PHYSICAL REVIEW A VOLUME 42, NUMBER 9 1 NOVEMBER 1990

Laser-induced detachment processes in an electric field

Bo Gao and Anthony F. Starace
Department of Physics and Astronomy, The University of Nebraska, Lincoln, Nebraska 68588-0111
(Received 28 June 1990)

An analytic momentum-space wave function for an electron in both laser and static uniform electric fields is presented. It is used to obtain analytic multiphoton detachment cross sections in a static, uniform electric field which include effects of static-field-induced electron-photon interactions. These general results are not restricted to weak laser intensities or to weak static-field strengths and depend only on (a) the electric dipole approximation and (b) the approximation that final-state electron-atom interactions are ignored. Four specific predictions of our general formulas for the most interesting case of linearly polarized light polarized along the static-field direction are presented for weakly bound electrons initially in an s state. First, the effects of a weak static electric field on N-photon detachment cross sections near threshold are shown to be described by two modulation factors, one for odd N and one for even N, which depend only on a scaled energy. Second, for photodetachment in a static electric field, effects of static-field-induced electron-photon interactions are demonstrated. Third, the lifetime against field ionization is presented. Fourth, the cross section for electric-field-induced stimulated emission is presented. Numerical results for the latter three effects are presented for the H^- ion. The simpler case of circularly polarized light directed along the static-electric-field direction is treated briefly. In particular, we show for this case that the static- and laser-field effects are uncoupled and that the near-threshold, weak static-electric-field modulation factors for the N-photon detachment cross sections are dependent on N.

I. INTRODUCTION

The effect of an external electric field on the optical-absorption properties of excitons, insulators, and semiconductors has for a long time now attracted much theoretical interest.[1–7] In the 1970s, this theoretical interest extended also to the effect of external electric fields on multiphoton detachment processes.[8–11] For single-photon absorption both by solids and by negative ions, there were explicit theoretical predictions of oscillatory behavior above the absorption thresholds.[3,10] The experimental observations of oscillatory resonance behaviors in the photoionization spectra of Rb both above the classical ionization threshold[12] and especially above the zero-field-ionization threshold[13] stimulated much more detailed theoretical investigations of atomic photoionization[14–20] as well as photodetachment[17,21] in the presence of a weak static electric field. Recent experimental observations[22] of electric-field-induced resonances in the photodetachment spectrum of H^- stimulated a resurgence of activity on single-photon detachment processes in the presence of a weak external electric field.[23–29] Most recently, theorists have returned to the investigation of static-field effects on single-photon and multiphoton[30-32] detachment processes, for the case in which the electric field can no longer be regarded as a weak perturbation, in order to provide detailed predictions which experiment may soon be capable of testing.

In this work we present a theory of single-photon and multiphoton detachment processes in the presence of a static uniform electric field for an s-electron bound initially in a short-range potential. We include specific applications to the H^- ion. Before describing our theoretical approach, it is useful to give an overview of those previous works that are most relevant to ours. Arutyunyan and Askar'yan[8] have given, as far as we know, the first qualitative overview of the general process of multiphoton detachment in the presence of a static uniform electric field. They focused on the dependence of the detachment process on the laser frequency and on the field intensities.

Nikishov[9] gave a general formal solution for the multiphoton transition rates using a gauge in which both fields are described by a vector potential. Nikishov[9] presents detailed results of the influence of the static field on the multiphoton detachment process, however, for only two simple cases: the case of laser photons that are linearly polarized perpendicular to the static field (in which case the field-coupling effects to be discussed in this paper are absent) and the case of the low-frequency limit.

Slonim and Dalidchik[10] have treated both single-photon and multiphoton detachment of negative ions in a static uniform electric field. For the single-photon case, they treated the coupling of the negative ion to the electromagnetic field perturbatively for arbitrarily polarized incident photons. The final state was described by a free electron moving in a static uniform electric field. Such a treatment for the single-photon case typifies nearly all more recent theoretical descriptions. For the multiphoton case, only circular polarization was considered. They treated the effects of both a static uniform electric field and a circularly polarized electromagnetic field nonperturbatively. The case of circularly polarized light does not involve oscillations of the multiphoton cross sections.

However, Slonim and Dalidchik did show for this case that the static field could increase the ratio of the three-photon detachment cross section to the two-photon detachment cross section.

Manakov and Fainshtein[11] also considered the decay of a weakly bound electron in both a static uniform electric field and a circularly polarized electromagnetic field. They showed that the presence of the electromagnetic field could increase the rate of decay by tunneling by several orders of magnitude.

In these previous theoretical works, the neglect of electron-atom interactions in the final state is a very common approximation, in particular for theoretical treatments of single-photon detachment of the negative hydrogen ion. The success of this approximation for photodetachment of H^- in a uniform electric field rests on the near-zero phase shift of the outgoing p wave in the final state, as well as on the very small mixing of s and p waves in the weak static electric fields considered in most of these previous works. As the external static electric field strength increases, however, so does the necessity for theoretical treatment of final-state electron-atom interactions. Formal treatments of such interactions have been given.[10,17(b)] Only very recently have their effects been calculated. A theoretical treatment of both static-field-induced electron-atom and electron-photon final-state interactions has been described briefly by Nicolaides and Mercouris.[30] For photodetachment of H^- in weak electric fields, however, no significant deviations from previous theoretical treatments[22,23,26] which ignore these final-state effects were found.[30] Fabrikant,[31] however, has shown that for an electric-field strength of 1.44 MV/cm, final-state electron-atom interactions produce a measurable decrease of the photodetachment cross section for H^- in the vicinity of the zero-field-ionization threshold.

In our work we present an exact expression for the momentum-space wave function of an electron acted upon by the combined field of a monochromatic laser and a static uniform electric field. This wave function is used to obtain analytic results for single-photon and multiphoton detachment of a weakly bound electron in an external uniform electric field. We treat both the case of linearly polarized light, which involves oscillations of the cross section, and, more succinctly, the case of circularly polarized light. Because of our use of an exact final-state wave function for an electron moving in both fields, our treatment includes implicitly the effects of static-field-induced electron-photon interactions, whose effects have not been studied previously for the interesting case of linearly polarized laser light. In order to isolate these effects of electron-photon interactions, we have ignored in this work final-state electron-atom interactions.

Using these general analytic results for the most interesting case of linearly polarized light polarized along the static-field direction, we predict four effects for detachment of negative ions which may be observable by experiment. All of our results are presented as simple analytic formulas for a weakly bound electron initially in an s state. Where our numerical results require specification of a particular system, we have illustrated our results for the H^- ion.

First, the effects of a weak static electric field on N-photon detachment cross sections near threshold are shown to be described by two modulation factors, one for odd N and one for even N, which depend only on a scaled energy. For $N=1$, the corresponding modulation factor agrees with previous work[23] on single-photon detachment of H^-. In the more general case of photodetachment of an arbitrary negative ion in a weak external electric field, a frame transformation treatment[24] has been shown to allow a separation of external field and electric-atom interaction effects. Such a treatment is shown here to apply also in the multiphoton case. Therefore the multiphoton detachment modulation factors presented in this paper may serve as input to more detailed calculations which include electron-atom final-state interactions. In any case, the N-photon detachment modulation factors presented here may be observed for currently common electric-field strengths in any experiment capable of measuring multiphoton detachment cross sections.

Second, for photodetachment of a negative ion (and, in particular of H^-) in a static uniform electric field, static-field-induced electron-photon interactions are shown here to cause a measurable decrease of the cross section, relative to results predicted by previous works,[22,23,26] in the energy region of the zero-field threshold. These predictions complement those of Fabrikant[31] regarding a decrease of the photodetachment cross section of H^- near threshold arising from final-state electron-atom interactions. In comparison with the decrease predicted by Fabrikant,[31] we show that static-field-induced electron-photon interactions produce a decrease which is comparable in magnitude and which occurs over a larger energy region. The predicted decrease of the photodetachment cross section near threshold may be observed experimentally for electric field strengths less than an order of magnitude larger than commonly used in present experimental work.

Third, we present as a trivial case of our general results the transition rate for field ionization. In particular we present the lifetime against field ionization for H^- as a function of the static-field strength.

Fourth, again for a negative ion in both laser and static fields, the cross section for electric-field-induced stimulated emission is given, and numerical results for H^- are presented. It should be possible to measure this cross section by photoelectron spectroscopy, even though the presence of the external electric field could present some extra complications.

Finally, we treat briefly the case of circularly polarized light traveling along the direction of the static electric field. We show that the effects of the laser field and the static field are given by independent factors in this case. As an application of our results, we obtain the weak static-electric-field modulation factors for the near-threshold N-photon detachment cross sections. We show that these factors, in contrast to the case of linearly polarized photons polarized along the direction of the static electric field, depend on the number of photons absorbed, N.

