Femtosecond squeezed-vacuum-state generation in mode-locked soliton lasers

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A mode-locked soliton laser, experiencing group-velocity dispersion and self-phase modulation via a nonlinear Kerr medium in an antiresonant ring interferometer reflector is analyzed in the linearized regime. It is demonstrated that this laser can produce an ultrashort squeezed vacuum from the open port of the antiresonant ring interferometer and that the spectral linewidth of the soliton output is broadened via the coupling of amplitude and phase in the Kerr medium.

INTRODUCTION

A pulsed squeezed state is expected to improve the signal-to-noise ratio in precision measurements of ultrafast phenomena. The use of squeezed states may be advantageous specifically when the expected signal is extremely small or when only a weak optical probe is allowed. Examples are electro-optic field-sensing measurements for ultrashort pulse propagation in electronic integrated circuits,[1] polarization rotation measurements for nerve pulse propagation in biological systems,[2] and measurements of particle-induced birefringence in vacuum.[3] Slusher *et al.* produced a pulsed squeezed state of 100 ps duration using a traveling-wave $KTiOPO_4$ (KTP) parametric amplifier pumped by second-harmonic light of a 1.06-μm cw mode-locked yttrium aluminum garnet (YAG) laser.[4] The degree of squeezing was, however, modest mainly due to the limited pump intensity.

In previous work on soliton like pulse generation in a colliding-pulse mode-locked (CPM) dye laser, a natural self-phase modulation (SPM) due to the optical Kerr effect in the dye solvents (up chirp) and time-dependent absorption saturation (down chirp), were employed together with negative group-velocity dispersion (GVD) caused by intracavity prisms.[5,6]

In this paper, a new generation scheme for a femtosecond squeezed vacuum state is proposed using the antiresonant ring interferometer. The scheme is free of excess noise of the pump wave, in contrast to the scheme proposed by Kitagawa and Yamamoto[7] for generation of number-phase squeezed states using SPM in a Kerr medium. Shirasaki *et al.*[8] analyzed a nonlinear Mach-Zehnder interferometer, in which two number-phase squeezed states interfere at an output half mirror to produce a squeezed vacuum state. We have discovered the fact that the excess noise of the pump wave incident from the pump port does not affect the squeezed vacuum state at the other (output) port in the range of validity of the linearization approximation when one input has only a vacuum state. This paper applies this discovery to a mode-locked laser with an antiresonant ring fiber reflector as shown in Fig. 1. This mode-locked laser can produce a squeezed vacuum state and a stable femtosecond solitonlike pulse. The two squeezed pulses interfere coherently at the coupler, and a squeezed vacuum state and a number-phase squeezed state with coherent excitation are emitted out of the open port and reflected into the main cavity, respectively (see Fig. 2).

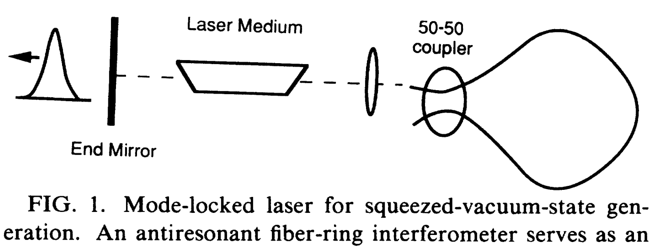

FIG. 1. Mode-locked laser for squeezed-vacuum-state generation. An antiresonant fiber-ring interferometer serves as an end reflector of the laser cavity.

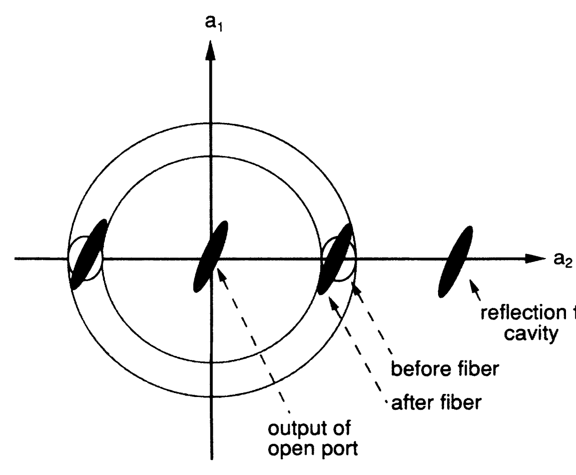

FIG. 2. Wigner densities for a clockwise and counterclockwise pulses in an antiresonant interferometer before and after self-phase modulation. A squeezed vacuum state is the output at the open port and the coherent excitation is reflected back into the cavity.

This system has several advantages over other squeezed-state generation schemes. Since the pulse duration is 10^{-4} to 10^{-5} of the repetition rate and the intensity for the pulse inside the cavity is usually about 2 orders of magnitude larger than the one outside the cavity, the peak intensity of the pulse can be extremely high and so efficient nonlinear interaction (squeezing) becomes possible. Unlike a degenerate parametric amplifier, it does not require any frequency conversion and reconversion processes. No pump pulse energy is wasted to produce a squeezed state. It is also a wavelength-tunable squeezer.

This paper is organized as follows. Section I looks at the ring interferometer reflector in the absence of dispersion. Section II is a brief review of the quantum theory of soliton propagation in fibers as presented in Ref. 9. Section III considers a fiber-ring interferometer reflector excited by solitons (i.e., after the beam splitter the two counter propagating pulses are solitons). Section IV studies the quantization of the laser medium. We consider the limit of a very broadband laser medium such that the spreading of the pulse via passage through the gain medium is negligible. Correspondingly the temporal modulation of the pulse by the saturable absorber is also unimportant. With this approximation, the noise in the resonator may be considered a stationary process. We make the further assumption that the laser medium and saturable absorber shape the pulse into the soliton shape required by the fiber. Section V combines the equations of the cavity with those of soliton propagation. In this way the equations are obtained for the four operators (photon number, phase, position, momentum) describing the soliton traveling in the resonator. These four equations have a close resemblance to the equations of noise in an oscillator, except, of course, that there is now an additional pair of equations describing position and momentum. The phase noise of the soliton is found to be enhanced by the Kerr modulation, in a way very similar to the way the amplitude-dependent index in a semiconductor laser, (the α parameter) enhances the phase noise and the linewidth of a semiconductor laser.

