Inverse-free-electron-laser beat-wave accelerator

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It has been proposed [J. L. Bobin, Opt. Commun. **55**, 413 (1985)] that the presence of a plasma can, under certain conditions, enhance the accelerating electric field in an inverse free-electron laser. In this scheme, the beat wave generated by a laser and an undulator is expected to couple to the plasma oscillations generated by the electron beam streaming through the plasma. We have undertaken an analytical and numerical study of the proposed acceleration scheme. Our results show that the electric field is dominantly the self-field of the electron beam, and the plasma makes a negligible contribution to the electric field. Based on our analysis, we propose an alternative method of acceleration that employs a high-current electron beam to generate a beat wave that is subsequently used to accelerate a higher-energy beam. We show that an accelerating electric field ~ 1 MeV/cm can be achieved with an electron beam of current density ~ 20 kA/cm^2. The results of the analytical study agree well with numerical results from a two-dimensional computer code. The parameters of a proof-of-principle experiment are presented.

I. INTRODUCTION

The acceleration of charged particles by plasma waves has been a subject of considerable interest in recent years.[1] The most prominent of such schemes is the plasma beat-wave accelerator[2] (PBWA), which uses two intense collinear laser beams to generate a beat wave at the plasma frequency in a high-density plasma. When the difference in the frequency of the two lasers is equal to the plasma frequency, the amplitude of the beat wave can grow to a very high value.[3,4] Furthermore, if each of the laser frequencies is much higher than the plasma frequency, the phase velocity of the beat wave may be made to approach c—the speed of light in vacuum. Thus, charged particles trapped in the troughs of the plasma waves may be accelerated to very high energies.

Another laser accelerator of considerable interest is the inverse free-electron laser[5-7] (IFEL). In an IFEL, energy is transferred from a laser to a relativistic electron beam in the presence of the magnetic field of an undulator. In the rest frame of the electron beam, the magnetostatic field of the undulator is transformed into an electromagnetic wave that beats with the laser. In order to accelerate the electrons in the beam, it is necessary to keep the relative phase of the electrons with respect to the beat-wave constant. In an IFEL, this synchronism is sustained by varying either the undulator period or the magnetic field, or both.

Since plasmas are capable of generating large electric fields, it may seem advantageous to introduce a plasma in an IFEL in order to enhance its acceleration possibilities. This has indeed been proposed by Bobin,[8] and provided the initial stimulus for the present study. Bobin's idea is to use the beat wave (of frequency ω and wave number k) generated by a laser (of frequency ω_l and wave number k_l) and the zero-frequency undulator (of wave number k_w) to accelerate an electron beam in the presence of a plasma. The beat wave, which obeys the Manly-Rowe relations $\omega=\omega_l$, $k=k_l+k_w$, is required to satisfy the "two-stream" dispersion equation

$$\frac{\omega_p^2}{\omega^2}+\frac{\omega_{pb}^2}{\gamma_b^3(\omega-kv_b)^2}=1 , \tag{1}$$

where ω_p and ω_{pb} are, respectively, the plasma frequencies of the background plasma and the electron beam, v_b is the axial velocity of the electron beam, and γ_b its Lorentz energy factor. Bobin reported electric field gradients substantially larger than are attained in an IFEL.

In Sec. II, we develop an analytical nonlinear fluid theory to examine Bobin's suggestion. Our results indicate that although the total electric field can be high in an IFEL containing a plasma, this electric field is mainly the self-field of the electron beam, and the contribution of the background plasma is very small. Hence, there does not appear to be any tangible benefit in introducing a plasma in an IFEL.

In spite of the flaw in Bobin's scheme, an interesting variant suggests itself. The electron beam, in the presence of an immobile neutralizing background, is a moving plasma. At high currents, this plasma can produce large electric fields under the influence of a laser and an undulator. This electric field is associated with the beat wave, which has a phase velocity $v_p=\omega_l/(k_l+k_w)$, and obeys the dispersion equation

$$v_b=\frac{\omega_l}{k_l+k_w}-\frac{\omega_{pb}}{\gamma_{pb}^{3/2}(k_l+k_w)} , \tag{2}$$

which is the "fast-wave" solution of Eq. (1) with ω_p set equal to zero. [Equation (2) is precisely the dispersion equation for an IFEL modified by the effect of space charge.] We now propose the following variant of the PBWA and IFEL, which for reasons that will become obvious, is called the inverse free-electron-laser beat-wave accelerator (IFELBWA). In an IFELBWA, we rely on the mechanism of an IFEL to bunch a dense but low-energy electron beam, and then use the electric field generated between the bunches to accelerate another higher-energy electron beam. A problem associated with this concept is that the phase velocity v_p of the beat wave is not close to c. The idea, therefore, may appear to be of little interest for accelerating particles to high energies. In Sec. III, we show that this is not an insuperable problem, and propose two methods by which the phase velocity of the beat wave can be enhanced. We then present results of a numerical simulation for the IFELBWA. We conclude in Sec. IV with a discussion of our results.

II. ANALYTICAL NONLINEAR FLUID THEORY

The theoretical underpinnings of the PBWA came from a paper by Rosenbluth and Liu,[3] who developed an elegant Lagrangian theory to analyze the problem of excitation of plasma waves by two laser beams. The same results were obtained by Mori[9] from an Eulerian theory. Here we extend the analytical calculation of Mori to a system of two cold interpenetrating beams. Each beam has a particle density n_α $(\alpha=1,2)$ and moves along the x axis with speed v_α. In the presence of a laser and an undulator, the motion of the beams is perturbed by the electromagnetic (em) fields of the laser:

$$\mathbf{B}_l=\hat{\mathbf{y}}B_l\sin(k_l x-\omega_l t)\ , \tag{3a}$$

$$\mathbf{E}_l=-\hat{\mathbf{z}}\frac{\omega_l}{ck_l}B_l\sin(k_l x-\omega_l t)\ , \tag{3b}$$

and the undulator,

$$\mathbf{B}_w=\hat{\mathbf{y}}B_w(\sin k_w x)\ , \tag{3c}$$

where B_l and B_w are constant amplitudes.

