Characteristics of Rabi oscillations in the two-mode squeezed state of the field

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The phenomenon of collapses and revivals of Rabi oscillations is studied for a two-level atom undergoing either one- or two-photon transitions in the two-mode squeezed state field inside a lossless cavity. The qualitative behavior of this phenomenon is found to be different as compared with corresponding coherent-state fields. Analytic expressions for excitation probabilities are obtained. Also, the statistical aspect of the field in terms of intensity-intensity correlations is discussed.

I. INTRODUCTION

The Jaynes-Cummings model[1] of a two-level atom interacting with a single mode of a radiation field provides a useful means of studying nonclassical[2] effects in the interaction between the electromagnetic field and matter. Recent experimental[3] advances have made it possible to realize such a model. The prominent nonclassical effect in this model is the phenomenon of collapses and revivals[4] of the Rabi oscillations in a field that is not in a pure number state. This phenomenon, which is due to the granular nature of the field, is absent if the field is considered classically. The nature of these oscillations depends on the statistical properties of the field. For example, in the single-photon transition,[5] the revivals are regular but overlapping in a coherent-state field, but are all irregular in a chaotic field. For the two-photon transition,[6] the revivals are regular and compact. Extensive study of this phenomenon has also been carried out for the field in a single-mode binomial state,[7] negative binomial state,[8] two-photon coherent state,[6,9] and mixed coherent-chaotic[5] case for a Rydberg atom undergoing one- as well as two-photon transitions. Also, some very interesting features, such as doublets of revivals, etc. have been brought out when the atom is initially in a coherent superposition of its states.[9,10]

In this work we investigate the phenomenon of collapses and revivals in a field having two correlated modes. In particular, we consider the two modes in a pair-coherent state,[11] i.e., the eigenstate of the product of annihilation operators for the two modes, or two-mode squeezed state. This pair-coherent state is different from other coherent states, such as the two-photon coherent state, the atomic coherent state.[11] Such a state can be produced, for example, in the interaction of a classically driven two-level atom undergoing two-phonon transitions, in which suppression of amplified spontaneous emission (ASE) takes place due to a four-wave-mixing (FWM) process, where photons are either created in pairs or destroyed in pairs (see Ref. 12 for experimental details). A detailed description of the pair-coherent state has been given elsewhere.[13]

The organization of the paper is as follows. In Sec. II we give some properties of the pair-coherent-state field, and in Sec. III we present the complete dynamics of a two-level atom undergoing either one-photon or two-photon transitions in the pair-coherent field. Some concluding remarks are given in Sec. IV.

II. SOME FEATURES OF FIELD IN PAIR COHERENT STATE

We consider a two-mode field and associate with the modes annihilation operators $\hat{a}_1$ and $\hat{a}_2$. The product operator $\hat{a}_1\hat{a}_2$, acting on a Fock state, simultaneously annihilates photons of modes $\hat{a}_1$ and $\hat{a}_2$. Thus $\hat{a}_1\hat{a}_2$ is the pair-annihilation operator for the two modes. A pair coherent state $|\zeta\rangle$ is then defined as[11,13]

$$\hat{a}_1\hat{a}_2|\zeta,q\rangle=\zeta|\zeta,q\rangle\ , \tag{1}$$

where ζ is a complex number and q is the degeneracy parameter,

$$(\hat{a}_1^\dagger a_1-\hat{a}_2^\dagger a_2)|\zeta,q\rangle=q|\zeta,q\rangle\ , \tag{2}$$

implying that, whenever photons are either created in pairs or destroyed in pairs, the difference in the number of photons remains constant. The parameter will be zero when pair creation starts from vacuum. Without loss of generality, q can be assumed to be positive. It can be shown that

$$|\zeta,q\rangle=N_q\sum_{n=0}^{\infty}\frac{\zeta^n}{[n!(n+q)!]^{1/2}}|n+q,n\rangle\ , \tag{3}$$

