Relativistic theory for continuous measurement of quantum fields

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We have proposed a formal theory for the continuous measurement of relativistic quantum fields. We have also derived the corresponding scattering equations. The proposed formalism reduces to known equations in the Markovian case. Two recent models for spontaneous quantum state reduction have been recovered in the framework of our theory. A possible example of the relativistic continuous measurement has been outlined in standard quantum electrodynamics. The continuous measurement theory possesses an alternative formulation in terms of interacting quantum and stochastic fields.

I. INTRODUCTION

Considering quantum theory, Bell[1] has claimed recently that a fundamental theory should not refer to the term "measurement." In fact, bearing in mind the standard theory[2] of quantum measurements, one would incorporate a spontaneous measurementlike process into the unitary quantum theory.[3-14] The resulting theory contains the proper nonlinearities and stochasticity in addition to the unitary evolution. And then one may delete the term "measurement" (now a necessary and distinguished word) from the syntax of the theory. Such a modified theory, in itself, would be able to show the collapse of the wave function and when and how collapses are expected to occur.

We need a plausible mathematical model of what may and should be called "continuous quantum measurement" (alternative terms such as "continual" or "permanent observation," "collapse," or "reduction" are often used). This paper is devoted to this technical aspect of the fundamental project. We develop a possible theory for continuous measurement of relativistic quantum fields.

The literature of previous work is rather large, so we only emphasize the main points. While a consistent nonrelativistic theory of *Markovian* continuous quantum measurements has been developed by Barchielli, Lanz, and Prosperi,[5] a flexible formalism was still lacking. Gisin,[6] independently, has introduced *quantum-stochastic differential equations* governing the evolution of the state vector under continuous measurements. Caves and Milburn[9] observed that the measured information can be *fed back* into the original dynamics. After all, it has been shown[11] that the state vector and the measured value satisfy a couple of stochastic differential equations. There are only a few papers related to non-Markovian continuous measurements[15,16] and to relativistic ones.[17-20]

In this paper a straightforward relativistic (not necessarily Markovian) generalization of the above-noted results is proposed. This task can be solved successfully in scattering theory, leaving interpolating fields to be discussed elsewhere.

Section II contains our proposal for a relativistic continuous quantum measurement theory (CQMT); in Sec. III, the corresponding scattering superoperator is derived. In Sec. IV specific equations for the Markovian case are traced; Sec. V presents some applications; Sec. VI contains the conclusions. Appendixes A and B are useful to study in advance, as they familiarize the reader with notations and with superoperator formalism. Appendix C is not technical; in it we point out that CQMT, defined previously in Sec. II, can be reformulated in terms of a given quantum-stochastic field theory (QSFT).

II. CONTINUOUS MEASUREMENT OF RELATIVISTIC QUANTUM FIELDS

Let us start with the scattering equation of the standard relativistic quantum field theory, in the interaction picture:

$$|\mathrm{out}\rangle = T\exp\left[-i\int \mathcal{H}\,dx\right]|\mathrm{in}\rangle \equiv S|\mathrm{in}\rangle\ , \tag{2.1}$$

where $\mathcal{H}(x)$ is the density of the interaction Hamiltonian, S denotes the unitary scattering operator, T stands for time-ordering.

Let $q(x)$ denote the Hermitian boson field (not necessarily a scalar) we choose to be continuously measured. Actually it may be one of the primary boson fields the given field theory is built of. Nevertheless, any composite field operator, e.g., a current, is acceptable too. We require that $q(x)$ be local, i.e., $[q_\alpha(x),q_\beta(y)]=0$ when $(x-y)^2<0$. Greek indices, usually suppressed, label field components.

The outcome of continuous measurement of $q(x)$ must be a c-number field $\bar{q}(x)$. In other words, $\bar{q}(x)$ is the measured (sampled or selected[8,9]) value of the quantum field $q(x)$. In the case of standard quantum measurements[2] the outcome is random hence, by analogy, $\bar{q}(x)$ will be considered as a real stochastic field.

In order to incorporate a mechanism of the continuous measurement of the field q, the unitary scattering (2.1) has to be modified as follows. Given a certain norm $\|q\|$ (specified later) on the space of the measured fields, we introduce the unnormalized "out" states:

$$\Psi_{\rm out}[\bar q]=T\exp\left[-i\int(\mathcal{H}+\bar J^r q)dx-\tfrac{1}{2}\|q-\bar q\|^2\right]|{\rm in}\rangle\ , \tag{2.2}$$

which depend functionally on the measured (sampled or selected) value $\bar q(x)$. The real "current" $\bar J^r$ will realize the $\bar q$-dependent (hence also stochastic) feedback.