For simplicity, we assume here that all variables depend on x and t only. In this one-dimensional (1D) model, the dynamics of the plasma fluids is governed by the momentum equation, continuity equation, and Gauss' law, given, respectively, by

$$\frac{d\mathbf{P}_\alpha}{dt}=\left[\frac{\partial}{\partial t}+v_{\alpha x}\frac{\partial}{\partial x}\right]\mathbf{P}_\alpha=e_\alpha\left[\mathbf{E}+\frac{\mathbf{v}_\alpha\times\mathbf{B}}{c}\right]\ , \tag{4}$$

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x}(n_\alpha v_{\alpha x})=0\ , \tag{5}$$

$$\frac{\partial E_x}{\partial x}=4\pi\sum_{\alpha=1}^{2}e_\alpha n_\alpha\ , \tag{6}$$

where $\mathbf{P}_\alpha=\gamma_\alpha m_\alpha \mathbf{v}_\alpha$ is the relativistic momentum of the charged particles of rest mass m_α and charge e_α, $\mathbf{E}=E_x\hat{\mathbf{x}}+\mathbf{E}_l$ is the total electric field, and $\mathbf{B}=\mathbf{B}_w+\mathbf{B}_l$ is the total magnetic field. By using the relation $\gamma_\alpha=(1-v_\alpha^2/c^2)^{-1/2}$, Eq. (4) can be reduced exactly to

$$\frac{\partial \mathbf{v}_\alpha}{\partial t}=-v_{\alpha x}\frac{\partial}{\partial \mathbf{x}}\mathbf{v}_\alpha+\frac{e_\alpha}{m_\alpha\gamma_\alpha}\left[\left[\overleftrightarrow{\mathbf{I}}-\frac{\mathbf{v}_\alpha\mathbf{v}_\alpha}{c^2}\right]\mathbf{E}+\frac{\mathbf{v}_\alpha\times\mathbf{B}}{c}\right]\ , \tag{7}$$

where $\overleftrightarrow{\mathbf{I}}$ is the unit dyadic.

Equations (5), (6), and (7), form a complete set of equations for unknowns n_α, $\mathbf{v}_\alpha$, and E_x. To simplify the task of solving this set of equations, we first write

$$n_\alpha=n_{\alpha0}+n_{\alpha1},\quad \mathbf{v}_\alpha=v_{\alpha0}\hat{\mathbf{x}}+\mathbf{v}_{\alpha1}\ , \tag{8}$$

where $n_{\alpha0}$ and $v_{\alpha0}$ are constants and are the initial equilibrium values of n_α and $\mathbf{v}_\alpha$, and $n_{\alpha1}$ and $\mathbf{v}_{\alpha1}$ are perturbed quantities, assumed to be small. We then make the expansion

$$\frac{1}{\gamma_\alpha}\simeq\frac{1}{\gamma_{\alpha0}}\left[1-\gamma_{\alpha0}^2\frac{\epsilon v_{\alpha0}v_{\alpha1x}+\epsilon^2 v_{\alpha1}^2/2+\epsilon^2\gamma_{\alpha0}^2 v_{\alpha1x}^2/(2c^2)}{c^2}\right]+O(\epsilon^3)\ , \tag{9}$$

where $\gamma_{\alpha0}$ is constant, and a small parameter ϵ is introduced for the purpose of bookkeeping and is to be set to unity eventually. Note that we have assumed that the quantities $\mathbf{v}_{\alpha1}$, $n_{\alpha1}$, and E_x are each of first order in ϵ.

The system described by Bobin is a special case of the system described above, in which one stream is a stationary plasma, with $v_{p0}=0$. We have assumed that the ions in the plasma do not respond fast enough and simply form a uniform neutralizing background. Substituting Eqs. (8) and (9) in (5)–(7) and keeping terms up to $O(\epsilon^2)$, we obtain the equations for the perturbed quantities $\mathbf{v}_{\alpha1}$, $n_{\alpha1}$, and E_x,

$$\begin{aligned}\frac{\partial v_{\alpha1x}}{\partial t}+(v_{\alpha0}+\epsilon v_{\alpha1x})\frac{\partial v_{\alpha1x}}{\partial x}=&\frac{e_\alpha}{m_\alpha\gamma_{\alpha0}^3}E_x-\frac{e_\alpha}{m_\alpha\gamma_{\alpha0}}\frac{\epsilon v_{\alpha1z}B_y}{c}\left[1-\frac{\gamma_{\alpha0}^2}{c^2}\left[\epsilon v_{\alpha0}v_{\alpha1x}+\frac{\epsilon^2 v_{\alpha1}^2}{2}+\frac{\epsilon^2\gamma_{\alpha0}^2 v_{\alpha0}^2 v_{\alpha1x}^2}{2c^2}\right]\right]\\&-\frac{e_\alpha}{m_\alpha\gamma_{\alpha0}}\left[3\epsilon v_{\alpha0}v_{\alpha1x}+\epsilon^2\left[v_{\alpha1x}^2+v_{\alpha1}^2/2-\frac{3\gamma_{\alpha0}^2 v_{\alpha0}^2 v_{\alpha1x}^2}{2c^2}\right]\right]\frac{E_x}{c^2}\\&-\frac{e_\alpha}{m_\alpha\gamma_{\alpha0}}\left[\epsilon v_{\alpha0}v_{\alpha1z}+\epsilon^2\left[1-\frac{\gamma_{\alpha0}^2 v_{\alpha0}^2}{c^2}\right]v_{\alpha1x}v_{\alpha1z}\right]\frac{E_z}{c^2}\ ,\end{aligned} \tag{10}$$