where $|m,n\rangle$ is such that $\hat{a}_1^\dagger\hat{a}_1|m,n\rangle=m|m,n\rangle$ and $\hat{a}_2^\dagger\hat{a}_2|m,n\rangle=n|m,n\rangle$, and the normalization constant N_q is determined by the condition $\langle\zeta,q|\zeta,q\rangle=1$. We obtain

$$N_q=\left[\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{n!(n+q)!}\right]^{-1/2}$$
$$=[(i|\zeta|)^{-q}J_q(2i|\zeta|)]^{-1/2}\ , \qquad (4)$$

where $J_q(x)$ is Bessel's function. The probability of finding n photons in mode $\hat{a}_2$ and $n+q$ photons in mode $\hat{a}_1$ is

$$p_n=|\langle n+q,n|\zeta,q\rangle|^2=N_q^2\frac{|\zeta|^{2n}}{n!(n+q)!}\ , \qquad (5)$$

which is sub-Poissonian. The mean numbers of photons $\langle n_1\rangle$ and $\langle n_2\rangle$ in modes $\hat{a}_1$ and $\hat{a}_2$ are given by

$$\langle\hat{a}_1^\dagger a_1\rangle=\langle\hat{n}_1\rangle=q+\langle\hat{n}_2\rangle\ , \qquad (6)$$

$$\langle\hat{a}_2^\dagger a_2\rangle=\langle\hat{n}_2\rangle=N_q^2\sum_n\frac{|\zeta|^{2n}n}{n!(n+q)!}=\frac{N_q^2}{N_{q+1}^2}|\zeta|^2\ , \qquad (7)$$

and thus for $q=0$, $\langle n_1\rangle=\langle n_2\rangle$ and $P_n=N_0^2|\zeta|^2/(n!)^2$. Many other features of $|\zeta,q\rangle$, including the possibility of its generation, are investigated in Ref. 13.

III. ATOMIC DYNAMICS AND FIELD STATISTICS

A. An atom undergoing a one-photon transition

We consider a two-level atom having ground state $|g\rangle$ and excited state $|e\rangle$ interacting resonantly with the mode $\hat{a}_1$ of the field which is in the pair-coherent state $|\zeta,0\rangle$ (where for simplicity we have $q=0$) of the modes $\hat{a}_1$ and $\hat{a}_2$. The evolution of the density matrix of the system of a two-level atom of transition frequency ω_0 interacting resonantly with a monochromatic electromagnetic field is given by ($\hbar=1$) (Refs. 5 and 7)

$$i\frac{\partial\rho}{\partial t}=[H,\rho]\ , \qquad (8)$$

where H is the Jaynes-Cummings Hamiltonian[1] in the rotating-wave approximation (RWA):

$$H=\omega_0 S_z+\omega_0 a_1^\dagger a_1+g(a_1^\dagger S_-+S_+a_1)\ . \qquad (9)$$

Here,

$$S_+=|e\rangle\langle g|\ ,$$
$$S_-=|g\rangle\langle e|\ ,$$

and

$$S_z=\tfrac{1}{2}(|e\rangle\langle e|-|g\rangle\langle g|)\ ,$$

where $|g\rangle$ ($|e\rangle$) is the ground (excited) state of the atom and g is the coupling between the atom and the field. The Hamiltonian of the Eq. (2) refers to a nondecaying atom in a lossless cavity (cavity $Q=\infty$). The cavity losses can be accounted for as in Refs. 5 and 6.

