PHYSICAL REVIEW A VOLUME 42, NUMBER 1 1 JULY 1990

Spectral properties of light in quantum optics

L. Knöll, W. Vogel,* and D.-G. Welsch
Sektion Physik, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, DDR-6900 Jena, German Democratic Republic
(Received 7 June 1989; revised manuscript received 29 December 1989)

The problem of spectral filtering of quantized light fields is studied, based on the recently developed quantum-optical theory of the action of passive, lossless optical systems [L. Knöll, W. Vogel, and D.-G. Welsch, Phys. Rev. A **36**, 3803 (1987)]. Expressions for the operator of the electric field strength of the light and the normally and time-ordered field-correlation functions are derived for the case of a Fabry-Pérot interferometer being present. Various kinds of field decomposition that are usually considered in classical optics are studied. The results are compared with the Fourier approach to spectral properties of light. It is shown that, dependent on the experimental scheme used, new quantum effects appear, which may prevent the observation of the Fourier structure of the light as predicted from classical optics. Quantitatively this is demonstrated for the example of spectral squeezing in resonance fluorescence, where significant discrepancies between the measured and the full Fourier spectrum are found.

I. INTRODUCTION

In classical as well as in quantum optics the spectral properties of light play an important role. Whereas in classical optics the situation is rather transparent, in quantum optics some problems may arise in cases when other than second-order spectral properties are studied.

Mathematically, intrinsic spectral properties may be defined by means of a Fourier analysis of the light field under study.[1,2] Practically, spectral measurements are performed by inserting frequency-sensitive devices in front of the points of observation. For this reason, in their approach to the second-order physical spectrum Eberly and Wódkiewicz[3] and other authors[4,5] started from Glauber's theory of photodetection.[6,7] As is usually done in classical optics, the spectrally filtered field detected by photocounting measurements is expressed by a convolution of the unfiltered field with the transmission response function of the spectral apparatus.[3–5] In his review on the theory of the (second-order) spectrum of the quantized light field Cresser[8] pointed out that in quantum optics this point needs more careful consideration. Nevertheless, this classical approach to the physical spectrum was transferred to quantum optics by various authors.[4,5,9–15]

In this way, in their work on the frequency-resolved intensity correlation of the resonance fluorescence light from a single atom Knöll, Weber, and Schäfer[14] found time-delayed commutator contributions to the spectrally filtered intensity correlation function. Adopting the quantum-optical concept of the description of the action of a spectral filter, as was developed by Knöll, Vogel, and Welsch and briefly reported in Refs. 16 and 17, Knöll and Weber[18] showed that the intensity correlation function used in Ref. 14 cannot be observed and that in the observable correlation function the time-delayed commutator terms do not occur. It should be pointed out that this result is fully confirmed by Cresser in a recently published paper.[19]

Another illustrative example is the problem of spectral squeezing in single-atom resonance fluorescence. Making use of a Fourier transform approach and the theory of Eberly and Wódkiewicz,[3] Collett, Walls, and Zoller[15] found that time-delayed commutator contributions arise in the normally ordered variance of the spectrally filtered field. Note that in the explicit calculations they ignored these terms. Similar to the case of the spectrally filtered intensity correlation function, these terms occur, because the pyramidal time ordering of the operators in the corresponding convolution integrals is violated, as was shown by Knöll, Vogel, and Welsch.[17] The normally ordered variance derived from a consistent quantum-theoretical treatment of the action of the spectral apparatus preserves the pyramidal time ordering and therefore does not give rise to time-delayed contributions.[17]

From a more general point of view the question is raised of how to describe the action of passive filter systems in quantum optics and to calculate filtered correlation properties of light. Some attempts have been made to solve this problem. For instance, in Refs. 20–24 the well-known quantum-mechanical noise theories are generalized and relations between the output field and the internal and/or the input field are derived.

An alternative approach to the problem of the quantum-mechanical description of the action of passive optical systems was developed by Knöll, Vogel, and Welsch[16,17,25] on the basis of the concepts of quantum field theory. The purpose of the present paper is to apply this theory to the problem of spectral measurements in quantum optics and to study the action of spectral filters on quantum light fields. In this context, we will give a detailed analysis of some results briefly reported and discussed in our earlier papers (Refs. 16 and 17). To illustrate the method we will study the action of a spectral filter of the Fabry-Pérot type. For the sake of clearness and to avoid rather lengthy derivations and formulas we will restrict the explicit calculations to the simplest model of a Fabry-Pérot interferometer, namely, a dielectric layer. We note that this model is sufficient to demonstrate the characteristic quantum features in the action of a spectral apparatus.

In Sec. II the Fourier transform approach to spectral properties of quantum light is discussed. In Sec. III an observational approach to spectral properties is given and expressions are derived for the field operators in the presence of a spectral filter. The resulting quantum-theoretical filter equation is analyzed in Sec. IV. Observable correlation functions of spectrally filtered light are studied in Sec. V, and in Sec. VI different kinds of field decompositions and their applicability in quantum optics are discussed. Section VII deals with the application of the general results to the problem of spectral squeezing in resonance fluorescence. A summary and some conclusions are given in Sec. VIII.

II. FOURIER TRANSFORM APPROACH TO SPECTRAL PROPERTIES IN QUANTUM OPTICS

In analogy to the method by which spectral properties are usually defined in classical optics we may start from the Fourier representation of the operator of the electric field strength (operators are marked by a caret)

$$\hat{\mathcal{E}}(t)=\hat{\mathcal{E}}^{(+)}(t)+\hat{\mathcal{E}}^{(-)}(t)\ , \tag{2.1}$$

where $\hat{\mathcal{E}}^{(+)}$ and $\hat{\mathcal{E}}^{(-)}$, respectively, are the positive and negative frequency parts. We therefore may write

$$\hat{\mathcal{E}}^{(+)}(t)=(2\pi)^{-1}\int d\omega\, e^{-i\omega t}\hat{\tilde{\mathcal{E}}}^{(+)}(\omega)\ , \tag{2.2}$$

$$\hat{\mathcal{E}}^{(-)}(t)=(2\pi)^{-1}\int d\omega\, e^{i\omega t}\hat{\tilde{\mathcal{E}}}^{(-)}(\omega)\ , \tag{2.3}$$

where $\hat{\tilde{\mathcal{E}}}^{(+)}(\omega)=0=\hat{\tilde{\mathcal{E}}}^{(-)}(\omega)$ if $\omega<0$.

By using Eqs. (2.2) and (2.3) the normally ordered correlation functions

$$\Gamma^{(m,n)}(t_1,\ldots,t_{m+n})=\left\langle \left[\prod_{i=1}^{m}\hat{\mathcal{E}}^{(-)}(t_i)\right]\times\left[\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}^{(+)}(t_j)\right]\right\rangle \tag{2.4}$$

can be spectrally decomposed as follows:[2]

$$\Gamma^{(m,n)}(t_1,\ldots,t_{m+n})=(2\pi)^{-(m+n)}\int d\omega_1 e^{i\omega_1 t_1}\cdots\times\int d\omega_{m+n}e^{-i\omega_{m+n}t_{m+n}}\times\tilde{\Gamma}^{(m,n)}(\omega_1,\ldots,\omega_{m+n})\ , \tag{2.5}$$

where the spectral correlation functions $\tilde{\Gamma}^{(m,n)}$ read

$$\tilde{\Gamma}^{(m,n)}(\omega_1,\ldots,\omega_{m+n})=\left\langle \left[\prod_{i=1}^{m}\hat{\tilde{\mathcal{E}}}^{(-)}(\omega_i)\right]\times\left[\prod_{j=m+1}^{m+n}\hat{\tilde{\mathcal{E}}}^{(+)}(\omega_j)\right]\right\rangle . \tag{2.6}$$

This concept was first used by Metha and Wolf[1] in order to define the second-order spectral correlation function. In particular, they showed that in the case of stationary light fields the second-order spectral correlation function can be written as follows

$$\tilde{\Gamma}^{(1,1)}(\omega,\omega')=2\pi\delta(\omega-\omega')W(\omega)\ , \tag{2.7}$$

where $W(\omega)$ is the quantum-theoretical analog of the classical Wiener-Khinthchine spectrum:[26,27]

$$W(\omega)=\int dt\, e^{-i\omega t}\Gamma^{(1,1)}(t,0)\ . \tag{2.8}$$

The above-mentioned approach, which is based on the Fourier representation of the operator of the electric field strength, and hence on the Fourier analysis of the field correlation functions $\Gamma^{(m,n)}$, is a formal, mathematical one. We denote the spectral properties found in this way as intrinsic spectral properties of the field under consideration.

As it will be shown in the following, these intrinsic spectral properties cannot be expected, in general, to be measurable field properties. The problem consists of the following. By using Eqs. (2.2)–(2.6) Fourier inversion gives

$$\tilde{\Gamma}^{(m,n)}(\omega_1,\ldots,\omega_{m+n})=\int dt_1 e^{-i\omega_1 t_1}\cdots\times\int dt_{m+n}e^{i\omega_{m+n}t_{m+n}}\times\Gamma^{(m,n)}(t_1,\ldots,t_{m+n})\ . \tag{2.9}$$

On the other hand, from the quantum theory of measurement in the time domain[28] only time-ordered correlation functions $\Gamma_T^{(m,n)}(t_1,\ldots,t_{m+n})$ are expected to be observable,

$$\Gamma_T^{(m,n)}(t_1,\ldots,t_{m+n})=\left\langle \left[T_-\prod_{i=1}^{m}\hat{\mathcal{E}}^{(-)}(t_i)\right]\times\left[T_+\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}^{(+)}(t_j)\right]\right\rangle , \tag{2.10}$$

where T_+ (T_-) is the positive (negative) time-ordering operator. However, the time integrations in Eq. (2.9) are not restricted in any way and consequently $\tilde{\Gamma}^{(m,n)}$ also contains contributions of $\Gamma^{(m,n)}$ from time domains violating the time ordering.

In practice, spectral measurements are performed by inserting frequency-sensitive devices in front of photodetectors. Since the counts in the photodetectors are functions of the setting frequencies of the spectral filters, "spectral" properties are observed. From the results derived in the following it will be seen that these observable spectral properties are indeed closely related to the Fourier transform of the time-ordered correlation functions $\tilde{\Gamma}_T^{(m,n)}$.

III. OBSERVATIONAL APPROACH TO SPECTRAL PROPERTIES IN QUANTUM OPTICS

The problem of observable (second-order) spectral properties was studied by Eberly and Wódkiewicz.[3] They assumed that at the point of observation not the entire light field $\mathscr{E}^{(+)}$ under study is seen, but a filtered version of it,

$$E^{(+)}(t)=\int dt' T_f(t-t')\mathscr{E}^{(+)}(t') , \tag{3.1}$$

where $T_f(t)$ is the transmission response function of the filter. As a suitable example for a spectral filter they considered a lossless and highly reflective Fabry-Pérot interferometer. Making use of the Airy formula[29] they derived the following expression for the transmission response function $T_f(t)$:

$$T_f(t)=\Theta(t)\Gamma_f e^{-(\Gamma_f+i\omega_f)t} , \tag{3.2}$$

where ω_f and Γ_f, respectively, are the setting frequency and passband width of the Fabry-Pérot interferometer. Note that the unit step function $\Theta(t)$ reflects the causality of the filter. However, Eq. (3.1) cannot be valid in the sense of an operator equation. For example, Eq. (3.1) establishes that in the case when the spectral filter is closed ($t\to-\infty$ and/or $\Gamma_f\to 0$) the operator of the electric field strength identically vanishes at any point of observation (behind the filter). Clearly, this result violates fundamental rules of quantum mechanics.

A. Quantum theoretical approach: general case

We treat the action of a spectral apparatus in the sense of a linear, lossless filter, which may be modeled by a dielectric with space-dependent refractive index $n(\mathbf{r})=\sqrt{\epsilon(\mathbf{r})}$. Now the problem is to find expressions for the field operators in the presence of such a filter. Using the concept developed in Ref. 25 (which is referred to as paper I) we arrive at the following results.

The Hamiltonian is [see paper I, Eq. (2.57)]

$$\hat{H}=\hat{H}_r+\hat{H}_s+\hat{H}_{\text{int}} , \tag{3.3}$$

where $\hat{H}_r$ is the radiation field Hamiltonian in the presence of the filter, viz.,

$$\begin{aligned} H_r &= \frac{1}{2}\int d\mathbf{r}\left[\epsilon_0\epsilon(\mathbf{r})\hat{\mathbf{A}}^2+\frac{1}{\mu_0}(\nabla\times\hat{\mathbf{A}})^2\right] \\ &= \sum_\lambda \hbar\omega_\lambda(\hat{a}_\lambda^\dagger\hat{a}_\lambda+\tfrac{1}{2}) \end{aligned} \tag{3.4}$$

[see paper I, Eqs. (2.33) and (2.58)] and the vector potential $\hat{\mathbf{A}}$ is given by

$$\hat{\mathbf{A}}=\hat{\mathbf{A}}^{(+)}+\hat{\mathbf{A}}^{(-)}, \quad \hat{\mathbf{A}}^{(-)}=(\hat{\mathbf{A}}^{(+)})^\dagger , \tag{3.5}$$

where

$$\hat{\mathbf{A}}^{(+)}(\mathbf{r},t)=\sum_\lambda \mathbf{A}_\lambda(\mathbf{r})\hat{a}_\lambda(t) . \tag{3.6}$$

In Eqs. (3.5) and (3.6) the positive (negative) frequency part $\hat{\mathbf{A}}^{(+)}$ ($\hat{\mathbf{A}}^{(-)}$) is assumed to arise from the operators $\hat{a}_\lambda$ ($\hat{a}_\lambda^\dagger$). The mode functions $\mathbf{A}_\lambda(\mathbf{r})$ satisfy the equations

$$\nabla\times\nabla\times\mathbf{A}_\lambda(\mathbf{r})-\epsilon(\mathbf{r})\frac{\omega_\lambda^2}{c^2}\mathbf{A}_\lambda(\mathbf{r})=\mathbf{0} , \tag{3.7}$$

$$\nabla\cdot\epsilon(\mathbf{r})\mathbf{A}_\lambda(\mathbf{r})=0 , \tag{3.8}$$

$$\int d\mathbf{r}\,\epsilon(\mathbf{r})\mathbf{A}_{\lambda'}^*(\mathbf{r})\mathbf{A}_\lambda(\mathbf{r})=\frac{\hbar}{2\epsilon_0\omega_\lambda}\delta_{\lambda,\lambda'} , \tag{3.9}$$

[see paper I, Eqs. (2.36), (2.38), and (2.53)] and the equal-time commutators of the photon creation and destruction operators $\hat{a}^\dagger$ and $\hat{a}$, respectively, are

$$[\hat{a}_{\lambda'},\hat{a}_\lambda^\dagger]=\delta_{\lambda,\lambda'}, \quad [\hat{a}_{\lambda'},\hat{a}_\lambda]=0=[\hat{a}_{\lambda'}^\dagger,\hat{a}_\lambda^\dagger] . \tag{3.10}$$