Thus far we have treated a fiber-ring reflector as the nonlinear medium producing the squeezing. Indeed, only such a ring is capable of producing appreciable squeezing of vacuum, because in order to do so the nonlinear phase shift has to be of the order of π or larger. Any attempt to "lump" the nonlinearity and the GVD into separate elements suffers from a lack of rigor if the phase shift per pass in the Kerr medium is large. Yet we speculate that squeezing in such a system may yet be possible, although further work is required to make sure. Such a modified system would be more compact than the fiber-ring reflector system and could be operated in the positive GVD region of optical fibers, i.e., $\lambda < 1.3\ \mu$m.

I. THE SYMMETRIC NONLINEAR INTERFEROMETER

An important issue covered in this paper is the "orthogonality" of the squeezing in a ring interferometer reflector under the linearization approximation. By "orthogonality" we mean the property that the noise accompanying the pump reemerges transformed (squeezed) as a reflection from the pump port, whereas the vacuum entering from the vacuum port emerges squeezed from the vacuum port. The excess pump noise does not perturb the squeezing of the vacuum. We approach the problem in two steps. First we look at the properties of the interferometer in the absence of dispersion. Then we consider a soliton as a pump using the quantization developed for solitons elsewhere.[9]

A ring interferometer reflector under pulse excitation can be "developed" into a symmetric Mach-Zehnder interferometer if one may neglect the interaction of the counterpropagating pulses. This is permissible at power levels that produce phase shifts of the order of π in forward propagation through the loop. Hence the system of Fig. 3(a) can be studied in the form of Fig. 3(b). The single input coupler is then represented by two beam splitters in sequence. The scattering equations for the beam splitters are in the notation illustrated in Fig. 3(b):

$$\hat{A}_3=\frac{1}{\sqrt{2}}(\hat{A}_1+\hat{A}_2)\ , \tag{1.1}$$

$$\hat{A}_4=\frac{1}{\sqrt{2}}(\hat{A}_1-\hat{A}_2)\ , \tag{1.2}$$

$$\hat{A}_7=\frac{1}{\sqrt{2}}(\hat{A}_5+\hat{A}_6)\ , \tag{1.3}$$

$$\hat{A}_8=\frac{1}{\sqrt{2}}(\hat{A}_5-\hat{A}_6)\ . \tag{1.4}$$

We use capital letters for the annihilation operators of

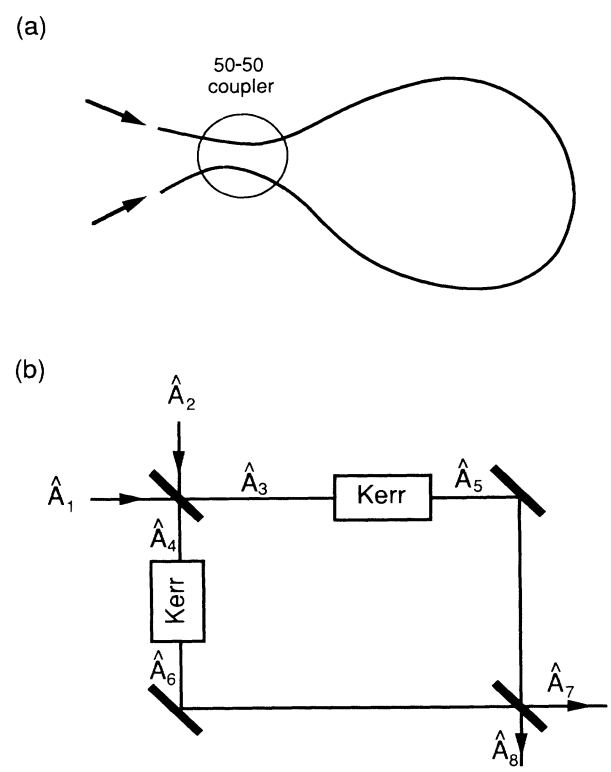

FIG. 3. (a) The fiber-ring interferometer, and (b) the folded-out Mach-Zehnder version.

the field of pulses. This is in anticipation of the linearization in which lower case letters will be used for the perturbations. The photon flux of the ith wave is $\hat{A}_i^\dagger \hat{A}_i$. The transformations by the nonlinear Kerr medium are

$$\hat{A}_5=\exp(i\kappa\hat{A}_3^\dagger\hat{A}_3)\hat{A}_3 , \tag{1.5}$$

$$\hat{A}_6=\exp(i\kappa\hat{A}_4^\dagger\hat{A}_4)\hat{A}_4 . \tag{1.6}$$

Here κ is proportional to the Kerr coefficient and the length of the fiber. We introduce now the linearization approximation by which we split all operators into a "large-signal" c-number part and a small-signal operator. The latter carries the commutation properties of the original operator. In the linearization only first-order quantities in the perturbation will be retained. The linearization of the action of the Kerr medium on operator $\hat{A}_i$, $i=3,4$ is, with

$$\hat{A}_i=A_i+\hat{a}_i , \tag{1.7}$$

$$\begin{aligned}&\exp[i\kappa(A_i^*+\hat{a}_i^\dagger)(A_i+\hat{a}_i)](A_i+\hat{a}_i)\\ &\approx\exp(i\kappa|A_i|^2)(1+i\kappa A_i^*\hat{a}_i+i\kappa A_i\hat{a}_i^\dagger)(A_i+\hat{a}_i)\\ &\approx\exp(i\kappa|A_i|^2)A_i+\exp(i\kappa|A_i|^2)(\mu_i\hat{a}+\nu_i\hat{a}^\dagger)\end{aligned} \tag{1.8}$$