In Sec. II we present our analytic momentum-space solution for an electron moving in the combined fields of a laser and a static uniform electric field. In Sec. III we use our analytic final-state wave function for the case of linearly polarized light to derive multiphoton detachment cross sections for a negative ion in the presence of a static uniform electric field. The gauge invariance of our result is discussed here briefly. In Sec. IV we examine the weak static-field limit of our general formulas. We derive also the modulation factors which are appropriate near the detachment threshold. In Sec. V we examine the weak laser-field limit of our general formulas. We present detailed results on single-photon detachment of H^- in a static uniform electric field and make comparisons with simpler theoretical treatments. The lifetime against field ionization is presented for H^-. Also, the electric-field-induced stimulated emission cross section for H^- is presented. In Sec. VI we present our results for circularly polarized light. Finally, in Sec. VII we summarize our results and discuss their implications. Preliminary reports of this work have been presented elsewhere,[32] including also the case of linearly polarized light polarized perpendicular to the direction of the static uniform electric field.[32(b)] A similar treatment for single-photon and multiphoton detachment processes in the presence of a static uniform magnetic field has been given by one of us.[33]

II. ANALYTIC WAVE FUNCTION FOR AN ELECTRON IN COMBINED LASER AND STATIC, UNIFORM ELECTRIC FIELDS

We consider the general situation of linearly polarized laser light with the polarization along any direction relative to the static uniform electric field. Letting the static field define the z axis, i.e., $\mathbf{E}_s = E_s\hat{\mathbf{z}}$, we can describe the field as follows:

$$\mathbf{E}_l = \mathbf{E}_0 \sin\omega t = (E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}} + E_{0z}\hat{\mathbf{z}})\sin\omega t \ .$$

In the radiation gauge, the wave function for an electron in the combined field satisfies the time-dependent Schrödinger equation,which, in the velocity gauge (V), has the following form in momentum space:

$$i\frac{\partial}{\partial t}\Psi^V(\mathbf{p},t) = \left[\frac{1}{2}\left[\mathbf{p} + \frac{\mathbf{E}_0}{\omega}\cos\omega t\right]^2 + iE_s\frac{\partial}{\partial p_z}\right]\Psi^V(\mathbf{p},t) \ . \tag{1}$$

Letting

$$\Psi^V(\mathbf{p},t) = \psi(\mathbf{p},t)\exp(-iv\sin 2\omega t - ist) \ , \tag{2}$$

where we have denoted the ponderomotive shift by

$$s \equiv E_0^2/4\omega^2 \ , \tag{3}$$

and where the coefficient of the second harmonic term is denoted by

$$v \equiv E_0^2/8\omega^3 \ , \tag{4}$$

we obtain the following equation for $\psi(\mathbf{p},t)$:

$$i\frac{\partial}{\partial t}\psi(\mathbf{p},t) = \left[\frac{1}{2}p^2 + \frac{\mathbf{E}_0\cdot\mathbf{p}}{\omega}\cos\omega t + iE_s\frac{\partial}{\partial p_z}\right]\psi(\mathbf{p},t) \ . \tag{5}$$

It is easy to verify that Eq. (5) has the following separable solution:

$$\psi_\alpha(\mathbf{p},t) = \psi_\alpha^x(p_x,t)\psi_\alpha^y(p_y,t)\psi_\alpha^z(p_z,t) \ , \tag{6}$$

where the separable components are given by

$$\psi_\alpha^x(p_x,t) = \delta(p_x - p_x^\alpha)\exp\left[-i\frac{E_{0x}p_x^\alpha}{\omega^2}\sin\omega t - i\epsilon_\alpha^x t\right] \ , \tag{7}$$

$$\psi_\alpha^y(p_y,t) = \delta(p_y - p_y^\alpha)\exp\left[-i\frac{E_{0y}p_y^\alpha}{\omega^2}\sin\omega t - i\epsilon_\alpha^y t\right] \ , \tag{8}$$

$$\psi_\alpha^z(p_z,t) = \frac{1}{\sqrt{2\pi E_s}}\exp\left\{i\left[\frac{1}{E_s}\left[\frac{p_z^3}{6} - \epsilon_\alpha^z p_z\right] - \frac{E_{0z}p_z}{\omega^2}\sin\omega t + \frac{E_{0z}E_s}{\omega^3}\cos\omega t - \epsilon_\alpha^z t\right]\right\} \ , \tag{9}$$

where $\epsilon_\alpha^x = p_x^{\alpha 2}/2$ and $\epsilon_\alpha^y = p_y^{\alpha 2}/2$.

Both $\psi_\alpha^x(p_x,t)$ and $\psi_\alpha^y(p_y,t)$ are momentum normalized and describe essentially free-electron motion in the laser field.[34] The nontrivial part of the wave function, $\psi_\alpha^z(p_z,t)$, is energy normalized. In Eq. (9) the first term in the exponential is what one would obtain in the absence of the laser field; the second term is the usual Volkov[34] phase factor. It is the third term that gives rise to the static-field-induced electron-photon interactions which we will discuss in more detail in Sec. IV. The quantum numbers $(p_x^\alpha, p_y^\alpha, \epsilon_z^\alpha)$ form a complete set of quantum numbers which specify the final state. Since p_z^α is no longer a good quantum number even asymptotically, the usual concept of the angular distribution of the photoelectrons is no longer a good one.

The important features of this solution are the following. First, different photon channels in the zero static-electric-field limit become coupled in a strong static uniform electric field by the term $(E_0E_s/\omega^3)\cos\omega t$ in the exponential. Second, unlike the solution used in previous works, our solution goes to a *stationary* state when the laser field is turned off. In other words, Eqs. (2)–(9) give the quasienergy solution for an electron in the combined laser and static electric fields.

The corresponding solution in the length gauge (L) is related to the solution in the velocity gauge by the standard-gauge transformation operator,[35–37]

$$\hat{T} \equiv \exp[i\mathbf{a}(t)\cdot\mathbf{r}] \ , \tag{10}$$

where

$$\mathbf{a}(t) \equiv \frac{\mathbf{E}_0}{\omega}\cos\omega t \ , \tag{11}$$

and where, in momentum space,

$$\mathbf{r} \equiv i\nabla_{\mathbf{p}} \ . \tag{12}$$

Noting that the gauge transformation operator $\hat{T}$ in momentum space acts as a momentum displacement operator,[38] we find the length and velocity form wave functions are related by

$$\Psi^L(\mathbf{p},t)=\hat{T}\Psi^V(\mathbf{p},t)=\Psi^V\left[\mathbf{p}-\frac{\mathbf{E}_0}{\omega}\cos\omega t,t\right] . \quad (13)$$

Using $\hat{T}$ to transform the Schrödinger equation, Eq. (1), we see that, as expected, $\Psi^L(\mathbf{p},t)$ satisfies the following equation:

$$i\frac{\partial}{\partial t}\Psi^L(p,t)=[\tfrac{1}{2}p^2+i(\mathbf{E}_s+\mathbf{E}_0\sin\omega t)\cdot\nabla_{\mathbf{p}}]\Psi^L(\mathbf{p},t) . \quad (14)$$

III. MULTIPHOTON DETACHMENT CROSS SECTIONS FOR A WEAKLY BOUND ELECTRON IN A STATIC, UNIFORM ELECTRIC FIELD

In this section we employ the analytic, momentum-space wave function derived above for an electron in the combined field of a laser and a static uniform electric field to calculate multiphoton detachment cross sections. First, though, we prove the gauge invariance of our results for a well-known representation of the initial-state wave function for a weakly bound electron. Limiting cases of the general formulas presented in this section are examined in detail in Secs. IV and V.

A. Gauge invariance of the S-matrix element

For interaction times sufficiently short that depletion effects do not enter, the multiphoton transition from an initial bound state, represented by $\Psi_i(\mathbf{p},t)=\phi_i(\mathbf{p})\exp(-i\epsilon_i t)$, to a final state Ψ_f^V, defined in the velocity gauge by Eqs. (1)–(9), may be described by the S-matrix element,[39]

$$S_{fi}^V=-i\int_{-\infty}^{\infty}\langle\Psi_f^V|V_f^V|\Psi_i\rangle dt , \quad (15)$$

where the electromagnetic and static electric potential terms in Eq. (1) define the interaction potential in the velocity gauge as

$$V_f^V\equiv\frac{\mathbf{p}\cdot\mathbf{E}_0}{\omega}\cos\omega t+\frac{E_0^2}{2\omega^2}\cos^2\omega t+iE_s\frac{\partial}{\partial p_z} . \quad (16)$$

Using the following properties of the initial- and final-state wave functions:

$$V_f^V\Psi_f^V=\left[i\frac{\partial}{\partial t}-\frac{1}{2}p^2\right]\Psi_f^V , \quad (17)$$

$$i\frac{\partial}{\partial t}\Psi_i=\epsilon_i\Psi_i , \quad (18)$$

where Eq. (17) follows from the time-dependent Schrödinger equation (1) and where Eq. (18) follows from the assumption that the initial state is stationary, we may write the S-matrix element in Eq. (15) as

$$S_{fi}^V=-i\int_{-\infty}^{\infty}\langle\Psi_f^V|\epsilon_i-\tfrac{1}{2}p^2|\Psi_i\rangle dt . \quad (19)$$

In a similar way, the S matrix in the length gauge may be written as

$$S_{fi}^L=-i\int_{-\infty}^{\infty}\langle\Psi_f^L|V_f^L|\Psi_i\rangle dt , \quad (20)$$

where the interaction potential is now defined as [cf. Eq. (14)]

$$V_f^L\equiv i(\mathbf{E}_s+\mathbf{E}_0\sin\omega t)\cdot\nabla_{\mathbf{p}} . \quad (21)$$

Since an equation analogous to Eq. (17) applies in the length gauge, Eq. (20) may be transformed to an equation similar to Eq. (19)

$$S_{fi}^L=-i\int_{-\infty}^{\infty}\langle\Psi_f^L|\epsilon_i-\tfrac{1}{2}p^2|\Psi_i\rangle dt . \quad (22)$$

Using now the gauge transformation in Eq. (13), which relates Ψ_f^L to Ψ_f^V, we have

$$S_{fi}^L=-i\int_{-\infty}^{\infty}\langle\Psi_f^V|\hat{T}(\epsilon_i-\tfrac{1}{2}p^2)|\Psi_i\rangle dt , \quad (23)$$

where $\hat{T}$ is the momentum displacement operator defined in Eq. (10).