Note that the presence of the filter is taken into account via the mode functions $\mathbf{A}_\lambda(\mathbf{r})$. $\hat{H}_s$ is the Hamiltonian of the atomic sources. The interaction of the field with the atomic sources is described by

$$\hat{H}_{\text{int}}=-\int d\mathbf{r}(\hat{\mathbf{J}}^\dagger\hat{\mathbf{A}}^{(+)}+\hat{\mathbf{A}}^{(-)}\hat{\mathbf{J}}) . \tag{3.11}$$

The atomic source operator $\hat{\mathbf{J}}$ defined by Eq. (3.1) in paper I is, in general, Hermitian. In the particular case of the rotating-wave approximation the operator $\hat{\mathbf{J}}$ only represents its non-Hermitian positive frequency part. Following paper I, the formal solution of the Heisenberg equations of motion for the vector components of the operator of the electric field strength $\hat{E}_\alpha(\mathbf{r},t)=\hat{E}_\alpha^{(+)}(\mathbf{r},t)+\hat{E}_\alpha^{(-)}(\mathbf{r},t)$ can be decomposed into a free-field and a source-field part (see paper I, Sec. III),

$$\hat{E}_\alpha^{(+)}(\mathbf{r},t)=\hat{E}_{\alpha,\text{free}}^{(+)}(\mathbf{r},t)+\hat{E}_{as}^{(+)}(\mathbf{r},t) , \tag{3.12}$$

where

$$\hat{E}_{\alpha,\text{free}}^{(+)}(\mathbf{r},t)=i\sum_\lambda \omega_\lambda A_{\alpha\lambda}(\mathbf{r})\hat{a}_{\lambda,\text{free}}(t) , \tag{3.13}$$

$$\hat{E}_{as}^{(+)}(\mathbf{r},t)=\int d\mathbf{r}'\int dt'\Theta(t-t')K_{\alpha\alpha'}(\mathbf{r},t;\mathbf{r}',t')\hat{J}_{\alpha'}(\mathbf{r}',t') , \tag{3.14}$$

the temporal evolution of $\hat{a}_{\lambda,\text{free}}$ is due to the Hamiltonian $\hat{H}_r$ of the free-radiation field, viz.,

$$\hat{a}_{\lambda,\text{free}}(t)=e^{-i\omega_\lambda(t-t')}\hat{a}_{\lambda,\text{free}}(t') . \tag{3.15}$$

The kernel K in Eq. (3.14) is defined by

$$K_{\alpha\alpha'}(\mathbf{r},t;\mathbf{r}',t')=-\frac{1}{\hbar}\sum_\lambda \omega_\lambda A_{\alpha\lambda}(\mathbf{r})A_{\alpha'\lambda}^*(\mathbf{r}')e^{-i\omega_\lambda(t-t')} . \tag{3.16}$$

Obviously, it fulfills the symmetry relation

$$K_{\alpha\alpha'}^*(\mathbf{r},t;\mathbf{r}',t')=K_{\alpha'\alpha}(\mathbf{r}',t';\mathbf{r},t) . \tag{3.17}$$

Clearly, Eqs. (3.12)–(3.16) have to be supplemented by the Heisenberg equations of motion for the source operators: $i\hbar(d/dt)\hat{\mathbf{J}}=[\hat{\mathbf{J}},\hat{H}]$. Combining Eqs. (3.12) and (3.14) we obtain the result

$$\begin{aligned} \hat{E}_\alpha^{(+)}(\mathbf{r},t)=\hat{E}_{\alpha,\text{free}}^{(+)}(\mathbf{r},t) \\ \times\int d\mathbf{r}'\int dt'\Theta(t-t')K_{\alpha\alpha'}(\mathbf{r},t;\mathbf{r}',t') \\ \times\hat{J}_{\alpha'}(\mathbf{r}',t') . \end{aligned} \tag{3.18}$$

This equation may be regarded as basic equation for describing the action of a spectral filter in quantum optics. Since it is simply the general solution of the inhomogeneous Maxwell equations it is clear that Eq. (3.18) formally looks like the equation known for the case without filtering. The difference between the two cases consists of the mode functions $\mathbf{A}_\lambda(\mathbf{r})$ to be chosen, which determine the free field part [Eq. (3.13)] and via the kernel K the source field [Eqs. (3.14) and (3.16)].

B. Quantum theoretical approach: One-dimensional Fabry-Pérot filter

In analogy to the model of a spectral filter used by Eberly and Wódkiewicz,[3] we now consider the lossless Fabry-Pérot filter modeled by a dielectric layer of refractive index $n_f=\sqrt{\epsilon_f}$ and thickness d in the x direction. Assuming the Fabry-Pérot filter is surrounded by vacuum the refractive index n as a function of x may be written as follows:

$$n=\sqrt{\epsilon}=\begin{cases} n_f & \text{if } -\dfrac{d}{2}\le x\le\dfrac{d}{2} \\ 1 & \text{if } |x|>\dfrac{d}{2}\,. \end{cases} \tag{3.19}$$

For simplicity, we restrict ourselves to the study of two typical cases of quasi-one-dimensional light propagation. Let us first consider the case when the propagation of light is perpendicular to the Fabry-Pérot filter [cf. Fig. 1(a)], so that Eq. (3.7) may be written as [see also paper I, Eq. (2.39)]

$$\frac{d^2A_k}{dx^2}+n^2k^2A_k=0,\quad k^2=\frac{\omega^2}{c^2}\,, \tag{3.20}$$

where $n=n(x)$ is given by Eq. (3.19). The mode functions $A_k(x)$ (labeled by the continuous mode index k) are determined from the solution of Eq. (3.20) together with the usual boundary conditions and Eq. (3.9), which now reads as

$$\int_{-\infty}^{+\infty}dx\, n^2(x)A_k^*(x)A_{k'}(x)=\frac{\hbar}{2\epsilon_0\omega}\delta(k-k')\,. \tag{3.21}$$

The mode functions outside the dielectric layer ($|x|>d/2$) may be written as follows:

$$A_k(x)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases} e^{ikx}+\bar{R}_f(\omega)e^{-ikx}, & x<-\dfrac{d}{2}\,, \\ \bar{T}_f(\omega)e^{ikx}, & x>\dfrac{d}{2},\quad k>0 \end{cases} \tag{3.22}$$

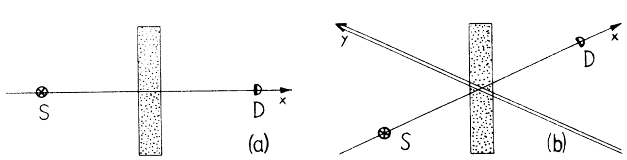

FIG. 1. Scheme of the geometrical arrangements considered (S, atomic light sources; D, photodetectors). (a) Perpendicular incidence of light. (b) Inclined incidence of light.

$$A_k(x)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases} \bar{T}_f(\omega)e^{ikx}, & x<-\dfrac{d}{2}\,, \\ e^{ikx}+\bar{R}_f(\omega)e^{-ikx}\,, & x>\dfrac{d}{2},\quad k<0\,. \end{cases} \tag{3.23}$$

$\bar{T}_f(\omega)$ and $\bar{R}_f(\omega)$, respectively, are the spectral transmission and reflection response functions of the Fabry-Pérot interferometer, which in agreement with the Airy formulas[29] read as

$$\bar{T}_f(\omega)=\tilde{T}_f(\omega)e^{i(n_f-1)(\omega/c)d}\,,\qquad \tilde{T}_f(\omega)=\frac{1-r^2}{1-r^2e^{2in_f(\omega/c)d}}\,, \tag{3.24}$$

$$\bar{R}_f(\omega)=\tilde{R}_f(\omega)e^{-i(\omega/c)d}\,,\qquad \tilde{R}_f(\omega)=-r+re^{2in_f(\omega/c)d}\tilde{T}_f(\omega)\,, \tag{3.25}$$

$$r^2=\left[\frac{n_f-1}{n_f+1}\right]^2\,. \tag{3.26}$$

The spectral response functions $\bar{T}_f(\omega)$ and $\bar{R}_f(\omega)$ are easily shown to satisfy the conditions

$$|\bar{T}_f(\omega)|^2+|\bar{R}_f(\omega)|^2=1\,, \tag{3.27}$$

$$\bar{T}_f(\omega)\bar{R}_f^*(\omega)+\bar{T}_f^*(\omega)\bar{R}_f(\omega)=0\,. \tag{3.28}$$

In the important case of a highly reflective filter ($r\approx1$), in the vicinity of a given resonance frequency ω_f of the Fabry-Pérot filter Eq. (3.24) can be simplified as

$$\tilde{T}_f(\omega)=\frac{\Gamma_f}{\Gamma_f-i(\omega-\omega_f)}\,, \tag{3.29}$$

where the resonance frequency ω_f and the passband width Γ_f are given by

$$\omega_f=f\frac{\pi c}{n_f d}\,,\qquad \Gamma_f=-\frac{c}{2n_f d}\ln r^2\simeq\frac{c(1-r^2)}{2n_f d}\,, \tag{3.30}$$

f being an integer. It is easily seen that for sufficiently small values of Γ_f the spectral transmission function becomes effective in discriminating against values of ω not equal to the resonance frequency ω_f.

In the one-dimensional model the kernel $K=K_f$ [defined according to Eq. (3.16)] reads as

$$K_f(x,t;x't')=-\frac{1}{\hbar}\int_{-\infty}^{+\infty}dk\,\omega A_k(x)A_k^*(x')e^{-i\omega(t-t')}\,. \tag{3.31}$$

In the case when $x>d/2$ and $x'<-d/2$ we find from substituting Eqs. (3.22) and (3.23) in Eq. (3.31) and by making use of Eq. (3.28) the following result:

$$K_f(x,t;x',t')=-\frac{1}{4\pi c\epsilon_0}\left[\int_0^\infty d\omega\,\overline{T}_f(\omega)e^{-i\omega(t-t'-l/c)}+\int_0^\infty d\omega\overline{T}_f^*(\omega)e^{i\omega(t'-t-l/c)}\right],\quad l=x-x' . \tag{3.22}$$

To calculate the integrals in Eq. (3.32) we note that in practical applications the spectral transmission function $\overline{T}_f(\omega)$ can be assumed to take on significant nonzero values only in the neighborhood of the setting frequency ω_f of the spectral filter [cf. Eqs. (3.29) and (3.30)]. We therefore may approximately extend the integrations in Eq. (3.32) to minus infinity. Writing the spectral transmission function $\overline{T}_f(\omega)$ in the form as given in Eq. (3.24) we finally arrive at [cf. Eq. (A18)]

$$K_f(x,t;x',t')=-\frac{1}{2c\epsilon_0}[T_f(t-t'-l_f/c)+T_f^*(t'-t-l_f/c)] , \tag{3.33}$$

where

$$T_f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\omega\,\overline{T}_f(\omega)e^{-i\omega t} \tag{3.34}$$

and

$$l_f=l+(n_f-1)d . \tag{3.35}$$

In particular, substituting Eq. (3.29) in Eq. (3.34) we easily find

$$T_f(t)=\Theta(t)\Gamma_f e^{-(i\omega_f+\Gamma_f)t} . \tag{3.36}$$

In order to calculate the kernel $\mathcal{H}$ for the case when no spectral apparatus is present we have to substitute for the mode functions in Eq. (3.31) plane waves,

$$\mathcal{H}(x,t;x',t')=-\frac{1}{4\pi\epsilon_0}\int_{-\infty}^{+\infty}dk\, e^{-i\omega(t-t')+ik(x-x')} . \tag{3.37}$$

Equation (3.37) may be rewritten in the form of Eq. (3.32) with $\overline{T}_f=1$. The kernel $\mathcal{H}(x,t;x',t')$ is needed in order to calculate integrals of the form $\int dx'\int dt'\mathcal{H}(x,t;x',t')\hat{J}(x',t')$, where the operator $\hat{J}$ is oscillating close by an optical frequency, so that we may again extend the frequency integration to minus infinity with the result

$$\mathcal{H}(x,t;x',t')=-\frac{1}{2c\epsilon_0}[\delta(t-t'-l/c)+\delta(t'-t-l/c)] . \tag{3.38}$$

Clearly, Eq. (3.38) may be understood to describe the limiting case of a spectral filter the passband width Γ_f of which goes to infinity ($\Gamma_f\to\infty$).

Comparing Eq. (3.33) with (3.38) the following relation between K_f and $\mathcal{H}$ is seen to be valid:

$$\Theta(t-t')\mathcal{H}_f(x,t;x',t')=\int dt''T_f(t-t''-\Delta t)\Theta(t''-t')\times\mathcal{H}(x,t'';x't') , \tag{3.39}$$

where the time retardation

$$\Delta t=(n_f-1)\frac{d}{c} \tag{3.40}$$

reflects the fact that the geometrical path through the Fabry-Pérot filter is different from the optical one. We note that Fabry-Pérot filters in quantum optics has also been studied by Cresser[19] and Ley and Loudon.[30]

From an inspection of Eqs. (3.22) and (3.23) we see that two kinds of mode functions have to be considered, which describe incoming waves from the left and right, each of which being partly reflected and transmitted. It should be pointed out that in the case under study [cf. Fig. 1(a)] the reflected part of a light signal arising from the sources is fully directed into the sources. From the point of view of classical optics one already expects that due to this reaction of the spectral apparatus on the sources the dynamics of the sources is, more or less, changed. In the following sections we show that in quantum optics there are additional source-quantity commutator effects, which may be of drastic consequence to observable field quantities.

In practical spectral measurements such back actions are of course tried to be avoided. We therefore consider also the case of inclined incidence of light [cf. Fig. 1(b)], that is, we assume the reflected part of light does not strike the sources (which may experimentally be realized by means of diaphragms). In this case we have to consider four types of mode functions describing incoming waves from the left and right on both the x and the y axis. Analogously to Eqs. (3.22) and (3.23) we may represent the mode functions at the points P on the x or y axis as follows:

$$A_{1k}(P)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases} e^{ikx}, & x<-\frac{d}{2}\\ \overline{T}_f(\omega)e^{ikx}, & x>\frac{d}{2}\\ 0, & y<-\frac{d}{2}\\ \overline{R}_f(\omega)e^{iky}, & y>\frac{d}{2}\end{cases} \tag{3.41}$$

$$A_{2k}(P)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases} 0, & x<-\frac{d}{2}\\ \overline{R}_f(\omega)e^{ikx}, & x>\frac{d}{2}\\ e^{iky}, & y<-\frac{d}{2}\\ \overline{T}_f(\omega)e^{iky}, & y>\frac{d}{2}\end{cases} \tag{3.42}$$

where $k>0$, and

$$A_{1k}(P)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases}\bar{T}_f(\omega)e^{ikx}, & x<-\frac{d}{2}\\ e^{ikx}, & x>\frac{d}{2}\\ \bar{R}_f(\omega)e^{iky}, & y<-\frac{d}{2}\\ 0, & y>\frac{d}{2}\end{cases} \tag{3.43}$$

$$A_{2k}(P)=\left[\frac{\hbar}{4\pi\epsilon_0\omega}\right]^{1/2}\times\begin{cases}\bar{R}_f(\omega)e^{ikx}, & x<-\frac{d}{2}\\ 0, & x>\frac{d}{2}\\ \bar{T}_f(\omega)e^{iky}, & y<-\frac{d}{2}\\ e^{iky}, & y>\frac{d}{2}\end{cases} \tag{3.44}$$

where $k<0$. The spectral transmission and reflection response functions may be taken from Eqs. (3.24) and (3.25), where d must now be understood as an effective thickness.