$$\frac{\partial v_{\alpha 1z}}{\partial t}+(v_{\alpha 0}+\epsilon v_{\alpha 1x})\frac{\partial v_{\alpha 1z}}{\partial x}=\frac{e_\alpha}{m_\alpha\gamma_{\alpha 0}}\left\{\left[\frac{v_{\alpha 0}}{c}B_y+E_z\right]\left[1-\frac{\gamma_{\alpha 0}^2}{c^2}\left[\epsilon v_{\alpha 0}v_{\alpha 1x}+\frac{\epsilon^2 v_{\alpha 1}^2}{2}+\frac{\epsilon^2\gamma_{\alpha 0}^2 v_{\alpha 0}^2 v_{\alpha 1z}^2}{2c^2}\right]\right]\right.$$
$$\left.-\frac{\epsilon}{c}\left[\frac{v_{\alpha 0}v_{\alpha 1z}}{c}E_x-v_{\alpha 1x}B_y\right]\left[1-\epsilon\frac{\gamma_{\alpha 0}^2 v_{\alpha 1x}}{c^2}\right]-\frac{\epsilon^2 v_{\alpha 1z}}{c^2}(v_{\alpha 1z}E_z+v_{\alpha 1x}E_x)\right\}, \tag{11}$$

$$\frac{\partial n_{\alpha 1}}{\partial t}+(v_{\alpha 0}+\epsilon v_{\alpha 1x})\frac{\partial n_{\alpha 1}}{\partial x}+(n_{\alpha 0}+\epsilon n_{\alpha 1})\frac{\partial v_{\alpha 1x}}{\partial x}=0\ , \tag{12}$$

and

$$\frac{\partial E_x}{\partial x}=\sum_{\alpha=1}^{2}4\pi e_\alpha n_{\alpha 1}\ . \tag{13}$$

To solve Eqs. (10)–(13), we make the following Bogoliubov-Mitropolsky expansions:[10]

$$\begin{aligned}
E_x&=e_0\cos\Psi_e+\sum_{i=1}\epsilon^i e_i(e_0,\Psi_e)\\
n_{\alpha 1}&=N_{\alpha 0}\sin\Psi_{an}+\sum_{i=1}\epsilon^i N_{\alpha i}(N_{\alpha 0},\Psi_{an})\\
v_{\alpha 1x}&=u_{\alpha 0}\sin\Psi_{au}+\sum_{i=1}\epsilon^i u_{\alpha i}(u_{\alpha 0},\Psi_{au})\\
v_{\alpha 1z}&=q_{\alpha 0}\cos\Psi_{aq}+\sum_{i=1}\epsilon^i q_{\alpha i}(q_{\alpha 0},\Psi_{aq})
\end{aligned} \tag{14}$$

with

$$\begin{aligned}
\frac{\partial W}{\partial t}&=\epsilon A_{1W}+\epsilon^2 A_{2W}+\cdots\\
\frac{\partial \Psi_W}{\partial t}&=-\omega_0+\epsilon B_{1W}+\epsilon^2 B_{2W}+\cdots\\
\frac{\partial W}{\partial x}&=\epsilon C_{1W}+\epsilon^2 C_{2W}+\cdots\\
\frac{\partial \Psi_W}{\partial x}&=k_0+\epsilon D_{1W}+\epsilon^2 D_{2W}+\cdots\ ,
\end{aligned} \tag{15}$$

where W stands for any of e_0, $N_{\alpha 0}$, or $u_{\alpha 0}$, and Ψ_e is the phase of the plasma wave. Relations similar to (15) also hold for $q_{\alpha 0}$ and $\Psi_{\alpha q}$, except that in the equations for $\Psi_{\alpha q}$ the frequency $\omega_{\alpha q}$ and wave number $k_{\alpha q}$ take the place, respectively, of the "natural" frequency ω_0 and wave number k_0 of the plasma wave. Solving Eqs. (10)–(13) is straightforward but tedious; a detailed derivation of the solutions is given in Appendix A. We summarize below the results.

At $O(1)$, we obtain the linear dispersion equation

$$\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2}{\gamma_{\alpha 0}^3(\omega_0-v_{\alpha 0}k_0)^2}=1\ , \tag{16}$$

of which Eq. (1) is a special case. [Here $\omega_{p\alpha}=(4\pi n_{\alpha 0}e_\alpha/m_\alpha)^{1/2}$ is the plasma frequency.] At $O(\epsilon)$ and $O(\epsilon^2)$, we get

$$\begin{aligned}
&C_{1e}=C_{2e}=D_{1e}=D_{2e}=0\ ,\\
&A_{1e}=-\frac{F}{\alpha}\cos\phi\ ,\\
&A_{2e}=-\frac{G}{e_0}\sin\phi\cos\phi\ ,\\
&B_{1e}=\frac{F}{a}\sin\phi\ ,\\
&B_{2e}=\frac{G}{e_0^2}\sin^2\phi+He_0^2\ ,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
a&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2}{\gamma_{\alpha 0}^3(\omega_0-v_{\alpha 0}k_0)^3}\ ,\\
F&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2[a_{\alpha w}B_l(1-v_{\alpha 0}/c)+a_{\alpha l}B_w]}{4\gamma_{\alpha 0}^2(\omega_0-v_{\alpha 0}k_0)^2}\ ,\\
G&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2 F}{2a^2\gamma_{\alpha 0}^2(\omega_0-v_{\alpha 0}k_0)^3}\left\{\left[a_{\alpha w}B_l\left[1-\frac{v_{\alpha 0}}{c}\right]+a_{\alpha l}B_w\right]-\frac{3F}{a\gamma_{\alpha 0}^2(\omega_0-v_{\alpha 0}k_0)}\right\}\ ,\\
H&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2 e_\alpha^2}{8am_\alpha^2\gamma_{\alpha 0}^5(\omega_0-v_{\alpha 0}k_0)^4}\left[\frac{k_0^2}{\gamma_{\alpha 0}^4(\omega_0-v_{\alpha 0}k_0)^2}+\frac{3k_0 v_{\alpha 0}}{c^2\gamma_{\alpha 0}^2(\omega_0-v_{\alpha 0}k_0)}-\frac{9v_{\alpha 0}^2}{c^4}\right]\ .
\end{aligned} \tag{18}$$

Then, from Eqs. (15) and (17), the evolution equations of the fundamental component of the plasma wave are seen to be

$$\dot{e}_0=-\frac{F}{a}\cos\phi-\frac{G}{e_0}\sin\phi\cos\phi \ , \tag{19a}$$

$$e_0\dot{\phi}=\frac{F}{a}\sin\phi+\frac{G}{e_0}\sin^2\phi+He_0^3 \ . \tag{19b}$$