The normalized out states have the form

$$|{\rm out};\bar q\rangle=\mathcal{N}^{-1/2}[\bar q]\Psi_{\rm out}[\bar q]\ , \tag{2.3}$$

where $\mathcal{N}$ is equal to the form of the unnormalized states:

$$\mathcal{N}[\bar q]=\Psi^\dagger_{\rm out}[\bar q]\Psi_{\rm out}[\bar q]\ . \tag{2.4}$$

Observing that $\mathcal{N}$ is a positive functional of the measured values $\bar q$, it is possible to define the probability distribution functional w of the measured field values $\bar q$ so as to be proportional to $\mathcal{N}$:

$$w[\bar q]=C^{-1}\mathcal{N}[\bar q]=C^{-1}\Psi^\dagger_{\rm out}[\bar q]\Psi_{\rm out}[\bar q]\ , \tag{2.5}$$

where C is a normalization factor so that $\int w[\bar q]d[\bar q]=1$.

We have postponed the explanation of certain details of the right-hand side (rhs) of Eq. (2.2). The norm $\|q\|$ is specified by

$$\|q\|^2\equiv(q|\Gamma|q)\ , \tag{2.6}$$

where Γ is the positive definite symmetric kernel characterizing the *strength* of measurement of q. When q has more components, e.g., q is a vector or a tensor, Γ acquires discrete (Greek) indices, too. And finally, we need to specify the real current $\bar J^r$ representing the retarded feedback of the measured information into the quantum dynamics:

$$\bar J^r(x)=\int G^r(x,y)\bar q(y)dy\ . \tag{2.7}$$

Here $G^r(x,y)$ is supposed to be a real retarded kernel, i.e., it must vanish for $x_0<y_0$ and also for $(x-y)^2<0$.

Equations (2.2)–(2.7) define the proposed relativistic continuous quantum measurement theory, measuring a given quantum field q, with strength Γ of measurement and with retarded feedback $\bar J^r$. The unitary scattering (2.1) is obviously recovered when $\Gamma\equiv 0$ and $G^r\equiv 0$.

This paper is not intended to give a systematic discussion of the internal consistency of the CQMT. Nevertheless, we anticipate that the retarded feedback [i.e., G^r in Eq. (2.7)] cannot be chosen arbitrarily. We shall return to this issue in the next section.

III. SCATTERING SUPEROPERATOR

Let us illustrate the notion of scattering superoperator in standard quantum field theory. If $\rho_{\rm in}$ stands for the initial density operator of the system, then, after scattering, the final-state density operator takes the form

$$\rho_{\rm out}=S\rho_{\rm in}S^\dagger\ , \tag{3.1}$$

where $S=T\exp(-i\int\mathcal{H}\,dx)$ is the unitary scattering operator; cf. Eq. (2.1). Now, the relation (3.1) can be written in the compact form

$$\rho_{\rm out}=\hat S\rho_{\rm in}\ , \tag{3.2}$$

where $\hat S$ is called scattering superoperator.

There is a general theorem, due to Gisin,[6,14] from which it follows that the above linear relation between asymptotic states holds in any reasonable theory including, e.g., nonunitary ones, too. In addition, $\hat S$ must be an automorphism of the space of density operators. This represents very strong mathematical constraints. The full classification of possible $\hat S$'s is lacking. However, for the special case of Markovian systems we have definite results (see Sec. IV).

In the superoperator formalism of Appendix B, the scattering superoperator of the standard quantum field theory takes the following form:

$$\hat S=S_+S_-^\dagger=\hat T\exp\left[-i\int\mathcal{H}_\Delta dx\right]\ . \tag{3.3}$$

This $\hat S$ transforms pure states into pure ones. Let us proceed to derive the scattering superoperator in the presence of continuous measurement. By introducing the asymptotic pure state projectors

$$P_{\rm in}\equiv|{\rm in}\rangle\langle{\rm in}|,\quad P_{\rm out}[\bar q]\equiv|{\rm out};\bar q\rangle\langle{\rm out};\bar q|\ , \tag{3.4}$$

Eqs. (2.2) and (2.3) yield the following relation between them:

$$P_{\rm out}[\bar q]=\mathcal{N}^{-1}[\bar q]T\exp\left[-i\int(\mathcal{H}+\bar J^r q)dx-\tfrac{1}{2}\|q-\bar q\|^2\right]P_{\rm in}\tilde T\exp\left[i\int(\mathcal{H}+\bar J^r q)dx-\tfrac{1}{2}\|q-\bar q\|^2\right]\ , \tag{3.5}$$

which can be written in the form

$$P_{\rm out}[\bar q]=\hat S[\bar q]P_{\rm in}\ . \tag{3.6}$$

In superoperator formalism, the *selected*[8,9] scattering superoperator $\hat S[\bar q]$ can be written as

$$\hat S[\bar q]=\mathcal{N}^{-1}[\bar q]\hat T\exp\left[-i\int(\mathcal{H}_\Delta+\bar J^r q_\Delta)dx-\tfrac{1}{4}\|q_\Delta\|^2-\|q_c-\bar q\|^2\right]\ . \tag{3.7}$$

It is nonlinear because the normalization factor $\mathcal{N}[\bar{q}]$ depends always on the initial state of the system.

Up to now we have discussed the scattering process in detail, i.e., by selecting (sampling) the measured values $\bar{q}$ of the quantum field q. A pure state P_{in} is scattered into a pure state $P_{\text{out}}[\bar{q}]$. We turn now to the averaged scattering process.