In general, S_{fi}^V in Eq. (19) and S_{fi}^L in Eq. (23) are not equal. We show here, however, that for a particular form of the initial-state wave function, the S matrix is gauge invariant. Specifically, if we choose the initial-state wave function to have the following form in coordinate space:

$$\Psi_i(\mathbf{r},t)=(Be^{-\kappa r}/r)e^{-i\epsilon_i t} , \quad (24)$$

where B is a normalization constant and where $\kappa\equiv(-2\epsilon_i)^{1/2}$, then in momentum space

$$\Psi_i(\mathbf{p},t)=\phi_i(\mathbf{p})e^{-i\epsilon_i t}$$
$$=\frac{B}{(2\pi)^{1/2}}\frac{1}{(p^2/2-\epsilon_i)}e^{-i\epsilon_i t} . \quad (25)$$

The form of the initial-state wave function given by Eq. (24) is a well-known approximation stemming from the effective range theory for an s electron.[40] It represents also the solution of an attractive spherical δ-function potential, whose effect may be described by a particular boundary condition at the origin.[41] In particular, a wave function of the form of Eq. (24) has been used to describe the H^- ion in an electric field[41] as well as to treat single-photon detachment of the H^- ion, both with[23,26] and without[42] a static uniform electric field.

Subsituting Eq. (25) for the initial-state wave function into Eq. (23), we obtain

$$S_{fi}^L=i\int_{-\infty}^{\infty}\langle\Psi_f^V|\hat{T}|B/(2\pi)^{1/2}\rangle e^{-i\epsilon_i t}dt \quad (26)$$

$$=i\int_{-\infty}^{\infty}\langle\Psi_f^V|B/(2\pi)^{1/2}\rangle e^{-i\epsilon_i t}dt \quad (27)$$

$$=S_{fi}^V . \quad (28)$$

Equation (27) follows from the fact that a constant is invariant to the momentum displacement operator $\hat{T}$. Equation (28) follows from substitution of Eq. (25) for the initial-state wave function into Eq. (19). Given the gauge invariance of the S-matrix element, one may choose to use either Eq. (19) or Eq. (22) on the basis of convenience. In what follows, we choose the velocity-gauge expression in Eq. (19).

B. Multiphoton detachment cross sections

In this section we evaluate the S-matrix element S_{fi}^V, that is given in the velocity gauge (V) by Eq. (19). In this calculation we use the analytic final-state wave function Ψ_f^V, defined by Eqs. (1)–(9) for an electron moving in the combined laser and static electric fields. We also use an initial-state wave function Ψ_i having the analytic form given in Eq. (25); i.e., we assume the initial state is unaffected by either the laser field or the static uniform electric field. For simplicity, we assume in this section that the laser field is linearly polarized along the static-field direction

$$\mathbf{E}_l=\mathbf{E}_0\sin\omega t=E_0\hat{\mathbf{z}}\sin\omega t \ . \tag{29}$$

In order to evaluate the momentum and time integrals in Eq. (19), we expand the harmonically time-dependent terms in Ψ_f^V as follows [cf. Eqs. (2) and (9)]:

$$\exp\left[-i\left[\frac{E_0E_s}{\omega^3}\right]\cos\omega t\right]=\sum_{n=-\infty}^{\infty}(-i)^nJ_n\left[\frac{E_0E_s}{\omega^3}\right]e^{-in\omega t} \ , \tag{30}$$

and

$$\exp\left[i\left[\frac{E_0p_z}{\omega^2}\right]\sin\omega t+iv\sin 2\omega t\right]=\sum_{n=-\infty}^{\infty}(-1)^nJ_n\left[\frac{E_0p_z}{\omega^2},-v\right]e^{-in\omega t} \ . \tag{31}$$

Equation (30) is the usual series of Bessel functions $J_n(x)$, generated by the exponential function on the left-hand side.[43] Equation (31) is a similar generating function equation for the generalized Bessel function $J_n(x,y)$, whose properties have been discussed thoroughly by Reiss.[44]

Substituting Eqs. (30) and (31) into Eqs. (2) and (9) for the final-state wave function Ψ_f^V, the momentum and time integrations in Eq. (19) may be performed to obtain the following result:

$$S_{fi}=\sum_N S_{fi}^{(N)}\delta(\epsilon_f+s-\epsilon_i-N\omega) \ . \tag{32}$$

Here $S_{fi}^{(N)}$, the S-matrix element corresponding to an N-photon transition, is defined by a sum of momentum space integrals

$$S_{fi}^{(N)}\equiv i(-1)^N(\pi/2E_s)^{1/2}\sum_{n=-\infty}^{\infty}i^nJ_n\left[\frac{E_0E_s}{\omega^3}\right]\int_{-\infty}^{\infty}F_l^{(N-n)}(p_x^f,p_y^f,p_z)\exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^zp_z)]dp_z \ , \tag{33}$$

where the function $F_l^{(N-n)}$ in the integral in Eq. (33) depends on the polarization of the incident light [here taken to be linear (l)] and is defined as

$$F_l^{(M)}(p_x,p_y,p_z)\equiv J_M\left[\frac{E_0p_z}{\omega^2},-v\right](p^2-2\epsilon_i)\phi_i(\mathbf{p})=J_M\left[\frac{E_0p_z}{\omega^2},-v\right]B(2/\pi)^{1/2} \ , \tag{34}$$

where the second line follows from use of Eq. (25) for $\phi_i(\mathbf{p})$. For an incident photon flux $cE_0^2/8\pi\omega$, the N-photon detachment cross section is given by

$$\sigma^{(N)}=\frac{8\pi\omega}{cE_0^2}\int W_{fi}^{(N)}dp_x^fdp_y^fd\epsilon_f^z \ , \tag{35}$$

where the transition rate $W_{fi}^{(N)}$ is defined by

$$W_{fi}^{(N)}\equiv(2\pi)^{-1}|S_{fi}^{(N)}|^2\delta(\epsilon_f+s-\epsilon_i-N\omega) \ . \tag{36}$$

Equations (33)–(36) are our general analytic results for multiphoton detachment in an external electric field. For any particular values of the field strengths E_0 and E_s and for any frequency ω, the N-photon S-matrix elements in Eq. (33) [and hence the N-photon detachment cross sections in Eq. (35)] can be calculated to any desired degree of accuracy. The key feature which distinguishes these results from prior work is the sum over Bessel functions with argument E_0E_s/ω^3 in Eq. (33). These Bessel functions arise from using Eq. (30) to expand the exponential of the third term in Eq. (9), which represents a coupling of the laser and static electric fields. If this term were not included (as in prior treatments), Eq. (33) would collapse to the result obtained by restricting n to $n=0$ in Eq. (33). We examine the effect of this coupling term in Sec. V. In Sec. IV we examine Eq. (33) in the limit of a weak static electric field.

IV. THE WEAK STATIC-FIELD LIMIT

Using an analytic solution for an electron in the combined field of a laser and a static uniform electric field, we have derived in Sec. III the multiphoton detachment cross sections for a weakly bound electron in a static uniform electric field for the most interesting case of linearly polarized light polarized along the static-field direction. So far no restrictions have been placed on either the laser intensity E_0 or the static-field strength E_s, other than those implied by our assumption that the initial bound state is unaffected by these fields. In this section and the next we examine in turn the two major limiting cases of our general results, given by Eqs. (33)–(36), namely the limits of $E_s\to0$ and $E_0\to0$, respectively. As an application of our results in this section for weak static fields, we present a derivation of the analytic factors describing the electric-field-induced modulation of the multiphoton cross sections near threshold.

A. Multiphoton detachment cross sections in the limit $E_s\to0$

To evaluate the N-photon S-matrix element $S_{fi}^{(N)}$ given in Eq. (33) in the limit that the static field E_s is small, we use first the following properties of Bessel functions of integer order:[45]

$$J_{-n}(z)=(-1)^n J_n(z)\ , \tag{37}$$

and

$$J_n(z)\to(z/2)^n/n!\quad (z\to 0)\ . \tag{38}$$

For $E_s\to 0$, these equations imply that only the $n=0$ term in Eq. (33) is significant. Hence

$$\lim_{E_s\to 0} S_{fi}^{(N)}=i(-1)^N B E_s^{-1/2} \times\int_{-\infty}^{\infty} J_N\left[\frac{E_0 p_z}{\omega^2},-v\right] \times\exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^z p_z)]dp_z\ , \tag{39}$$

where we have used Eq. (34) to replace $F_l^{(N)}$.

The remaining dependence of $S_{fi}^{(N)}$ in Eq. (39) on E_s as $E_s\to 0$ may be treated by the method of stationary phase. Due to the rapid oscillations of the exponential function, the contributions of the integrand to the integral will be small except for values of p_z satisfying the stationary phase condition

$$\frac{d}{dp_z}(p_z^3/6-\epsilon_f^z p_z)=p_z^2/2-\epsilon_f^z=0\ . \tag{40}$$

These critical values of p_z are $\pm p_z^f$, where we have defined

$$p_z^f\equiv(2\epsilon_f^z)^{1/2}\ . \tag{41}$$

[Note that even though p_z^f, defined by Eq. (41), can be associated with the z component of the electron's momentum in the zero static-field limit, it is only a parameter in the presence of the static field. In particular, it may take both real and imaginary values.]