where

$$\mu_i\equiv 1+i\kappa|A_i|^2,\quad \nu_i=i\kappa A_i^2 .$$

The last expression is cast in the usual notation of squeezing, with the squeezing parameters μ_i and ν_i obeying the constraint

$$|\mu_i|^2-|\nu_i|^2=1 . \tag{1.9}$$

Consider now the case when a pump excitation enters through port 1 and vacuum enters through port 2. Then the signal in the two arms is symmetric $|A_3|^2=|A_4|^2$ and the c-number excitation of the nonlinearity is symmetric in the two arms. Defining

$$\Phi=\kappa|A_3|^2=\kappa|A_4|^2 \tag{1.10}$$

we obtain for the outputs

$$\hat{A}_7=A_i\exp i\Phi+\exp(i\Phi)(\mu\hat{a}_1+\nu\hat{a}_1^\dagger) , \tag{1.11}$$

$$\hat{A}_8=\exp(i\Phi)(\mu\hat{a}_2+\nu\hat{a}_2^\dagger) . \tag{1.12}$$

The pump and its squeezed fluctuations emerge from port 7, the squeezed vacuum emerges from port 8. The remarkable result is that, even in the nonlinear interferometer, the noises entering through the two input ports do not interact. They are squeezed individually by the symmetric signal (pump), assuming, of course, that the linearization approximation is valid.

II. REVIEW OF SOLITON PROPAGATION

The nonlinear Schrödinger equation is quantized by replacing the positive frequency amplitudes by photon annihilation operators. In the time domain, the equation is

$$-i\frac{\partial\hat{U}}{\partial z}=\frac{1}{2}\frac{\partial^2\hat{U}}{\partial x^2}+\hat{U}^\dagger\hat{U}\hat{U} \tag{2.1}$$

where we use a capital $\hat{U}$ for the annihilation operator, in anticipation of the linearization, linearized quantities will be represented by a lowercase letter. The operator $\hat{U}$ obeys the commutation relations

$$[\hat{U}(z,x)\hat{U}^\dagger(z,x')]=\delta(x-x') . \tag{2.2}$$

Note that x expresses time and z propagation distance along the fiber.

The classical fundamental soliton solution of (2.1) is[10]

$$\begin{aligned}U(z,x)&=\frac{n_0}{2}\exp\left[i\frac{n_0^2}{8}z-i\frac{p_0^2}{2}z+ip_0x+i\theta_0\right]\\ &\quad\times\operatorname{sech}\left[\frac{n_0}{2}(x-x_0-p_0z)\right]\\ &\equiv U_0(z,x)\exp\left[i\frac{n_0^2}{8}z-i\frac{p_0^2}{2}z+ip_0x+i\theta_0\right] .\end{aligned} \tag{2.3}$$

Here the parameters are chosen so as to conform with the quantization. The field amplitude U is so normalized that $\int|U|^2dx=n_0$ is the photon number, p_0 is the momentum, x_0 is the position (or time of the peak of the soliton), and θ_0 is the phase. Without loss of generality, we assume $p_0=x_0=\theta_0=0$. This simply says we choose coordinates in which the soliton is followed so that the envelope can be made independent of z.

The equation (2.1) is linearized by writing the operator $\hat{U}$ as

$$\hat{U}(z,x)=[U_0(x)+\hat{u}(z,x)]\exp\left[i\frac{n_0^2}{8}z\right] . \tag{2.4}$$

The commutation relations of $\hat{U}$ are taken over by $\hat{u}(z,x)$.

To first order, the operator $\hat{u}(z,x)$ obeys the equation

$$-i\frac{\partial\hat{u}}{\partial z}=\frac{n_0^2}{8}\hat{u}+\frac{1}{2}\frac{\partial^2\hat{u}}{\partial x^2}+2U_0^2\hat{u}+U_0^2\hat{u}^\dagger . \tag{2.5}$$

The excitation $\hat{u}$ can be written as

$$\begin{aligned}\hat{u}(z,x)=&\Delta\hat{n}(z)f_n(x)+i\Delta\hat{\theta}(z)f_\theta(x)+i\Delta\hat{p}(z)f_p(x)\\ &+\Delta\hat{x}(z)f_x(x)+\Delta\hat{u}_c(z,x)\end{aligned} \tag{2.6}$$

where $\Delta\hat{n}$, $\Delta\hat{\theta}$, $\Delta\hat{p}$, and $\Delta\hat{x}$ are the perturbation operators associated with the photon number, phase, momentum, and position and $\Delta\hat{u}_c(z,x)$ is the continuum part. The four functions used in the expansion are

$$f_n(x)=\left[\frac{1}{n_0}-\frac{x}{2}\tanh\left[\frac{n_0}{2}x\right]\right]U_0(x) , \tag{2.7}$$

$$f_\theta(x)=U_0(x) , \tag{2.8}$$

$$f_p(x)=xU_0(x) , \tag{2.9}$$

$$f_x(x)=\frac{n_0}{2}\tanh\left[\frac{n_0}{2}x\right]U_0(x) . \tag{2.10}$$

Their shapes are illustrated in Fig. 4. Further, it has been shown[9] that one may project out any one of the operators by multiplication by an adjoint function and integration.

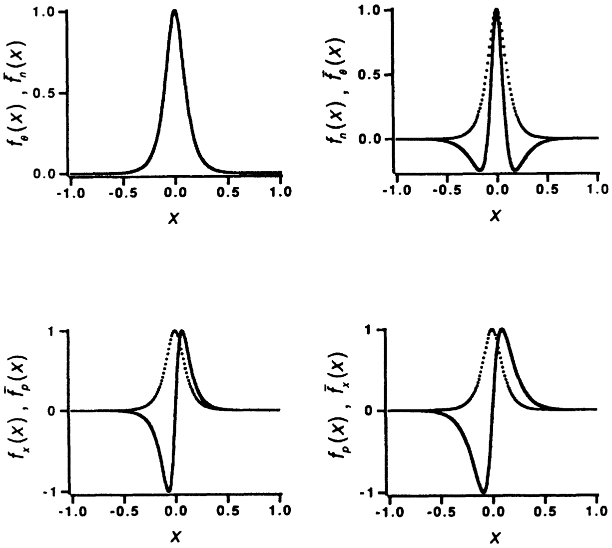

FIG. 4. The perturbation functions due to change of n_0, θ_0, p_0, and x_0.