The kernel $K(P,t;P',t')$ reads as

$$K(P,t;P',t')=-\frac{1}{\hbar}\sum_{i=1}^{2}\int_{-\infty}^{+\infty}dk\,\omega A_{ik}(P)A_{ik}^{*}(P')\times e^{-i\omega(t-t')}\ . \tag{3.45}$$

In particular in the case when the points P and P', respectively, correspond to points x and x' on the x axis with $x>d/2$ and $x'<d/2$ we may write

$$K(P,t;P',t')=K_f(x,t;x',t')\ , \tag{3.46}$$

where $K_f(x,t;x',t')$ is given in Eq. (3.23) [cf. Eq. (A18)].

IV. QUANTUM-THEORETICAL FILTER EQUATION

For the further discussion of the quantum theoretical basic equation (3.18) let us first consider the simple one-dimensional case with perpendicular incidence. According to Fig. 1(a) we assume that the sources are on the left of the filter $(x'<-d/2)$ and the points of observation are on the right of it $(x>d/2)$. Furthermore, we define appropriate operators

$$\hat{\mathcal{E}}_s^{(+)}(x,t)=-\frac{1}{2c\epsilon_0}\int dx'\hat{J}(x',t-l/c)\ , \tag{4.1}$$

$$\hat{\mathcal{E}}_{\rightarrow,\text{free}}^{(+)}(x,t)=\hat{\mathcal{E}}_{\rightarrow,\text{free}}^{(+)}\left[t-\frac{x}{c}\right]=\int_0^{\infty}dk\,i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}\hat{a}_{k,\text{free}}\left[t-\frac{x}{c}\right]\ , \tag{4.2}$$

$$\hat{\mathcal{E}}_{\leftarrow,\text{free}}^{(+)}(x,t)=\hat{\mathcal{E}}_{\leftarrow,\text{free}}^{(+)}\left[t+\frac{x}{c}\right]=\int_{-\infty}^{0}dk\,i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}\hat{a}_{k,\text{free}}\left[t+\frac{x}{c}\right]\ . \tag{4.3}$$

Clearly, Eqs. (4.2) and (4.3) define free-field operators for the fields traveling to the right and left, respectively. They completely agree with the free-field operators in the absence of the spectral filter. Formally, the source field $\hat{\mathcal{E}}_s^{(+)}(x,t)$ defined in Eq. (4.1) has the same structure as in the absence of the filter. However, the source operator $\hat{J}(x',t-l/c)$ must be calculated from the Heisenberg equations of motion which, in general, take into account the back action of the spectral filter on the sources. Together with Eqs. (3.13)–(3.15), (3.22)–(3.25), (3.34), (3.38)–(3.40), and (4.1)–(4.3) we may rewrite the basic equation (3.18) for the field behind the filter $(x>d/2)$ as follows:

$$\hat{E}^{(+)}(x,t)=\hat{E}_{\text{free}}^{(+)}(x,t)+\hat{E}_s^{(+)}(x,t)\ , \tag{4.4}$$

where

$$\hat{E}_s^{(+)}(x,t)=\int dt'T_f(t-t'-\Delta t)\hat{\mathcal{E}}_s^{(+)}(x,t')\ , \tag{4.5}$$

$$\hat{E}_{\text{free}}^{(+)}(x,t)=\int dt'T_f(t-t'-\Delta t)\hat{\mathcal{E}}_{\rightarrow,\text{free}}^{(+)}\left[t'-\frac{x}{c}\right]+\hat{\mathcal{E}}_{\leftarrow,\text{free}}^{(+)}\left[t+\frac{x}{c}\right]+\int dt'R_f(t-t')\hat{\mathcal{E}}_{\leftarrow,\text{free}}^{(+)}\left[t'-\frac{x-d}{c}\right]\ , \tag{4.6}$$

where the reflection response function of the Fabry-Pérot filter $R_f(t)$ reads as

$$R_f(t)=(2\pi)^{-1}\int d\omega\,\tilde{R}_f(\omega)e^{-i\omega t}=-r\delta(t)+rT_f\left[t-2\Delta t-2\frac{d}{c}\right]\ . \tag{4.7}$$

Clearly in the absence of the filter $[T_f(t-\Delta t)\rightarrow\delta(t),\ R_f(t)\rightarrow 0]$ Eqs. (4.5) and (4.6) read as

$$\hat{E}_s^{(+)}(x,t)\equiv\hat{\mathcal{E}}_s^{(+)}(x,t)\ , \tag{4.8}$$

$$\hat{E}_{\text{free}}^{(+)}(x,t)\equiv\hat{\mathcal{E}}_{\text{free}}^{(+)}(x,t)=\hat{\mathcal{E}}_{\rightarrow,\text{free}}^{(+)}\left[t-\frac{x}{c}\right]+\hat{\mathcal{E}}_{\leftarrow,\text{free}}^{(+)}\left[t+\frac{x}{c}\right]\ . \tag{4.9}$$

In classical optics any light field may be thought to be attributed to sources and hence, free-field terms may be omitted. Consequently, in classical optics any light field may be identified with a source field:

$$E^{(+)}(x,t)=E_s^{(+)}(x,t)\ ,\qquad \mathcal{E}^{(+)}(x,t)=\mathcal{E}_s^{(+)}(x,t)\ . \tag{4.10}$$

In this sense and if we take no account of the influence of the filter on the source dynamics, we can combine Eqs. (4.5), (4.8), and (4.10) which yields

$$E^{(+)}(x,t)=\int dt'T_f(t-t'+\Delta t)\mathcal{E}^{(+)}(x,t')\ , \tag{4.11}$$

which [apart from the time correction Δt disregarded in Eq. (3.1)] formally agrees with the classical filter equation (3.1).

In quantum optics, however, the situation changes drastically. The operator of the electric field strength cannot be related, in principle, to the operator of the electric field strength of a source field alone, but it must also be related to a free-field operator, which is obviously needed for the correct description of the effects of quantum noise. From a more general point of view, the free-field operator, which, in general, does not commute with the source-field operator, ensures the quantum-mechanical consistency of the theory.

Combining Eqs. (4.4) and (4.5) yields

$$\hat{E}^{(+)}(x,t)=\hat{E}^{(+)}_{\text{free}}(x,t)+\int dt' T_f(t-t'-\Delta t)\hat{\mathcal{E}}^{(+)}_s(x,t') \; . \tag{4.12}$$

The structure of the free-field operator $\hat{E}^{(+)}_{\text{free}}(x,t)$ of course depends on the properties of the spectral filter. As can be seen from Eq. (4.6) the free-field is not only determined by the transmission of the spectral filter but also by its reflection. This can be easily understood from the argument that the filter is a symmetric equipment with respect to the free field. As expected, $\hat{E}^{(+)}_{\text{free}}$ consists of the incident free field from the right [second term in Eq. (4.6)], that part of it which is reflected by the Fabry-Pérot filter (third term) and that part of the incident free-field from the left which is transmitted by the filter (first term). In particular, in the case when the filter is closed $[T_f(t)\to 0,\ R_f(t)\to -\delta(t)]$, from Eqs. (4.4)–(4.6) we arrive at

$$\hat{E}^{(+)}(x,t)=\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t+\frac{x}{c}\right]-\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t-\frac{x-d}{c}\right] . \tag{4.13}$$

The minus sign obviously results from the phase shift due to total reflection. Equation (4.13) reflects the fact that also in the case when the filter is closed and the free field is in the vacuum state, at any point of observation (behind the filter) the full vacuum fluctuations are of course present.

At this point we note that it may be useful to introduce two other decompositions of the operator of the electric field strength $\hat{E}(x,t)$ behind the filter. As it is easily seen we can rewrite Eqs. (4.4)–(4.6) as follows:

$$\hat{E}^{(+)}(x,t)=\hat{E}^{(+)}_{\text{trans}}(x,t)+\hat{E}'^{(+)}_{\text{free}}(x,t) \; , \tag{4.14}$$

$$\hat{E}^{(+)}_{\text{trans}}(x,t)=\int dt' T_f(t-t'-\Delta t)\hat{\mathcal{E}}^{(+)}_{\text{inc}}(x,t') \; , \tag{4.15}$$

where

$$\hat{E}'^{(+)}_{\text{free}}(x,t)=\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t+\frac{x}{c}\right] + \int dt' R_f(t-t')\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t'-\frac{x-d}{c}\right] , \tag{4.16}$$

$$\hat{\mathcal{E}}^{(+)}_{\text{inc}}(x,t)=\hat{\mathcal{E}}^{(+)}_{\to,\text{free}}\left[t-\frac{x}{c}\right]+\hat{\mathcal{E}}^{(+)}_s(x,t) \; . \tag{4.17}$$

Here, $\hat{\mathcal{E}}^{(+)}_{\text{trans}}(x,t)$ may be interpreted as the operator of the transmitted field, which is the convolution of the filter transmission response function with the total incoming field from the left, and $\hat{E}'^{(+)}_{\text{free}}(x,t)$ describes that part of the free field which is related to the free field coming from the right and the reflected part of it. Clearly, in classical optics the second term in Eq. (4.14) can be omitted because it reflects (for the case under study) a pure vacuum effect. The only relevant field which is affected by the spectral filter is the real incoming field $\mathcal{E}^{(+)}_{\text{inc}}(x,t)=\mathcal{E}^{(+)}(x,t)$, so that we again arrive at Eq. (4.11). Furthermore, we see that any kind of decomposition of $\mathcal{E}^{(+)}_{\text{inc}}(x,t)$ (outside the light source) into source-field and free-field contributions is merely a question of appropriateness in the description of the light.

From Eqs. (4.14)–(4.17) that part of $\hat{E}^{(+)}(x,t)$ which describes the field traveling to the right is readily seen to be

$$\hat{E}^{(+)}_{\to}(x,t)=\hat{E}^{(t)}_{\text{trans}}(x,t) + \int dt' R_f(t-t')\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t'-\frac{(x-d)}{c}\right] . \tag{4.18}$$

We therefore may decompose $\hat{\mathbf{E}}^{(+)}(x,t)$ in the following way:

$$\hat{E}^{(+)}(x,t)=\hat{E}^{(+)}_{\to}(x,t)+\hat{E}^{(+)}_{\leftarrow}(x,t) \; , \tag{4.19}$$

where

$$\hat{E}^{(+)}_{\leftarrow}(x,t)=\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t+\frac{x}{c}\right] \tag{4.20}$$

represents the (free) field traveling to the left.

Let us now consider the case with inclined incidence [cf. Fig. 1(b)]. Using the notation $\hat{a}_k=\hat{a}_{1k}$ and $\hat{b}_k=\hat{a}_{2k}$ for the photon destruction operators we define $\hat{\mathcal{E}}^{(+)}_s(x,t)$, $\hat{\mathcal{E}}^{(+)}_{\to,\text{free}}(x,t)$, and $\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}(x,t)$ according to Eqs. (4.1)–(4.3), and additionally

$$\hat{\mathcal{E}}^{(+)}_{\Leftarrow,\text{free}}(y,t)=\hat{\mathcal{E}}^{(+)}_{\Leftarrow,\text{free}}\left[t-\frac{y}{c}\right] =\int_0^\infty dk\; i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}\hat{b}_{k,\text{free}}\left[t-\frac{y}{c}\right] , \tag{4.21}$$

which represents the free-field operator for the field traveling to the left in the y direction. It can easily be proved that with the replacement

$$\int dt' R_f(t-t')\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t'-\frac{(x-d)}{c}\right] \to \int dt' R_f(t-t')\hat{\mathcal{E}}^{(+)}_{\Leftarrow,\text{free}}\left[t'-\frac{(x-d)}{c}\right] \tag{4.22}$$

Eqs. (4.4)–(4.6), (4.14)–(4.17), and (4.18)–(4.20) also remain valid in the case of inclined incidence.

As mentioned above, the main difference between classical and quantum optics consists of the fact that in quantum optics source-field and free-field terms are, in general, noncommutable quantities. To clarify the resulting consequences and the usefulness of the above-mentioned decompositions for practical spectral measurements we now turn to the problem of studying time-ordered field correlation functions.

V. SPECTRALLY RESOLVED CORRELATION FUNCTIONS

Time-ordered correlation functions of spectrally filtered light represent measurable spectral properties of the field. Since in many cases of practical interest the operators are subjected also to normal ordering we study correlation functions of the following type:

$$G_T^{(m,n)}=\left\langle \left[T_- \prod_{i=1}^{m} \hat{E}_{\alpha_i}^{(-)}(\mathbf{r}_i,t_i) \right] \times \left[T_+ \prod_{j=m+1}^{m+n} \hat{E}_{\alpha_j}^{(+)}(\mathbf{r}_j,t_j) \right] \right\rangle . \tag{5.1}$$

For example, from Glauber's theory of light detection[6,7] the photocount distribution function is determined by the correlation functions $G_T^{(m,m)}$. Correlation functions with $n \neq m$ may be observed in photo-detection experiments after homodyne mixing the light under study with a reference beam. The homodyne detection scheme for observing squeezed light is an example.[31] In this case correlation functions $G_T^{(2,0)}$ and $G_T^{(0,2)}$ also contribute to the detection signal.

Since the $\hat{E}_\alpha^{(\pm)}$ are the field operators in the presence of the spectral filter, $G_T^{(m,n)}$ describes measurable spectral properties of the quantum light field. As shown in paper I [see Eq. (5.7) of paper I], Eq. (5.1) can be rewritten in the form

$$G_T^{(m,n)}=\left\langle O\left\{ \left[\prod_{i=1}^{m} \hat{E}_{\alpha_i}^{(-)}(\mathbf{r}_i,t_i) \right] \left[\prod_{j=m+1}^{m+1} \hat{E}_{\alpha_j}^{(+)}(\mathbf{r}_j,t_j) \right] \right\} \right\rangle . \tag{5.2}$$

The ordering symbol O introduced in paper I acts on products of the operators $\hat{E}_\alpha^{(\pm)}(\mathbf{r},t)$ as follows.

(i) Order the free-field operators $\hat{E}_{\alpha_j,\text{free}}^{(+)}(\mathbf{r}_j,t_j)$ to the right of the source-field operators $\hat{E}_{\alpha_j S}^{(+)}(\mathbf{r}_j,t_j)$.