Because of the coalescence of the two points of stationary phase as $p_z^f\to 0$, care must be taken in expanding the function J_N about the critical points. As discussed in detail by Schulman,[46] the proper form of the expansion is as follows:

$$J_N\left[\frac{E_0 p_z}{\omega^2},-v\right]=\sum_{j=0}^{\infty} a_j[p_z^2-(p_z^f)^2]^j + \sum_{j=0}^{\infty} b_j p_z[p_z^2-(p_z^f)^2]^j\ . \tag{42}$$

As $E_s\to 0$, only those values of p_z close to the critical points, i.e., $\pm p_z^f$, will contribute significantly to the integral in Eq. (39). We thus represent J_N by only the $j=0$ terms in Eq. (42), i.e.,

$$J_N\left[\frac{E_0 p_z}{\omega^2},-v\right]\approx a_0+b_0 p_z\ . \tag{43}$$

Evaluating Eq. (43) at the two critical points, $p_z=\pm p_z^f$, and using the following property of the generalized Bessel function:[47]

$$J_N\left[-\frac{E_0 p_z}{\omega^2},-v\right]=(-1)^N J_N\left[\frac{E_0 p_z}{\omega^2},-v\right]\ , \tag{44}$$

we find that

$$a_0=\frac{1+(-1)^N}{2}J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right]\ , \tag{45}$$

$$b_0=\frac{1-(-1)^N}{2p_z^f}J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right]\ . \tag{46}$$

Substituting Eq. (43) into Eq. (39), where a_0 and b_0 are given by Eqs. (45) and (46), we may evaluate the integral analytically to obtain the following result:[46]

$$\lim_{E_s\to 0} S_{fi}^{(N)}=i(-1)^N B E_s^{-1/2}\int_{-\infty}^{\infty}(a_0+b_0 p_z)\exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^z p_z)]dp_z$$

$$=i(-1)^N 2^{3/2}\pi B J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right](2E_s)^{-1/6}\mathrm{Ai}[-(2/E_s^2)^{1/3}\epsilon_f^z]\quad (N\text{ even}) \tag{47}$$

$$=(-1)^{N+1}2^{3/2}\pi B(p_z^f)^{-1}J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right](2E_s)^{1/6}\mathrm{Ai}'[-(2/E_s^2)^{1/3}\epsilon_f^z]\quad (N\text{ odd})\ . \tag{48}$$

The N-photon detachment cross sections in the presence of a weak static uniform electric field are obtained to the lowest order in the static-electric-field strength E_s by substituting the S matrix elements in Eqs. (47) and (48) into Eq. (36) and then using Eq. (35). Our results are

$$\sigma^{(N)}=\frac{64\pi^3 B^2\omega}{cE_0^2}\int_{-\infty}^{\epsilon_i+N\omega-s}\left|J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right]\right|^2(2E_s)^{-1/3}\{\mathrm{Ai}[-(2/E_s^2)^{1/3}\epsilon_f^z]\}^2 d\epsilon_f^z\quad (N\text{ even})\ , \tag{49}$$

$$\sigma^{(N)}=\frac{64\pi^3 B^2\omega}{cE_0^2}\int_{-\infty}^{\epsilon_i+N\omega-s}\left|J_N\left[\frac{E_0 p_z^f}{\omega^2},-v\right]\Big/p_z^f\right|^2(2E_s)^{1/3}\{\mathrm{Ai}'[-(2/E_s^2)^{1/3}\epsilon_f^z]\}^2 d\epsilon_f^z\quad (N\text{ odd})\ . \tag{50}$$

B. Electric-field modulation factors

In a weak electric field, the multiphoton detachment cross sections are affected most significantly by the electric field only in the detachment threshold region. We examine now this region, i.e., the region where $|\epsilon_f|=|\epsilon_i+N\omega-s|\sim(E_s^2/2)^{1/3}\ll 1$, in order to extract the electric-field-induced modulation of the N-photon cross sections given by Eqs. (49) and (50). Since both the Airy function and its derivative decay exponentially for positive arguments, the main contributions to the integrals in Eqs. (49) and (50) come from the region where $|\epsilon_f^x|\sim(E_s^2/2)^{1/3}\ll 1$. We can therefore use the following small argument expansions for the generalized Bessel functions:[48]

$$\lim_{|p_z^f|\to 0} J_N\left[\frac{E_0 p_z}{\omega^2},-v\right] =(-1)^{N/2}J_{N/2}(v) \quad (N \text{ even}) \tag{51}$$

$$=(-1)^{(N-1)/2}\frac{E_0 p_z^f}{2\omega^2}\times[J_{(N-1)/2}(v)+J_{(N+1)/2}(v)] \quad (N \text{ odd}), \tag{52}$$

where the Bessel functions on the right-hand sides of Eqs. (51) and (52) are ordinary Bessel functions of integer order. Substituting Eqs. (51) and (52) into Eqs. (49) and (50), we obtain the near-threshold approximations to the multiphoton cross sections in the presence of a weak static electric field,

$$\sigma^{(N)}=\frac{32\pi^3B^2\omega}{cE_0^2}(2E_s)^{1/3}J_{N/2}^2(v)\times\int_{-\infty}^{\xi}[\mathrm{Ai}(-\xi')]^2d\xi' \quad (N \text{ even}), \tag{53}$$

$$\sigma^{(N)}=\frac{16\pi^3B^2}{c\omega^3}E_s[J_{(N-1)/2}(v)+J_{(N+1)/2}(v)]^2\times\int_{-\infty}^{\xi}[\mathrm{Ai}'(-\xi')]^2d\xi' \quad (N \text{ odd}). \tag{54}$$

In Eqs. (53) and (54) we have defined the scaled energy variable

$$\xi\equiv(\epsilon_i+N\omega-s)(2/E_s^2)^{1/3}. \tag{55}$$

In order to extract the electric-field-induced modulations of the near-threshold region, we compare our results for the multiphoton detachment cross sections in Eq. (53) and (54) with the corresponding multiphoton detachment cross sections in the absence of an electric field. Multiphoton detachment cross sections using the Volkov solution[34] for the final-state wave function have been derived by Reiss.[49] Using the near-threshold approximations in Eqs. (51) and (52), one may obtain the near-threshold approximations to the field-free multiphoton detachment cross sections.[50] Taking the ratio of our results in Eqs. (53) and (54) to these field-free results, we obtain the following electric-field modulation factors:

$$\frac{\sigma^{(N)}(E_s)}{\sigma^{(N)}(E_s=0)}=\pi\xi^{-1/2}\int_{-\infty}^{\xi}[\mathrm{Ai}(-\xi')]^2d\xi'(N \text{ even}) \tag{56}$$

$$=3\pi\xi^{-3/2}\int_{-\infty}^{\xi}[\mathrm{Ai}'(-\xi')]^2d\xi'(N \text{ odd}). \tag{57}$$

These two modulation factors are plotted in Fig. 1. Note that they depend on the static field E_s only through the scaled energy variable ξ defined by Eq. (55).

The fact that there are only two modulation factors, depending on whether the number of photons N is even or odd, may be understood as follows. For a weak static uniform electric field, the effects of the laser field are not coupled to those of the static field. The electron-photon coupling remains of short range, as in the static-field-free case. Therefore the detached electron's orbital angular momentum can still be regarded as a good quantum number as far as the electron-photon interaction is concerned. For the case of multiphoton detachment of an s electron by linearly polarized light in the near-threshold energy region, the Wigner threshold law[51] and electric dipole selection rules indicate s-waves will dominate the cross sections for even N while p waves will dominate the cross sections for odd N. Hence there are only two modulation factors, corresponding to final s or p waves.

This dependence of the electric-field-induced modulation factors on only the photodetached electron's orbital angular momentum enables us to make connection with previous work for the case of $N=1$. Thus, our odd N modulation factor agrees with that calculated by Rau and Wong[23] for photodetachment of H^-. Our even N modulation factor, which for $N=2$ corresponds to an $s\to p\to s$ transition under our assumption that we have an s-electron initially, agrees with that obtained for the $p\to s$ transition in S^- photodetachment by Wong, Rau, and Greene[24]

Lastly, we point out the clear separation of the laser-atom interaction from the effects of the static uniform

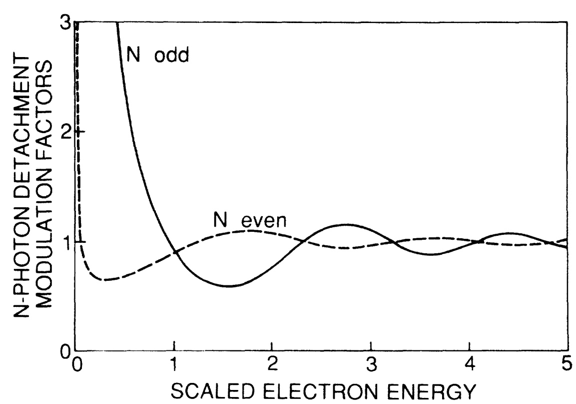

FIG. 1. Electric-field modulation factors [Eqs. (56) and (57)] for the near-threshold multiphoton detachment cross section of an s electron by linearly polarized light polarized along the static-field direction in the limit of a weak static electric field plotted as a function of the scaled (dimensionless) electron energy ξ [cf. Eq. (55)].

electric field. For weak electric fields, the laser-atom interaction is of short range. At large distances, the effect of the static uniform electric field is only to redistribute the oscillator strength among the static-field-dependent asymptotic channels. For this reason, we expect a frame transformation approach to be as applicable in the multiphoton case as in the single-photon case treated previously.[24]

V. THE WEAK LASER-FIELD LIMIT

In Sec. IV we considered the weak electric-field limit of our general formulas [i.e., Eqs. (33)–(36)] for multiphoton detachment in the presence of a static uniform electric field for the interesting case of linearly polarized light polarized along the static-field direction. Here we consider the weak laser-field limit of our general formulas in order to isolate as clearly as possible the effects of static-field-induced electron-photon interactions in a laser intensity regime for which it is possible to make comparison with previous work. In particular, we illustrate these effects numerically for the case of photodetachment of the H^- ion in the presence of a static uniform electric field. We also examine the processes of field ionization and of static-field-induced stimulated emission, and we present numerical results for these processes for the H^- ion.

