Incomplete "collapse" and partial quantum Zeno effect

Asher Peres and Amiram Ron
Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel
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If the interaction generating a quantum measurement is too weak to completely "collapse" the wave function, a *partial* quantum Zeno effect may result. An experimental test is proposed.

It was shown long ago by von Neumann[1] that it is possible to steer a quantum system from any arbitrary pure state into any other arbitrary pure state, by a sequence of measurements in rapid succession (the meaning of the word "measurement" is discussed in the last chapter of von Neumann's book). In particular, if all these measurements test whether the momentary quantum state is the same as the initial preparation state, that initial state is "frozen," irrespective of the dynamical properties of the free (unmeasured) quantum system. This effect has become a popular subject of discussion after it acquired the name "quantum Zeno paradox."[2,3]

There is, however, nothing paradoxical here. Simply, the frequent interactions of the measuring apparatus with the quantum system alter the dynamical properties of the latter, and in particular its transition rates.[4] Therefore, the quantum Zeno effect is nothing more than an ordinary dynamical effect, which can be completely discussed without invoking ill-defined terms such as "measurement" or "collapse" of the wave function.

Recently, the existence of the Zeno effect was dramatically verified in an experiment by Itano *et al.*[5] In that experiment, Rabi oscillations between two atomic levels, $|1\rangle$ and $|2\rangle$, were monitored by means of a sequence of brief laser pulses. These pulses were tuned in such a way that they could excite the atom from level $|1\rangle$ (but not level $|2\rangle$) into an unstable level $|3\rangle$, which then immediately decayed back into level $|1\rangle$, thereby emitting an observable fluorescence photon. It was found that, as the time separation between the pulses decreased, the probability for a transition between the atomic levels $|1\rangle$ and $|2\rangle$ was depressed, and tended to zero. The agreement between theory and experiment was excellent.

The purpose of the present article is to examine the behavior of the Zeno effect when the intensity of the laser pulses is reduced to the point that the probability of exciting the atom from level $|1\rangle$ to level $|3\rangle$ becomes significantly less than unity. In the language of measurement theory, this means that the measurement is no longer of the idealized von Neumann type, whereby the final states of the measuring apparatus are orthogonal.[6,7] Therefore, the infamous "collapse" postulate becomes ambiguous and useless; a detailed dynamical description of the measuring process is needed, as shown below.

In addition to the three atomic states mentioned above, let us denote by $|0\rangle$ the ground state of the photon field. That field too is a dynamical system, coupled to the atom. Suppose that the initial state, before the laser pulse, is $|\Psi_i\rangle = |1\rangle \otimes |0\rangle$. As the lifetime τ of the excited state $|3\rangle$ is *very brief* on the time scale of the experiment, the state soon after the end of the pulse will be $|\Psi_f\rangle = |1\rangle \otimes |\Phi\rangle$, where $|\Phi\rangle$ is an excited state of the electromagnetic field, including various amplitudes for 0,1,2, ... photons. On the other hand, the laser pulse does not affect the atomic state $|2\rangle$, so that the initial state $|2\rangle \otimes |0\rangle$ remains unchanged.

In general, the initial state of the atom will not be pure and it must be represented by a density matrix ρ. The quantum evolution generated by the pulse is

$$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \otimes |0\rangle\langle 0| \rightarrow \begin{pmatrix} \rho_{11} \otimes |\Phi\rangle\langle\Phi| & \rho_{12} \otimes |\Phi\rangle\langle 0| \\ \rho_{21} \otimes |0\rangle\langle\Phi| & \rho_{22} \otimes |0\rangle\langle 0| \end{pmatrix} . \tag{1}$$

Since our problem is to monitor only the Rabi oscillations of the atom, we disregard the photon field in (1) by tracing it out. The net result is[7]

$$\begin{pmatrix} \rho_{11} \, \rho_{12} \\ \rho_{21} \, \rho_{22} \end{pmatrix} \rightarrow \begin{pmatrix} \rho_{11} & S\rho_{12} \\ S^* \rho_{21} & \rho_{22} \end{pmatrix} , \tag{2}$$

where $S = \langle 0|\Phi\rangle$ is the survival amplitude of the electromagnetic vacuum state—that is, the amplitude for the *absence* of resonance fluorescence.