The average out state is a mixed state:

$$\rho_{\text{out}} \equiv \int P_{\text{out}}[\bar{q}] w[\bar{q}] d[\bar{q}] \ . \tag{3.8}$$

Let us calculate the average of the selected scattering superoperator (3.7) too. Using Eq. (2.5) one obtains

$$\hat{S}_{\text{CQMT}} \equiv \int \hat{S}[\bar{q}] w[\bar{q}] d[\bar{q}] = C^{-1} \int \hat{S}[\bar{q}] \mathcal{N}[\bar{q}] d[\bar{q}] \ . \tag{3.9}$$

By substituting Eq. (3.7) the functional $\mathcal{N}$ cancels. Recalling Eq. (2.7), we observe $\bar{J}^r$ is a linear functional of $\bar{q}$, thence the Gaussian functional integration over $\bar{q}$ is easy to perform:

$$\hat{S}_{\text{CQMT}} = C^{-1} \hat{T} \exp\left[-i \int \mathcal{H}_\Delta dx - \tfrac{1}{2} i (q_\Delta | G^r | q_c) - \tfrac{1}{2} i (q_c | G^a | q_\Delta) - \tfrac{1}{4} (q_\Delta | \Gamma' | q_\Delta) \right] , \tag{3.10}$$

with the convention $G^a(x,y) \equiv G^r(y,x)$; and a transposition of possible discrete indices is understood too. The new kernel

$$\Gamma'(x,y) = \Gamma(x,y) + \int \int G^a(x,x') \Gamma^{-1}(x',y') G^r(y',y) dx' dy' \tag{3.11}$$

reflects the way the feedback modifies the strength Γ.

$\hat{S}_{\text{CQMT}}$ is the scattering superoperator in the presence of continuous measurement. Its linearity would seem obvious and, consequently, the scattering relation

$$\rho_{\text{out}} = \hat{S}_{\text{CQMT}} P_{\text{in}} \ , \tag{3.12}$$

which follows from Eqs. (3.6), (3.8), and (3.9), could be generalized for mixed initial states as well:

$$\rho_{\text{out}} = \hat{S}_{\text{CQMT}} \rho_{\text{in}} \ , \tag{3.13}$$

in accordance with Gisin's theorem.[6,14]

However, the case is slightly more complicated. The constant C must be a number independent of the initial state $|\text{in}\rangle$, otherwise the scattering superoperator S_{CQMT} ceases to be linear, as seen from Eq. (3.10). If feedback is absent (i.e., $D^r \equiv 0$) then C is always a pure constant, as shown at the end of Appendix C. The requirement that C must be a number presents a stringent constraint on the feedback mechanism. The generic problem of introducing causal nonlocal feedback is unsolved; for a particular (not completely pursued) example see Sec. V. Classes of Markovian feedbacks, both relativistic and nonrelativistic, are shown to work (Secs. IV and V).

IV. MARKOVIAN MEASUREMENT

In this section we consider a special (Markovian) case of the relativistic CQMT defined in previous sections. In a Markovian theory the strength Γ as well as the feedback function G^r are assumed to contain a $\delta(x_0 - y_0)$ factor:

$$\Gamma(x,y) = \gamma(x_0; \mathbf{x}, \mathbf{y}) \delta(x_0 - y_0) \ , \tag{4.1}$$

$$G^r(x,y) = g(x) \delta(x-y) \ . \tag{4.2a}$$

In relativistic Markovian theory the above local feedback would be the only causal one. However, in the nonrelativistic Markovian approximation, instantaneous remote signals are allowed, hence we shall use the more general form

$$G^r(x,y) = g^r(x_0; \mathbf{x}, \mathbf{y}) \delta(x_0 - y_0) \tag{4.2b}$$

with an arbitrary real function g^r. We also retain the convention $g^a(x_0; \mathbf{x}, \mathbf{y}) \equiv g^r(x_0; \mathbf{y}, \mathbf{x})$, with possible index transposition understood.

In Markovian theory the scattering superoperator (3.10) can be rewritten into the form

$$\hat{S}_{\text{CQMT}} = \hat{T} \exp\left[\int_{-\infty}^{\infty} \hat{L}(t) dt \right] , \tag{4.3}$$

where $\hat{L}(t)$ is the linear *evolution superoperator:*

$$\hat{L}(t) = -i \int \mathcal{H}_\Delta(t, \mathbf{x}) d\mathbf{x} - \tfrac{1}{2} i (q_\Delta | g^r | q_c)_t - \tfrac{1}{2} i (q_c | g^a | q_\Delta)_t - \tfrac{1}{4} (q_\Delta | \gamma' | q_\Delta)_t \ . \tag{4.4}$$

In accordance with Eqs. (3.11) and (4.1)–(4.2b) we introduced

$$\gamma'(t; \mathbf{x}, \mathbf{y}) = \gamma(t; \mathbf{x}, \mathbf{y}) + \int \int g^a(t; \mathbf{x}, \mathbf{x}') \gamma^{-1}(t; \mathbf{x}', \mathbf{y}') g^r(t; \mathbf{y}', \mathbf{y}) d\mathbf{x}' d\mathbf{y}' \ . \tag{4.5}$$

The normalization constant C in Eq. (3.10) will turn out to be 1.