A. Multiphoton detachment cross sections in the limit of $E_0 \to 0$

We present here our results for the S-matrix element in Eq. (33) for an N-photon detachment process in the presence of a static uniform electric field in the limit that the laser field is weak, i.e., $E_0 \to 0$. Eqs. (37) and (38) give the limiting forms for the Bessel functions $J_n(E_0E_s/\omega^3)$ in this limit. The generalized Bessel functions in Eq. (34) satisfy similar relations[52]

$$J_{-M}(u,-v)=(-1)^M J_M(u,v)\ , \tag{58}$$

$$J_M(z^{1/2}\mu,-zv)\xrightarrow{z\to 0} z^{M/2}(-v/2)^{M/2}\sum_{k=0}^{M/2}\frac{(-\mu^2/2v)^2}{(2k)!(M/2-k)!}\quad (M\ \text{even}) \tag{59}$$

$$\xrightarrow{z\to 0} z^{M/2}(-\nu/2)^{M/2}\sum_{k=0}^{(M-1)/2}\frac{(-\mu^2/2\nu)^{k+1/2}}{(2k+1)![(M-1)/2-k]!}\quad (M\ \text{odd})\ . \tag{60}$$

Detailed examination of these limiting formulas shows that for the terms $0\le n\le N$ in the summation in Eq. (33), the products $J_n(E_0E_s/\omega^3)J_{N-n}(E_0p_z/\omega^2,-v)$ are each of order E_0^N, whereas for the terms having $-\infty\le n\le -1$ or $N+1\le n\le\infty$, the dependence on E_0 as $E_0\to 0$ of these products is at least of order E_0^{N+2}. Hence, the limiting form of Eq. (33) becomes

$$S_{fi}^{(N)}\approx i(-1)^N(B/E_s^{1/2})\sum_{n=0}^{N} i^n J_n\left[\frac{E_0E_s}{\omega^3}\right]\int_{-\infty}^{\infty} J_{N-n}(E_0p_z/\omega^2,-v)\exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^z p_z)]dp_z\quad (E_0\to 0)\ . \tag{61}$$

We emphasize that effects arising from the static uniform electric field are treated here nonperturbatively. In a perturbative treatment only the $n=0$ term would be included in Eq. (61). The multiphoton detachment cross sections in the limit $E_0\to 0$ are obtained by substituting Eq. (61) into Eqs. (35) and (36).

B. Photodetachment in an electric field

We focus now on the particular case of $N=1$ in order to make connection with previous work on photodetachment of H^- in a static uniform electric field. Using the limiting forms of the ordinary and generalized Bessel functions given in Eqs. (38), (59), and (60), Eq. (61) gives the following result in the weak laser-field limit:

$$S_{fi}^{(N=1)}\approx -\frac{2^{2/3}\pi BE_0E_s^{1/6}}{\omega^2}\left[\frac{d}{d\xi'}-\frac{(E_s^2/2)^{1/3}}{\omega}\right]\text{Ai}(-\xi')\quad (E_0\to 0)\ , \tag{62}$$

where

$$\xi'\equiv(2/E_s^2)^{1/3}\epsilon_f^z\ . \tag{63}$$

The photodetachment cross section in the presence of a static uniform electric field in the weak laser-field limit is then given by [cf. Eqs. (35) and (36)],

$$\sigma^{(1)}=\frac{16\pi^3B^2E_s}{c\omega^3}\int_{-\infty}^{\xi}\left[\left[\frac{d}{d\xi'}-\frac{(E_s^2/2)^{1/3}}{\omega}\right]\times\text{Ai}(-\xi')\right]^2 d\xi'\ , \tag{64}$$

where the upper limit of integration ξ is defined by Eq. (55), in which the ponderomotive potential s [cf. Eq. (3)] can be ignored in the limit of weak laser fields. Equation (64) differs from what has been obtained by others[23,26] using a perturbative treatment of the laser-atom interaction because of the static-field-dependent second term inside the square brackets of the integral. This second term results from the $n=1$ term in the definition of the S matrix [cf. Eq. (61)]. The perturbative approach used in previous treatments gives an approximate expression for the S-matrix equal to the result obtained by keeping only the $n=0$ term in Eq. (61).

The difference of our result in Eq. (64) from previous perturbative calculations may be interpreted as follows. In a strong external static electric field the detached electron may interact with the laser to absorb or emit photons, which is unlike the case of a free electron or of an electron in a weak external static electric field. Thus the laser-electron interaction becomes of long-range rather than only over the short range of the atomic potential. Because of the long range of the laser-electron interaction in a strong external static electric field, a perturbative treatment of the laser-electron interaction becomes increasingly inappropriate as the external static-electric-field strength increases. Furthermore, corrections to a lowest-order perturbative treatment are difficult to calculate. Our use of the exact wave function for an electron in the combined laser and static uniform electric fields implicitly treats this nonperturbative, long-range nature of the electron-laser interaction in the presence of a strong external static electric field.

For H^- the constant B in Eq. (64) is properly chosen to have the value 0.315 52, as explained in detail by Du and Delos.[26] Briefly, we note that this is not the value which normalizes the approximate ground-state wave function given in Eq. (24). Rather B is the constant which normalizes the exact ground-state wave function according to the effective range theory,[40] i.e.,

$$B^2=(k_b/2\pi)(1-k_b r_{\rm eff})^{-1} , \tag{65}$$

where

$$k_b^2/2 \equiv |\epsilon_i| . \tag{66}$$

Its value for H^- is obtained from the variational calculation of Ohmura and Ohmura,[42] who found that

$$k_b=0.235\,588\,3 , \tag{67}$$

and

$$r_{\rm eff}=2.646 . \tag{68}$$

Figure 2 compares the photodetachment cross section

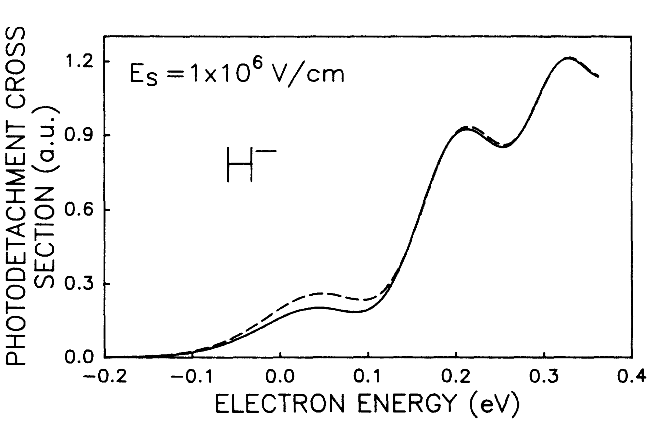

FIG. 2. Photodetachment cross section of H^- by linearly polarized light in the presence of a uniform electric field directed along the axis of linear polarization. Plotted vs detached electron energy for an electric field strength of 10^6 V/cm. Solid line: present results. Dashed line: results of the theory of Du and Delos, Ref. 26.

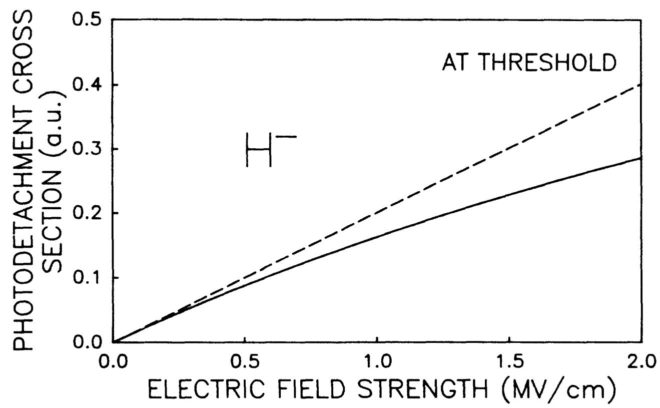

FIG. 3. Same as Fig. 2 except here the threshold value of the cross section is plotted vs electric field strength.

predicted by our Eq. (64) for an electric field strength of 10^6 V/cm with that predicted by the theory of Du and Delos.[26] Near threshold, we predict the plateau regions to be lower in magnitude. Figure 3 compares threshold cross sections predicted by the two theories as a function of electric-field strength. For fields less than 10^5 V/cm the two theories agree, whereas for higher field strengths the difference between the predictions increases, giving clear evidence of field-coupling effects. As the electric field strength increases, it becomes more likely for the electron to become detached by field ionization, as is discussed in the next section.

C. Field ionization

In the weak laser-field limit, the transition rate for field ionization of the initial state may be calculated from the S-matrix element for $N=0$, which is obtained from Eq. (61) as

$$S_{fi}^{(N=0)} \approx 2^{3/2}\pi i B(2E_s)^{-1/6} \times \mathrm{Ai}[-(2/E_s^2)^{1/3}\epsilon_f^z] \quad (E_0 \to 0) . \tag{69}$$

From Eq. (36), the transition rate is thus

$$W_{fi}^{(N=0)} \approx 4\pi^2 B^2(2E_s)^{1/3} \times \int_{-\infty}^{\xi} [\mathrm{Ai}(-\xi')]^2 d\xi' \quad (E_0 \to 0) , \tag{70}$$

where

$$\xi \equiv (2/E_s^2)^{1/3}\epsilon_i = -(2/E_s^2)^{1/3}|\epsilon_i| . \tag{71}$$

[Note again that ξ here differs from that in Eq. (55) by our neglect of the ponderomotive shift s given by Eq. (3). This is appropriate in the limit of weak laser fields.]