Separating $\hat{u}$ into two Hermitian operators

$$\hat{u}=\hat{u}_1+i\hat{u}_2 , \tag{2.11}$$

one has

$$\Delta\hat{n}=\int \bar{f}_n\hat{u}_1 dx , \tag{2.12}$$

$$\Delta\hat{\theta}=\int \bar{f}_\theta\hat{u}_2 dx , \tag{2.13}$$

$$\Delta\hat{p}=\int \bar{f}_p\hat{u}_2 dx , \tag{2.14}$$

$$\Delta\hat{x}=\int \bar{f}_x\hat{u}_1 dx . \tag{2.15}$$

The four adjoint functions used in the projection are

$$\bar{f}_n(x)=2U_0(x) , \tag{2.16}$$

$$\bar{f}_\theta(x)=2\left[\frac{1}{n_0}-\frac{x}{2}\tanh\left[\frac{n_0}{2}x\right]\right]U_0(x) , \tag{2.17}$$

$$\bar{f}_p(x)=\tanh\left[\frac{n_0}{2}x\right]U_0(x) , \tag{2.18}$$

$$\bar{f}_x(x)=\frac{2}{n_0}xU_0(x) . \tag{2.19}$$

Their shapes are also illustrated in Fig. 4.

With (2.12)–(2..19), it is easy to prove that the operators obey the familiar commutation relations:

$$[\Delta\hat{n},\Delta\hat{\theta}]=i , \tag{2.20}$$

$$[\Delta\hat{x},n_0\Delta\hat{p}]=i . \tag{2.21}$$

The very important physical consequence of this formalism is that one may measure any one of the operators by a suitable choice of a local oscillator excitation of a balanced homodyne detection. Indeed, projections of the kind appearing in (2.12)–(2.15) have been shown[9] to be the result of a homodyne detection with the local oscillator wave form proportional to the projection functions

The operators obey the following equations of motion:

$$\frac{\partial}{\partial z}\Delta\hat{n}=0 , \tag{2.22}$$

$$\frac{\partial}{\partial z}\Delta\hat{\theta}=\frac{n_0}{4}\Delta\hat{n} , \tag{2.23}$$

$$\frac{\partial}{\partial z}\Delta\hat{p}=0 , \tag{2.24}$$

$$\frac{\partial}{\partial z}\Delta\hat{x}=\Delta\hat{p} \tag{2.25}$$

with the solutions

$$\Delta\hat{n}(z)=\Delta\hat{n}(0) , \tag{2.26}$$

$$\Delta\hat{\theta}(z)=\Delta\hat{\theta}(0)+\frac{n_0}{4}z\Delta\hat{n}(0) , \tag{2.27}$$

$$\Delta\hat{p}(z)=\Delta\hat{p}(0) , \tag{2.28}$$

$$\Delta\hat{x}(z)=\Delta\hat{x}(0)+\hat{z}\Delta\hat{p}(0) \tag{2.29}$$

where z is the propagation distance. Note that the linearization establishes linear relations between the input and output operators; the system transformation is fully determined by the classical properties of the soliton, namely, by the photon number of the soliton and the propagation distance.

The physical significance of (2.26)–(2.29) is simple. The photon perturbation and the frequency perturbation are preserved during soliton propagation. The phase perturbation is coupled to the photon number perturbation by the Kerr effect, and the position perturbation is coupled to the momentum (center frequency) perturbation because a change in frequency is associated with a change of soliton velocity via GVD.

If one assumes at $z=0$ the quantum soliton state is a coherent pulse state, then the variances of the four operators at $z=0$ are

$$\langle \Delta\hat{n}^2(0)\rangle=\tfrac{1}{4}\int |\bar{f}_n(x)|^2dx=n_0 , \tag{2.30}$$

$$\langle \Delta\hat{\theta}^2(0)\rangle=\tfrac{1}{4}\int |\bar{f}_\theta(x)|^2dx\approx\frac{0.607}{n_0} , \tag{2.31}$$

$$\langle \Delta\hat{p}^2(0)\rangle=\tfrac{1}{4}\int |\bar{f}_p(x)|^2dx=\frac{n_0}{12} , \tag{2.32}$$

$$\langle \Delta\hat{x}^2(0)\rangle=\tfrac{1}{4}\int |\bar{f}_z(x)|^2dx\approx\frac{3.29}{n_0^3} . \tag{2.33}$$

The variances at nonzero z can be calculated according to (2.26)–(2.29).

III. THE RING INTERFEROMETER

The ring interferometer is shown in Fig. 3(a) in its fiber realization. If the interaction of the counterpropagating pulses is neglected, then the interferometer can be "developed" into a conventional Mach-Zehnder interferometer, Fig. 3(b). The linearization of the fiber propagation law linearizes the interferometer. The symmetry of the ring balances the interferometer. Suppose that the scattering transform of the first mirror is

$$\hat{U}_3 = \frac{1}{\sqrt{2}}(\hat{U}_1 + \hat{U}_2) , \tag{3.1}$$

$$\hat{U}_4 = \frac{1}{\sqrt{2}}(\hat{U}_1 - \hat{U}_2) , \tag{3.2}$$

the transformations through the Kerr media are

$$\hat{U}_5 = O\hat{U}_3 O^\dagger ,$$

$$\hat{U}_6 = O\hat{U}_4 O^\dagger$$

(here, O is the propagator of $\hat{U}$ in the fiber), and after the output mirror,

$$\hat{U}_7 = \frac{1}{\sqrt{2}}(\hat{U}_5 + \hat{U}_6) , \tag{3.3}$$

$$\hat{U}_8 = \frac{1}{\sqrt{2}}(\hat{U}_5 - \hat{U}_6) . \tag{3.4}$$

In the absence of a nonlinear medium, the scattering transform of the second mirror sends $\hat{U}_1$ out of port 7, and $\hat{U}_2$ out of port 8. In the linearized approximation, the nonlinear medium does not change this response. In other words, the fluctuations in the two arms do not interact. Thus, even if the noise level of the soliton injected into port 1 is much higher than that of vacuum fluctuations, which is the case when it is regenerated in an oscillator, the squeezed vacuum emerging from port 8 is unaffected by that noise.