Recalling that the scattering superoperator $\hat{S}_{\mathrm{CQMT}}$ relates asymptotic states via Eq. (3.13), the Markovian $\hat{S}_{\mathrm{CQMT}}$ (4.3) allows one to interpolate between the in and out states. The interpolating state $\rho(t)$ obeys the following evolution ("master" or Liouville) equation:

$$\frac{d\rho(t)}{dt}=\hat{L}(t)\rho(t)\ . \tag{4.6}$$

Lindblad[21] classified all finite-dimensional $\hat{L}$'s; our evolution operator (4.4) is formally of the Lindblad type. Since the evolution equation (4.6) retains the normalization of the state, the choice $C=1$, mentioned earlier, has thus been confirmed.

Of course, in a Markovian theory the selective evolution of the pure quantum state $|t;\bar{q}\rangle$ is also easy to define. It is natural to generalize Eqs. (2.2)–(2.7) of the generic CQMT, in order to interpolate between the asymptotic states $|\mathrm{in}\rangle\equiv|t=-\infty\rangle$ and $|\mathrm{out};\bar{q}\rangle\equiv|t=\infty;\bar{q}\rangle$. The corresponding equations are as follows:

$$\Psi_t[\bar{q}]=T\exp\left[-i\int_{x_0<t}(\mathcal{H}+\bar{J}^r q)dx-\tfrac{1}{2}\int_{x_0<t}\|q-\bar{q}\|^2_{x_0}dx_0\right]|\mathrm{in}\rangle\ , \tag{4.7}$$

$$|t;\bar{q}\rangle=\mathcal{N}_t^{-1/2}[\bar{q}]\Psi_t[\bar{q}]\ , \tag{4.8}$$

$$\mathcal{N}_t[\bar{q}]=\Psi_t^\dagger[\bar{q}]\Psi_t[\bar{q}]\ , \tag{4.9}$$

$$w_t[\bar{q}]=\mathcal{N}_t[\bar{q}]=\Psi_t^\dagger[\bar{q}]\Psi_t[\bar{q}]\ , \tag{4.10}$$

$$\|q\|_t^2\equiv(q|\gamma|q)_t\ , \tag{4.11}$$

$$\bar{J}^r(t,\mathbf{x})=\int g^r(t;\mathbf{x},\mathbf{y})\bar{q}(t,\mathbf{y})d\mathbf{y}\ . \tag{4.12}$$

These equations represent the interpolating joint Markovian processes for the state $|t;\bar{q}\rangle$ and for the measured value $\bar{q}(t)$. Still the Markovian nature of the processes mentioned is rather implicit. This would become more transparent in terms of quickly repeated imprecise measurements[5,8,11] (elsewhere called the "hitting process"[13]).

The Markovian process (4.7)–(4.12) can be cast into the form of stochastic differential equations (cf. Ref. 11). For technical reasons, we introduce the pure state projectors

$$P(t)\equiv|t\rangle\langle t| \tag{4.13}$$

instead of the current state vectors $|t;\bar{q}\rangle$, and replace variable $\bar{q}$ by a new one $\bar{Q}$ via the relation

$$d\bar{Q}(t,\mathbf{x})\equiv\bar{q}(t,\mathbf{x})dt\ . \tag{4.14}$$

Furthermore, we introduce complex-valued Wiener processes $\xi(t,\mathbf{x})$ with dispersions

$$d\xi(t,\mathbf{x})d\xi^\dagger(t,\mathbf{y})=\tfrac{1}{2}[\gamma(t;\mathbf{x},\mathbf{y})+ig^r(t;\mathbf{x},\mathbf{y})-ig^a(t;\mathbf{x},\mathbf{y})]dt\ . \tag{4.15}$$

Then P and $\bar{Q}$ obey the following coupled stochastic Itô equations:

$$dP(t)=\hat{L}(t)P(t)dt+\left[\int d\xi(t,\mathbf{x})[q(t,\mathbf{x})-\langle q(t,\mathbf{x})\rangle_t]d\mathbf{x}\,P(t)+\mathrm{H.c.}\right]\ , \tag{4.16}$$

$$d\bar{Q}(t,\mathbf{x})=\langle q(t,\mathbf{x})\rangle_t dt+\int\gamma^{-1}(t;\mathbf{x},\mathbf{y})d\xi(t,\mathbf{y})d\mathbf{y}\ , \tag{4.17}$$

where $\langle q(t,\mathbf{x})\rangle_t$ stands for $\mathrm{tr}[q(t,\mathbf{x})P(t)]$ and $\hat{L}$ is given by Eq. (4.4).