Equations (19) are very similar to the equations derived in Refs. 3 and 9, except that we have kept terms proportional to G. Numerical estimates show that in most cases of interest, $G \ll He_0^4$ unless e_0 is very small. In these cases, it is reasonable to neglect the terms proportional to G in Eqs. (19). Setting $G=0$, it is then easy to see[3,9] that the system exhibits phase locking at $\phi=(2n+1)\pi$, were n is an integer, and e_0 grows linearly in time initially with a growth rate $\Gamma=F/a$. The maximum amplitude attained is given by

$$e_{0\,\max}=\left[\frac{4F}{aH}\right]^{1/3} . \tag{20}$$

Based on the results obtained above, we now examine the acceleration scheme proposed by Bobin. In this case, we first set $v_{p0}=0$. As an example, we calculate the electric field with the parameters given in Ref. 8, namely $\gamma_b=100$, $\lambda_l=10$ μm, $a_l=3.1\times10^{-2}$ (corresponding to a laser intensity of 2.6×10^{13} W/cm^2), $\lambda_w=2.51$ cm, $a_w=3.5$ (corresponding to an undulator field of 1.5 T), and $\omega_{pb}=2.5\times10^{12}$ Hz. The required plasma density, which satisfies Eq. (1), is calculated to be 7.8×10^{15} cm^{-3}. The growth rate is given by $\Gamma=1.8\times10^{14}$ V/cm/sec and the maximum electric field is $e_{0\,\max}=0.84$ MV/cm. However, we note that this electric field contains two contributions—one because of the density modulation of the stationary plasma and the other because of the self-field of the beam. The part contributed from the plasma is given by $e_{0p}=-(4\pi e/k_0)N_{p0}$, where N_{p0} is the lowest-order modulation in the plasma density. By using the relations (A5) in the Appendix, it is easy to show that

$$e_{0p}=\frac{\omega_p^2}{\omega_0^2}e_0 \ , \tag{21}$$

which is much smaller than e_0 because usually $\omega_p \ll \omega_0$. In fact, this conclusion can be anticipated from the following simple physical argument. Since the stationary plasma has zero equilibrium velocity, the electrons in this plasma do not respond much to the magnetic field of the undulator. In other words, the stationary plasma couples very weakly to the beat of the laser and the undulator, which primarily acts to bunch the electron beam.

III. INVERSE FREE-ELECTRON LASER BEAT-WAVE ACCELERATOR

We now study the possibility of using a low-energy electron beam in the presence of an undulator and a high-intensity laser beam to generate a beat wave that can be used to accelerate another high-energy electron beam. As in other analyses of beat-wave acceleration schemes, we will assume for simplicity that the high-energy beam to be accelerated is essentially a test beam of very low current, and hence nonperturbing. The appropriate dispersion equation is then given by (2). The maximum electric field attained is given by Eq. (20), specialized to the case of one beam.

As mentioned earlier, the phase velocity of the beat wave, $v_p=\omega_l/(k_l+k_w)$, is not usually close to c. We now propose two possible methods to enhance v_p. In the first method, we simply choose the laser frequency and the undulator period such that $k_l[=(\omega_l^2/c^2-\omega_{pb}^2/\gamma_{pb}c^2)^{1/2}]+k_w \simeq \omega_l/c$. The second method, which is more suitable if high-intensity microwaves are used instead of a laser, involves using a wave guide. In this method, the phase velocity of the beat wave is manipulated by changing the size of the wave guide. In the proof-of-principle experiment described in Sec. IV, we use this method.

In order to evaluate the potential of the IFELBWA, we consider an example. We consider a relativistic beam of current density of 20 kA/cm^2, which gives $\omega_{pb}=6.3\times10^{10}$ Hz, with the beam energy $\gamma_{pb}=78.3$. The undulator parameters are taken to be $\lambda_w=2.4$ cm, $a_w=3.2$. The radiation frequency which satisfies Eq. (2) is $\omega_l=1.7\times10^{11}$ Hz, and we take $a_l=0.09$ (corresponding to a radiation intensity of 1.8×10^8 W/cm^2). Then the calculated maximum field gradient is $e_{0\,\max}=0.9$ MeV/cm. Since the growth rate is $\Gamma=4.9\times10^9$ MeV/cm/sec, the distance over which $e_{0\,\max}$ is attained is 5.4 cm.

We now describe numerical simulations of a proof-of-principle experiment in support of the IFELBWA concept. Since the electron dynamics is similar to that of an IFEL, existing FEL codes, with minor modifications, can be employed to do the simulations. Here we report the results from our single-frequency two-dimensional (2D) code. This code solves the equations of motion of test particles,

$$\frac{d\gamma_j}{dx}=-\frac{\omega_l a_l a_w}{v_x\gamma_j}\sin\Psi_j+\frac{2\omega_p^2 v_{x0}}{\omega_l c^2}(\langle\cos\Psi\rangle\sin\Psi_j-\langle\sin\Psi\rangle\cos\Psi_j) \ , \tag{22}$$

$$\frac{d\Psi_j}{dx}=k_w+k_l-\frac{\omega_l}{c}\bigg/\left[1-\frac{1+a_w^2-2a_w a_l\cos\Psi_j}{\gamma_j^2}\right]^{1/2}+\frac{d\phi}{dx} \ , \tag{23}$$

and the wave equation,

$$\left[2ik_l\frac{\partial}{\partial x}+\nabla_\perp^2\right](a_l e^{i\phi})=-\frac{\omega_p^2 a_w}{c^2}\left\langle\frac{\exp[-i(\Psi-\phi)]}{\gamma}\right\rangle \ , \tag{24}$$

with the cylindrical wave-guide boundary conditions. Here the subscript j identifies individual test particles, the angular brackets denote ensemble average of the quantities enclosed, and Ψ_j is the phase of the jth test particle with respect to the radiation wave. Note that Ψ_j is the counterpart of the phase Ψ_e in the fluid model. A detailed description of the code can be found in Ref. 11. Whereas we describe the radiation field in 2D by including the radial variation, the electron beam is described in 1D under the approximation that the space-charge field has only an x component. The local maximum acceleration field is measured by the second term on the right-hand side of Eq. (22), i.e.,

$$(E_x)_{\max} = \max\left[\frac{2\omega_p^2 v_{x0}}{\omega_l c^2}(\langle\cos\Psi\rangle\sin\Psi_j - \langle\sin\Psi\rangle\cos\Psi_j)\right]\left[\frac{mc^2}{e}\right]. \tag{25}$$

which is to be compared with $e_{0\max}$ calculated analytically. To simulate the IFELBWA, the test particles are divided into two groups. One group has lower initial energy and serves as the driver. The other group has higher initial energy and is to be accelerated. Both groups of particles are distributed uniformly in phase between $-\pi$ and π.

