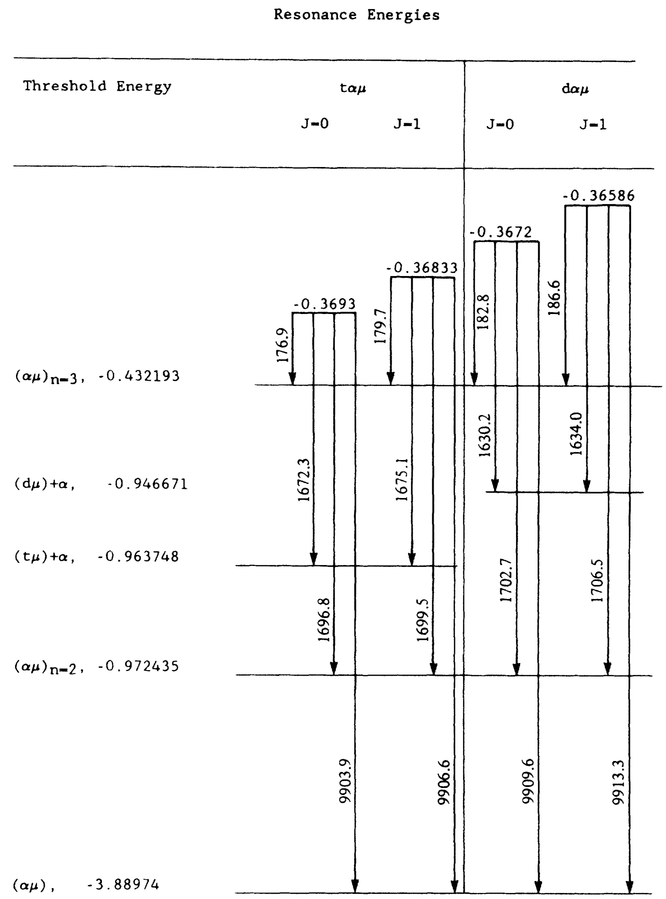

FIG. 5. Energy-level diagram for the real part of the resonance energy compared to various thresholds. For clarity, the vertical energy axis is not scaled. Energies are given in the units of $m_\mu R_\infty/m_e$. The energy released in various open channels is given in eV.

broadened widths are 5.53 and 10.96 eV in $J=0$ of $t\alpha\mu$ and $d\alpha\mu$, respectively, while they are 2.01 and 5.24 eV in $J=1$ of $t\alpha\mu$ and $d\alpha\mu$, respectively.

In conclusion, we have shown the existence of resonances in $t\alpha\mu$ and $d\alpha\mu$ systems and their potential importance in muon-catalyzed fusion. There could also be higher resonances and resonances in $J=2$ state and this is being investigated.

TABLE II. Positions and widths of $t\alpha\mu$ and $d\alpha\mu$ resonance states. The units for $E_r-i\Gamma/2$ are $m\mu R_\infty/m_e=2813.2584$ eV.

System	J	Angle[a]	$E_r-i\Gamma/2$	Position[b]	Width[c]
$t\alpha\mu$	0	9.375	$-0.369388-i0.415\times10^{-3}$	9903.563	2.34
$t\alpha\mu$	1	3.750	$-0.368333-i0.150\times10^{-3}$	9906.531	0.85
$d\alpha\mu$	0	11.25	$-0.367248-i0.640\times10^{-3}$	9909.582	3.60
$d\alpha\mu$	1	7.500	$-0.365918-i0.306\times10^{-3}$	9913.326	1.72

[a]Angles are in degrees.
[b]Positions are with respect to the ground state of $\alpha\mu$ and are in eV.
[c]Widths are in eV.

TABLE III. Average interparticle distances in resonant states. The units are in terms of the muon Bohr radius (a_μ).

System	J	$\langle r_{12}\rangle$	$\langle r_{13}\rangle$	$\langle r_{23}\rangle$
$t\alpha\mu$	0	13.75	7.45	8.39
$t\alpha\mu$	1	26.50	6.66	24.74
$d\alpha\mu$	0	14.89	7.37	9.60
$d\alpha\mu$	1	28.46	6.33	26.49

TABLE IV. Laboratory energies (keV) of $\alpha\mu$ required to produce resonances in $J=0$ state.

State of $\alpha\mu$	t	d
$n=1$	23.419	30.16
$n=2$	4.024	5.199
$n=3$	0.418	0.557

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[1]S. Hara and T. Ishihara, Phys. Rev. A **40**, 4232 (1989).

[2]I. Shimamura, Phys. Rev. A **40**, 4863 (1989).

[3]C. Y. Hu and A. K. Bhatia, in *Proceedings of an International Symposium on Muon Catalyzed Fusion μCF-89,* edited by J. D. Davies (Rutherford Appleton Laboratory, Chilton Didcot Oxon, 1990), p. 73; and (unpublished).

[4]A. K. Bhatia and R. J. Drachman, Comments At. Mol. Phys. **22**, 281 (1989).

[5]A. V. Kravtsov, N. P. Popov, G. E. Solyakin, Yu. A. Aristov, M. P. Faifman, and N. F. Truskova, Phys. Lett. **83A**, 379 (1981).

[6]Yu. A. Aristov, A. V. Kravtsov, N. P. Popov, G. E. Solyakin, N. F. Truskova, and M. P. Faĭfman, Yad. Fiz. **33**, 1066 (1981) [Sov. J. Nucl. Phys. **33**, 564 (1981)].

[7]A. V. Kravtsov, A. I. Mikhailov, and N. P. Popov, J. Phys. B At. Mol. Phys. **19**, 2579 (1986).

[8]T. Matsuzaki, K. Ishida, and K. Nagamine, Muon Catal. Fusion **2**, 217 (1988).

[9]Y. K. Ho, Phys. Rep. **99**, 1 (1983), and references cited therein.

[10]C. Y. Hu, Phys. Rev. A **36**, 4135 (1987).

[11]C. Y. Hu, Phys. Rev. A **34**, 2536 (1986).

[12]Y. K. Ho (private communication).

[13]A. K. Bhatia and R. J. Drachman, this issue, Phys. Rev. A **42**, 5117 (1990).