The Itô stochastic differential equations (4.16) and (4.17) offer a powerful formalism of the Markovian CQMT. If one wishes to impose the relativistic causality then the feedback functions $g^r(t;\mathbf{x},\mathbf{y}),g^a(t;\mathbf{x},\mathbf{y})$ must be replaced by $g(t,\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$ and $g^T(t,\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$, respectively, according to Eq. (4.2a).

V. APPLICATIONS

Markovian nonrelativistic measurement. According to a concept outlined in the Introduction, a certain universally and spontaneously measured field is supposed to exist. This may be the relativistic energy-momentum tensor T_{ab}. Now we have only a nonrelativistic picture of the required theory of spontaneous measurement (reduction). In a recent paper,[12] the nonrelativistic mass distribution f has been suggested for the role of a universal spontaneously measured quantity. The quantum field $f(t,\mathbf{x})$ is equal to a certain nonrelativistic limit of the component $T_{00}(t,\mathbf{x})$.

Let us specify the strength (4.1) and feedback (4.2b) of the continuous measurement:

$$\gamma(\mathbf{x},\mathbf{y})=G_N|\mathbf{x}-\mathbf{y}|^{-1}\ , \tag{5.1}$$

$$g^r(\mathbf{x},\mathbf{y})=g^a(\mathbf{x},\mathbf{y})=-\gamma(\mathbf{x},\mathbf{y})\ , \tag{5.2}$$

where G_N is Newton's constant. Then, replacing q by f, Eq. (4.4) yields the evolution operator of the form

$$\hat{L}(t)=-iH_\Delta(t)-i(f_\Delta|\gamma|f_c)_t-\tfrac{1}{2}(f_\Delta|\gamma|f_\Delta)_t\ , \tag{5.3}$$

where $H=\int\mathcal{H}\,dx$. In the usual operator formalism the evolution equation (4.6) of the density operator ρ reads

$$\frac{d\rho(t)}{dt}=\hat{L}(t)\rho(t)=-i[H(t)+H'(t),\rho(t)]-\tfrac{1}{2}\int\int\gamma(\mathbf{x},\mathbf{y})[f(t,\mathbf{x}),[f(t,\mathbf{y}),\rho(t)]]d\mathbf{x}\,d\mathbf{y}\ , \tag{5.4}$$

where $H'(t)=-\frac{1}{2}(f|\gamma|f)_t$ is just the Newtonian gravitational interaction induced by the feedback. In Ref. 12 no feedback was used; apart from this, the same results have been obtained there. It is not necessary to write down the Itô equations (4.16) and (4.17) of the selective evolution since they can be found also in Ref. 12.

Markovian relativistic measurement. Let us choose a certain, yet physically not identified, Hermitian scalar field φ for the universally measured field. Let us assume that φ couples to basic matter fields. Then a universal spontaneous measurement (reduction) theory can be constructed, which is relativistically invariant.

Let the strength (4.1) of measurement be the simplest one:

$$\Gamma(x,y)=\kappa\delta(x-y)\ , \tag{5.5}$$

with constant κ, let the feedback (4.2a) be absent. Then, substituting φ in place of q, the evolution operator (4.4) takes the form

$$\hat{L}(t)=\int[-i\mathcal{H}_\Delta(t,\mathbf{x})-\tfrac{1}{4}\kappa\varphi_\Delta^2(t,\mathbf{x})]d\mathbf{x}\ . \tag{5.6}$$

In ordinary operator formalism the evolution equation (4.6) of the density operator is as follows:

$$\frac{d\rho(t)}{dt}=\hat{L}(t)\rho(t)=-i[H(t),\rho(t)] -\tfrac{1}{4}\kappa\int\int[\varphi(t,\mathbf{x}),[\varphi(t,\mathbf{x}),\rho(t)]]d\mathbf{x}\ . \tag{5.7}$$

Let us write down the Itô equations of selective evolution. According to Eq. (4.15) we introduce the real scalar Wiener process ξ with dispersion

$$d\xi(t,\mathbf{x})d\xi(t,\mathbf{y})=\tfrac{1}{2}\kappa\delta(\mathbf{x}-\mathbf{y})dt\ , \tag{5.8}$$

then the selected pure state P satisfies Eq. (4.16):

$$dP(t)=\hat{L}(t)P(t)dt+\int\{\varphi(t,\mathbf{x})-\langle\varphi(t,\mathbf{x})\rangle_t,P(t)\}\times d\xi(t,\mathbf{x})d\mathbf{x} \tag{5.9}$$

and the measured value $\bar{\varphi}$ evolves according to Eq. (4.17):

$$d\bar{\Phi}(t,\mathbf{x})=\langle\varphi(t,\mathbf{x})\rangle_t dt+\kappa^{-1}d\xi(t,\mathbf{x})\ , \tag{5.10}$$

where $\bar{\Phi}$ and $\bar{\varphi}$ are related by $d\bar{\Phi}(t,\mathbf{x})=\bar{\varphi}(t,\mathbf{x})dt$; cf. Eq. (4.14).