The output operators are, when O is linearized,

$$\Delta\hat{n}_7 = \Delta\hat{n}_1 , \tag{3.5}$$

$$\Delta\hat{x}_7 = \Delta\hat{x}_1 + z_0 \Delta\hat{p}_1 , \tag{3.6}$$

$$\Delta\hat{\theta}_7 = \Delta\hat{\theta}_1 + \frac{n_0}{4} z_0 \Delta\hat{n}_1 , \tag{3.7}$$

$$\Delta\hat{p}_2 = \Delta\hat{p}_1 , \tag{3.8}$$

and a similar set of expressions for the operators emerging from port 8 that are all determined from the injection into port 2. Here z_0 is the fiber length and the photon number n_0 refers to the photon number of the soliton pulse propagating in one direction of the ring. One should note that we have included explicitly the phase shifts due to linear propagation and classical Kerr action on the soliton itself, with respect to which all the phases are defined. These phase shifts can be taken into account by proper choice of the carrier frequency of the oscillator.

IV. THE EQUATIONS OF THE LASER MEDIUM

Note that the expectation value of the field in the cavity is different from that in the fiber by a factor of $\sqrt{2}$. We shall denote the cavity fields by $\hat{V}$ and $\hat{v}$ to distinguish them from the fiber fields. According to Ref. 11 and in the notation of Ref. 12, the cavity internal field operator $\hat{V}$ is described by the equation

$$\frac{\partial}{\partial t}\hat{V} = -\frac{\gamma}{2}\hat{V} - igN\hat{\Sigma} + \sqrt{\gamma}\hat{S}_A \tag{4.1}$$

where γ is the photon decay rate, g is the gain factor proportional to the dipole matrix element, N is the total number of atoms, $\hat{\Sigma}$ is the dipole moment operator, and $\hat{S}_A$ is the reservoir fluctuation operator. $\hat{S}_A$ is so normalized that $\langle \hat{S}^\dagger_A \hat{S}_A \rangle$ represents the average photon flux incident upon the cavity. The equation for the dipole moment is[12]

$$\frac{\partial}{\partial t}\hat{\Sigma} = -\Gamma\hat{\Sigma} + ig\hat{\sigma}\hat{V} + \left[\frac{2\Gamma}{N}\right]^{1/2} \hat{S}_\Sigma \tag{4.2}$$

where $\hat{\sigma}$ is the population inversion operator, Γ is the dipole moment decay rate, and $\hat{S}_\Sigma$ is the dipole moment noise operator. The equation for the population inversion operator is

$$\frac{\partial}{\partial t}\hat{\sigma} = \hat{\Lambda} - \frac{\hat{\sigma}}{\tau_{sp}} + i2g(\hat{V}^\dagger\hat{\Sigma} - \hat{\Sigma}^\dagger\hat{V}) + \hat{S}_P \tag{4.3}$$

where $\hat{\Lambda}$ is the pump rate per atom, τ_{sp} is the spontaneous lifetime, and $\hat{S}_P$ is the pump noise operator.

In (4.1)–(4.3), the variables are functions of the short-term variable x, expressing the time dependence of an individual phase, and the long-term variable t, expressing the overall evolution of the pulse train envelope.

When the dipole decay rate Γ is much larger than the photon decay rate, and population decay rate $1/\tau_{sp}$, the dipole operator can be eliminated adiabatically. The resulting equations are

$$\frac{\partial\hat{V}}{\partial t} = -\frac{\gamma}{2}\hat{V} + \frac{g^2}{\Gamma}\hat{N}\hat{V} - ig\left[\frac{2N}{\Gamma}\right]^{1/2}\hat{S}_\Sigma + \sqrt{\gamma}\hat{S}_A , \tag{4.4}$$

$$\frac{\partial\hat{N}}{\partial t} = \hat{P} - \frac{\hat{N}}{\tau_{sp}} - \frac{4g^2}{\Gamma}\hat{N}\hat{V}^\dagger\hat{V} + i2g\left[\frac{2N}{\Gamma}\right]^{1/2}(\hat{V}^\dagger\hat{S}_\Sigma - \hat{S}^\dagger_\Sigma\hat{V}) + N\hat{S}_P , \tag{4.5}$$

where

$$\hat{N} \equiv N\hat{\sigma} \tag{4.6}$$

is the total inversion operator and

$$\hat{P} \equiv N\hat{\Lambda} \tag{4.7}$$

is the total pumping rate.

Denote the steady state value of $\hat{N}$ by N_0 and that of $\hat{V}$ by

$$V_0 = \sqrt{2}U_0 = \sqrt{2}\frac{n_0}{2}\text{sech}\left[\frac{n_0}{2}x\right] .$$

We assume that the bandwidth of the medium is sufficiently broad so that it accommodates the full spectrum of the pulse. Then the time dependence of the absorption of the saturable absorber becomes negligible and the noise associated with the absorber can be incorporated in the stationary noise sources already included in the above equations. In the steady state, the gain must be equal to the loss so that

$$\frac{\gamma}{2} = \frac{g^2}{\Gamma}N_0 . \tag{4.8}$$

The noise is, by assumption, small so that it does not enter into the relations describing the steady state. The average population is given by

$$P-\frac{N_0}{\tau_{sp}}-\frac{4g^2}{\Gamma T_R}N_0\int|V_0|^2dx=0 \tag{4.9}$$

where T_R is roundtrip time. Here $\int|V_0|^2dx\equiv n_c=2n_0$ is the photon number of a single pulse in the cavity.