The proof-of-principle experiment we simulate is depicted schematically in Fig. 1. Since the FEL is a good source of microwave radiation, we divide the system into two sections. The first section is taken to be an FEL. The radius of the wave guide (R_0) in this FEL section is chosen so that the FEL dispersion relation,

$$v_{pb} = \frac{\omega_l}{k_0} + \frac{\omega_{pb}}{\gamma_{pb}^{3/2} k_0}, \tag{26}$$

is satisfied, where $k_0 = k_l + k_w$ and $k_l = (\omega_l^2/c^2 - k_\perp^2)^{1/2}$. The length of this section is chosen to be long enough to bring the FEL into saturation. The electron beam and radiation are then coupled into the second section where the radius of the wave guide is reduced to R_i so that the IFEL dispersion relation (2) is satisfied. The wave guide

TABLE I. Parameters of the proof-of-principle experiment.

Undulator period λ_w	2.65 cm
Undulator length (the second section)	60 cm
R_0	0.75 cm
R_i	0.35 cm
Radiation wave number k_l	7.0 cm^{-1}
Radiation amplitude a_l	0.034
Lower-energy electron-beam parameters	
Initial energy	600 keV
Current density J	8400 A/cm^2
Beam radius r_b	0.2 cm
Higher-energy electron-beam parameters	
Initial energy	3.58 MeV
Current density	10^{-9} A/cm^2

can have a continuous adiabatic taper in order to enhance the phase velocity of the beat wave as the higher-energy electron beam is accelerated. Here we choose a uniform wave guide for simplicity.

The parameters used in the simulations are listed in Table I and are close to those of the Columbia FEL facility.[12] At the end of the first section, the saturated radiation amplitude is seen to be $a_l \simeq 0.03$. The maximum electric field calculated from Eq. (21) is then $e_{0\max} = 0.5$ MV/cm and the growth rate is $\Gamma = 1.5\times10^9$ MV/cm/sec, which predicts that it takes about 10 cm for the beat wave to grow to $e_{0\max}$.

Figure 2 shows a plot of the acceleration field $(E_x)_{\max}$ in the accelerating section obtained from the numerical simulation. We see that the maximum acceleration field is about 0.5 MV/cm and it takes about 14 cm for the wave to grow to such an amplitude. Thus the numerical simulations are in good agreement with the analytical results reported in the previous paragraph. The energy distribution of the higher-energy electron beam at the end of

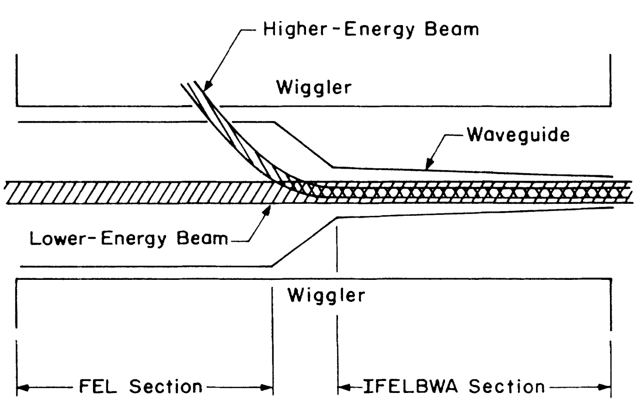

FIG. 1. Schematic diagram of the proof-of-principle experimental setup.

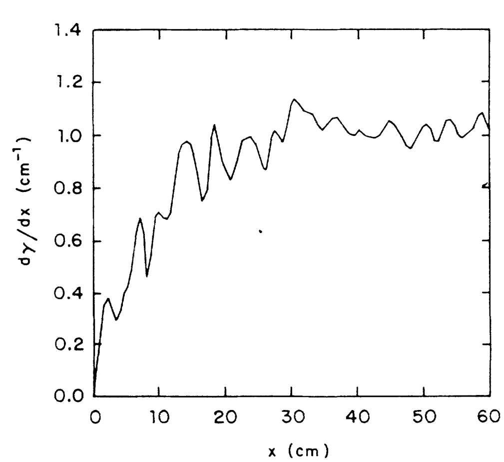

FIG. 2. Accelerating electric field amplitude as a function of x calculated from computer simulations.

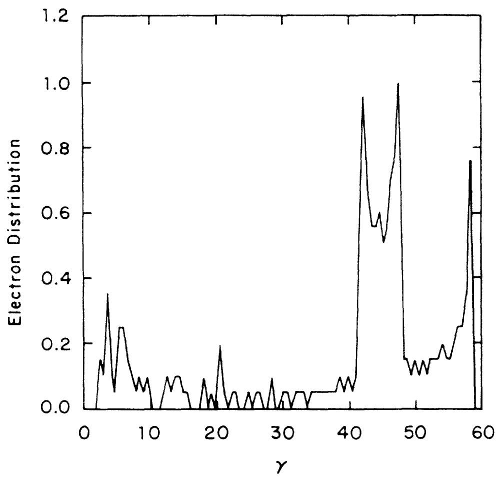

FIG. 3. Energy distribution of the higher-energy electron beam at the end of the accelerating section. Note that most electrons are accelerated to energies $\gamma \sim 40-48$.

the 60-cm accelerating section is plotted in Fig. 3. We note that most of the particles in the test beam have been accelerated to about 23 MeV; some of the electrons are even accelerated up to 29.6 MeV, corresponding to an average acceleration rate of 0.43 MeV/cm, which is again close to the predicted maximum value.