(ii) Substituting for $\hat{E}_{\alpha_j s}^{(+)}(\mathbf{r}_j,t_j)$ Eq. (3.14) and performing T_+ time ordering of the resulting products of source quantity operator $\hat{J}_{\alpha_j'}(\mathbf{r}_j',t_j')$ before integrating with respect to t_j'. Accordingly, the action of O on products of the operators $\hat{E}_{\alpha_i}^{(-)}(\mathbf{r}_i,t_i)$ in Eq. (5.2) is

$$O\left\{ \prod_{i=1}^{m} \hat{E}_{\alpha_i}^{(-)}(\mathbf{r}_i,t_i) \right\} = \left[O\left\{ \prod_{i=1}^{m} \hat{E}_{\alpha_i}^{(+)}(\mathbf{r}_i,t_i) \right\} \right]^\dagger . \tag{5.3}$$

In practical measurements spectral properties that are closely related to source fields are often desired to be observed. We therefore assume that at the points of observation the following conditions are fulfilled:

$$\langle \cdots \hat{E}_{\alpha,\text{free}}^{(+)} \rangle = 0 = \langle \hat{E}_{\alpha,\text{free}}^{(-)} \cdots \rangle . \tag{5.4}$$

These conditions enable us to eliminate the free-field operators in Eq. (5.1) and to express the correlation functions $G_T^{(m,n)}$ in terms of source correlation functions alone. From Eq. (5.2) together with Eq. (5.4) the result is (cf. paper I)

$$\begin{aligned} G_T^{(m,n)} &= \left\langle O\left\{ \left[\prod_{i=1}^{m} \hat{E}_{\alpha_i s}^{(-)}(\mathbf{r}_i,t_i) \right] \left[\prod_{j=m+1}^{m+n} \hat{E}_{\alpha_j s}^{(+)}(\mathbf{r}_j,t_j) \right] \right\} \right\rangle \\ &= \int d\mathbf{r}_1' \int dt_1' \Theta(t_1-t_1') K_{\alpha_1\alpha_1'}^*(\mathbf{r}_1,t_1;\mathbf{r}_1',t_1') \cdots \\ &\quad \times \int d\mathbf{r}_{m+n}' \int dt_{m+n}' \Theta(t_{m+n}-t_{m+n}') K_{\alpha_{m+n}\alpha_{m+n}'}(\mathbf{r}_{m+n},t_{m+n};\mathbf{r}_{m+n}',t_{m+n}') \\ &\quad \times \left\langle \left[T_- \prod_{i=1}^{m} \hat{J}_{\alpha_i}^\dagger(\mathbf{r}_i',t_i') \right] \left[T_+ \prod_{j=m+1}^{m+n} \hat{J}_{\alpha_j'}(\mathbf{r}_j',t_j') \right] \right\rangle . \end{aligned} \tag{5.5}$$

Equation (5.5) establishes that in the calculation of correlation functions of the type defined in Eq. (5.1), taking into account Eq. (5.4), the total field operators $\hat{E}_\alpha^{(+)}$ and $\hat{E}_\alpha^{(-)}$, respectively, may formally be replaced by the source-field operators $\hat{E}_{\alpha s}^{(+)}$ and $\hat{E}_{\alpha s}^{(-)}$ defined in Eq. (3.14), and the positive and negative time ordering originally concerning the operators $\hat{E}_\alpha^{(+)}$ and $\hat{E}_\alpha^{(-)}$, respectively, being transferred to the corresponding atomic source operators $\hat{J}_{\alpha'}$ and $\hat{J}_{\alpha'}^\dagger$.[25] The time ordering becomes superfluous only in the case of the second-order correlation function $G_T^{(1,1)}$ and hence in the case of the usual second-order spectrum.

As known from Sec. III, the properties of the spectral filters used are contained in the kernel K defined in Eq. (3.16). Even in the simplest case when only one filter is used the calculation of the kernel K may be, in general, rather difficult. This problem becomes still more difficult in such cases when more than one filter is used in the observation scheme. Let us assume that the filters are of the Fabry-Pérot filter type as considered in the preceding sections. Furthermore, let us suppose that the filters are situated in the far-field region of the source field under study. In this case the one-dimensional approach is expected to yield a sufficiently good approximation for describing the actual situation. If the field observed at point $\mathbf{r}_i$ (behind the filters) is related to the sources (in front of the filters) via the ith spectral filter we therefore may write

$$K_{\alpha_i\alpha_i'}(\mathbf{r}_i,t_i;\mathbf{r}_i't_i')=K_{f_i\alpha_i\alpha_i'}(\mathbf{r}_i,t_i;\mathbf{r}_i',t_i') , \quad (5.6)$$

where [cf. Eq. (3.39)]

$$\Theta(t_i-t_i')K_{f_i\alpha_i\alpha_i'}(\mathbf{r}_i,t_i;\mathbf{r}_i',t_i') = \int dt_i'' T_{f_i}(t_i-t_i''-\Delta t_i)\Theta(t_i''-t_i') \times \mathcal{K}_{\alpha_i\alpha_i'}(\mathbf{r}_i,t_i'';\mathbf{r}_i',t_i') . \quad (5.7)$$

Inserting Eqs. (5.6) and (5.7) into Eq. (5.5), remembering the definition of $\mathcal{K}$ in Eq. (3.38), and using Eq. (4.5) we may write Eq. (5.5) as follows:

$$G_T^{(m,n)}=\left\langle O\left\{\left[\prod_{i=1}^{m}\int dt_i' T_{f_i}^*(t_i-t_i'-\Delta t_i)\hat{\mathcal{E}}_{\alpha_i s}^{(-)}(\mathbf{r}_i,t_i')\right]\left[\prod_{j=m+1}^{m+n}\int dt_j' T_{f_j}(t_j-t_j'-\Delta t_j)\hat{\mathcal{E}}_{\alpha_j s}^{(+)}(\mathbf{r}_j,t_j')\right]\right\}\right\rangle . \quad (5.8)$$

Introducing the correlation function

$$\Gamma_0^{(m,n)}=\left\langle O\left\{\left[\prod_{i=1}^{m}\hat{\mathcal{E}}_{\alpha_i s}^{(-)}(\mathbf{r}_i,t_i)\right]\left[\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}_{\alpha_j s}^{(+)}(\mathbf{r}_j,t_j)\right]\right\}\right\rangle , \quad (5.9)$$

we may rewrite Eq. (5.8) as follows

$$\begin{aligned} G_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n}) &= \int dt_1' T_{f_1}^*(t_1-t_1'-\Delta t_1)\cdots \\ &\quad\times\int dt_{m+n}' T_{f_{m+n}}(t_{m+n}-t_{m+n}'-\Delta t_{m+n}) \\ &\quad\times\Gamma_0^{(m,n)}(\mathbf{r}_1,t_1',\ldots,\mathbf{r}_{m+n},t_{m+n}') . \end{aligned} \quad (5.10)$$

Substituting in Eqs. (3.13), (3.14), and (3.16) for the mode functions $A_{\alpha\lambda}(\mathbf{r})$ those parts of them which (at the points of observation) describe the free-space field structure, we define, according to Eq. (3.12), the field operator

$$\hat{\mathcal{E}}_\alpha^{(+)}(\mathbf{r},t)=\hat{\mathcal{E}}_{\alpha,\mathrm{free}}^{(+)}(\mathbf{r},t)+\hat{\mathcal{E}}_{\alpha s}^{(+)}(\mathbf{r},t) . \quad (5.11)$$

From this definition it is obvious that $\hat{\mathcal{E}}_\alpha^{(+)}$ looks like the positive frequency part of the operator of the electric field strength in the case when the spectral filter is absent. Note that this equivalence is only a formal one, because the correct source-field dynamics must, of course, be calculated by taking into account the presence of the filter. Making use of Eq. (5.11) together with Eq. (5.4) and remembering the definitions of the O ordering, we may rewrite Eq. (5.9) as follows:

$$\Gamma_0^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n}) = \left\langle O\left\{\left[\prod_{i=1}^{m}\hat{\mathcal{E}}_{\alpha i}^{(-)}(\mathbf{r}_i,t_i)\right]\left[\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}_{\alpha_j}^{(+)}(\mathbf{r}_j,t_j)\right]\right\}\right\rangle . \quad (5.12)$$

Applying the method given in paper I, in Eq. (5.12) we now return to time-ordered operator products:

$$\begin{aligned}\Gamma_0^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n}) &= \Gamma_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n}) \\ &\quad+\langle\hat{\delta}_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})\rangle , \end{aligned} \quad (5.13)$$

where

$$\Gamma_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n}) = \left\langle\left[T_-\prod_{i=1}^{m}\hat{\mathcal{E}}_{\alpha_i}^{(-)}(\mathbf{r}_i,t_i)\right]\left[T_+\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}_{\alpha_j}^{(+)}(\mathbf{r}_j,t_j)\right]\right\rangle , \quad (5.14)$$

and $\hat{\delta}_T^{(m,n)}$ is expressed in terms of commutators of the atomic source quantity operators. The simplest example is $\hat{\delta}_T^{(0,2)}$, an explicit expression of which the reader finds in the Appendix [Eq. (A53) together with Eq. (A51)]. This example already shows that the appearance of the commutator term $\hat{\delta}_T^{(m,n)}$ is closely related to the back action of the Fabry-Pérot filter to the sources. That is, in an experimental setup outlined in Fig. 1(a) this back action not only gives rise to a more or less pronounced modification of the source dynamics via the interaction of the reflected light with the sources (as already expected from the point of view of classical optics) but it prevents, on principle, that the observable correlation functions $G_T^{(m,n)}$ can be related to the correlation functions $\Gamma_T^{(m,n)}$ [see Eqs. (5.10) and (5.13)].

If the back action is suppressed [inclined incidence of light according to Fig. 1(b)] the quantum-mechanical commutator term $\hat{\delta}_T^{(m,n)}$ vanishes, so that from Eqs. (5.10) and (5.13) we find

$$G_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})=\int dt_1' T_{f_1}^*(t_1-t_1'-\Delta t_1)\cdots \times\int dt_{m+n}' T_{f_{m+n}}(t_{m+n}-t_{m+n}'-\Delta t_{m+n})\Gamma_T^{(m,n)}(\mathbf{r}_1,t_1',\ldots,\mathbf{r}_{m+n},t_{m+n}') \quad (5.15)$$

or, expressing $\Gamma_T^{(m,n)}$ by its Fourier transform $\tilde{\Gamma}_T^{(m,n)}$,

$$G_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})=\int d\omega_1\frac{1}{2\pi}e^{i\omega_1(t_1-\Delta t_1)}\tilde{T}^*_{f_1}(\omega_1)\cdots\times\int d\omega_{m+n}\frac{1}{2\pi}e^{-i\omega_{m+n}(t_{m+n}-\Delta t_{m+n})}\tilde{T}_{f_{m+n}}(\omega_{m+n})\times\tilde{\Gamma}_T^{(m,n)}(\mathbf{r}_1,\omega_1,\ldots,\mathbf{r}_{m+n},\omega_{m+n})\,. \quad (5.16)$$

It should be pointed out that Eq. (5.12) remains valid when the field operators $\hat{\mathcal{E}}_\alpha^{(\pm)}$ are replaced by their incoming parts $\hat{\mathcal{E}}_{\alpha,\mathrm{inc}}^{(\pm)}$ [according to the definition in Eq. (4.17)]. The calculation yields [cf. the example in the Appendix, Eq. (A55)]

$$\Gamma_0^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})=\Gamma_{\mathrm{inc}}^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})+\langle\hat{\delta}^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})\rangle \quad (5.17)$$

instead of Eq. (5.13), where

$$\Gamma_{\mathrm{inc}}^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})=\left\langle\left[\prod_{i=1}^{m}\hat{\mathcal{E}}_{\alpha_i,\mathrm{inc}}^{(-)}(\mathbf{r}_i,t_i)\right]\left[\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}_{\alpha_j,\mathrm{inc}}^{(+)}(\mathbf{r}_j,t_j)\right]\right\rangle\,. \quad (5.18)$$

Note that in Eq. (5.18) there is no need for time ordering. Provided that the back action of the filter to the sources can be suppressed (inclined incidence), so that $\hat{\delta}^{(m,n)}=0$ is valid, in Eqs. (5.15) and (5.16) we may substitute for $\Gamma_T^{(m,n)}$ and $\tilde{\Gamma}_T^{(m,n)}$, respectively, the correlation functions of the incoming field $\Gamma_{\mathrm{inc}}^{(m,n)}$ and $\tilde{\Gamma}_{\mathrm{inc}}^{(m,n)}$. Remembering the definition of the transmitted field [Eq. (4.15)] we therefore may write

$$G_T^{(m,n)}(\mathbf{r}_1,t_1,\ldots,\mathbf{r}_{m+n},t_{m+n})=\left\langle\left[\prod_{i=1}^{m}\hat{E}_{\alpha_i,\mathrm{trans}}^{(-)}(\mathbf{r}_i,t_i)\right]\left[\prod_{j=m+1}^{m+n}\hat{E}_{\alpha_j,\mathrm{trans}}^{(+)}(\mathbf{r}_j,t_j)\right]\right\rangle\,. \quad (5.19)$$

(see also Sec. VI). This equation closely corresponds to the result of classical optics.

We point out that the results given above are also valid in the case, when the free-field part of the incoming field is not in the vacuum state.

The results show that measurable spectral properties expressed in terms of correlation functions $G_T^{(m,n)}$ can be related to the Fourier transforms of both the time-ordered correlation functions $\Gamma_T^{(m,n)}$ of the full field and the correlation functions of the incoming field $\Gamma_{\mathrm{inc}}^{(m,n)}$, provided that the back action of the spectral apparatus to the sources can be suppressed (inclined incidence of light). In particular, they cannot be related to the Fourier transforms of the correlation functions $\Gamma^{(m,n)}$ representing the full Fourier structure of the field, with the exception of the case when the field under study may be approximated by a free-field alone. Moreover, in this case O and time orderings do not play a role, independent of the angle of incidence of the light.

It is worth noting that in the case of back action of the spectral apparatus being present it may be suitable to represent the correlation function $G_T^{(m,n)}$ in a somewhat different form as given above. For this purpose let us consider the (quasi)linear model with perpendicular incidence of light. From a careful inspection of Eq. (A50) we see that even in the case when $\alpha=1$ is valid (perpendicular incidence of light), this equation may be rewritten as

$$T_+^{(r)}(\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2))=O(\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2))\,, \quad (5.20)$$

where $x_1\geq d/2$ and $x_2\geq d/2$. Here and in the following the notation $T_+^{(r)}$ ($T_-^{(r)}$) indicates positive (negative) time ordering with respect to the retarded time arguments $t-x/c$. Furthermore, from the derivation of Eq. (A50) it is clear that this equation may also be applied to $\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}$ (instead of $\hat{\mathcal{E}}^{(+)}$) if the term proportional T_f^* is disregarded. Since in the case of the condition given in Eq. (6.1a) being fulfilled the expectation value of the O-ordered product of $\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2)$ is equal to the expectation value of the O-ordered product of $\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}(x_2,t_2)$, we also may write

$$\langle T_+^{(r)}(\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}(x_2,t_2))\rangle=\langle O(\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2))\rangle\,, \quad (5.21)$$

where $x_1\geq d/2$ and $x_2\geq d/2$. Clearly, Eqs. (5.20) and (5.21) are also valid for higher-order operator products. Hence, combining Eqs. (5.10), (5.12), and (5.21) yields

$$G_T^{(m,n)}(x_1,t_1,\ldots,x_{m+n},t_{m+n})=\int dt_1'T_f^*(t_1-t_1'-\Delta t)\cdots\int dt_{m+n}'T_f(t_{m+n}-t_{m+n}'-\Delta t)\times\left\langle\left[T_-^{(r)}\prod_{i=1}^{m}\hat{\mathcal{E}}_{\mathrm{inc}}^{(-)}(x_i,t_i')\right]\left[T_+^{(r)}\prod_{j=m+1}^{m+n}\hat{\mathcal{E}}_{\mathrm{inc}}^{(+)}(x_j,t_j')\right]\right\rangle\,. \quad (5.22)$$

Remember that $\hat{\mathcal{E}}_{\mathrm{inc}}^{(\pm)}(x,t)$ is a function of the retarded time argument $t-x/c$.