If the laser pulse is very strong, $S \rightarrow 0$ and the effect of the pulse simply is to obliterate the off-diagonal elements of ρ. Their disappearance is the so-called "collapse" of the wave function. If, on the other hand, the pulse is only moderately strong, there is a non-negligible survival amplitude $\langle 0|\Phi\rangle$ and we must explicitly use the evolution law (2). The appropriate formalism can be found in Ref. 7.

Our problem thus is to evaluate the survival amplitude $S = \langle 0|\Phi\rangle$. We shall assume that the duration of the laser pulse is so short that one can neglect the occurrence of transitions between states $|1\rangle$ and $|2\rangle$ while the laser is acting. Moreover, let us assume for simplicity that the initial state is pure, namely $|\Psi_i\rangle = |1\rangle \otimes |0\rangle$. [A mixed state such as in Eq. (1) can always be considered as a statistical average of pure states.] The state after the laser pulse acts during a time t will be

$$|\Psi(t)\rangle = a_1(t)\,|1\rangle \otimes |0\rangle + a_3(t)\,|3\rangle \otimes |0\rangle + \dots , \tag{3}$$

where the ellipsis represents other terms. The evolution of this $\Psi(t)$ is generated by a Hamiltonian whose relevant terms can be written as

$$H = H_0 + H_{\rm em} + V. \quad (4)$$

Here,

$$H_0 = \sum_k (E_k \, |k\rangle\langle k|) + \hbar\Omega \, \sin(\omega t) \, (|1\rangle\langle 3| + |3\rangle\langle 1|), \quad (5)$$

where E_k is the energy of level $|k\rangle$; ω is the frequency of the laser; $\hbar\Omega$ is the value of the matrix element generating transitions between states $|1\rangle$ and $|3\rangle$ (that value is proportional to the amplitude of the laser field); $H_{\rm em}$ is the Hamiltonian of the free electromagnetic field; and V is the interaction between that field and the atom, which causes the spontaneous decay from $|3\rangle$ to $|1\rangle$. The laser field itself is treated classically and is not considered as a quantized dynamical variable (we can safely neglect the reaction of the atom on the laser).

The relevant equations of motion thus are

$$i\hbar\dot{a}_1 = E_1 \, a_1 + \hbar\Omega \, \sin(\omega t) \, a_3 \,, \quad (6)$$

and

$$i\hbar\dot{a}_3 = E_3 \, a_3 + \hbar\Omega \, \sin(\omega t) \, a_1 \, - (\langle 0| \otimes \langle 3|)V|\Psi\rangle. \quad (7)$$

For times that are neither very short nor very long,[8] the last term can be treated phenomenologically as causing an exponential decay, and therefore replaced by $-ia_3/2\tau$, where $1/\tau$ is the Einstein A coefficient. This is the Weisskopf-Wigner approximation. It obviously violates unitarity, because we have restricted our attention to a two-dimensional subspace of the Hilbert space of states. It is customary to bypass this difficulty by using a density matrix formalism, keeping track of the atomic state only. Here, we preferred to use a pure-state analysis,[9] to show more clearly how *rare* fluorescence photons cause a *partial* decoherence when they are traced out.

We now define $b_n = a_n \, \exp(iE_n t/\hbar)$ and obtain

$$i\dot{b}_1 = \Omega \, \sin(\omega t) \, \exp(i\omega_{31} t) \, b_3 \,, \quad (8)$$

and

$$i\dot{b}_3 = \Omega \, \sin(\omega t) \, \exp(-i\omega_{31} t) \, b_1 - ib_3/2\tau, \quad (9)$$

where $\omega_{31} = (E_3 - E_1)/\hbar$. As we are interested in the case $\omega \simeq \omega_{31}$, we can neglect the rapidly oscillating terms $\exp[i(\omega_{31} + \omega)]t$ and we have

$$\dot{b}_1 = -\tfrac{1}{2} \Omega \, e^{-i\delta t} \, b_3 \,, \quad (10)$$

$$\dot{b}_3 = \tfrac{1}{2} \Omega \, e^{i\delta t} \, b_1 - b_3/2\tau, \quad (11)$$

where $\delta = \omega - \omega_{31}$ is the detuning of the laser with respect to the ω_{31} transition frequency. Eliminating b_3, we obtain

$$\ddot{b}_1 + \left(\frac{1}{2\tau} + i\delta\right) \dot{b}_1 + \frac{\Omega^2}{4} \, b_1 = 0. \quad (12)$$

It follows that b_1 evolves as a sum of two exponentials. The value of b_1 at the end of the pulse is the coefficient S which appears in Eq. (2) and controls the partial Zeno effect. [The phase factor $\exp(iE_1 t/\hbar)$ is irrelevant, since it would also appear in the absence of the laser pulse.]