To conserve the commutator brackets, $\hat{S}_A$ and $\hat{S}_\Sigma$ have to satisfy the following commutation relations:[12]

$$[\hat{S}_A(t,x),\hat{S}_A^\dagger(t',x')]=\delta(t-t')\delta(x-x') , \tag{4.10}$$

$$[\hat{S}_\Sigma(t,x),\hat{S}_\Sigma^\dagger(t',x')]=-\langle\hat{\sigma}\rangle\delta(t-t')\delta(x-x') . \tag{4.11}$$

We can now linearize the equations around the steady state. The equation for the small-signal field operator is

$$\frac{\partial}{\partial t}\hat{v}=\frac{g^2}{\Gamma}\Delta\hat{N}V_0-ig\left[\frac{2N}{\Gamma}\right]^{1/2}\hat{S}_\Sigma+\sqrt{\gamma}\hat{S}_A \tag{4.12}$$

and the equation for the perturbation of the population, $\Delta\hat{N}$, is

$$\frac{\partial}{\partial t}\Delta\hat{N}=-\frac{\Delta\hat{N}}{\tau_{sp}}-\frac{4g^2}{\Gamma}|V_0|^2\Delta\hat{N}-\frac{4g^2}{\Gamma}N_0(V_0^*\hat{v}+V_0\hat{v}^\dagger)+2ig\left[\frac{2N}{\Gamma}\right]^{1/2}(V_0^*\hat{S}_\Sigma-V_0\hat{S}_\Sigma^\dagger)+N\hat{S}_P . \tag{4.13}$$

In media with long relaxation time (long compared with the cavity roundtrip time T_R), the gain is pulled down by the cumulative effect of all pulses within the relaxation time. The gain change caused by one individual pulse is small. Its effect on the pulse shape is small and we shall neglect it. Within this approximation the short-term time dependence (upon x) can be ignored in (4.13) and the equation becomes

$$\frac{\partial}{\partial t}\Delta\hat{N}=-\frac{\Delta\hat{N}}{\tau_{sp}}-\frac{4g^2}{\Gamma}\frac{n_c}{T_R}\Delta\hat{N}-\frac{4g^2}{\Gamma}N_0\frac{1}{T_R}\int(V_0^*\hat{v}+V_0\hat{u}^\dagger)dx+i2g\left[\frac{2N}{\Gamma}\right]^{1/2}\frac{1}{T_R}\int(V_0^*\hat{S}_\Sigma-V_0\hat{S}_\Sigma^\dagger)dx+N\hat{S}_P . \tag{4.14}$$

If the relaxation rate

$$\frac{1}{T_N}\equiv\frac{1}{\tau_{sp}}+\frac{4g^2}{\Gamma}\frac{n_c}{T_R} \tag{4.15}$$

is fast, then one may eliminate $\Delta\hat{N}$ adiabatically, obtaining the equation for the field:

$$\frac{\partial}{\partial t}\hat{v}=-\frac{\gamma}{2}\frac{4g^2T_N}{\Gamma T_R}V_0\int(V_0^*\hat{v}+V_0\hat{v}^\dagger)dx+\hat{S} \tag{4.16}$$

with

$$\hat{S}=\frac{g^2}{\Gamma}V_0T_N\left[i2g\left[\frac{2N}{\Gamma}\right]^{1/2}\frac{1}{T_R}\times\int(V_0^*\hat{S}_\Sigma-\hat{S}_\Sigma^\dagger V_0)dx+N\hat{S}_p\right]-ig\left[\frac{2N}{\Gamma}\right]^{1/2}\hat{S}_\Sigma+\sqrt{\gamma}\hat{S}_A . \tag{4.17}$$

When $1/\tau_{sp}$ is small compared with the induced emission rate in (4.15), the contribution of the noise operator $\hat{S}_P$ can be ignored. In this limit of high excitation, Eqs. (4.16) and (4.17) become

$$\frac{\partial}{\partial t}\hat{v}=-\gamma\frac{V_0}{2n_c}\int(V_0^*\hat{v}+V_0\hat{u}^\dagger)dx+\hat{S} , \tag{4.18}$$

$$\hat{S}=\sqrt{\gamma}\left[\hat{S}_A+i\frac{V_0}{2n_c}\frac{1}{\sqrt{\langle\hat{\sigma}\rangle}}\int(V_0^*\hat{S}_\Sigma-V_0\hat{S}_\Sigma^\dagger)dx-i\frac{\hat{S}_\Sigma}{\sqrt{\langle\hat{\sigma}\rangle)}}\right] . \tag{4.19}$$

For solitons, V_0 can be assumed to be real and Eq. (4.18) can be separated into equations for two Hermitian operators $\hat{v}_1$ and $\hat{v}_2$, the in-phase and quadrature components $\hat{v}=\hat{v}_1+i\hat{v}_2$.

The photon number in the fiber ring splits into two halves, so that the Kerr shift accumulated within the fiber length is half of that which would be observed if the entire pulse traveled through one single fiber. Because the two arms of the ring interferometer act independently and the noise in each of the input ports is squeezed independently, we may view the insertion of the ring into the laser like an insertion of a single fiber of the same total length but with half of the Kerr coefficient. Thus all functions $f_i(x)$ and $\bar{f}_i(x)$ developed for the Schrödinger equation of the form of (2.1) have to be redefined in terms of a Schrödinger equation with a factor of $\frac{1}{2}$ in front of the nonlinear term. The new $\bar{f}_i$'s are

$$\bar{f}_n(x)=2V_0(x) \tag{4.20}$$

$$\bar{f}_\theta(x)=2\left[\frac{1}{n_c}-\frac{x}{4}\tanh\left[\frac{n_c}{4}x\right]\right]V_0(x) , \tag{4.21}$$