Let us now consider $\hat{E}^{(\pm)}(x,t)$ between the source represented by $\hat{J}(x',t')$ and the filter which means $x'<x\leq-d/2$,

where x' represents all points where the source is present. Analogous to Eq. (4.19) we decompose $\hat{E}^{(\pm)}(x,t)$ for $x'<x\leq -d/2$ into a part $\hat{E}^{(\pm)}_{\rightarrow}$ traveling to the right and a part $\hat{E}^{(\pm)}_{\leftarrow}$ traveling to the left. Calculations similar to those resulting in Eqs. (4.18)–(4.20) show that

$$\hat{E}^{(\pm)}_{\rightarrow}(x,t)=\hat{\mathcal{E}}^{(\pm)}_{\text{inc}}(x,t) \quad \text{for } x'<x\leq -d/2 \; . \tag{5.23}$$

Combining Eqs. (5.22) and (5.23), after a transformation of integration variables we arrive at the remarkable result

$$G_T^{(m,n)}(x_1,t_1,\ldots,x_{m+n},t_{m+n})=\int dt_1' T_f^* \left[t_1-t_1'-\frac{x_1+d/2}{c}-\Delta t\right] \times \int dt'_{m+n} T_f \left[t_{m+n}-t'_{m+n}-\frac{x_{m+n}+d/2}{c}-\Delta t\right] \Gamma_{\rightarrow}(t_1',\ldots,t'_{m+n}) \; , \tag{5.24}$$

where

$$\Gamma_{\rightarrow}(t_1',\ldots,t'_{m+n}) =\left\langle \left[T_- \prod_{i=1}^{m} \hat{E}^{(-)}_{\rightarrow}\left(-\frac{d}{2},t_i'\right)\right] \times \left[T_+ \prod_{j=m+1}^{m+n} \hat{E}^{(+)}_{\rightarrow}\left(-\frac{d}{2},t_j'\right)\right]\right\rangle . \tag{5.25}$$

This result shows that $G_T^{(m,n)}$ may be represented by convolutions of the transmission functions with the correlation function $\Gamma_{\rightarrow}$, which represents the time-ordered product of the field operators traveling to the right and taken at the input port of the filter located at $x=-d/2$. The result expresses the physical point of view that the information about the field on the right-hand side of the filter (remember $\langle\cdots\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\rangle=0=\langle\hat{\mathcal{E}}^{(-)}_{\leftarrow,\text{free}}\rangle$) is contained in correlation functions of field operators traveling to the right at the input port of the filter which have to be time ordered since they do not commute at different times. In case of inclined incidence we arrive at similar results but now the field operators $\hat{E}^{(+)}_{\rightarrow}(-d/2,t)$ commute for different times.

Note that an alternative approach to study spectral properties of quantum light fields is to measure the (unfiltered) field by a photodetector and to perform a spectral analysis of the classical (stochastic) photocurrent.[15] Clearly, the spectral properties of the classical current can be related to its full Fourier structure. It can be shown that the spectral correlation functions of the photocurrent are also related to the Fourier transform of normally and time-ordered field correlation functions.[32,33] Thus the spectral filtering of a light field followed by photodetection yields the same spectral information on the field under study as the postdetection filtering of the photocurrent. However, our approach allows to use the spectrally filtered light in further applications, whereas the other one is only to determine the spectral properties via annihilation of the field.

VI. DISCUSSION OF THE FIELD DECOMPOSITIONS

It is obvious that in classical optics there is no essential difference between the field decompositions presented in Sec. IV. All of them can be used for expressing the observable field correlation functions in terms of correlation functions of the resulting field parts. In quantum optics the situation may be quite different and a more careful consideration is needed.

In many cases of typical scattering problems, when the properties of the field scattered by atomic sources are sought and the dynamics of the sources is known the decomposition of the field into free field and source field may be useful [cf. Eqs. (4.4)–(4.6)]. However, in contrast to classical optics, in quantum optics the observable field correlation functions cannot be expressed, in general, in terms of correlation functions of the source field. As shown in Sec. V the way must be to express the field correlation functions in terms of (time-ordered) correlation functions of the atomic source operators [cf. Eq. (5.5)].

Now let us consider the decomposition of the field into the field transmitted by the spectral apparatus and the residual free-field part [Eqs. (4.14)–(4.17) for the case of perpendicular incidence according to Fig. 1(a), and Eqs. (4.14)–(4.17) together with Eqs. (4.21) and (4.22) for the case of inclined incidence according to Fig. 1(b)]. For practical spectral measurements we may assume that this free-field part only gives rise to vacuum effects (the used output port of the spectral apparatus is not used as input port), so that we may write

$$\langle\cdots\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\rangle=0=\langle\hat{\mathcal{E}}^{(-)}_{\leftarrow,\text{free}}\cdots\rangle \; , \tag{6.1a}$$

$$\langle\cdots\hat{\mathcal{E}}^{(+)}_{\Leftarrow,\text{free}}\rangle=0=\langle\hat{\mathcal{E}}^{(-)}_{\Leftarrow,\text{free}}\cdots\rangle \; . \tag{6.1b}$$

Hence, decomposing in Eq. (5.2) the field operators in the way described and remembering the definition of the ordering symbol O we easily see that Eq. (5.2) may be rewritten as follows:

$$G_T^{(m,n)}=\left\langle O\left\{\left[\prod_{i=1}^{m}\hat{E}^{(-)}_{\text{trans}}(x_i,t_i)\right] \times \left[\prod_{j=m+1}^{m+n}\hat{E}^{(+)}_{\text{trans}}(x_j,t_j)\right]\right\}\right\rangle . \tag{6.2}$$

Clearly, in classical optics the ordering symbol O is meaningless, so that the field correlation functions detected are simply given by the correlation functions of the transmitted field, which for its part is the convolution of the incoming field with the transmission response function of the spectral apparatus [cf. Eqs. (4.15) and (4.17)]. Moreover, from Sec. V [Eq. (5.19)] we already know that in quantum optics the ordering symbol O is also meaningless, provided that the back action of the spectral apparatus to the sources can be suppressed [inclined incidence of light according to Fig. 1(b), see also Eq. (A38)].

If back action must be taken into account [perpendicular incidence of light according to Fig. 1(a)] the situation changes drastically. In this case, the resemblance of Eq. (6.2) with the classical result is a purely formal one, because the O-ordering prescription now prevents that observable field correlation functions can be attributed to correlation functions of the transmitted field (an exception is of course the usual second-order spectrum). In other words, the transmitted field hardly has importance in higher (than second-) order spectral measurements. To clarify this point in more detail let us assume perpendicular incidence [according to Fig. 1(a)] and consider the correlation function

$$G_T^{(0,2)} = \langle T_+ \hat{E}^{(+)}(x_1,t_1)\hat{E}^{(+)}(x_2,t_2)\rangle = \langle O\{\hat{E}^{(+)}_{\text{trans}}(x_1,t_1)\hat{E}^{(+)}_{\text{trans}}(x_2,t_2)\}\rangle , \tag{6.3}$$

which, for example, is of interest for spectral squeezing. Substituting in the first row of Eq. (6.3) for the operators $\hat{E}^{(+)}$ Eq. (4.14) and rearranging the resulting operator products in such a way that the operators $\hat{E}'^{(+)}_{\text{free}}$ are on the right of the operators $\hat{E}^{(+)}_{\text{trans}}$ by making use of the method given in paper I for calculating the commutator relations needed, we may rewrite Eq. (6.3) as follows (cf. Appendix):

$$G_T^{(0,2)} = \langle T_+ \hat{E}^{(+)}_{\text{trans}}(x_1,t_1)\hat{E}^{(+)}_{\text{trans}}(x_2,t_2)\rangle = \langle \hat{\Delta}_T(x_1,t_1;x_2,t_2)\rangle , \tag{6.4}$$

where the operator $\hat{\Delta}_T$ [given in Eq. (A29) for $\alpha=1$] is closely related to commutators of the atomic source operators $\hat{J}$. In general, these commutators (and their mean values) do not vanish, which clearly shows that the correlation function $G_T^{(0,2)}$ cannot be expressed in terms of the correlation function of the transmitted field alone. In particular, let us consider a pointlike atomic source located at $-s$ $(s>d/2)$. As shown in the Appendix [Eqs. (A31)–(A34)] we obtain the result

$$\hat{\Delta}_T(x_1,t_1;x_2,t_2) = \Gamma_f^2 \left|\frac{1}{2\epsilon_0 c}\right|^2 \frac{(-1)^f}{2}\int dt_i' \int dt_j' \Theta(t_j^r - t_j')\Theta(t_j'-t_i')[\Theta(t_i'-t_i^r-2s_f/c)-\Theta(t_i^r+2s_f/c-t_i')] \times \exp\{-i\omega_f[t_i+t_j-t_i'-t_j'-(x_{f_i}+x_{f_j})/c]\} \times \exp[-\Gamma_f(t_j^r-t_j'+|t_i^r+2s_f|c-t_i'|)][\hat{J}(t_i'),\hat{J}(t_j')] , \tag{6.5}$$

where the indices i and j are determined from the requirements $t_i=\max(t_1,t_2)$ and $t_j=\min(t_1,t_2)$, and

$$t_{i(j)}^r = t_{i(j)} - (x_{f_{i(j)}} + s_f)/c , \tag{6.6}$$

$$x_{fi(j)} = x_{i(j)} + (n_f - 1)d/2 , \tag{6.7}$$

$$s_f = s + (n_f - 1)d/2 . \tag{6.8}$$

From an inspection of Eq. (6.5) it is clearly seen that in general the mean value $\langle\hat{\Delta}_T\rangle$ cannot be expected to be vanishing. In particular in the case, when the inequality $t_i^r+2s_f/c>t_j^r$ is valid, we find $\langle\hat{\Delta}\rangle \propto \exp(-2\Gamma_f s_f/c)$, which shows that (in this case) neglect of the term $\langle\hat{\Delta}\rangle$ in Eq. (6.4) may be justified if the sources are sufficiently far away from the spectral apparatus: $s_f \gg c/2\Gamma_f$. Clearly, for high spectral resolution this requirement becomes crucial. Furthermore, in the case when the value of t_i^r becomes smaller than the value of t_j^r $(t_i^r<t_j^r)$, the allowed values of t_i' can exceed the value of t_i^r and, therefore, the causality is not ensured. On the other hand, from the meaning of the ordering symbol O in the second line of Eq. (6.3) it is clear that in any case $G_T^{(0,2)}$ is built up only by causally determined contributions. Equation (6.4) therefore shows that the (time-ordered) correlation function of the transmitted field may contain unphysical, noncausal contributions, which are needed for compensating the corresponding terms in $\langle\hat{\Delta}\rangle$.

Finally, let us study the decomposition of the field behind the filter into the field traveling to the right and the field traveling to the left [cf. Eqs. (4.18)–(4.20) for the case of perpendicular incidence of light according to Fig. 1(a), and Eqs. (4.18)–(4.20) together with Eqs. (4.21), (4.22) for the case of inclined incidence of light according to Fig. 1(b). From Eqs. (A22) and (A25) in the Appendix we find

$$[\hat{E}^{(+)}_{\leftarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]=0 \quad \text{if } t_1>t_2 , \tag{6.9}$$

$$[\hat{E}^{(+)}_{\rightarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]=0 . \tag{6.10}$$

Substituting in Eq. (5.2) for the field operators the decomposition given in Eq. (4.20) and making use of Eqs. (6.9) and (6.10) together with Eq. (6.1) we arrive at the result

$$G_T^{(m,n)} = \left\langle \left[\prod_{i=1}^{m}\hat{E}^{(-)}_{\rightarrow}(x_i,t_i)\right]\left[\prod_{j=m+1}^{m+n}\hat{E}^{(+)}_{\rightarrow}(x_j,t_j)\right]\right\rangle , \tag{6.11}$$

which is valid for both perpendicular and inclined incidence.

We see that the correlation functions detected are simply given by the corresponding correlation functions of that part of the light field, which behind the spectral apparatus travels to the right into the photodetectors. Since the operators $\hat{E}^{(+)}_{\rightarrow}$ (and the operators $\hat{E}^{(-)}_{\rightarrow}$) are commuting quantities, time orderings are needless. We therefore may regard this kind of field decomposition as a quasi-free-field decomposition, because it shows features similar to the free-field case. The difference between the two cases becomes apparent if one tries to eliminate in Eq. (6.11) that part of $\hat{E}^{(\pm)}_{\rightarrow}$ which describes the free field reflected at the right (unused input) port of the spectral apparatus. Clearly, in the free-field case this (reflected) part can simply be omitted, so that the correlation functions detected are given by the corresponding correlation functions of the remaining transmitted field. However, in cases when the light field under study is attributed to sources the replacement of $\hat{E}^{(\pm)}_{\rightarrow}$ by $\hat{E}^{(\pm)}_{\text{trans}}$ in Eq. (6.11) is only allowed if back actions of the spectral apparatus to the sources can be suppressed [inclined incidence of light, see Eq. (A37)].

VII. SPECTRAL SQUEEZING IN RESONANCE FLUORESCENCE

To demonstrate the difference between intrinsically spectral properties and observable ones let us study the problem of spectral squeezing in resonance fluorescence (inclined incidence).

Squeezing may be defined by the requirement that

$$\langle :\Delta\hat{\mathscr{E}}_{\alpha\phi})^2:\rangle = \langle :\hat{\mathscr{E}}_{\alpha\phi}(\mathbf{r},t),\hat{\mathscr{E}}_{\alpha\phi}(\mathbf{r},t):\rangle < 0 \ , \tag{7.1}$$

where

$$\hat{\mathscr{E}}_{\alpha\phi}(\mathbf{r},t) = \tfrac{1}{2}[\hat{\mathscr{E}}^{(+)}_{\alpha}(\mathbf{r},t)e^{i(\omega_L t+\phi)} + \text{H.c.}] \ , \tag{7.2}$$

which for $\phi=0$ and $\phi=\pi/2$ coincides with the in-phase and out-of-phase component of the electric field, respectively. In Eq. (7.1), the symbol : : means normal ordering, and $\langle \hat{A},\hat{B}\rangle = \langle \hat{A}\hat{B}\rangle - \langle \hat{A}\rangle\langle\hat{B}\rangle$.