Our result for the transition rate [Eq. (70)] may be related easily to previous work of Demkov and Drukarev[41] in the limit of a weak static electric field, i.e., $E_s \to 0$. In this limit, the following asymptotic expansion of the Airy function is appropriate:[53]

$$\mathrm{Ai}(-\xi') \to \tfrac{1}{2}\pi^{-1/2}(-\xi')^{-1/4} \times \exp[-\tfrac{2}{3}(-\xi')^{3/2}] \quad (-\xi' \to \infty) . \tag{72}$$

Substituting this expression into Eq. (70) and carrying out the integration, we obtain

$$W_{fi}^{(N=0)} \to \frac{\pi B^2 E_s}{2|\epsilon_i|} \exp\left[-\frac{(2^5|\epsilon_i|^3)^{1/2}}{3E_s} \right] \quad (E_0, E_s \to 0) \ . \tag{73}$$

If now we choose for B the value which normalizes the approximate ground-state wave function in Eq. (24), i.e., $B^2=(2|\epsilon_i|)^{1/2}/(2\pi)$, then Eq. (73) equals exactly 2Γ, where Γ is the half width obtained by Demkov and Drukarev.[54]

However, as discussed briefly at the end of Sec. V B above (as well as in more detail elsewhere[26]), it is more appropriate to choose B according to effective range theory.[40] Thus, substituting Eqs. (65) and (66) into Eq. (73), we find that the lifetime of a weakly bound electron in a static uniform electric field is given by

$$\tau(\text{a.u.}) \equiv W^{-1} = \frac{2k_b(1-k_b r_{\text{eff}})}{E_s} \exp\left[-\frac{2k_b^3}{3E_s} \right] . \tag{74}$$

Using the Ohmura and Ohmura[42] parameters for H^- given by Eqs. (67) and (68), we have plotted $\log\tau(\text{sec})$ versus E_s in Fig. 4. This figure shows clearly that field ionization of the ground state of H^- becomes increasingly likely as E_s increases beyond the range of 1–2 MeV.

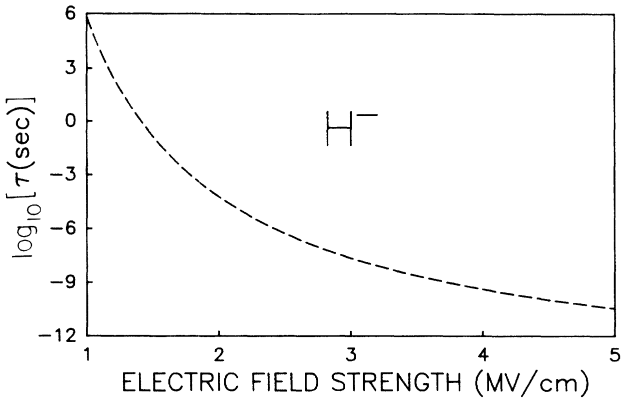

FIG. 4. Dependence of the H^- ground-state lifetime τ [cf. Eq. (74)] in an external static uniform electric field as a function of the static-field strength E_s.

D. Static-field-induced stimulated emission

We examine in this section our general S-matrix element in Eq. (33) for negative values of N in the limit of a weak laser field, $E_0 \to 0$. Consider the case that $N=-|N|$. Using the symmetry properties of the ordinary and generalized Bessel functions [cf. Eqs. (37) and (58)], Eq. (33) may be rewritten in the following form:

$$S_{fi}^{(-|N|)} \equiv iBE_s^{-1/2} \sum_{n=-\infty}^{\infty} (-i)^n J_n \left[\frac{E_0 E_s}{\omega^3} \right] \int_{-\infty}^{\infty} J_{|N|-n} \left[\frac{E_0 p_z}{\omega^2}, v \right] \exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^z p_z)] dp_z \ . \tag{75}$$

Using now the same arguments adduced in Sec. V B, Eq. (73) reduces in the weak laser-field limit to

$$S_{fi}^{(-|N|)} \equiv iBE_s^{-1/2} \sum_{n=0}^{|N|} (-i)^n J_n \left[\frac{E_0 E_s}{\omega^3} \right] \int_{-\infty}^{\infty} J_{|N|-n} \left[\frac{E_0 p_z}{\omega^2}, v \right] \exp[-iE_s^{-1}(p_z^3/6-\epsilon_f^z p_z)] dp_z \ . \tag{76}$$

Equations (75) and (76) represent, respectively, the general S-matrix element and its weak laser-field approximation for detachment of a weakly bound electron in the combined fields of a laser and a static uniform electric field with emission of $|N|$ photons. Since these emitted photons are identical to the incident photons, this process can only be detected experimentally by photoelectron spectroscopy.

In the weak laser-field limit, the most likely stimulated emission process is the one in which only a single photon is emitted. Setting $|N|=1$ in Eq. (76) and making the same weak laser-field arguments as were made to derive Eq. (62) above, we obtain

$$S_{fi}^{(-1)} \approx \frac{2^{2/3}\pi B E_0 E_s^{1/6}}{\omega^2} \times \left[\frac{d}{d\xi'} + \frac{(E_s^2/2)^{1/3}}{\omega} \right] \text{Ai}(-\xi') \quad (E_0 \to 0) \ , \tag{77}$$

where ξ' is defined in Eq. (63). This S-matrix element gives, upon use of Eqs. (35) and (36), the following cross section:

$$\sigma^{(-1)} = \frac{16\pi^3 B^2 E_s}{c\omega^3} \int_{-\infty}^{\xi} \left[\left[\frac{d}{d\xi'} + \frac{(E_s^2/2)^{1/3}}{\omega} \right] \times \text{Ai}(-\xi') \right]^2 d\xi' \ , \tag{78}$$

where

$$\xi \equiv -(|\epsilon_i|+\omega)(2/E_s^2)^{1/3} \ , \tag{79}$$

and where, once again, we have ignored the ponderomotive potential s in defining ξ, as is appropriate in the limit of weak laser fields. In the weak static-field limit, $E_s \to 0$, the upper limit of integration ξ [cf. Eq. (79)] is large and negative. Hence the integrand may be approximated by the square of $Ai'(-\xi)$, which has the following asymptotic form:[55]

$$\mathrm{Ai}'(-\xi')\rightarrow -\tfrac{1}{2}\pi^{-1/2}(-\xi')^{1/4} \times \exp[-\tfrac{2}{3}(-\xi')^{3/2}] \quad (-\xi'\rightarrow\infty) . \tag{80}$$

Substituting the square of Eq. (80) for the integrand in Eq. (78), carrying out the integration, and using Eq. (65) to replace B, we obtain

$$\sigma^{(-1)}=\frac{\pi k_b}{c\omega^3}(1-k_b r_{\mathrm{eff}})^{-1}E_s \exp\left[-\frac{2k_{-1}^3}{3E_s}\right] , \tag{81}$$

where k_b is defined by Eq. (66) and where we have defined k_{-1} by

$$k_{-1}^2/2\equiv|\epsilon_i|+\omega . \tag{82}$$

Figure 5 shows the dependence of the stimulated emission cross section of the H^- ion on the static-electric-field strength E_s for two laser wavelengths $\lambda=1064$ and 10 550 nm. The parameters k_b and r_{eff} are obtained from Ohmura and Ohmura[42] [cf. Eqs. (67) and (68)]. These figures show that $\sigma^{(-1)}$ is very small, but increases rapidly with increasing electric field strength. As expected from Eqs. (81) and (82), the stimulated emission cross section can be much larger for longer wavelengths, as is shown by comparison of Figs. 5(a) and 5(b). Note also that stimulated photoemission from a ground state would not be possible in the absence of the static electric field.

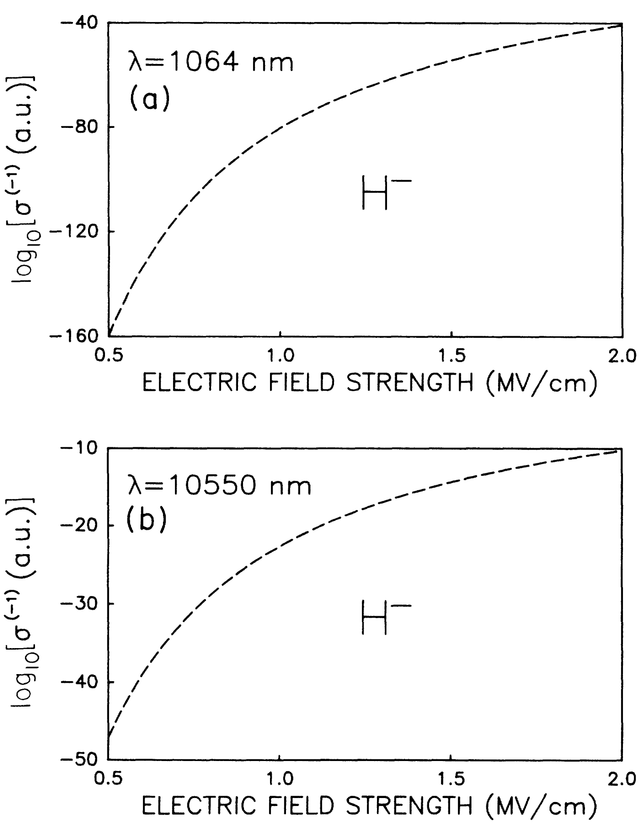

FIG. 5. Dependence of the stimulated emission cross section $\sigma^{(-1)}$ [cf. Eq. (81)] for the H^- ion on the static uniform electric-field strength for two laser wavelengths: (a) $\lambda=1064$ nm, (b) $\lambda=10\,550$ nm. The laser field is assumed to be linearly polarized along the static-field direction.