To solve Eq. (8), we consider an atom initially to be either in an excited state or a ground state, and the field state given by

$$|\zeta\rangle=N_0\sum_n\left[\frac{|\zeta|^{2n}}{(n!)^2}\right]^{1/2}|n,n\rangle\ .$$

Also, at $t=0$, the atom and the field are decoupled. The solution of Eq. (8) in this case is given by[5,7]

$$\begin{aligned}\rho(t)=\sum_{n,m}[&\cos(gt\sqrt{m+1})\cos(gt\sqrt{n+1})|m,\tfrac{1}{2}\rangle\langle m|\rho_f(0)|n\rangle\langle n,\tfrac{1}{2}|\\&+\sin(gt\sqrt{m+1})\sin(gt\sqrt{n+1})|m+1,-\tfrac{1}{2}\rangle\langle m|\rho_f(0)|n\rangle\langle n+1,-\tfrac{1}{2}|\\&+i\cos(gt\sqrt{m+1})\sin(gt\sqrt{n+1})|m,\tfrac{1}{2}\rangle\langle m|\rho_f(0)|n\rangle\langle n+1,-\tfrac{1}{2}|\\&-i\cos(gt\sqrt{m+1})\sin(gt\sqrt{n+1})|m+1,-\tfrac{1}{2}\rangle\langle m|\rho_f(0)|n\rangle\langle n,\tfrac{1}{2}|]\ ,\end{aligned} \qquad (10)$$

where $\rho_f(0)$ is the initial density matrix operator of the field, which for the state $|5\rangle$ is

$$\rho_f(0)=|\zeta\rangle\langle\zeta|=\sum_{n,m}P_{nm}|n\rangle\langle n|m\rangle\langle m|$$
$$=\sum_{n,m}P_{nn}|n\rangle\langle n|\ , \qquad (11)$$

with

$$P_{nn}=N_0^2\left[\frac{|\zeta|^{2n}}{(n!)^2}\right]\ . \qquad (12)$$

$P_n=P_{nn}$ is the photon-number distribution function.

The probability $P_e(t)$ of finding the atom in an excited state is defined as[5]

$$P_e(t)=\tfrac{1}{2}+\langle S_z\rangle\ , \qquad (13)$$

where

$$\langle S_z\rangle=\tfrac{1}{2}\sum_n P_n\cos(2gt\sqrt{n+1})\ . \qquad (14)$$

This expression is the key result which describes the collapse and revival phenomenon in the population inversion. Here P_n is the sub-Poissonian distribution for the pair-coherent-state field with mean photon number $\langle n\rangle\sim|\zeta|$ (for $|\zeta|\gg 1$). Also, for $\langle n\rangle\gg 1$, the distribution is localized around $\langle n\rangle$, and has a width (Δn) much narrower than that of $\langle n\rangle^{1/2}$. The revivals in $P_e(t)$ occur when the oscillator r with n near $\langle n\rangle$ becomes in phase. The period of revivals T_r can be estimated[4] as the time when the two neighboring oscillators with $n=\langle n\rangle$

and $n=\langle n\rangle+1$ acquires a 2π phase difference

$$2g(\langle n\rangle+1)^{1/2}T_r-2g(\langle n\rangle)^{1/2}T_r\cong 2\pi ,$$

which for $\langle n\rangle \gg 1$ leads to

$$T_r=2\pi(\langle n\rangle)^{1/2}/g=2\pi(|\zeta|)^{1/2}/g . \tag{15}$$

We have carried out a more sophisticated analysis in which the collapse-revival phenomenon is self-explanatory, exactly in the same manner as that described in Ref. 4 for the ordinary coherent state. The asymptotic expression for $P_e(t)$ (in the limit $\langle n\rangle \gg 1$) so obtained is found to be in good agreement with the numerical summation of Eq. (13). With this analysis we have found the period of revival T_r to be

$$T_r=2\pi\sqrt{|\zeta|}/g , \tag{16}$$

and the width of the Gaussian envelope of the kth revival, or, equivalently, the time of collapse T_c of the kth revival to be

$$T_c=2\left[\frac{1+\pi^2k^2}{2\pi^2 2|\zeta|}\right]^{1/2} . \tag{17}$$

For the sake of comparison we note that the period of revival T_r^c and the period of time T_c^c for an ordinary coherent state[4] is given by