Clearly, in the case when the light under study is filtered before homodyning, the normally ordered variance $\langle :(\Delta\hat{E}_{\phi})^2:\rangle$ is observed,

$$\langle :(\Delta\hat{E}_{\phi})^2:\rangle = \langle :\hat{E}_{\alpha\phi}(\mathbf{r},t),\hat{E}_{\alpha\phi}(\mathbf{r},t):\rangle \ , \tag{7.3}$$

where $\hat{E}_{\alpha\phi}$ is defined in Eq. (7.2) with $\hat{E}^{(\pm)}_{\alpha}$ instead of $\hat{\mathscr{E}}^{(\pm)}_{\alpha}$. For simplicity we consider the case when the spectral filter (Fabry-Pérot filter type) is tuned to the frequency $\omega_f=\omega_L$. Making use of the general result given in Eq. (5.8) and representing the T_f as Fourier integrals we may rewrite Eq. (7.3) as follows:

$$\langle :(\Delta\hat{E}_{\phi})^2:\rangle = \left[\frac{1}{2\pi}\right]^2 \int d\omega_1 \int d\omega_2 e^{-i(\omega_1+\omega_2)(t-\Delta t)}\tilde{T}_f(\omega_L+\omega_1)\tilde{T}_f(\omega_L+\omega_2)\tilde{S}_{\phi_f}(\mathbf{r},\omega_1,\omega_2) \ , \tag{7.4}$$

where $\phi_f=\phi+\Delta t\omega_L$ and

$$\tilde{S}_{\phi}(\mathbf{r},\omega_1,\omega_2) = \tfrac{1}{4}\int dt_1 \int dt_2 e^{i(\omega_1 t_1+\omega_2 t_2)}\langle O\{[\hat{\mathscr{E}}^{(+)}_{as}(\mathbf{r},t_1)e^{i(\omega_L t_1+\phi)}+\text{H.c.}],[\hat{\mathscr{E}}^{(+)}_{as}(\mathbf{r},t_2)e^{i(\omega_L t_2+\phi)}+\text{H.c.}]\}\rangle \ . \tag{7.5}$$

$\tilde{S}_{\phi}$ may be expressed by $\tilde{\Gamma}^{(m,n)}_{T}$ in the following way:

$$\tilde{S}_{\phi}(\mathbf{r},\omega_1,\omega_2) = \tfrac{1}{4}[\tilde{\Gamma}^{(1,1)}_{T}(\omega_L-\omega_1,\omega_L+\omega_2) + \tilde{\Gamma}^{(1,1)}_{T}(\omega_L-\omega_2,\omega_L+\omega_1) + \tilde{\Gamma}^{(0,2)}_{T}(\omega_L+\omega_1,\omega_L+\omega_2)e^{2i\phi} + \tilde{\Gamma}^{(2,0)}_{T}(\omega_L-\omega_1,\omega_L-\omega_2)e^{-2i\phi}] \ . \tag{7.6}$$

In steady state we have

$$\tilde{S}_{\phi}(\mathbf{r},\omega_1,\omega_2) = \delta(\omega_1+\omega_2)\tilde{\sigma}_{\phi}(\mathbf{r},\omega_1) \tag{7.7}$$

and we find

$$\langle :(\Delta\hat{E}_{\phi})^2:\rangle = \frac{1}{2\pi}\int \frac{d\omega}{2\pi}\frac{\Gamma_f^2}{\Gamma_f^2+\omega^2}\tilde{\sigma}_{\phi}(\mathbf{r},\omega) \ . \tag{7.8}$$

It is seen that the normally ordered variance measured behind the spectral apparatus is closely related to $\tilde{\sigma}_{\phi}(\mathbf{r},\omega)$. In particular, if the resolution of the spectral apparatus is sufficiently low ($\Gamma_f\to\infty$), the total normally ordered variance is obtained by frequency integration of $\tilde{\sigma}_{\phi}$:

$$\lim_{\Gamma_f\to\infty} \langle :(\Delta\hat{E}_{\phi})^2:\rangle = \langle :[\Delta\hat{\mathscr{E}}_{\phi}(\mathbf{r},t)]^2:\rangle = \frac{1}{2\pi}\int\frac{d\omega}{2\pi}\tilde{\sigma}_{\phi}(\mathbf{r},\omega) \ . \tag{7.9}$$

In the opposite limiting case of extremely high resolution ($\Gamma_f\to 0$) we obtain

$$\lim_{\Gamma_f\to 0}\frac{4\pi}{\Gamma_f}\langle :(\Delta\hat{E}_{\phi})^2:\rangle = \tilde{\sigma}_{\phi}(\mathbf{r},\,\omega=0) \ . \tag{7.10}$$

On the other hand, by Fourier transform we can define a quantity $\tilde{S}'_{\phi}$,

$$\tilde{S}'_{\phi}(\mathbf{r},\omega_1,\omega_2) = \int dt_1 \int dt_2 e^{i(\omega_1 t_1+\omega_2 t_2)} \times \langle :\hat{\mathscr{E}}_{\alpha\phi}(\mathbf{r},t_1),\hat{\mathscr{E}}_{\alpha\phi}(\mathbf{r},t_2):\rangle \ , \tag{7.11}$$

which is related to $\tilde{\Gamma}^{(m,n)}$ in the same functional form as $\tilde{S}_{\phi}(\mathbf{r},\omega_1,\omega_2)$ to $\tilde{\Gamma}^{(m,n)}_{T}$ [see Eq. (7.6)].

In steady state we derive

$$\tilde{S}'_{\phi}(\mathbf{r},\omega_1,\omega_2) = \delta(\omega_1+\omega_2)\tilde{\sigma}'_{\phi}(\mathbf{r},\omega_1) \ , \tag{7.12}$$

$$\langle :[\Delta\hat{\mathscr{E}}_{\phi}(\mathbf{r},t)]^2:\rangle = \frac{1}{2\pi}\int\frac{d\omega}{2\pi}\tilde{\sigma}'_{\phi}(\mathbf{r},\omega) \ . \tag{7.13}$$

The intrinsic squeezing spectrum may now be defined [based on the Fourier decomposition of the field $\hat{\mathscr{E}}_{\phi}(\mathbf{r},t)$] by $\tilde{\sigma}'_{\phi}$ in close analogy to the definition of the quantum theoretical analogue of the Wiener-Khintchine spectrum as introduced by Metha and Wolf[1] [cf. Eqs. (2.7) and (2.8)].

Clearly, the normally ordered variance $\langle :[\Delta\hat{\mathscr{E}}_{\phi}(\mathbf{r},t)]^2:\rangle$ of the (unfiltered) field represents a measurable quantity, which gives the information on squeezing. It can be related to the frequency integral of either the measurable spectrum $\tilde{\sigma}_\phi$ or the intrinsic spectrum $\tilde{\sigma}'_\phi$, see Eqs. (7.9) and (7.13). The relation between the measurable spectral quantity $\tilde{\sigma}_\phi(\mathbf{r},\omega)$ and the intrinsic one $\tilde{\sigma}'_\phi(\mathbf{r},\omega)$ is given by

$$\tilde{\sigma}'_\phi(\mathbf{r},\omega)=\tilde{\sigma}_\phi(\mathbf{r},\omega)+\tilde{\Delta}_\phi(\mathbf{r},\omega)\ , \tag{7.14}$$

where obviously the relation

$$\int d\omega\,\tilde{\Delta}_\phi(\mathbf{r},\omega)=0 \tag{7.15}$$

is valid. The quantity $\tilde{\Delta}_\phi(\mathbf{r},\omega)$ is defined by

$$\delta(\omega_1+\omega_2)\tilde{\Delta}_\phi(\mathbf{r},\omega_1)=\tfrac{1}{4}\int dt_1\int dt_2 e^{i(\omega_1 t_1+\omega_2 t_2)}(D_{\alpha\alpha}(\mathbf{r},t_1;\mathbf{r},t_2)\exp\{i[\omega_L(t_1+t_2)+2\phi]\}$$
$$+D^*_{\alpha\alpha}(\mathbf{r},t_2;\mathbf{r},t_1)\exp\{-i[\omega_L(t_1+t_2)+2\phi]\})\ , \tag{7.16}$$

where

$$D_{\alpha_1\alpha_2}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)=-\int d\mathbf{r}'_1\int dt'_1\int d\mathbf{r}'_2\int dt'_2\Theta(t_2-t'_2)\Theta(t'_2-t'_1)\Theta(t'_1-t_1)\mathcal{K}_{\alpha_1\alpha'_1}(\mathbf{r}_1,t_1;\mathbf{r}'_1,t'_1)$$
$$\times\mathcal{K}_{\alpha_2\alpha'_2}(\mathbf{r}_2,t_2;\mathbf{r}'_2,t'_2)\langle[\hat{J}_{\alpha'_1}(\mathbf{r}'_1,t'_1),\hat{J}_{\alpha'_2}(\mathbf{r}'_2,t'_2)]\rangle\ , \tag{7.17}$$

represent so-called time-delayed contributions studied in Ref. 8 and paper I.

We now consider the resonance fluorescence from a single (two-level) atom located at $\mathbf{r}=0$ and driven by a coherent, monochromatic plane-wave field. As known, $\hat{\mathcal{E}}^{(+)}_{as}$ in the radiation zone may be written as[34]

$$\hat{\mathcal{E}}^{(+)}_{as}(\mathbf{r},t)=f_\alpha(\mathbf{r})\hat{b}\left[t-\frac{r}{c}\right]\exp\left[-i\omega_L\left[t-\frac{r}{c}\right]\right]\ , \tag{7.18}$$

where $\hat{b}(t)$ is the slowly varying atomic lowering operator and $f_\alpha(\mathbf{r})$ is the characteristic dipole-field functions. Equation (7.18) may be derived from Eq. (3.14) together with

$$\mathcal{K}_{\alpha\alpha'}(\mathbf{r},t;\mathbf{r}',t')\hat{J}_{\alpha'}(\mathbf{r}',t')=f_\alpha(\mathbf{r}-\mathbf{r}')[\delta(t-t'-|\mathbf{r}-\mathbf{r}'|/c)+\delta(t'-t-|\mathbf{r}-\mathbf{r}'|/c)]\delta(\mathbf{r}')\hat{b}(t')e^{-i\omega_L t'} \tag{7.19}$$

[also cf. Eq. (3.38)]. The second δ function in the brackets in Eq. (7.19) of course does not contribute to the time integral in Eq. (3.14) but it is needed in order to ensure the validity of the symmetry relation given in Eq. (3.17). Inserting Eq. (7.18) in Eq. (7.5) and Eq. (7.19) in Eq. (7.17) and considering the steady-state case, from Eqs. (7.7), (7.16), and (7.14) we arrive at

$$\tilde{\sigma}_\phi(\mathbf{r},\omega)=2\pi\,\mathrm{Re}\left[\int_0^\infty d\tau\cos(\omega\tau)[|f_\alpha(\mathbf{r})|^2\langle\hat{b}^\dagger(0),\hat{b}(\tau)\rangle+f^2_\alpha(\mathbf{r})e^{2i(\phi+\omega_L r/c)}\langle\hat{b}(\tau),\hat{b}(0)\rangle]\right]\ , \tag{7.20}$$

$$\tilde{\sigma}'_\phi(\mathbf{r},\omega)=\sigma_\phi(\mathbf{r},\omega)+\pi\,\mathrm{Re}\left[f^2_\alpha(\mathbf{r})e^{2i(\phi-\omega r/c)}\int_0^\infty d\tau\, e^{-i\omega\tau}\langle[\hat{b}(0),\hat{b}(\tau)]\rangle\right]\ . \tag{7.21}$$

In order to show that $\tilde{\sigma}_\phi$ and $\tilde{\sigma}'_\phi$ can substantially differ from one another we confine ourselves to the case $\omega=0$, for which maximum spectral squeezing is attainable in resonance fluorescence.[15] The atomic correlation functions in Eqs. (7.20) and (7.21) may be calculated from standard methods (atomic Bloch equations and quantum regression theorem). Straightforward calculation yields

$$\tilde{\sigma}_\phi(\mathbf{r},0)=\frac{2\pi}{\gamma}f^2_\alpha(\mathbf{r})\left[\left[\frac{\Omega}{\gamma}\right]^2+\frac{1}{2}\right]^{-3}\times\left\{\frac{1}{2}\left[\frac{\Omega}{\gamma}\right]^6+\left[\frac{\Omega}{\gamma}\right]^4+\left[\frac{1}{2}\left[\frac{\Omega}{\gamma}\right]^6+\frac{1}{4}\left[\frac{\Omega}{\gamma}\right]^2\right]\times\cos[2(\phi+\omega_L r/c)]\right\}\ , \tag{7.22}$$

$$\tilde{\sigma}'_\phi(\mathbf{r},0)=\tilde{\sigma}_\phi(\mathbf{r},0)+\frac{2\pi}{\gamma}f^2_\alpha(\mathbf{r})\left[\left[\frac{\Omega}{\gamma}\right]^2+\frac{1}{2}\right]^{-3}\times\left[\frac{1}{2}\left[\frac{\Omega}{\gamma}\right]^4+\frac{1}{4}\left[\frac{\Omega}{\gamma}\right]^2\right]\times\cos(2\phi)\ , \tag{7.23}$$

where γ is the radiative decay rate and Ω the Rabi frequency. In Eqs. (7.22) and (7.23), f_α is assumed to be real because phases may be included in ϕ.

In Fig. 2 the maximum values of the in-phase spectral functions $\tilde{\sigma}_1(\mathbf{r},\,\omega=0)$ and $\tilde{\sigma}'_1(\mathbf{r},\,\omega=0)$ and the minimum values of the out-of-phase spectral function $\tilde{\sigma}_2(\mathbf{r},\,\omega=0)$ and $\tilde{\sigma}'_2(r,\,\omega=0)$ are shown in dependence on Ω/γ. The maximum intrinsically spectral squeezing attainable at $\omega=0$ is seen to be $\tilde{\sigma}'_2(r,\,\omega=0)/(2\pi/\gamma)f^2_\alpha=-0.235$. In contrast to this the maximum spectral squeezing effect that might be observable is substantially smaller: $\tilde{\sigma}_2(\mathbf{r},\,\omega=0)/(2\pi/\gamma)f^2_\alpha=-0.070$.

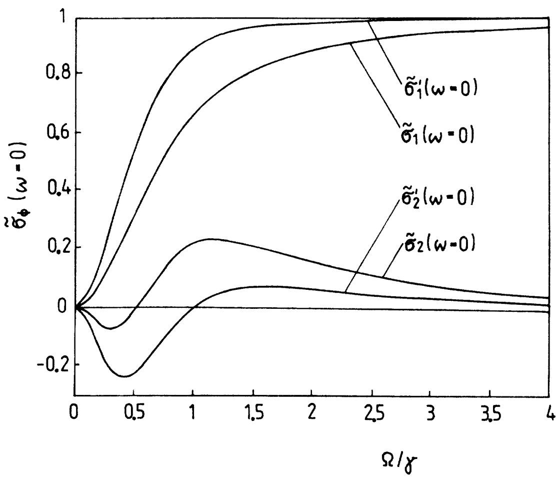

FIG. 2. Maximum values of the normalized in-phase spectral functions $\tilde{\sigma}_1(\omega=0)$ and $\tilde{\sigma}'_1(\omega=0)$ and minimum values of the normalized out-of-phase spectral functions $\tilde{\sigma}_2(\omega=0)$ and $\tilde{\sigma}'_2(\omega=0)$ versus Ω/γ (Ω, Rabi frequency; γ, radiative damping constant).

VIII. SUMMARY AND CONCLUSIONS

Based on the recently developed theory of the action of passive, lossless optical systems in quantum optics (cf. Ref. 25) we have studied the problem of spectral filtering of quantized light fields. The spectral filter (Fabry-Pérot filter type) is modeled by a macroscopic dielectric. Formulas have been derived for the field operators and the time- and normally ordered correlation functions of the filtered light in the presence of atomic sources.

We have shown, that, apart from the second-order (Wiener-Khintchine) spectrum, measurable, spectral properties of nonclassical light fields differ, in principle, from intrinsic, spectral characteristics derived from a full Fourier analysis of the field under study, and hence from a Fourier analysis of field correlation functions which are not ordered in time. This difference reflects the fact that in the presence of sources the full-field commutation rules become different from the free-field commutation rules.