Note that as Eqs. (5.7)–(5.10) represent a relativistically invariant theory, they are valid in an arbitrary Lorentz frame. In our opinion, the above construction is equivalent to the former proposals.[17,18]

Relativistic measurement, nonlocal feedback. We are going to illustrate that in standard quantum electrodynamics certain mechanisms resemble, at least formally, the continuous measurement of the electromagnetic four-current $j(x)$. In Ref. 22 the following scattering superoperator has been derived for the *reduced dynamics* of the charges:

$$\hat{S}_{\mathrm{RQED}}=\hat{T}\exp\{\tfrac{1}{2}i[(j_+|G^F|j_+)-(j_-|G^{\bar{F}}|j_-) -(j_+|G^+|j_-)-(j_-|G^-|j_+)]\}\ , \tag{5.11}$$

where G's are standard photonic Green functions of quantum electrodynamics and RQED denotes reduced quantum electrodynamics. In physical representation (see Appendix B) the following expression can be obtained:

$$\hat{S}_{\mathrm{RQED}}=\hat{T}\exp[\tfrac{1}{2}i(j_\Delta|G^r|j_c)+\tfrac{1}{2}i(j_c|G^a|j_\Delta) +\tfrac{1}{4}i(j_\Delta|D^c|j_\Delta)]\ , \tag{5.12}$$

where the physical Green functions are defined by

$$iD^c(x,y)=(2\pi)^{-4}\int 2\pi\delta(p^2)e^{-ip(x-y)}dp\ , \tag{5.13}$$

$$G^r(x,y)=(2\pi)^{-4}\int[p^2-ip_0\epsilon]^{-1}e^{-ip(x-y)}dp =-(1/2\pi)\Theta(x_0-y_0)\delta((x-y)^2)\ . \tag{5.14}$$

With the choice $\Gamma'=iD^c$ the scattering superoperator $\hat{S}_{\mathrm{RQED}}$ (5.12) shows structural similarity to the scattering superoperator $\hat{S}_{\mathrm{CQMT}}$ (3.10) of the relativistic CQMT. The opposite signs are due to the $(+,-,-,-)$ convention for summing up Lorentz indices. Had we chosen $(-,-,-,+)$, the scattering superoperator (5.12) of the reduced dynamics of the charges in quantum electrodynamics would be completely identical to the scattering superoperator (3.10) of a system, where charges are free of any photonic interaction, however, their current j is continuously measured in the sense of Sec. II, with nonlocal retarded feedback included.

However, this case is not so simple. As is seen from Eq. (5.13), the kernel Γ' is degenerate, i.e., it is positive semidefinite. Therefore the aforementioned reinterpretation of electromagnetic interaction in terms of continuous measurement of the current j requires more care. Nevertheless, the above formal similarities of formulas are not at all accidental. There is a certain field, though not j itself, which seems to be continuously measured.[23]

VI. SUMMARY

We have proposed a possible theory for the continuous measurement of relativistic quantum fields. We have also derived the corresponding scattering equations. The proposed formalism reduces to known equations in the Markovian case. Two recent models for spontaneous quantum state reduction have been recovered in the framework of our theory. A possible example of the relativistic continuous measurement has been outlined in standard quantum electrodynamics. The continuous measurement theory possesses an alternative formulation in terms of interacting quantum and stochastic fields.

The proposed theory should be considered as a first approach to the problem of relativistic continuous measurement. Hence we did not go beyond the level of accuracy of formal field theories. This formal level is not yet exhausted. In future investigations the construction of interpolating quantum fields seems to be straightforward enough. Presumably the generic form of causal feedback of measured information will represent more serious problems.

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APPENDIX A: SPECIAL NOTATIONS

For the intelligibility of the appendixes first we introduce a set of notations. Latin letters x and y always denote four-coordinates; dx and dy stand for the corresponding four-volume elements. Time components are denoted by x_0 and y_0, respectively, or by t; $\mathbf{x}$ and $\mathbf{y}$ stand for spatial components, $d\mathbf{x}$ and $d\mathbf{y}$ are spatial volume elements.

Given real bosonic (not necessarily scalar) fields $q(x)$ and $p(x)$, the "matrix element" of a real kernel $C(x,y)$ will be denoted in the following compact way:

$$\sum_{\alpha,\beta}\int\int C_{\alpha\beta}(x,y)q_\alpha(x)p_\beta(y)dx\,dy\equiv(q|C|p) , \quad \text{(A1)}$$

where Greek indices label field components.