$$\bar{f}_p(x)=\tfrac{1}{2}\tanh\left[\frac{n_c}{4}x\right]V_0(x) , \tag{4.22}$$

$$\bar{f}_x(x)=\frac{4}{n_c}xV_0(x) . \tag{4.23}$$

Subsequent multiplication by the adjoint functions in (2.20)–(2.23) and integration over the short time scale of the time parameter x produce equations of motion for the four operators in their long-term evolution described by the t parameter:

$$\frac{\partial}{\partial t}\Delta\hat{n}=-\gamma\Delta\hat{n}+\hat{S}_n , \tag{4.24}$$

$$\frac{\partial}{\partial t}\Delta\hat{\theta}=\hat{S}_\theta , \tag{4.25}$$

$$\frac{\partial}{\partial t}\Delta\hat{p}=\hat{S}_p \ , \tag{4.26}$$

$$\frac{\partial}{\partial t}\Delta x=\hat{S}_x \ . \tag{4.27}$$

There are no "restoring forces" for $\Delta\theta$, Δp, and Δx. The four noise operators are

$$\hat{S}_n\equiv\int dx\sqrt{\gamma}\hat{S}_{A1}\bar{f}_n(x) \ , \tag{4.28}$$

$$\hat{S}_\theta=\int dx\sqrt{\gamma}\left[\hat{S}_{A2}-\frac{\hat{S}_{\Sigma 1}}{\sqrt{\langle\hat{\sigma}\rangle}}\right]\bar{f}_\theta(x) \ , \tag{4.29}$$

$$\hat{S}_p=\int dx\sqrt{\gamma}\left[\hat{S}_{A2}-\frac{\hat{S}_{\Sigma 1}}{\sqrt{\langle\hat{\sigma}\rangle}}\right]\bar{f}_p(x) \ , \tag{4.30}$$

$$\hat{S}_x=\int dx\sqrt{\gamma}\left[\hat{S}_{A1}+\frac{\hat{S}_{\Sigma 2}}{\sqrt{\langle\hat{\sigma}\rangle}}\right]\bar{f}_x(x) \ . \tag{4.31}$$

V. SOLITON EVOLUTION AND THE SPECTRA

Upon every pass, the fluctuations of the operators are acted upon by the ring reflector. If the effect per pass is small, it can be added to the time rate of change ascribed to the laser medium. The effect is converted into a rate by division by the roundtrip time T_R. We write

$$\frac{\partial}{\partial t}\Delta\hat{n}(t)=-\gamma\Delta\hat{n}(t)+\hat{S}_n(t) \ , \tag{5.1}$$

$$\frac{\partial}{\partial t}\Delta\hat{\theta}(t)=\frac{z_0}{T_R}\frac{n_c}{8}\Delta\hat{n}(t)+\hat{S}_\theta(t) \ , \tag{5.2}$$

$$\frac{\partial}{\partial t}\Delta\hat{p}(t)=\hat{S}_p(t) \ , \tag{5.3}$$

$$\frac{\partial}{\partial t}\Delta\hat{x}(t)=\frac{z_0}{T_R}\Delta\hat{p}(t)+\hat{S}_x(t) \ . \tag{5.4}$$

Here z_0 is the normalized fiber length.

Using (4.11) and (4.12) and assuming at $t=0$ the quantum state is the vacuum state, it is easy to prove that in the case of high excitation,

$$\langle\hat{S}_n(t)\hat{S}_n(t')\rangle=\gamma\langle\Delta\hat{n}^2(0)\rangle\delta(t-t') \ , \tag{5.5}$$

$$\langle\hat{S}_\theta(t)\hat{S}_\theta(t')\rangle=2\gamma\langle\Delta\hat{\theta}^2(0)\rangle\delta(t-t') \ , \tag{5.6}$$

$$\langle\hat{S}_p(t)\hat{S}_p(t')\rangle=2\gamma\langle\Delta\hat{p}^2(0)\rangle\delta(t-t') \ , \tag{5.7}$$

$$\langle\hat{S}_x(t)\hat{S}_x(t')\rangle=2\gamma\langle\Delta\hat{x}^2(0)\rangle\delta(t-t') \ . \tag{5.8}$$

There is no cross correlation between these noise operators.

Equations (5.1)–(5.4) are easier to solve in the Fourier transform space. The noise spectra are

$$\langle\Delta\bar{\hat{n}}^\dagger(\Omega)\Delta\bar{\hat{n}}(\Omega')\rangle=\frac{1}{2\pi}\frac{\gamma}{\Omega^2+\gamma^2}\langle\Delta\hat{n}^2(0)\rangle\delta(\Omega-\Omega') \ , \tag{5.9}$$

$$\langle\Delta\bar{\hat{\theta}}^\dagger(\Omega)\Delta\bar{\hat{\theta}}(\Omega')\rangle =\frac{1}{2\pi}\left[\frac{n_c^2}{64}\left[\frac{z_0}{T_R}\right]^2\frac{\gamma}{\Omega^2(\Omega^2+\gamma^2)}\langle\Delta\hat{n}^2(0)\rangle +\frac{2\gamma}{\Omega^2}\langle\Delta\hat{\theta}^2(0)\rangle\right]\delta(\Omega-\Omega') \ , \tag{5.10}$$

$$\langle\Delta\bar{\hat{p}}^\dagger(\Omega)\Delta\bar{\hat{p}}(\Omega')\rangle=\frac{1}{2\pi}\frac{2\gamma}{\Omega^2}\langle\Delta\hat{p}^2(0)\rangle\delta(\Omega-\Omega') \ , \tag{5.11}$$

$$\langle\Delta\bar{\hat{x}}^\dagger(\Omega)\Delta\bar{\hat{x}}(\Omega')\rangle=\frac{1}{2\pi}\left[\left[\frac{z_0}{T_R}\right]^2\frac{2\gamma}{\Omega^4}\langle\Delta\hat{p}^2(0)\rangle +\frac{2\gamma}{\Omega^2}\langle\Delta\hat{x}^2(0)\rangle\right]\delta(\Omega-\Omega') \ . \tag{5.12}$$