We have studied two different situations, namely, inclined incidence of light as an example without back action of the spectral apparatus to the source, and perpendicular incidence as an example for back action.

In the first case, observable spectral properties (expressed in terms of normally and time-ordered correlation functions) can be related (similar as in classical optics) to the Fourier transforms of (normally ordered) correlation functions of the incoming part of the complete field under study, without need of time ordering. However, with respect to the complete field (which differs from the incoming field in vacuum field contributions), they can only be related to the Fourier transforms of (normally and) time-ordered correlation functions.

In the second case the situation changes drastically. In this case, the back action of the spectral apparatus to the sources gives rise to source-quantity commutator effects and time ordering with respect to the retarded time arguments of the incoming field becomes reasonable. In particular, the transmitted light field (that is the convolution of the incoming field with the transmission response function of the spectral apparatus) does not therefore represent the field relevant for photodetection.

As an example of application we have discussed the problem of spectral squeezing of resonance fluorescence radiation originating from a two-level atom for the case without back action of the spectral apparatus to the radiating atom. We have shown that in this case the measurable, spectral squeezing may substantially differ from the intrinsic one defined by Fourier transformation of the complete fluorescence radiation field. In particular, in the case when the spectral filter is tuned to the frequency of maximum observable squeezing, the observed squeezing effect is substantially lower than the intrinsic one. In other spectral ranges the situation must be inverse, because the two squeezing spectra integrated over the whole frequency range yield equal total squeezing. The difference between the two spectra illustrates the fact that in the case of sources being present the vacuum fluctuations may play a substantial role even for normally ordered correlation functions.

APPENDIX: DERIVATION OF COMMUTATION RELATIONS AND RELATIONS BETWEEN *O* AND *T*-ORDERED OPERATOR PRODUCTS

Starting from Eq. (3.13) and making use of Eqs. (3.22), (3.23), and Eqs. (3.41)–(3.44), respectively, for the cases of perpendicular and inclined incidence of light [cf. Figs. 1(a) and 1(b)] we define

$$E_k^{lmn}(x)=i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}\times[\Theta(l)\Theta(-kx)e^{ikx}+\Theta(m)\Theta(kx)\overline{T}_f(\omega)e^{ikx}+\Theta(n)\Theta(-kx)\overline{R}_f(\omega)e^{-ikx}]\ , \tag{A1a}$$

$$E_{ik}^{lmn}(x)=i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}\times[\Theta(l)\delta_{i1}\Theta(-kx)e^{ikx}+\Theta(m)\delta_{i1}\Theta(kx)\overline{T}_f(\omega)e^{ikx}+\Theta(n)\delta_{i2}\Theta(kx)\overline{R}_f(\omega)e^{ikx}]\ , \tag{A1b}$$

$$\hat{E}^{(+)}_{lmn}(x,t)=\int_{-\infty}^{+\infty}dk\ E_k^{lmn}(x)\hat{a}_{k,\text{free}}(t)\ , \tag{A2a}$$

$$\hat{E}^{(+)}_{lmn}(x,t)=\sum_{i=1}^{2}\int_{-\infty}^{+\infty}dk\ E_{ik}^{lmn}(x)\hat{a}_{ik,\text{free}}(t)\ , \tag{A2b}$$

where the values of l, m, and n are $+1$ or -1. Comparing these definitions (for $x>d/2$) with the field decompositions given in Sec. IV, we see that for perpendicular incidence of light [Eqs. (A1a) and (A2a)] $\hat{E}^{(+)}_{1,1,1}$,

$\hat{E}^{(+)}_{-1,1,-1}$, $\hat{E}^{(+)}_{1,-1,-1}$, and $\hat{E}^{(+)}_{-1,-1,1}$, respectively, are the full free field, the transmitted free field, the free field incident from the right, and its reflected part. In the case of inclined incidence [Eqs. (A1b) and (A2b)], the meaning of $\hat{E}^{(+)}_{lmn}$ is analogous. Note that $\hat{E}^{(+)}_{-1,-1,1}$ is now the reflected part of the free field incident on the y axis from the right.

Using the results given in paper I [cf. Eq. (4.13) in paper I and its derivation] we may represent the commutators for the various free-field parts and the source-field part in the following form:

$$[\hat{E}^{(+)}_{lmn}(x_1,t_1),\hat{E}^{(+)}_{s}(x_2,t_2)]=-\int dx'_1\int dt'_1\int dx'_2\int dt'_2\Theta(t_2-t'_2)\Theta(t'_2-t'_1)K^{lmn}(x_1,t_1;x'_1,t'_1)$$
$$\times K^{111}(x_2,t_2;x'_2,t'_2)[\hat{J}(x'_1,t'_1),\hat{J}(x'_2,t'_2)]\ , \quad \text{(A3)}$$

$$K^{lmn}(x,t;x',t')=-\frac{1}{i\hbar}\int_{-\infty}^{+\infty}dkE_k^{lmn}(x)A_k^*(x')e^{-i\omega(t-t')}$$
$$=-\frac{1}{4\pi\epsilon_0}\int_0^{\infty}dk[\Theta(m)\bar{T}_f(\omega)e^{-i\omega[t-t'-(x-x')/c]}+\Theta(l)\bar{T}^*_f(\omega)e^{-i\omega[t-t'+(x-x')/c]}$$
$$+\Theta(m)\bar{T}_f(\omega)\bar{R}^*_f(\omega)e^{-i\omega[t-t'-(x+x')/c]}+\Theta(n)\bar{R}_f(\omega)\bar{T}^*_f(\omega)e^{-i\omega[t-t'-(x+x')/c]}]\ , \quad \text{(A4a)}$$

$$K^{lmn}(x,t;x',t')=-\frac{1}{i\hbar}\sum_{i=1}^{2}\int_{-\infty}^{+\infty}dk\ E_{ik}^{lmn}(x)A_{ik}^*(x')e^{-i\omega(t-t')}$$
$$=-\frac{1}{4\pi\epsilon_0}\int_0^{\infty}dk[\Theta(m)\bar{T}_f(\omega)e^{-i\omega[t-t'-(x-x')/c]}+\Theta(l)\bar{T}^*_f(\omega)e^{-i\omega[t-t'+(x-x')/c]}]\ . \quad \text{(A4b)}$$

In Eqs. (A4), the functions $A_k(x)$ and $A_{ik}(x)$, respectively, have been taken from Eqs. (3.22), (3.23), and (3.41)–(3.44). Note that $x>d/2$ and $x'<d/2$.

In order to treat Eqs. (A4a) and (A4b) in an uniform manner it is convenient to introduce the parameter α defined by the relation $\alpha=1$ for the case of perpendicular incidence of light [Eq. (A4a)], and $\alpha=0$ for the case of inclined incidence [Eq. (A4b)]. Making use of Eqs. (3.24), (3.25), and (3.28), from Eqs. (A4a) and (A4b) we derive

$$K^{lmn}(x,t;x',t')=-\frac{1-r^2}{4\pi\epsilon_0 c}\int_0^{\infty}d\omega\left[\Theta(l)\frac{e^{-i\omega\delta_+}}{1-r^2e^{-i\sigma\omega}}+\Theta(m)\frac{e^{-i\omega\delta_-}}{1-r^2e^{i\sigma\omega}}\right.$$
$$\left.+\alpha[\Theta(m)-\Theta(n)]\frac{r}{1+r^2}\left[\frac{e^{-i\omega D_+}}{1-r^2e^{-i\sigma\omega}}-\frac{e^{-i\omega D_-}}{1-r^2e^{i\sigma\omega}}\right]\right]\ , \quad \text{(A5)}$$

where

$$\delta_\pm=t-t'\pm l_f/c\ , \quad \text{(A6)}$$

$$D_\pm=D\pm\sigma/2\ , \quad \text{(A7)}$$

$$D=t-t'-(x+x')/c\ , \quad \text{(A8)}$$

$$\sigma=2n_fd/c\ , \quad \text{(A9)}$$

l_f being given in Eq. (3.35).

Since we are dealing with optical frequencies we may (approximately) extend the integral in Eq. (A5) to minus infinity. Making use of the relation

$$\int_{-\infty}^{+\infty}d\omega\frac{e^{i\omega z}}{1-r^2e^{\pm i\omega\sigma}}=2\pi\sum_{j=0}^{\infty}r^{2j}\delta(z\pm j\sigma) \quad \text{(A10)}$$

and remembering the relation

$$e^{-\Gamma_f\sigma}=r^2 \quad \text{(A11)}$$

[cf. Eq. (3.30) together with Eq. (A9)] we therefore obtain the result

$$K^{lmn}(x,t;x',t')=-\frac{1-r^2}{2\epsilon_0 c}\sum_{j=0}^{\infty}\left[\Theta(l)e^{\Gamma_f\delta_+}\delta(j\sigma+\delta_+)+\Theta(m)e^{-\Gamma_f\delta_-}\delta(j\sigma-\delta_-)\right.$$
$$\left.+\alpha[\Theta(m)-\Theta(n)]\frac{r}{1+r^2}[e^{\Gamma_fD_+}\delta(j\sigma+D_+)-e^{-\Gamma_fD_-}\delta(j\sigma-D_-)]\right]\ , \quad \text{(A12)}$$

which, after some simple manipulation, may be rewritten as follows:

$$K^{lmn}(x,t;x',t')=-\frac{1-r^2}{2\epsilon_0 c}\sum_{j=-\infty}^{+\infty}\left[\Theta(l)e^{\Gamma_f\delta_+}\Theta(-\delta_+)\delta(j\sigma+\delta_+)+\Theta(m)e^{-\Gamma_f\delta_-}\Theta(\delta_-)\delta(j\sigma-\delta_-)\right.$$
$$\left.+\alpha[\Theta(m)-\Theta(n)]\frac{1}{1+r^2}[e^{\Gamma_f D}\Theta(-D)\delta(j\sigma+D_+)-e^{-\Gamma_f D}\Theta(D)\delta(j\sigma-D_-)]\right] . \tag{A13}$$

Making use of the identity

$$\sum_{j=-\infty}^{+\infty}e^{i(2\pi jz/\sigma)}=\sigma\sum_{j=-\infty}^{+\infty}\delta(j\sigma-z) \tag{A14}$$

and taking into account that the relation $\exp(\Gamma_f\sigma/2)=r^{-1}$ is valid we arrive at

$$K^{lmn}(x,t;x',t')=-\frac{1-r^2}{2\epsilon_0 c\sigma}\sum_j\left[(\Theta(l)e^{\Gamma_f\delta_+}\Theta(-\delta_+)e^{-i\omega_j\delta_+}+\Theta(m)e^{-\Gamma_f\delta_-}\Theta(\delta_-)e^{-i\omega_j\delta_-}\right.$$
$$\left.+\alpha[\Theta(m)-\Theta(n)]\frac{(-1)^j}{1+r^2}e^{-\Gamma_f|D|-i\omega_j D}[\Theta(-D)-\Theta(D)]\right] , \tag{A15}$$

where

$$\omega_j=j\frac{2\pi}{\sigma} \tag{A16}$$

are the resonance frequencies of the Fabry-Pérot filter [cf. Eq. (3.30) together with Eq. (A9)]. Note that in Eq. (A15) the sum over j may be restricted to positive (optical) frequencies ω_j. The (off-resonant) negative-frequency terms formally appear in consequence of the extension of the original frequency integral to minus infinity.

In particular, in the case, when the bandwidth of the (source) light studied is small compared with the difference of neighboring resonance frequencies $\Delta\omega_f=2\pi/\sigma$ we may restrict ourselves to a single resonance frequency $\omega_f=f2\pi/\sigma$. Furthermore, assuming a highly reflective filter ($r\simeq 1$) we may simplify Eq. (A15) as follows:

$$K^{lmn}(x,t;x',t')=-\frac{1}{2\epsilon_0 c}\left[\Theta(l)T_f^*(t'-t-l_f/c)+\Theta(m)T_f(t-t'-l_f/c)\right.$$
$$\left.+\alpha[\Theta(m)-\Theta(n)]\frac{(-1)^f}{2}[T_f^*(t'-t+(x+x')/c)-T_f(t-t'-(x+x')/c)]\right] , \tag{A17}$$

where $T_f(t)$ is given in Eq. (3.36).

Remembering the relation $K_f(x,t;x',t')=K^{111}(x,t;x',t')$ [cf. Eqs. (3.31), (3.46), (A4), and (A1)], from Eq. (A17) we easily obtain

$$K_f(x,t;x',t')=-\frac{1}{2\epsilon_0 c}[T_f(t-t'-l_f/c)+T_f^*(t'-t-l_f/c)] . \tag{A18}$$

Combining Eqs. (A3) and (A17) and taking into account Eq. (3.36) we derive

$$[\hat{E}^{(+)}_{lmn}(x_1,t_1),\hat{E}^{(+)}_s(x_2,t_2)]=-\left[\frac{1}{2\epsilon_0 c}\right]^2\int dx_1'\int dt_1'\int dx_2'\int dt_2'\Theta(t_2'-t_1')$$
$$\times\{\Theta(l)T_f^*(t_1'-t_1-l_f^{(1)}/c)+\Theta(m)T_f(t_1-t_1'-l_f^{(1)}/c)$$
$$+\alpha[\Theta(m)-\Theta(n)]\frac{(-1)^f}{2}[T_f^*(t_1'-t_1+(x_1+x_1')/c)$$
$$-T_f(t_1-t_1'-(x_1+x_1')/c)]\}$$
$$\times T_f(t_2-t_2'-l_f^{(2)}/c)[\hat{J}(x_1',t_1'),\hat{J}(x_2',t_2')] , \tag{A19}$$

where

$$l_f^{(i)}=x_i-x_i'+(n_f-1)d>0 . \tag{A20}$$

From Eq. (A19) together with Eq. (3.36) it is readily seen that the relation

$$[\hat{E}^{(+)}_{1,-1,-1}(x_1,t_1),\hat{E}^{(+)}_s(x_2,t_2)]=0,\quad t_1>t_2 \tag{A21}$$

is valid. Remembering that according to Eqs. (A2) and (4.20) the relations $\hat{E}^{(+)}_{1,-1,-1}=\hat{\mathcal{E}}^{(+)}_{\leftarrow,\mathrm{free}}(t_1+x_1/c)=\hat{E}^{(+)}_{\leftarrow}(x_1,t_1)$ are valid and taking into account that the free-field part of $\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)$ and $\hat{\mathcal{E}}^{(+)}_{\leftarrow,\mathrm{free}}(t_1+x_1/c)$ commute, from Eq. (A21) we obtain

$$[\hat{E}^{(+)}_{\leftarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]=0,\quad t_1>t_2\ . \tag{A22}$$

Combining Eqs. (A2) and (4.18), (4.22), (4.15), (4.17) we may write

$$\begin{aligned}[\hat{E}^{(+)}_{\rightarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]= {} & [\hat{E}^{(+)}_{s}(x_1,t_1),\hat{E}^{(+)}_{s}(x_2,t_1)]+[\hat{E}^{(+)}_{-1,1,1}(x_1,t_1),\hat{E}^{(+)}_{s}(x_2,t_2)]\\ & -[\hat{E}^{(+)}_{-1,1,1}(x_2,t_2),\hat{E}^{(+)}_{s}(x_1,t_1)]\ .\end{aligned} \tag{A23}$$