In Markovian theory, one is faced with kernels of the form

$$C_{\alpha\beta}(x,y)=c_{\alpha\beta}(x_0;\mathbf{x},\mathbf{y})\delta(x_0-y_0) . \quad \text{(A2)}$$

We introduce a separate notation for the matrix element of real spatial kernel $c_{\alpha\beta}(t;\mathbf{x},\mathbf{y})$:

$$\sum_{\alpha,\beta}\int\int c_{\alpha\beta}(t;\mathbf{x},\mathbf{y})q_\alpha(t,\mathbf{x})p_\beta(t,\mathbf{y})d\mathbf{x}\,d\mathbf{y}\equiv(q|c|p)_t . \quad \text{(A3)}$$

The following relation is then fulfilled:

$$\int(q|c|p)_t dt=(q|C|p) . \quad \text{(A4)}$$

APPENDIX B: SUPEROPERATOR FORMALISM

This formalism is a simplified version of the closed-time-path technique, a detailed presentation of which is given in Ref. 24. Let ρ stand for the density operator corresponding to a given, pure or mixed, state of the system. An operator, say q, multiplies ρ from the left or, alternatively, from the right; it depends on the mathematical term in question. In superoperator formalism, one appends a label (usually $+$ or $-$) to each operator and the label tells the direction of multiplication. In our case e.g.,

$$q_+\rho=q\rho, \quad q_-\rho=\rho q . \quad \text{(B1)}$$

By convention, each labeled multiplier can formally be grouped on the left and they can be combined together. For example, it is customary to switch on the so-called physical representation q_Δ, q_c:

$$q_\Delta=q_+-q_-, \quad q_c=\tfrac{1}{2}(q_++q_-) . \quad \text{(B2)}$$

By using the relations (B1) it is easy to see the effects of the following simple superoperators:

$$q_\Delta\rho=[q,\rho], \quad q_c\rho=\tfrac{1}{2}\{q,\rho\} . \quad \text{(B3)}$$

In superoperator formalism the notion of the usual time-ordering has to be generalized as well. $\hat{T}$, the time-ordering superoperator, will prescribe time-ordering (T) for field operators with label $(+)$ and, respectively, anti-time-ordering $(\tilde{T})$ for operators of label $(-)$.

APPENDIX C: QUANTUM-STOCHASTIC FIELD THEORY (QSFT)

Consider the equations of relativistic CQMT specified in Sec. II, and define the following *a priori* distribution of the c-number stochastic field $\bar{q}$:

$$w_{\text{in}}[\bar{q}]=\exp(-\|\bar{q}\|^2) . \quad \text{(C1)}$$

Introduce the stochastic field

$$\bar{J}^\Gamma(x)=\int\Gamma(x,y)\bar{q}(y)dy . \quad \text{(C2)}$$

For completeness, let us invoke the definition (2.7) of the retarded stochastic current too:

$$\bar{J}^r(x)=\int G^r(x,y)\bar{q}(y)dy . \quad \text{(C3)}$$

Now, observe that on the left-hand side (lhs) of Eq. (2.2) one can cancel a trivial c-number factor by introducing a new state vector $w_{\text{in}}^{-1/2}[\bar{q}]\Psi_{\text{out}}[\bar{q}]$ instead of $\Psi_{\text{out}}[\bar{q}]$:

$$\Psi_{\text{out}}[\bar{q}]=T\exp\left[-i\int[\mathcal{H}+(\bar{J}^r+i\bar{J}^\Gamma)q]dx-\tfrac{1}{2}\|q\|^2\right]|\text{in}\rangle . \quad \text{(C4)}$$

The normalized out state is of the same form as in Eq. (2.3)

$$|\text{out};\bar{q}\rangle=\mathcal{N}^{-1/2}[\bar{q}]\Psi_{\text{out}}[\bar{q}] , \quad \text{(C5)}$$

with

$$\mathcal{N}[\bar{q}]=\Psi^\dagger_{\text{out}}[\bar{q}]\Psi_{\text{out}}[\bar{q}] , \quad \text{(C6)}$$

which differs from $\mathcal{N}[q]$ of Sec. II by a factor $w_{\text{in}}[\bar{q}]$. Let us now introduce the notion of *a posteriori* distribution $w_{\text{out}}[\bar{q}]$ of the stochastic field $\bar{q}$. By definition, let it be identical to the distribution (2.5), hence

$$w_{\text{out}}[\bar{q}]=C^{-1}\mathcal{N}[\bar{q}]w_{\text{in}}[\bar{q}]=C^{-1}\mathcal{N}[\bar{q}]\exp(-\|\bar{q}\|^2) . \quad \text{(C7)}$$

Note that $\bar{q}$ was called the measured value through the entire paper. Construction (C1)–(C7) is mathematically equivalent to the CQMT of Sec. II, nevertheless here no reference to "measurement" is needed. We propose the following interpretation.

The classical stochastic field $\bar{q}$ possesses the initial distribution (C1). It creates a non-Hermitian interaction "Hamiltonian" as seen in Eq. (C4). Then, this scattering will have a back-action on the classical field $\bar{q}$, leading to its final probability distribution (C7).

Such a theory may be called quantum-stochastic field theory: the quantum and stochastic fields interact with each other. We wish to make a distinction here: In contrast to this scheme, in the ordinary (unitary) stochastic quantum field theories (SQFT) the *a priori* statistics of the stochastic field $\bar{q}$ does not change since $\bar{q}$ is considered all the time as an external stochastic field.