$$T_r^c=2\pi|\zeta|/g , \tag{18}$$

$$T_c^c=2\left[\frac{1+\pi^2k^2}{2\pi^2|\zeta|^2}\right]^{1/2} . \tag{19}$$

In Fig. 1 we have plotted $P_e(t)$ using Eq. (13) for two different values of $|\zeta|^2$ for a pair-coherent-state field. Clearly, the predictions of revival time and collapse time are in good agreement with the numerical results. In Fig. 2 we have given a comparison of $P_e(t)$ between a pair-coherent field and an ordinary coherent field for $|\zeta|^2=40$.

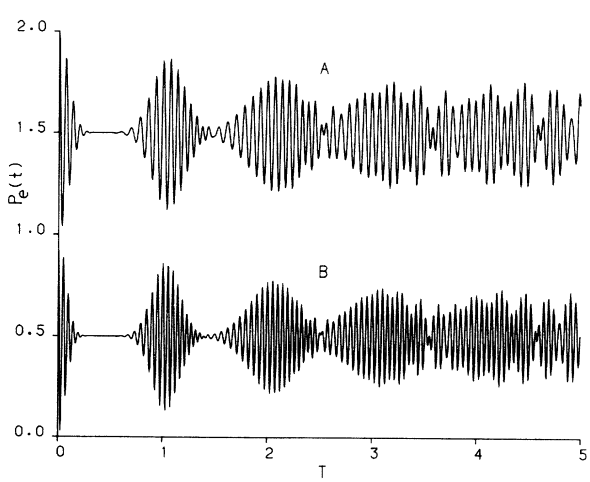

FIG. 1. Excitation probability $P_e(t)$ as a function of scaled time $T=t/T_r$ ($T_r=2\pi\sqrt{|\zeta|}/g$) for an atom undergoing a one-photon transition in initial pair-coherent states. Curve A $[P_e(t)+1]$ is for $|\zeta|^2=50$, curve B $[P_e(t)]$ is for $|\zeta|^2=100$.

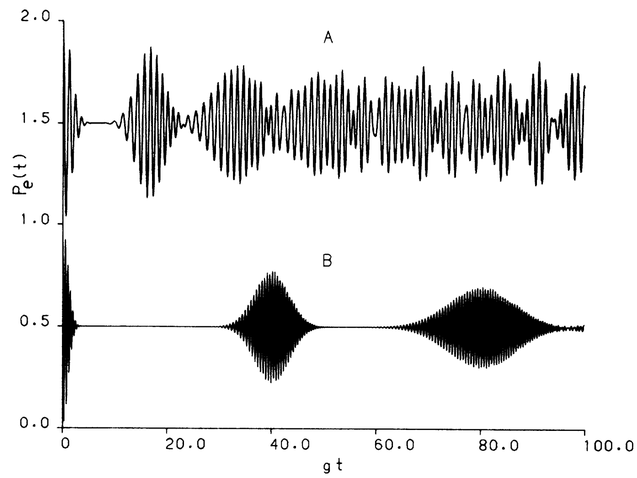

FIG. 2. Excitation probability $P_e(t)$ as a function of time for an atom undergoing a one-photon transition with $|\zeta|^2=40$. Curve A $[P_e(t)+1]$ is for the pair-coherent state, curve B $[P_e(t)]$ is for the corresponding coherent state.

We also observe that the period of Rabi oscillation within a revival increases for the pair-coherent field, in comparison with the coherent field. Thus the effect of sub-Poissonian statistics[13] on a pair-coherent-state field is clearly visible in $P_e(t)$.