VI. RESULTS FOR CIRCULARLY POLARIZED LIGHT

Our analytic wave function for an electron in the combined fields of a laser and a static uniform electric field, which was presented in Sec. II, has been given for a laser field of arbitrary linear polarization. Our applications to various detachment processes have been presented, however, only for the case of linearly polarized light polarized along the static-electric-field direction. This case is physically the most interesting one because of the electric-field modulation of the multiphoton cross sections as well as because of the coupling of the electromagnetic and static-field effects, as discussed in detail above. The case of linearly polarized light polarized perpendicular to the static-electric-field direction is discussed elsewhere.[32(b)] However, our results are approximate due to our neglect of electron-atom interactions in the final state, whose effects on detachment by two or more photons may be considerable.

In contrast, consider the case of a circularly polarized laser field in which the photon wave vector is directed along the static electric field. This case is physically less interesting: as we show below, the electric-field-induced modulation factors for the multiphoton detachment cross sections do not exhibit oscillations; furthermore, there is no interplay of the static and electromagnetic fields. On the other hand, the effects of electron-atom interactions are much less important in this case since no intermediate or final s wave, which is essentially the only wave scattered by a spherically symmetric short-range potential, plays a direct role. In fact, in the absence of any external static field, Becker, McIver, and Confer[56] have shown that for an electron bound by a δ-function potential, the Keldysh[57] theory for multiphoton detachment gives "virtually the exact ionization rate for circular polarization." Even though the l mixing by the static electric field is such that the electron-atom interaction still plays a role for detachment by a circularly polarized light in the presence of a static electric field, its effect can be expected to be small for moderate electric field strengths, as has been shown by Fabrikant.[31] Hence, we expect our predictions for the case of circularly polarized light to be very accurate even for two or more photon detachment processes, even though we have neglected electron-atom interactions in the final state. For this reason, we present our results for circular polarization very briefly below. We note that this case has also been treated by Slonim and Dalidchik using a Green's-function approach.[10]

A. The S-matrix elements

Consider the following circularly polarized laser field traveling along the direction of an external static uniform electric field, $\mathbf{E}_s=E_s\hat{\mathbf{z}}$:

$$\mathbf{A}=(cE_0/\sqrt{2}\omega)(\cos\omega t\hat{\mathbf{x}}\pm\sin\omega t\hat{\mathbf{y}}) , \tag{83}$$

where the plus and minus signs correspond to left and right polarization, respectively. It is straightforward to show that the exact wave function for an electron in this combined field has the following separable form:

$$\Psi_f(\mathbf{p},t)=\psi_f^x(p_x,t)\psi_f^y(p_y,t)\psi_f^z(p_z,t) \;, \qquad (84)$$

where

$$\psi_f^z(p_z,t)=\frac{1}{\sqrt{2\pi E_s}}\exp\left[\frac{i}{E_s}\left[\frac{p_z^3}{6}-\epsilon_f^z p_z\right]-i\epsilon_f^z t\right] , \qquad (85)$$

and where

$$\psi_f^x(p_x,t)\psi_f^y(p_y,t)=\delta(p_x-p_x^f)\delta(p_y-p_y^f) \times\exp\left[-i\frac{E_0 p_\perp^f}{\sqrt{2}\omega^2}\sin(\omega t\mp\varphi) -i(\epsilon_f^x+\epsilon_f^y+s)t\right] . \qquad (86)$$

In Eq. (86), s is the ponderomotive shift [cf. Eq. (3)], and $p_\perp^f$ and φ are defined as the following combinations of the final-state momentum components:

$$p_\perp^f=[(p_x^f)^2+(p_y^f)^2]^{1/2} \;, \qquad (87)$$

$$\tan\varphi=p_y^f/p_x^f \;. \qquad (88)$$

Unlike the case of linearly polarized light [cf. Eq. (9)], for circularly polarized light the effect of the static uniform electric field is completely separable from that of the laser field, as long as final-state electron-atom interactions are ignored.

Following procedures analogous to those discussed above for the derivation of Eq. (33), we find that the S-matrix element for detachment of an electron having the initial-state wave function given in Eq. (25) by N circularly polarized photons is

$$S_{fi}^{(N)}=i(-1)^N 2^{4/3}\pi B E_s^{-1/6} e^{\pm iN\varphi} J_N\left[\frac{E_0 p_\perp^f}{2^{1/2}\omega^2}\right] \times\text{Ai}[-(2/E_s^2)^{1/3}\epsilon_f^z] \;. \qquad (89)$$

Note here also that in contrast to the linear polarization case [cf. Eq. (33)], the S-matrix element in Eq. (89) factors into terms dependent on only one of the two fields, E_0 or E_s.

B. Multiphoton detachment cross section

Substituting the S-matrix element in Eq. (89) into Eqs. (35) and (36), we obtain the multiphoton detachment cross sections in the presence of a static uniform electric field for the case of circularly polarized light traveling along the direction of the static electric field

$$\sigma^{(N)}=\frac{64\pi^3 B^2\omega}{cE_0^2}(2E_s)^{-1/3} \times\int_{-\infty}^{\epsilon_i+N\omega-s}\left[J_N\left[\frac{E_0}{\omega^2}(\epsilon_i+N\omega-s-\epsilon_f^z)^{1/2}\right] \times\text{Ai}[-(2/E_s^2)^{1/3}\epsilon_f^z]\right]^2 d\epsilon_f^z \;. \qquad (90)$$

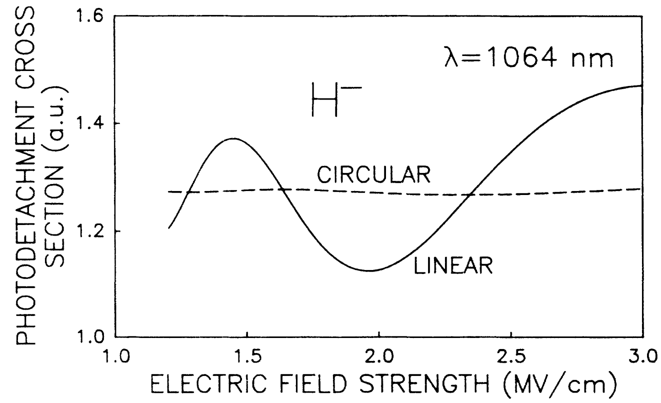

FIG. 6. Photodetachment cross sections for the H^- ion in a static uniform electric field using circularly polarized and linearly polarized light of wavelength $\lambda=1064$ nm.

Unlike the multiphoton cross sections obtained from Eqs. (35) and (36) using the S-matrix element in Eq. (33) for linearly polarized light, the cross sections given in Eq. (90) for circularly polarized light are numerically tractable without taking either the weak laser or the weak static-field limit. For comparison with the case of linear polarization, however, we show in Fig. 6 our prediction using the weak laser limit of Eq. (90) for single-photon detachment of the H^- ion by circularly polarized light having a wavelength $\lambda=1064$ nm as a function of the static-electric-field strength. Also shown are our results using Eq. (64) for the case of linearly polarized laser light. In contrast to the case of linear polarization, the photodetachment cross section for the case of circularly polarized light is nearly independent of the static-electric-field strength.

C. Electric-field-modulation factors

Following our treatment in Sec. IV B for the case of linearly polarized light, we find that near the threshold for multiphoton detachment of an electron initially in an s state by circularly polarized light, the electric-field modulation factors (for both left and right circular polarization) are

$$\frac{\sigma^{(N)}(E_s)}{\sigma^{(N)}(E_s=0)}=\frac{(2N+1)!!\pi}{(2N)!!\xi^{N+1/2}} \times\int_{-\infty}^{\xi}(\xi-\xi')^N[\text{Ai}(-\xi')]^2 d\xi' \;. \qquad (91)$$

Here ξ is the scaled energy variable defined in Eq. (55).

Note that in contrast to the case of linear polarization [in which the modulation factors in Eqs. (56) and (57) depend only on $(-1)^N$], the modulation factors for circularly polarized light depend on N. This may be understood as simply due to the final-state orbital angular momentum of the detached electron, i.e., $l_f=N$ and $m_{l_f}=\pm N$. In fact, the modulation factor given by Eq. (91) should apply in the weak electric-field limit to any process which produces an electron in the same final angular momentum state. Figure 7 shows the modulation factors given by Eq. (91) for $N=1$, 2, and 3.

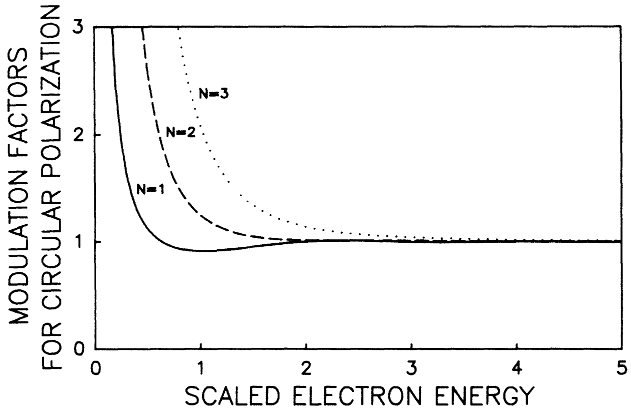

FIG. 7. Electric field modulation factors for the near-threshold multiphoton detachment cross section of an s electron by N circularly polarized photons traveling along the direction of the static electric field vs the scaled (dimensionless) electron energy ξ, defined in Eq. (55).

VII. SUMMARY AND CONCLUSIONS

In this paper we have employed an analytic, momentum-space wave function for an electron in the combined fields of a laser and a static uniform electric field to examine a number of detachment processes for an electron initially bound weakly in an s state. Our major approximations are the electric dipole approximation and the neglect of final-state electron-atom interactions. No restrictions on the magnitudes of the laser or the static field are required other than those implied by our assumption that the initial state is unaffected by these fields. For the case of photons linearly polarized along the direction of the static field, we have shown that the effects of the laser and static fields are coupled. The static field thus enables the electron to interact with the laser over a much larger range than just in the region of the atomic potential which provides the initial binding. In contrast, for the case of circularly polarized photons traveling along the direction of the static field, the effects of the laser and static fields are uncoupled. Note that, in general, whenever the laser field has a component along the direction of the static electric field, the effects of the static and laser fields will be coupled.