These are rather interesting results. The spectrum of the photon fluctuations is Lorentzian. The phase fluctuations have a contribution that corresponds to a random walk of the phase. This contribution is the standard phase noise of an oscillator. There is a contribution due to the photon fluctuations producing phase fluctuations in the Kerr medium. In the frequency range smaller than γ, they add to the standard random walk. This is analogous to the frequency noise enhancement in a semiconductor laser due to the α parameter that couples gain fluctuations to index fluctuations.[13] The position fluctuations are to be expected in a passively mode-locked system in which there is no pulse-timing reference. The position fluctuations are enhanced by the momentum fluctuations. In fact, the result found here is identical with the result found by Gordon and Haus in their investigation of long-distance soliton propagation in the presence of amplifier noise which led to the Gordon-Haus limit.[14]

DISCUSSION

We have developed the theory of the noise and squeezing in a laser that produces counterpropagating solitons in the fiber-ring reflector. The fiber soliton was necessary to allow for a large phase shift Φ so that the squeezing parameters μ and ν can be made large.

The main advantage of the proposed scheme is that the fiber length required for squeezing could be made much shorter, using the internal intensity of the laser, than when a fiber interferometer is excited by pulses derived from the output of another laser. One may consider the system shown in Fig. 5. Here the squeezing is performed in a Kerr medium inside the ring reflector that does not

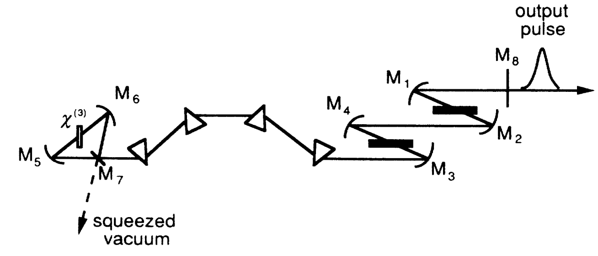

FIG. 5. A modified laser and squeezer.

necessarily possess negative GVD. One may imagine it to be a section of fiber as well. The net negative group-velocity dispersion is produced by the prism arrangement in the main cavity. If the phase shift per pass and GVD effect per pass are small, a solitonlike pulse will form inside the cavity. However, by definition, the squeezing in the ring cannot be large. If the phase shift is indeed large, then the equations of the system become difference equations and analytic solutions become more difficult. However, it is likely that squeezing ought to be possible also within this configuration. Certainty about this awaits further analysis.

ACKNOWLEDGMENTS

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[1]M. C. Nuss, P. M. Mankiewich, R. E. Howard, T. E. Harvey, C. D. Brandle, B. L. Straughn, and P. R. Smith (unpublished).

[2]L. B. Cohen, R. D. Keynes, and D. Landowne, J. Phys. **224**, 727 (1972).

[3]L. Maini, R. Petronzio, and E. Zavatlini, Phys. Lett. B **175**, 359 (1986).

[4]R. E. Slusher, P. Grangier, A. Laporta, B. Yurke, and M. J. Potasek, Phys. Rev. Lett. **59**, 2566 (1987).

[5]J. C. Diels, W. Dietel, J. J. Fontaine, W. Rudolph, and B. Wilhelmi, J. Opt. Soc. Am. B **2**, 680 (1985); O. E. Martinez, R. J. Fork, and J. P. Gordon, *ibid.* **2**, 753, 1985; F. Salin, P. Grangier, G. Roger, and A. Burn, Phys. Rev. Lett. **60**, 569 (1988); F. W. Wise, I. A. Walmsley, and C. L. Tang, Opt. Lett. **13**, 129 (1988); H. Avaramopoulos, P. M. W. French, J. A. R. Williams, G. H. C. New, and J. R. Taylor, IEEE J. Quantum Electron. **QE-24**, 1884 (1988).

[6]C. Y. Wang, Y. Ishida, and Y. Yamamoto, Opt. Lett. (to be published).

[7]M. Kitagawa and Y. Yamamoto, Phys. Rev. A **34**, 3974 (1986).

[8]M. Shirasaki, H. A. Haus, and D. Liu Wong, J. Opt. Soc. Am. B **6**, 82 (1989).

[9]H. A. Haus and Y. Lai, J. Opt. Soc. Am. B **7**, 386 (1990).

[10]V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **34**, 61 (1971) [Sov. Phys.—JETP **34**, 62 (1972)].

[11]M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974).

[12]Y. Yamamoto and H. A. Haus, Phys. Rev. A **41**, 5164 (1990).

[13]C. H. Henry, IEEE J. Quantum Electron. **QE-18**, 259 (1982).

[14]J. P. Gordon and H. A. Haus, Opt. Lett. **11**, 665 (1986).

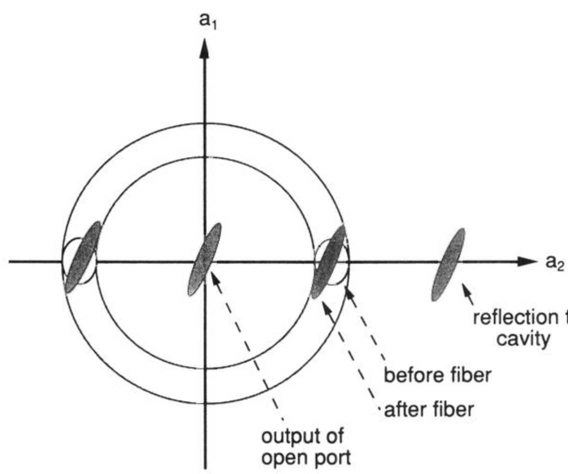

FIG. 2. Wigner densities for a clockwise and counterclockwise pulses in an antiresonant interferometer before and after self-phase modulation. A squeezed vacuum state is the output at the open port and the coherent excitation is reflected back into the cavity.