The intensity-intensity correlation function $g^{(2)}(t)$, defined as

$$g^{(2)}(t)=\frac{\langle (a^\dagger)^2a^2\rangle}{\langle a^\dagger a\rangle^2}-1 , \tag{20}$$

determines the fluctuation in photon-number distribution. From Eq. (10) it follows that[7]

$$\langle a^\dagger a\rangle=\tfrac{1}{2}+\sum_n nP_n-\tfrac{1}{2}\sum_n P_n\cos(2gt\sqrt{n+1}) , \tag{21}$$

$$\langle (a^\dagger)^2a^2\rangle=\sum_n n^2P_n-\sum_n nP_n\cos(2gt\sqrt{n+1}) , \tag{22}$$

and $g^{(2)}(t)<0$ implies antibunching. We evaluate $g^{(2)}(t)$ using Eqs. (21) and (22). In Fig. 3 we have plotted $g^{(2)}(t)$ as a function of time for a pair-coherent field as well as for an ordinary coherent field. Note that the extent of antibunching is greater in the pair-coherent field than in the coherent field due to the intrinsic sub-Poissonian nature of the pair-coherent field. Since for the same $|\zeta|^2$ the mean photon number of the pair-coherent field is smaller than that of the coherent field, we find an increased overlap of revivals in the former, along with a larger period of Rabi oscillation within a revival.

B. An atom undergoing a two-photon transition

In the case of a two-photon process in infinitely high-Q cavities, the collapses and revivals of Rabi oscillations are both compact and regular in a coherent field, in comparison with the single-photon transition in which the re-

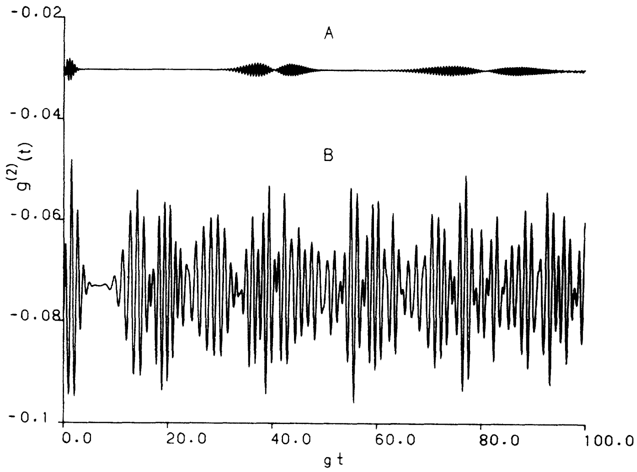

FIG. 3. Intensity intensity correlation function $g^{(2)}(t)$ as a function of time for an atom undergoing a one-photon transition with $|\zeta|^2=40$. Curve A $[g^{(2)}(t)-0.03]$ is for the initial coherent-state field and curve B ($g^{(2)}$) is for the pair-coherent-state field.

vivals are only partial. Here we consider an atom making two-photon transitions between the ground state $|g\rangle$ (energy E_g) and the excited state $|e\rangle$ (energy E_e) while interacting with two cavity fields with frequencies ω_1 and ω_2. The transition between the two states is mediated by an intermediate state $|i\rangle$ (energy E_i) such that $E_i-E_g=\omega_1-\Delta$, $E_e-E_i=\omega_2+\Delta$. If we assume $|\Delta|$ is much larger than the one-photon Rabi frequency of the oscillations between $|i\rangle$ and $|g\rangle$ and between $|i\rangle$ and $|e\rangle$, then intermediate state $|i\rangle$ can be eliminated adiabatically. Now the transition between $|e\rangle$ and $|g\rangle$ can be described by an effective Hamiltonian H, given by[6,14]

$$H=2\omega S_z+(\omega+\varepsilon)a_1^\dagger a_1+(\omega-\varepsilon)a_2^\dagger a_2 + g(a_1^\dagger a_2^\dagger S_- + S_+ a_1 a_2)\ , \tag{23}$$

where $\omega=(\omega_1+\omega_2)/2$, $\varepsilon=(\omega_1-\omega_2)/2$, $S_+=|e\rangle\langle g|$, $S_-=|g\rangle\langle e|$, and $S_z=\frac{1}{2}(|e\rangle\langle e|-|g\rangle\langle g|)$, and a_i ($a_i^\dagger$) is the cavity-field annihilation (creation) operator of the cavity-field modes ($i=1,2$) in the pair-coherent-state field. In the above Hamiltonian we have ignored the Stark shift of two levels arising from virtual transitions to the intermediate states, but this can be accounted for easily.