Our general formulas have been examined in several important limits. For the limit of a weak static field, we have shown that the effect of the electric field near the multiphoton detachment threshold is describable by a modulation factor. For linearly polarized light, there are two modulation factors, one for even N and one for odd N. For circularly polarized light, the modulation factor depends on the number of photons absorbed, N. In all cases, these modulation factors may be understood in terms of the photodetached electron's final orbital angular momentum, which is an approximately good quantum number in the weak static-electric-field limit.

For the limit of a weak laser field, we have examined the photodetachment cross section for a negative ion for both linearly and circularly polarized photons and presented numerical results for the H^- ion. In particular, we have shown that static-field-induced electron-photon interactions in the case of linearly polarized photons produce a measurable lowering of the near-threshold photodetachment cross section for static fields of order 1.0 MV and greater. Also for weak laser fields, we have presented expressions for the lifetime against field ionization and the cross section for static-field-induced stimulated emission for weakly bound electrons initially in an s state, and numerical results for H^- were presented.

ACKNOWLEDGMENTS

We thank János Bergou, Ilya I. Fabrikant, V. Z. Slonim, and Krzysztof Wódkiewicz for providing some useful references and for helpful discussions. This work was supported in part by National Science Foundation Grant No. PHY-8908605.

[1]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **34**, 1138 (1958) [Sov. Phys.—JETP **34**, 788 (1958)].

[2]W. Franz, Z. Naturforsch., Teil A **13**, 484 (1958).

[3]J. Callaway, Phys. Rev. **130**, 549 (1963); **134**, A998 (1964).

[4]K. Tharmalingam, Phys. Rev. **130**, 2204 (1963).

[5]C. B. Duke, Phys. Rev. Lett. **15**, 625 (1965); C..B. Duke and M. E. Alferieff, Phys. Rev. **145**, 583 (1966).

[6]D. E. Aspnes, Phys. Rev. **147**, 554 (1966).

[7]D. F. Blossey, Phys. Rev. B **2**, 3976 (1970).

[8]I. N. Arutyunyan and G. A. Askar'yan, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 378 (1970) [JETP Lett. **12**, 259 (1970)].

[9]A. I. Nikishov, Zh. Eksp. Teor. Fiz. **62**, 562 (1972) [Sov. Phys.—JETP **35**, 298 (1972)].

[10]V. Z. Slonim and F. I. Dalidchik, Zh. Eksp. Teor. Fiz. **71**, 2057 (1976) [Sov. Phys.—JETP **44**, 1081 (1976)].

[11]N. L. Manakov and A. G. Fainshtein, Zh. Eksp. Teor. Fiz. **79**, 751 (1980) [Sov. Phys.—JETP **52**, 382 (1980)].

[12]S. Feneuille, S. Liberman, J. Pinard, and P. Jacquinot, C. R. Acad. Sci. Ser. B **284**, 291 (1977); S. Liberman and J. Pinard, Phys. Rev. A **20**, 507 (1979); S. Feneuille, S. Liberman, J. Pinard, and A. Taleb, Phys. Rev. Lett. **42**, 1404 (1979).

[13]R. R. Freeman, N. P. Economou, G. C. Bjorklund, and K. T. Lu, Phys. Rev. Lett. **41**, 1463 (1978); R. R. Freeman and N. P. Economou, Phys. Rev. A **20**, 2356 (1979).

[14]A. R. P. Rau, J. Phys. B **12**, L193 (1979); A. R. P. Rau and K. T. Lu, Phys. Rev. A **21**, 1057 (1980).

[15]E. Luc-Koenig and A. Bachelier, Phys. Rev. Lett. **43**, 921 (1979); J. Phys. B **13**, 1743 (1980); **13**, 1769 (1980).

[16]V. D. Kondratovich and V. N. Ostrovskii, Zh. Eksp. Teor. Fiz. **79**, 395 (1980) [Sov. Phys.—JETP **52**, 198 (1980)]; Yu. N. Demkov, V. D. Kondratovich and V. N. Ostrovskii, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 425 (1981) [JETP Lett. **34**, 403 (1981)]; V. D. Kondratovich and V. N. Ostrovskii, J. Phys. B **17**, 1981 (1984); **17**, 2011 (1984).

[17](a) I. I. Fabrikant, Zh. Eksp. Teor. Fiz. **79**, 2070 (1980) [Sov. Phys.—JETP **52**, 1045 (1980)]; (b) **83**, 1675 (1982) [**56**, 967 (1983)].

[18]U. Fano, Phys. Rev. A **24**, 619 (1981).

[19]D. A. Harmin, Phys. Rev. A **24**, 2491 (1981); Phys. Rev. Lett. **49**, 128 (1982); Phys. Rev. A **26**, 2656 (1982).

[20]W. L. Glab, K. Ng, D. Yao, and M. H. Nayfeh, Phys. Rev. A **31**, 3677 (1985); G. B. Hillard and W. L. Glab, Physica **C145**, 229 (1987).

[21]W. P. Reinhardt, J. Phys. B **16**, L635 (1983); in *Atomic Excitation and Recombination in External Fields,* edited by M. H. Nayfeh and C. W. Clark (Gordon and Breach, New York, 1985), pp. 85–103.

[22]H. C. Bryant, A. Mohagheghi, J. E. Stewart, J. B. Donahue, C. R. Quick, R. A. Reeder, V. Yuan, C. R. Hummer, W. W. Smith, S. Cohen, W. P. Reinhardt, and L. Overman, Phys. Rev. Lett. **58**, 2412 (1987).

[23]A. R. P. Rau and H. Y. Wong, Phys. Rev. A **37**, 632 (1988).

[24]H. Y. Wong, A. R. P. Rau, and C. H. Greene, Phys. Rev. A **37**, 2393 (1988).

[25]C. H. Greene and N. Rouze, Z. Phys. D **9**, 219 (1988).

[26]M. L. Du and J. B. Delos, Phys. Rev. A **38**, 5609 (1988).

[27]M. L. Du, Phys. Rev. A **40**, 4983 (1989).

[28]V. D. Kondratovich and V. N. Ostrovskii, J. Phys. B **23**, 21 (1990); cf. Sec. 4.

[29]I. I. Fabrikant, J. Phys. B **23**, 1139 (1990).

[30]C. A. Nicolaides and Th. Mercouris, Chem. Phys. Lett. **159**, 45 (1989).

[31]I. I. Fabrikant, Phys. Rev. A **40**, 2373 (1989).

[32](a) Bo Gao and A. F. Starace, Bull. Am. Phys. Soc. **34**, 1384 (1989); (b) Bo Gao, Ph.D. thesis, The University of Nebraska-Lincoln, 1989.

[33]Bo Gao, Phys. Rev. A **41**, 5039 (1990).

[34]D. M. Volkov, Z. Phys. **94**, 250 (1935).

[35]S. Olariu, I. Popescu, and C. B. Collins, Phys. Rev. D **20**, 3095 (1979).

[36]R. R. Schlicher, W. Becker, J. Bergou, and M. O. Scully, in *Quantum Electrodynamics and Quantum Optics,* edited by A. O. Barut (Plenum, New York, 1984), pp. 405–441.

[37]C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Photons and Atoms: Introduction to Quantum Electrodynamics* (Wiley, New York, 1989), Section IV B 3.

[38]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics,* 2nd ed. (Pergamon, Oxford, 1965), p. 45.

[39]See, e.g., H. R. Reiss, Phys. Rev. A **22**, 1786 (1980), Eq. (3).

[40]H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950).

[41]Yu. N. Demkov and G. F. Drukarev, Zh. Eksp. Teor. Fiz. **47**, 918 (1964) [Sov. Phys.—JETP **20**, 614 (1965)].

[42]T. Ohmura and H. Ohmura, Phys. Rev. **118**, 154 (1960).

[43]See, e.g., H. R. Reiss, Ref. 39, Eq. (25).

[44]H. R. Reiss, Ref. 39, Appendix B. See especially Eq. (B11).

[45]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965). See Eqs. 9.1.5 and 9.1.7.

[46]L. S. Schulman, *Techniques and Applications of Path Integration* (Wiley, New York, 1981), pp. 132–134. See especially Eqs. (16.7), (16.9), and (16.11). Note also the following misprints: first, the numerator $2\sqrt{\beta}$ within the large square root on the right-hand side of Eq. (16.10) should be replaced by $\pm 2\sqrt{\beta}$. Second, the right-hand side of Eq. (16.11) should be multiplied by 2π.

[47]H. R. Reiss, Ref. 39, Appendix B, Eq. (B7).

[48]H. R. Reiss, Ref. 39, Appendix B, Eqs. (B14) and (B15).

[49]H. R. Reiss, Ref. 39, Eq. (45). Note that the normalization of Reiss's ground-state wave function differs from ours by a factor of $(2\pi)^{3/2}$.

[50]Bo Gao, Ref. 32(b), Appendix D, Section D.1(a).

[51]E. P. Wigner, Phys. Rev. **73**, 1002 (1948).

[52]Cf. H. R. Reiss, Ref. 39, Eqs. (B7), (C2), and (C3).

[53]Ref. 45, Eq. 10.4.59.

[54]Yu. N. Demkov and G. F. Drukarev, Ref. 41, Eq. (13).

[55]Ref. 45, Eq. 10.4.61.

[56]W. Becker, J. K. McIver, and M. Confer, Phys. Rev. A **40**, 6904 (1989).

[57]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [Sov. Phys.—JETP **20**, 1307 (1965)].