IV. CONCLUSIONS

This study was prompted by the suggestion in Ref. 8 that the introduction of a stationary plasma can enhance the accelerating capability of an IFEL. We have developed an analytical nonlinear theory using the Eulerian multifluid equations to investigate that suggestion. However, our conclusions do not support the results of Ref. 8. We find that the electric field produced is dominated by the self-field of the electron beam, and the background plasma makes only a very small contribution to the total field. We believe that the effect of the electron beam is not correctly taken into account in the nonlinear calculations of Ref. 8, ad the electric field generated is therefore ascribed erroneously to the presence of the plasma. Our conclusions suggest that there is no virtue in introducing a stationary plasma in an IFEL.

The calculations presented in this paper indicate that a moving plasma can be the source of a strong electric field in the presence of an undulator and a laser. This has led us to propose the IFELBWA, which may be viewed as a variant of the PBWA. Whereas in a PBWA two lasers are used to generate a beat wave in a stationary plasma, in an IFELBWA a laser and a magnetostatic undulator generate a beat in a moving plasma, which is the relativistic electron beam in an IFEL. We should also emphasize the similarities and differences between an IFEL and an IFELBWA, since the essential components of both are similar. The phase velocity of the beat wave generated is the same in an IFEL and an IFELBWA. In an IFEL, this beat wave is used to trap the electrons and accelerate the beam by varying the undulator parameters in such a way as to cause the beat wave to propagate slightly faster than the beam. In an IFELBWA, the beam is a moving plasma, and the electric field generated between its bunches is used to accelerate another higher-energy electron beam. Of course, in order to be effective at higher energies, it is necessary to enhance the speed of the beat wave in an IFELBWA. In the paper, we suggested methods by which this enhancement may be accomplished.

The IFELBWA is not sensitively dependent on beam quality for its effectiveness. All that is necessary is that the beam be of sufficiently good quality to get the IFEL mechanism in place. A weakness of the idea is that the density of the beam is constrained by the maximum current, and this limits the available plasma density to a value several orders of magnitude smaller than can be available to a PBWA. Altogether, the IFELBWA should be viewed as an idea which compliments the IFELA and the PBWA in many aspects, and provides an interesting test bed to evaluate many physics issues pertaining to the PBWA. As in the case of the PBWA, there are some additional questions that need to be addressed. These include the possible generation of instabilities when the current of the higher-energy beam becomes significant, multifrequency effects due to the emission of radiation through the FEL mechanism, sideband generation and particle detrapping, 2D effects, and effects of higher harmonics. Related technical challenges include the generation of electron beams of sufficiently high current, and the development of beam-injection techniques.

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APPENDIX A

In this appendix, we give a detailed derivation of the solutions of Eqs. (10)–(13) by the Bogoliubov-Mitropolsky method of averaging. Substituting the expansions (14) and (15) in Eqs. (10)–(13) and comparing coefficients of ϵ^i's, we obtain, to $O(\epsilon^0)$,

$$(v_{\alpha 0}k_0-\omega_0)u_{\alpha 0}\cos\Psi_{\alpha u}=\frac{e_\alpha e_0}{m_\alpha \gamma_{\alpha 0}^3}\cos\Psi_e\ , \qquad \text{(A1)}$$

$$-N_{\alpha 0}\omega_0\cos\psi_{\alpha n}+n_{\alpha 0}k_0u_{\alpha 0}\cos\Psi_{\alpha u} +k_0N_{\alpha 0}v_{\alpha 0}\cos\Psi_{\alpha n}=0\ , \qquad \text{(A2)}$$

$$e_0k_0\sin\Psi_e+4\pi\sum_{\alpha=1}^{2}e_\alpha N_{\alpha 0}\sin\Psi_{\alpha n}=0\ , \qquad \text{(A3)}$$

and

$$q_{\alpha 0}\cos\Psi_{\alpha q} = -\frac{c}{\gamma_{\alpha 0}}[a_{\alpha w}\cos k_w x + a_{\alpha l}\cos(k_l x - \omega_l t)] , \tag{A4}$$

where $a_\alpha = e_\alpha B/(m_\alpha c^2 k)$ is the normalized vector potential of the laser or the undulator. By comparing the phase and amplitudes on both sides of Eqs. (A1)–(A3), we arrive at the following results:

$$\begin{aligned}
&\Psi_{\alpha u} = \Psi_{\alpha n} = \Psi_e ,\\
&u_{\alpha 0} = \frac{e_\alpha e_0}{m_\alpha \gamma_{\alpha 0}^3 (v_{\alpha 0} k_0 - \omega_0)} ,\\
&N_{\alpha 0} = \frac{n_{\alpha 0} k_0 u_{\alpha 0}}{\omega_0 - k_0 v_{\alpha 0}} ,\\
&e_0 = -\sum_{\alpha=1}^{2} \frac{4\pi e_\alpha N_{\alpha 0}}{k_0} .
\end{aligned} \tag{A5}$$

Eliminating $u_{\alpha 0}$ and $N_{\alpha 0}$ from Eqs. (A5) yields the linear dispersion equation

$$\sum_{\alpha=1}^{2} \frac{\omega_{p\alpha}^2}{\gamma_{\alpha 0}^3 (\omega_0 - k_0 v_{\alpha 0})^2} = 1 . \tag{A6}$$

Furthermore, because of the relations (A5), there is only one independent linear phase and amplitude, which we arbitarily choose to be Ψ_e and e_0 for convenience. Therefore, we need to consider only the case $W = e_0$ in Eqs. (15) in the text.