The evolution of the density matrix of the system is described by the equation

$$i\frac{\partial\rho}{\partial t}=[H,\rho]\ . \tag{24}$$

We solve Eq. (24) by evaluating the eigenvalues $E^{\pm}_{n_1,n_2}$ and the eigenfunction $|\psi^{\pm}_{n_1 n_2}\rangle$ of the Hamiltonian H. Since the atom absorbs and emits one pair of photons in an ideal cavity, the basis vectors in the dressed-state representation are $|n_1,n_2,e\rangle$ and $|n_1+1,n_2+1,g\rangle$, so that

$$|\psi^{\pm}_{n_1 n_2}\rangle=(1/\sqrt{2})(|n_1,n_2,e\rangle\pm|n_1+1,n_2+1,g\rangle)\ , \tag{25}$$

$$H|\psi^{\pm}_{n_1 n_2}\rangle=E^{\pm}_{n_1 n_2}|\psi^{\pm}_{n_1 n_2}\rangle\ , \tag{26}$$

$$E^{\pm}_{n_1 n_2}=\frac{\omega}{2}(n_1+n_2+1)+\varepsilon(n_1-n_2)+\lambda^{\pm}_{n_1 n_2}\ , \tag{27}$$

where

$$\lambda^{\pm}_{n_1 n_2}=\pm g[(n_1+1)(n_2+1)]^{1/2}\ . \tag{28}$$

To obtain an expression for $P_e(t)$, let the atom and the field be decoupled at $t=0$. If initially the atom is in the excited state $|e\rangle$, and the field is in the superposition of number state $|n\rangle$, then

$$\rho(0)=\sum_{\substack{m_1,n_1\\m_2,n_2}} C_{m_1 m_2,n_1 n_2}|m_1 m_2 e\rangle\langle n_1 n_2 e|\ , \tag{29}$$

where

$$C_{m_1 m_2,n_1 n_2}=\langle m_1 m_2 e|\rho(0)|n_1 n_2 e\rangle\ . \tag{30}$$

In terms of the eigenstate of H,

$$\rho(0)=\sum_{\substack{m_1,n_1\\m_2,n_2}} C_{m_1 m_2,n_1 n_2}[\tfrac{1}{2}(|\psi^+_{m_1 m_2}\rangle+|\psi^-_{m_1 m_2}\rangle)(\langle\psi^+_{n_1 n_2}|+\langle\psi^-_{n_1 n_2}|)]\ , \tag{31}$$

so that[14]

$$\rho(t)=\sum_{\substack{m_1,n_1\\m_2,n_2}} C_{m_1 m_2 n_1 n_2}e^{it[(m_1-n_1+m_2-n_2)\omega/2+(m_1+n_1-m_2-n_2)\varepsilon]}(A_{m_1 m_2}|m_1 m_2 e\rangle+B_{m_1 m_2}|m_1+1,m_2+1,g\rangle)$$
$$\times(A^*_{n_1 n_2}\langle n_1 n_1 e|+B^*_{n_1 n_2}\langle n_1+1,n_2+1,g|)\ , \tag{32}$$

in which

$$A_{m_1 m_2} = i\sin(\lambda_{m_1 m_2} t)\,,$$
$$B_{m_1 m_2} = \cos(\lambda_{m_1 m_2} t)\,. \tag{33}$$

Using Eq. (32) we can determine all the dynamic and statistical properties of the atom and the field for any initial field state.