Making use of Eqs. (3.14), (A18), and (A19), from Eq. (A23) we derive

$$\begin{aligned}&[\hat{E}^{(+)}_{\rightarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]\\ &\quad=\left[\frac{1}{2\epsilon_0 c}\right]^2\int dx_1'\int dt_1'\int dx_2'\int dt_2' T_f(t-t_1'-l_f^{(1)}/c)T_f(t_2-t_2'-l_f^{(2)}/c)[\hat{J}(x_1',t_1'),\hat{J}(x_2',t_2')]\\ &\qquad\times[1-\Theta(t_2'-t_1')-\Theta(t_1'-t_2')]\ .\end{aligned} \tag{A24}$$

Taking into account the relation $\Theta(t_2'-t_1')+\Theta(t_1'-t_2')=1$, we arrive at the result

$$[\hat{E}^{(+)}_{\rightarrow}(x_1,t_1),\hat{E}^{(+)}_{\rightarrow}(x_2,t_2)]=0\ . \tag{A25}$$

Combining Eqs. (A2), (4.15), and (4.17) we may write

$$\hat{E}^{(+)}_{\mathrm{trans}}(x,t)=\hat{E}^{(+)}_{s}(x,t)+\hat{E}^{(+)}_{-1,1,-1}(x,t) \tag{A26}$$

and hence

$$\begin{aligned}\hat{E}^{(+)}_{\mathrm{trans}}(x_1,t_1)\hat{E}^{(+)}_{\mathrm{trans}}(x_2,t_2)= {} & \hat{E}^{(+)}_{s}(x_1,t_1)\hat{E}^{(+)}_{s}(x_2,t_2)+\hat{E}^{(+)}_{-1,1,-1}(x_1,t_1)\hat{E}^{(+)}_{-1,1,-1}(x_2,t_2)\\ & +\hat{E}^{(+)}_{s}(x_1,t_1)\hat{E}^{(+)}_{-1,1,-1}(x_2,t_2)+\hat{E}^{(+)}_{s}(x_2,t_2)\hat{E}^{(+)}_{-1,1,-1}(x_1,t_1)\\ & +[\hat{E}^{(+)}_{-1,1,-1}(x_1,t_1),\hat{E}^{(+)}_{s}(x_2,t_2)]\ .\end{aligned} \tag{A27}$$

Substituting in the first term on the right-hand side of Eq. (A27) for the source-term operators $\hat{E}^{(+)}_{s}$ Eq. (3.14) together with Eq. (A18) and Eq. (3.36), substituting for the commutator Eq. (A19), and remembering the definition of the ordering symbol O we derive after some algebra

$$T_+\hat{E}^{(+)}_{\mathrm{trans}}(x_1,t_1)\hat{E}^{(+)}_{\mathrm{trans}}(x_2,t_2)=O(\hat{E}^{(+)}_{\mathrm{trans}}(x_1,t_1)\hat{E}^{(+)}_{\mathrm{trans}}(x_2,t_2))-\hat{\Delta}(x_1,t_1;x_2,t_2)\ , \tag{A28}$$

where

$$\begin{aligned}\hat{\Delta}_T(x_1,t_1;x_2,t_2)=\alpha\left[\frac{1}{2\epsilon_0 c}\right]^2\frac{(-1)^f}{2}\int dx_i'\int dt_i'\int dx_j'\int dt_j'\Theta(t_j'-t_i')[T_f^*(t_i'-t_i+(x_i+x_i')/c)\\ -T_f(t_i-t_i'-(x_i+x_i')/c)]\\ \times T_f(t_j-t_j'-l_f^{(j)}/c)[\hat{J}(x_i',t_i'),\hat{J}(x_j',t_j')]\ .\end{aligned} \tag{A29}$$

Here, the indices i and j are determined from the requirements $t_i=\max(t_1,t_2)$ and $t_j=\min(t_1,t_2)$.

In particular, in the case of a pointlike source located at $-s$ $(s>d/2)$ we may write

$$\hat{J}(x,t)=\delta(x+s)\hat{J}(t)\ . \tag{A30}$$

Combining Eqs. (A29), (A30), (A20), and (3.36) we arrive at the following result:

$$\begin{aligned}\hat{\Delta}_T(x_1,t_1;x_2,t_2)=\alpha\Gamma_f^2\left[\frac{1}{2\epsilon_0 c}\right]^2\frac{(-1)^f}{2}\int dt_i'\int dt_j'\Theta(t_j^r-t_j')\Theta(t_j'-t_i')[\Theta(t_i'-t_i^r-2s_f/c)-\Theta(t_i^r+2s_f/c-t_i')]\\ \times\exp\{-i\omega_f[t_i+t_j-t_i'-t_j'-(x_{fi}+x_{fj})/c]\}\\ \times\exp[-\Gamma_f(t_j^r-t_j'+|t_i^r+2s_f/c-t_i'|)][\hat{J}(t_i'),\hat{J}(t_j')]\ ,\end{aligned} \tag{A31}$$

where

$$t^r_{i(j)} = t_{i(j)} - (x_{fi(j)} + s_f)/c \ , \qquad (A32)$$

$$x_{fi(j)} = x_{i(j)} + (n_f - 1)d/2 \ , \qquad (A33)$$

$$s_f = s + (n_f - 1)d/2 \ . \qquad (A34)$$

We note that in the case $\alpha=0$ (inclined incidence) from Eq. (A19)

$$[\hat{E}^{(+)}_{-1,-1,1}(x_1,t_1), \hat{E}^{(+)}_{s}(x_2,t_2)] = 0 \qquad (A35)$$

is seen to be valid. Remembering that

$$\hat{E}^{(+)}_{\rightarrow}(x,t) = \hat{E}^{(+)}_{\text{trans}}(x,t) + \hat{E}^{(+)}_{-1,-1,1}(x,t) \ , \qquad (A36)$$

and taking into account that $\hat{E}^{(+)}_{-1,-1,1}$ is a free-field operator which commutes with the free-field part of $\hat{E}^{(+)}_{\text{trans}}$, from Eq. (A25) together with Eqs. (A35) and (A36) we therefore derive for the case when $\alpha=0$,

$$[\hat{E}^{(+)}_{\text{trans}}(x_1,t_1), \hat{E}^{(+)}_{\text{trans}}(x_2,t_2)] = 0 \ , \qquad (A37)$$

so that Eq. (A28) can be simplified as follows:

$$\hat{E}^{(+)}_{\text{trans}}(x_1,t_1)\hat{E}^{(+)}_{\text{trans}}(x_2,t_2) = O(\hat{E}^{(+)}_{\text{trans}}(x_1,t_1)\hat{E}^{(+)}_{\text{trans}}(x_2,t_2)) \ . \qquad (A38)$$

We now define

$$\mathcal{E}^{lm}_k(x) = i\left[\frac{\hbar\omega}{4\pi\epsilon_0}\right]^{1/2}[\Theta(l)\Theta(-kx)e^{ikx} + \Theta(m)\Theta(kx)e^{ikx}] \ , \qquad (A39a)$$

$$\mathcal{E}^{lm}_{ik}(x) = \delta_{i1}\mathcal{E}^{lm}_k(x) \ , \qquad (A39b)$$

$$\hat{\mathcal{E}}^{(+)}_{lm}(x,t) = \int_{-\infty}^{+\infty} dk\ \mathcal{E}^{lm}_k(x)\hat{a}_{k,\text{free}}(t) \ , \qquad (A40a)$$

$$\hat{\mathcal{E}}^{(+)}_{lm}(x,t) = \sum_{i=1}^{2}\int_{-\infty}^{+\infty} dk\ \mathcal{E}^{lm}_{ik}(x)\hat{a}_{ik,\text{free}}(t) \ , \qquad (A40b)$$

where the values of l and m are again $+1$ or -1. Applying Eq. (5.11) to the field propagating in the x direction $[\hat{\mathcal{E}}^{(+)}_{\alpha}(\mathbf{r},t) \rightarrow \hat{\mathcal{E}}^{(+)}(x,t)]$ and making use of Eqs. (A39) and (A40) yields ($x > d/2$)

$$\hat{\mathcal{E}}^{(+)}(x,t) = \hat{\mathcal{E}}^{(+)}_{\text{free}}(x,t) + \hat{\mathcal{E}}^{(+)}_{s}(x,t) \ , \qquad (A41)$$

where

$$\hat{\mathcal{E}}^{(+)}_{\text{free}}(x,t) = \hat{\mathcal{E}}^{(+)}_{\rightarrow,\text{free}}\left[t - \frac{x}{c}\right] + \hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t + \frac{x}{c}\right] \ , \qquad (A42)$$

$$\hat{\mathcal{E}}^{(+)}_{\rightarrow,\text{free}}\left[t - \frac{x}{c}\right] = \hat{\mathcal{E}}^{(+)}_{-1,1}(x,t) \ , \qquad (A43)$$

$$\hat{\mathcal{E}}^{(+)}_{\leftarrow,\text{free}}\left[t + \frac{x}{c}\right] = \hat{\mathcal{E}}^{(+)}_{-1,1}(x,t) \ , \qquad (A44)$$

$$\hat{\mathcal{E}}^{(+)}_{\text{free}}(x,t) = \hat{\mathcal{E}}^{(+)}_{1,1}(x,t) \ , \qquad (A45)$$

and, according to Eqs. (3.14) and (3.38),

$$\hat{\mathcal{E}}^{(+)}_{s}(x,t) = \int dx' \int dt' \Theta(t-t')\mathcal{K}(x,t;x',t')\hat{J}(x',t') \ . \qquad (A46)$$

In complete analogy to Eq. (A3) we may write

$$[\hat{\mathcal{E}}^{(+)}_{lm}(x_1,t_1), \hat{\mathcal{E}}^{(+)}_{s}(x_2,t_2)] = -\int dx'_1 \int dt'_1 \int dx'_2 \int dt'_2 \Theta(t_2-t'_2)\Theta(t'_2-t'_1)K^{lm}(x_1,t_1;x'_1,t'_1) \times \mathcal{K}(x_2,t_2;x'_2,t'_2)[\hat{J}(x'_1,t'_1),\hat{J}(x'_2,t'_2)] \ , \qquad (A47)$$

where now [in analogy to the calculations leading to Eq. (A17)]

$$K^{lm}(x,t;x',t') = -\frac{1}{2\epsilon_0 c}[\Theta(m)\delta(t-t'-(x-x')/c) + \Theta(l)T^*_f(t'-t-l_f/c) + \alpha\Theta(m)R^*_f(t-t'-(x+x'+d)/c)] \ . \qquad (A48)$$

Combining Eqs. (A41) and (A45) we may write

$$\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2) = \hat{\mathcal{E}}^{(+)}_{s}(x_1,t_1)\hat{\mathcal{E}}^{(+)}_{s}(x_2,t_2) + \hat{\mathcal{E}}^{(+)}_{1,1}(x_1,t_1)\hat{\mathcal{E}}^{(+)}_{1,1}(x_2,t_2) + \hat{\mathcal{E}}^{(+)}_{s}(x_1,t_1)\hat{\mathcal{E}}^{(+)}_{1,1}(x_2,t_2) + \hat{\mathcal{E}}^{(+)}_{s}(x_2,t_2)\hat{\mathcal{E}}^{(+)}_{1,1}(x_1,t_1) + [\hat{\mathcal{E}}^{(+)}_{1,1}(x_1,t_1), \hat{\mathcal{E}}^{(+)}_{s}(x_2,t_2)] \ . \qquad (A49)$$

Making use of Eqs. (A46), (3.38), (A47), and (A48) ($l=m=1$), and remembering the definition of the ordering symbol O, from Eq. (A49) we derive

$$\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2) = O(\hat{\mathcal{E}}^{(+)}(x_1,t_1)\hat{\mathcal{E}}^{(+)}(x_2,t_2)) - \hat{\delta}^{(0,2)}(x_1,t_1;x_2,t_2) + \frac{1}{2\epsilon_0 c}\int dx'_1 \int dt'_1 \int dx'_2 \int dt'_2 \Theta(t_2-t'_2)\Theta(t'_2-t'_1)\mathcal{K}(x_2,t_2;x'_2,t'_2) \times T^*_f(t'_1-t_1-l^{(1)}_f/c)[\hat{J}(x'_1,t'_1),\hat{J}(x'_2,t'_2)] \ , \qquad (A50)$$

where

$$\hat{\delta}^{(0,2)}(x_1,t_1;x_2,t_2) = -\frac{\alpha}{2\epsilon_0 c}\int dx'_1 \int dt'_1 \int dx'_2 \int dt'_2 \Theta(t_2-t'_2)\Theta(t'_2-t'_1)R^*_f(t'_1-t_1+(x_1+x'_1+d)/c) \times [\hat{J}(x'_1,t'_1),\hat{J}(x'_2,t'_2)] \ . \qquad (A51)$$

Since $T_f(t)=0$ if $t<0$, we easily see that

$$T_+(\hat{\mathscr{E}}^{(+)}(x_1,t_1)\hat{\mathscr{E}}^{(+)}(x_2,t_2))=O(\hat{\mathscr{E}}^{(+)}(x_1,t_1)\hat{\mathscr{E}}^{(+)}(x_2,t_2))-\hat{\delta}_T^{(0,2)}(x_1,t_1;x_2,t_2)\ , \tag{A52}$$

where

$$\hat{\delta}_T^{(0,2)}(x_1,t_1;x_2,t_2)=\hat{\delta}^{(0,2)}(x_i,t_i;x_j,t_j)\ , \tag{A53}$$

the indices i and j being determined from the requirements $t_i=\max(t_1,t_2)$ and $t_j=\min(t_1,t_2)$.

Clearly, in the case when $\alpha=0$ is valid (inclined incidence), Eq. (A52) simply reads as

$$T_+(\hat{\mathscr{E}}^{(+)}(x_1,t_1)\hat{\mathscr{E}}^{(+)}(x_2,t_2))=O(\hat{\mathscr{E}}^{(+)}(x_1,t_1)\hat{\mathscr{E}}^{(+)}(x_2,t_2))\ . \tag{A54}$$

From the derivation of Eq. (A50) it is seen that the term proportional to T_f^* arises from $\hat{\mathscr{E}}^{(+)}_{\leftarrow,\text{free}}$ in Eq. (A42) [cf. Eqs. (A44) and (A48)]. That is, performing the calculations given above for the case of $\hat{\mathscr{E}}^{(+)}_{\text{inc}}$ [defined in Eq. (4.17)] instead of $\hat{\mathscr{E}}^{(+)}$, we arrive at Eq. (A50), however, without the term proportional to T_f^*:

$$\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_1,t_1)\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_2,t_2)=O(\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_1,t_1)\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_2,t_2))-\hat{\delta}^{(0,2)}(x_1,t_1;x_2,t_2)\ . \tag{A55}$$

In particular, in the case when $\alpha=0$ is valid (inclined incidence) we simply obtain

$$\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_1,t_1)\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_2,t_2)=O(\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_1,t_1)\hat{\mathscr{E}}^{(+)}_{\text{inc}}(x_2,t_2))\ . \tag{A56}$$

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*Present address: Sektion Mathematik/Physik, Pädagogische Hochschule "Liselotte Herrmann," Güstrow, 2600, German Democratic Republic.

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