To $O(\epsilon)$, Eq. (13) becomes

$$C_{1e}\cos\Psi_e - e_0 D_{1e}\sin\Psi_e + k_0 \frac{\partial e_1}{\partial \Psi_e} = 4\pi \sum_{\alpha-1}^{2} e_\alpha N_{\alpha 1} . \tag{A7}$$

We shall see later that in order to suppress the secular growth of the plasma wave, e_1 and $\sum_{\alpha=1}^{2} 4\pi e_\alpha N_{\alpha 1}$ should be independent of the fundamental phase component Ψ_e. Therefore, Eq. (A7) is satisfied only when

$$C_{1e} = D_{1e} = 0 . \tag{A8}$$

Then, the coefficients of ϵ^1 in Eqs. (10) and (12) can be written as

$$\begin{aligned}
&(k_0 v_{\alpha 0} - \omega_0)\frac{\partial u_{\alpha 1}}{\partial \Psi_e} + A_{\alpha 1 u}\sin\Psi_e + u_{\alpha 0} B_{\alpha 1 u}\cos\Psi_e + k_0 u_{\alpha 0}^2 \sin\Psi_e \cos\Psi_e \\
&\qquad = \frac{e_\alpha}{\gamma_{\alpha 0} m_\alpha}\left[\frac{e_1}{\gamma_{\alpha 0}^2} - \left(\frac{B_y q_{\alpha 0}\cos\Psi_{\alpha q}}{c} + \frac{3 v_{\alpha 0} u_{\alpha 0} e_0 \sin\Psi_e \cos\Psi_e}{c^2} + \frac{v_{\alpha 0} q_{\alpha 0} E_z \cos\Psi_e}{c^2}\right)\right] ,
\end{aligned} \tag{A9}$$

and

$$(k_0 v_{\alpha 0} - \omega_0)\frac{\partial N_{\alpha 1}}{\partial \Psi_e} + n_{\alpha 0} k_0 \frac{\partial u_{\alpha 1}}{\partial \Psi_e} + 2k_0 u_{\alpha 0} N_{\alpha 0} \sin\Psi_e \cos\Psi_e + A_{\alpha 1 n}\sin\Psi_e + B_{\alpha 1 n} N_{\alpha 0}\cos\Psi_e = 0 , \tag{A10}$$

Here the relation between $N_{\alpha 0}$, $u_{\alpha 0}$, and e_0, as well as that between $A_{\alpha 1 n}, B_{\alpha 1 n}, A_{\alpha 1 u}, B_{\alpha 1 u}$ and A_{1e}, B_{1e} are given by Eqs. (A5). Using these relations, we solve Eqs. (A7), (A9), and (A10) and obtain an equation for e_1, which is

$$\begin{aligned}
\left[\frac{\partial^2}{\partial \Psi_e^2} + 1\right] e_1 = &\sum_{\alpha=1}^{2} \frac{3\omega_{p\alpha}^2 u_{\alpha 0} e_0}{2\gamma_{\alpha 0}(\omega_0 - k_0 v_{\alpha 0})^2}\left[\frac{v_{\alpha 0}}{c^2} + \frac{k_0}{\gamma_{\alpha 0}^2 (v_{\alpha 0} k_0 - \omega_0)}\right]\sin 2\Psi_e \\
&- \sum_{\alpha=1}^{2} \frac{\omega_{p\alpha}^2 [a_{\alpha l} B_w + a_{\alpha w} B_l (1 - v_{\alpha 0}/c)]}{2\gamma_{\alpha 0}^2 (\omega_0 - k_0 v_{\alpha 0})^2}\sin(\Psi_e - \phi) + \sum_{\alpha=1}^{2} \frac{2\omega_{p\alpha}^2}{\gamma_{\alpha 0}^3 (k_0 v_{\alpha 0} - \omega_0)^3}(A_{1e}\sin\Psi_e + e_0 B_{1e}\cos\Psi_e) ,
\end{aligned} \tag{A11}$$

where $\phi \equiv \Psi_e - (k_w + k_l)x + \omega_l t$ is the phase difference between the plasma wave and the beat wave. In order to eliminate secular growth of e_1, we require that the coefficients of $\sin\Psi_e$ and $\cos\Psi_e$ on the right-hand side of Eq. (A11) be zero, which gives

$$\begin{aligned}
&A_{1e} = -\frac{F}{a}\cos\phi ,\\
&B_{1e} = \frac{F}{a}\sin\phi ,
\end{aligned} \tag{A12}$$

where

$$\begin{aligned}
&F \equiv \sum_{\alpha=1}^{2} \frac{\omega_{p\alpha}^2 [a_{\alpha w} B_l (1 - v_{\alpha 0}/c) + a_{\alpha l} B_w]}{4\gamma_{\alpha 0}^2 (\omega_0 - k_0 v_{\alpha 0})^2} ,\\
&a \equiv \sum_{\alpha=1}^{2} \frac{\omega_{p\alpha}^2}{\gamma_{\alpha 0}^3 (\omega_0 - k_0 v_{\alpha 0})^3} .
\end{aligned} \tag{A13}$$

The solution of Eq. (A11) is

$$e_1=\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2 e_\alpha e_0^2}{2m_\alpha\gamma_{\alpha0}^4(\omega_0-k_0v_{\alpha0})^3}\left[\frac{v_{\alpha0}}{c^2}+\frac{k_0}{\gamma_{\alpha0}^2(v_{\alpha0}k_0-\omega_0)}\right]\sin2\Psi_e\ . \tag{A14}$$

Similarly, comparing the coefficients of ϵ^1 in Eq. (A11) yields

$$q_{\alpha1}=\frac{e_\alpha v_{\alpha0}e_0\sin\Psi_e}{m_\alpha c\gamma_{\alpha0}^2(v_{\alpha0}k_0-\omega_0)}[a_{\alpha w}\cos k_w x+a_{\alpha l}\cos(k_l x-\omega_l t)]\ . \tag{A15}$$