For the field initially in the pair-coherent state defined by Eq. (3), we have

$$C_{m_1 m_2, n_1 n_2} = N_q^2 \frac{|\zeta|^2}{[m!(m+q)!n!(n+q)!]^{1/2}}\,. \tag{34}$$

The probability $P_e(t)$ of finding an atom in the excited state $|e\rangle$ can be evaluated using Eq. (32), and is found to be given by

$$\begin{aligned} P_e(t) &= \langle n_1 n_2 e|\rho|n_1 n_2 e\rangle \\ &= \sum_n C_{n_1 n_2, n_1 n_2} \sin^2(\lambda_{n_1 n_2} t) \\ &= \tfrac{1}{2} + \sum_n C_{n_1 n_2, n_1 n_2} \cos(2\lambda_{n_1 n_2} t)\,, \end{aligned} \tag{35}$$

where

$$C_{n_1 n_2, n_1 n_2} = N_q^2 \frac{|\zeta|^2}{n!(n+q)!}\,. \tag{36}$$

The expression of $P_e(t)$ is a series which can be summed up numerically for any parameter values. However, we can obtain an analytic expression for it by noting that in the limit $|\zeta| \gg 1$, the maximum contribution to the sum in Eq. (35) comes from n near $|\zeta|$, so that for $n_1, n_2 \sim |\zeta| \gg 1$, we have $(q \sim 0)$

$$[(n_1+1)(n_2+1)]^{1/2} \cong n + 1 + q/2\,, \tag{37}$$

and consequently the expression for $P_e(t)$ is

$$P_e(t) = N_q^2 \mathrm{Re}[e^{2igt} J_q(2i|\zeta|e^{igt})]\,. \tag{38}$$

The asymptotic form of $J_q(z)$ is a complicated series; however, a simplified expression for $P_e(t)$ can be obtained when $q=0$ with the substitution

$$J_0(z) = \sqrt{2/\pi z}\,\cos(z - \pi/4)\,, \tag{39}$$

and we obtain, after simplification,

$$\begin{aligned} P_e(t) \cong{}& \frac{e^{-2|\zeta|}}{\sqrt{2\pi|\zeta|}} \cos[2|\zeta|\sin(gt) + 2gt] \\ &\times \cosh[2|\zeta|\cos(gt)]\,. \end{aligned} \tag{40}$$

Note that the asymptotic expression of $P_e(t)$ in the case of a coherent field with mean photon number $|\zeta|^2$ is given by[6]

$$P_e(t) = \exp[=2|\zeta|^2\sin^2(gt)]\cos[|\zeta|^2\sin(2gt) + 3gt]\,. \tag{40a}$$

In Fig. 4 we plot $P_e(t)$ for $|\zeta|^2=40$ for both a pair-coherent field (curve A) and a coherent field with $q=0$. Interestingly, as in the coherent-state field, revivals are regular and compact in the pair-coherent field. The separation of revivals in both cases is equal to π/g, and the envelope of the revivals is Gaussian and matches well with the approximate results mentioned above. However, the widths of the revivals in the pair-coherent state is greater than those in the coherent state because of the sub-Poissonian nature of the former. After verification we find very good matching of analytical approximate results with the exact numerical summation of the series. To see the effect of parameter q on $P_e(t)$, we plot $P_e(t)$ in Fig. 5 for various values of q and keep $|\zeta|^2$ the same. We find that as q increases, revivals no longer remain regular and compact.