Finally we show an interesting relation between the QSFT (C1)–(C7) and a given ordinary unitary SQFT. Let us modify the scattering equation (C4) of the QSFT. Let us neglect the feedback term as well as the last term $-\frac{1}{4}\|q\|^2$ in the exponent and, furthermore, omit the factor i of the term $\bar{J}^\Gamma q$. One obtains

$$\Psi_{\text{out}}[\bar{q}]=|\text{out};\bar{q}\rangle =T\exp\left[-i\int(\mathcal{H}+\bar{J}^\Gamma q)dx\right]|\text{in}\rangle\ . \quad \text{(C8)}$$

This is unitary scattering in the presence of the external stochastic current $\bar{J}^\Gamma$ (C2). To approve consistency with what we stated about unitary SQFT's, observe that Eq. (C7) yields now the trivial result $w_{\text{out}}[\bar{q}]=w_{\text{in}}[\bar{q}]$ since $\mathcal{N}[\bar{q}]\equiv 1$.

The corresponding (selected) scattering superoperator can be written in the form

$$\hat{S}[\bar{q}]=\hat{T}\exp\left[-i\int(\mathcal{H}_\Delta+\bar{J}^\Gamma q_\Delta)dx\right]\ . \quad \text{(C9)}$$

This superoperator is linear [cf. Eq. (3.7)] and it needs no normalizing factor. Now one can take stochastic average over the external stochastic field $\bar{q}$; invoking Eq. (C1) we get

$$\hat{S}_{\text{SQFT}}\equiv\int\hat{S}[\bar{q}]w_{\text{in}}[\bar{q}]d[\bar{q}] =\hat{T}\exp\left[-i\int\mathcal{H}_\Delta dx-\tfrac{1}{4}\|q_\Delta\|^2\right]\ . \quad \text{(C10)}$$

By comparing $\hat{S}_{\text{SQFT}}$ and $\hat{S}_{\text{CQMT}}$ (3.10) one observes that they are identical apart from the absence of the feedback. The constant C, normalizing $\hat{S}_{\text{CQMT}}$, has turned out to be 1. Since the CQMT of Sec. II is, by construction, equivalent to the QSFT (C1)–(C7) their scattering superoperators $\hat{S}_{\text{CQMT}}$ and $\hat{S}_{\text{QSFT}}$ are obviously the same. All this can be summarized in the form

$$\hat{S}_{\text{CQMT}}\equiv\hat{S}_{\text{QSFT}}=\hat{S}_{\text{SQFT}}\ . \quad \text{(C11)}$$

We formulate the following conclusion: at the level of asymptotic density operators ρ_{in} and ρ_{out}, the scattering in CQMT (or in the corresponding quantum-stochastic field theory) can be reproduced by an ordinary stochastic quantum field theory, i.e., by unitary scattering in the proper external stochastic field (provided there is an absence of feedback).

[1]J. S. Bell, in *Sixty-Two Years of Uncertainty: Historical, Philosophical and Physics Inquires into the Foundation of Quantum Physics*, edited by A. I. Miller (Plenum, New York, 1990).

[2]J. von Neumann, *Matematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).

[3]D. Bohm and J. Bub, Rev. Mod. Phys. **38**, 453 (1966).

[4]P. Pearle, Phys. Rev. D **13**, 857 (1976); Phys. Rev. A **39**, 2277 (1989).

[5]A. Barchielli, L. Lanz, and G. M. Prosperi, Nuovo Cimento B **72**, 79 (1982); A. Barchielli, Phys. Rev. A **34**, 1642 (1986).

[6]N. Gisin, Phys. Rev. Lett. **52**, 1657 (1984); **53**, 1776 (1984).

[7]P. Pearle, Phys. Rev. Lett. **53**, 1775 (1984).

[8]G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[9]C. M. Caves and G. J. Milburn, Phys. Rev. A **36**, 5543 (1987).

[10]L. Diósi, J. Phys. A **21**, 2885 (1988).

[11]L. Diósi, Phys. Lett. A **129**, 419 (1988).

[12]L. Diósi, Phys. Rev. A **40**, 1165 (1989).

[13]G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990); G. C. Ghirardi, Renata Grassi, and A. Rimini, *ibid.* **42**, 1057 (1990).

[14]N. Gisin, Helv. Phys. Acta **62**, 363 (1989).

[15]C. M. Caves, Phys. Rev. D **33**, 1643 (1986); **35**, 1815 (1987).

[16]A. Barchielli, Phys. Rev. D **34**, 2527 (1986).

[17]A. Barchielli, L. Lanz, and G. M. Prosperi, in *Proceedings of an International Symposium on the Foundations of Quantum Mechanics*, edited by S. Kamefuchi *et al.* (Physical Society of Japan, Tokyo, 1986).

[18]P. Pearle, in Ref. 1.

[19]E. J. Squires (unpublished).

[20]A. Frigerio and M. Ruzzier, Ann. Inst. Henri Poincaré **51**, 67 (1989).

[21]G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[22]L. Diósi, Found. Phys. **20**, 63 (1990).

[23]L. Diósi (unpublished).

[24]K. Chou, Z. Su, B. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).