Having solved the first-order equations, we next proceed to look at the coefficients of ϵ^2. To avoid secular growth, we must also have

$$C_{2e}=D_{2e}=0\ . \tag{A16}$$

Comparing the coefficients of ϵ^2 in Eqs. (10), (12), and (13), we obtain three linear equations for $u_{\alpha2}$, $N_{\alpha2}$, and e_2:

$$\begin{aligned}\frac{\partial u_{\alpha2}}{\partial\Psi_e}=&\frac{e_\alpha e_2}{m_\alpha\gamma_{\alpha0}^3(v_{\alpha0}k_0-\omega_0)}+\frac{1}{\omega_0-v_{\alpha0}k_0}(B_{1e}+k_0u_{\alpha0}\sin\Psi_e)\frac{\partial u_{\alpha1}}{\partial\Psi_e}\\&+\frac{A_{1e}}{\omega_0-v_{\alpha0}k_0}\frac{\partial u_{\alpha1}}{\partial e_0}-\frac{e_\alpha}{m_\alpha\gamma_{\alpha0}^3(\omega_0-v_{\alpha0}k_0)^2}(A_{2e}\sin\Psi_e+e_0B_{2e}\cos\Psi_e)\\&+\frac{e_\alpha e_0u_{\alpha1}}{m_\alpha\gamma_{\alpha0}(\omega_0-v_{\alpha0}k_0)}\left[\frac{3v_{\alpha0}}{c^2}+\frac{k_0}{\gamma_{\alpha0}^2(v_{\alpha0}k_0-\omega_0)}\right]\cos\Psi_e+\frac{3e_\alpha v_{\alpha0}u_{\alpha0}e_1}{m_\alpha c^2\gamma_{\alpha0}(\omega_0-v_{\alpha0}k_0)}\sin\Psi_e\\&+\frac{e_\alpha q_{\alpha1}}{m_\alpha c\gamma_{\alpha0}(\omega_0-v_{\alpha0}k_0)}\left[B_w\sin k_wx+B_l\left[1-\frac{v_{\alpha0}}{c}\sin(k_lx-\omega_lt)\right]\right]\\&+\frac{3e_\alpha u_{\alpha0}^2e_0}{2m_\alpha\gamma_{\alpha0}c^2(\omega_0-v_{\alpha0}k_0)}\left[1-\frac{\gamma_{\alpha0}^2v_{\alpha0}^2}{c^2}\right]\sin^2\Psi_e\cos\Psi_e-\frac{u_{\alpha0}}{2}a_{\alpha l}a_{\alpha w}\cos(\Psi_e-\phi)\cos\Psi_e\\&+\frac{u_{\alpha0}a_{\alpha w}a_{\alpha l}}{2(\omega_0-v_{\alpha0}k_0)}\left[v_{\alpha0}(k_l+k_w)+k_lc\left[\frac{1}{\gamma_{\alpha0}^2}-\frac{v_{\alpha0}^2}{c^2}\right]\right]\sin(\Psi_e-\phi)\sin\Psi_e\ ,\end{aligned} \tag{A17}$$

$$\begin{aligned}\frac{\partial N_{\alpha2}}{\partial\Psi_e}=&\frac{n_{\alpha0}k_0}{\omega_0-v_{\alpha0}k_0}\frac{\partial u_{\alpha2}}{\partial\Psi_e}+\frac{1}{\omega_0-v_{\alpha0}k_0}(B_{1e}+k_0u_{\alpha0}\sin\Psi_e)\frac{\partial N_{\alpha1}}{\partial\Psi_e}+\frac{A_{1e}}{\omega-v_{\alpha0}k_0}\frac{\partial N_{\alpha1}}{\partial e_0}+\frac{k_0^2n_{\alpha0}u_{\alpha0}}{(\omega_0-v_{\alpha0}k_0)^2}\sin\Psi_e\frac{\partial u_{\alpha1}}{\partial\Psi_e}\\&-\frac{e_\alpha n_{\alpha0}k_0}{m_\alpha\gamma_{\alpha0}^3(\omega_0-v_{\alpha0}k_0)^3}(A_{2e}\sin\Psi_e+e_0B_{2e}\cos\Psi_e)+\frac{k_0}{\omega_0-v_{\alpha0}k_0}(u_{\alpha0}N_{\alpha1}+N_{\alpha0}u_{\alpha1})\cos\Psi_e\ ,\end{aligned} \tag{A18}$$

and

$$\frac{\partial e_2}{\partial\Psi_e}=\sum_{\alpha=1}^{2}\frac{4\pi e_\alpha}{k_0}N_{\alpha2}\ . \tag{A19}$$

Using Eq. (A17) in Eq. (A18), solving for $(\partial N_{\alpha2})/(\partial\Psi_e)$, and then substituting the result in the derivative of Eq. (A19) gives an equation for e_2. This equation can be written in the form

$$\left[\frac{\partial^2}{\partial\Psi_e^2}+1\right]e_2=2a\left[\left[A_{2e}+\frac{G}{e_0}\sin\phi\cos\phi\right]\sin\Psi_e+e_0\left[B_{2e}-\frac{G}{e_0^2}\sin^2\phi-He_0^2\right]\cos\Psi_e\right]+\cdots\ , \tag{A20}$$

where the + sign indicates additional terms containing dc, second harmonic, and third harmonic contributions, and

$$\begin{aligned}G&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2F}{2a^2\gamma_{\alpha0}^2(\omega_0-v_{\alpha0}k_0)^3}\left\{\left[a_{\alpha w}B_l\left[1-\frac{v_{\alpha0}}{c}\right]+a_{\alpha l}B_w\right]+\frac{3F}{a\gamma_{\alpha0}^2(\omega_0-v_{\alpha0}k_0)}\right\}\ ,\\H&\equiv\sum_{\alpha=1}^{2}\frac{\omega_{p\alpha}^2e_\alpha^2}{8am_\alpha^2\gamma_{\alpha0}^5(\omega_0-v_{\alpha0}k_0)^4}\left[\frac{k_0^2}{\gamma_{\alpha0}^4(\omega_0-v_{\alpha0}k_0)^2}+\frac{3k_0v_{\alpha0}}{c^2\gamma_{\alpha0}^2(\omega_0-v_{\alpha0}k_0)}-\frac{9v_{\alpha0}^2}{c^4}\right]\ .\end{aligned} \tag{A21}$$

Again, to exclude the secular growth of e_2, we must require the coefficients of $\sin\Psi_e$ and $\cos\Psi_e$ on the right-hand side of Eq. (A20) to be zero. Namely,

$$A_{2e} = -\frac{G}{e_0}\sin\phi\cos\phi \ ,$$
$$B_{2e} = \frac{G}{e_0^2}\sin^2\phi + He_0^2 \ . \tag{A22}$$

Thus, we have solved for the fundamental components of Eqs. (10)–(13) in the text, up to the order of ϵ^2.

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