We henceforth restrict the discussion to the simple case of no detuning ($\delta = 0$). The two exponentials in the solution of (12) are

$$\exp\left[-(1/2\tau) \pm \sqrt{(1/2\tau)^2 - (\Omega/2)^2}\right] t. \quad (13)$$

Here, if there were no decay, $\Omega/2$ would be the Rabi frequency for the $|1\rangle \leftrightarrow |3\rangle$ oscillations. In the experiment of Itano *et al.*,[5] the laser pulse was strong and $\Omega\tau \gg 1$. Therefore the solution of (12) behaved as $e^{-t/2\tau} \cos(\Omega t/2)$. At the end of each pulse, $t \gg 2\tau$ and S was essentially zero.

In the present article, we are interested in weaker pulses, such that $\Omega\tau \ll 1$. In that case, the solution of (12) behaves as $\exp(-\Omega^2 \tau t/2)$. By adjusting the strength of the pulses, the factor $\Omega^2 \tau t$ can be continuously controlled, and the survival amplitude $S = \langle 0|\Phi\rangle$ can be given any desired value between 0 and 1.

We are finally ready to compute the result of a sequence of n laser pulses, equally spaced during a half-period of the Rabi oscillations between levels $|1\rangle$ and $|2\rangle$ as in the experiment of Itano *et al.*[5]. We write the density matrix as

$$\begin{pmatrix} \rho_{11} \, \rho_{12} \\ \rho_{21} \, \rho_{22} \end{pmatrix} = \tfrac{1}{2} (\mathbf{I} + \sigma \cdot \mathbf{R}). \quad (14)$$

Initially, $\mathbf{R} = (0,0,1)$. In each one of the n time intervals, the vector $\mathbf{R}$ rotates by an angle $\theta = \pi/n$, and then its x and y components (corresponding to the off-diagonal elements of ρ) are reduced by a factor S. If phases are chosen so that R_y remains zero, the result is given by[10]

TABLE I. Predicted values for $|1\rangle \to |2\rangle$ or $|2\rangle \to |1\rangle$ transition probabilities, for various values of n (the number of pulses) and S (the amplitude for no fluorescence). The laser pulses are assumed to be so brief that optical pumping $|1\rangle \leftrightarrow |2\rangle$ is negligible during each pulse.

n	$S = 0$	$S = 0.2$	$S = 0.4$	$S = 0.6$	$S = 0.8$	$S = 1$
1	1.0	1.0	1.0	1.0	1.0	1.0
2	0.5000	0.6000	0.7000	0.8000	0.9000	1.0
4	0.3750	0.4560	0.5530	0.6720	0.8190	1.0
8	0.2346	0.3054	0.3948	0.5154	0.6962	1.0
16	0.1334	0.1840	0.2541	0.3590	0.5419	1.0
32	0.0716	0.1028	0.1488	0.2261	0.3834	1.0
64	0.0371	0.0543	0.0814	0.1304	0.2458	1.0

$$\begin{pmatrix} R_x \\ R_z \end{pmatrix} \to \begin{pmatrix} R'_x \\ R'_z \end{pmatrix} = \begin{pmatrix} S\cos\theta & S\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^n \begin{pmatrix} R_x \\ R_z \end{pmatrix}, \quad (15)$$

where $\theta = \pi/n$. The transition probability is[5] $(1-R'_z)/2$, and is given in Table I for various values of n and S. The two columns on the left are identical to those of Table I of Ref. 5. The column on the right corresponds to the trivial case of very weak laser pulses. The other columns, for S=0.2 to 0.8, illustrate a *partial* quantum Zeno effect, due to an incomplete "collapse." An experimental test, extending the results of Ref. 5, should be feasible.

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[10] See Eqs. (45)–(47) of Ref. 7, where **R** is called **m**.