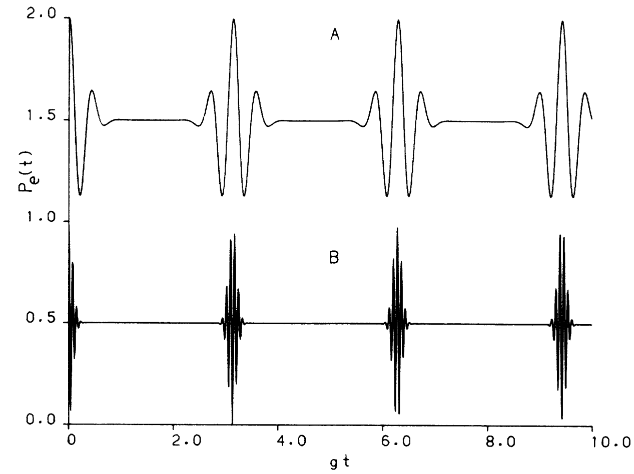

FIG. 4. Same as Fig. 2 but for an atom undergoing a two-photon transition.

Next, we study the statistical properties of the photon-

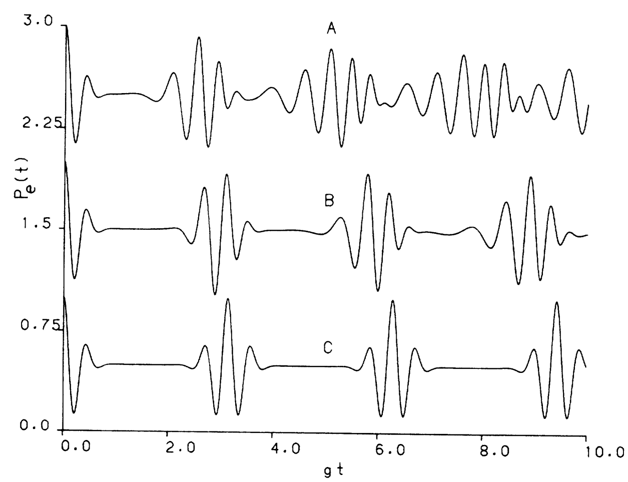

FIG. 5. Excitation probability $P_e(t)$ as a function of time for a two-photon transition in a pair-coherent-state field with $|\zeta|^2=40$. Curve A $[P_e(t)+2]$ is for $q=10$, curve B $[P_e(t)+1]$ is for $q=5$, and curve C $[P_e(t)]$ is for $q=0$.

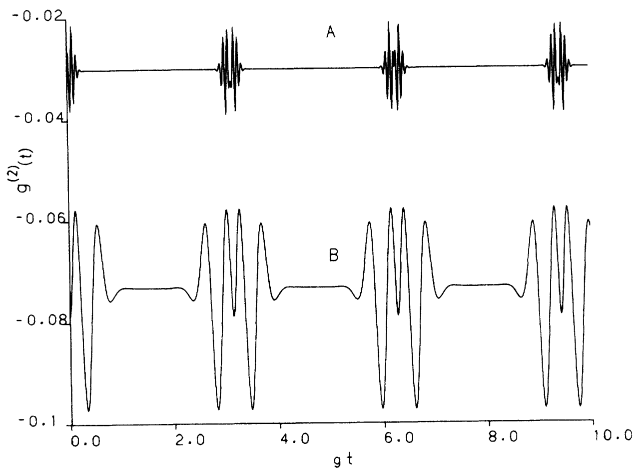

FIG. 6. Second-order coherence function $g^{(2)}(t)$ [defined by Eq. (41)] as a function of time for an atom undergoing a two-photon transition with $|\zeta|^2=40$. Curve A $[g^{(2)}(t)-0.03]$ is for the initial coherent-state field, curve B $(g^{(2)})$ is for the pair-coherent-state field.

number distribution by evaluating the second-order coherence function (Fig. 6), defined by[13,14]

$$g_{11}^{(2)}=\frac{\langle (a_1^\dagger)^2 a_1^2\rangle-\langle a_1^\dagger a_1\rangle^2}{\langle a_1^\dagger a_1\rangle^2}\ , \tag{41}$$

$$g_{22}^{(2)}=\frac{\langle (a_2^\dagger)^2 a_2^2\rangle-\langle a_2^\dagger a_2\rangle^2}{\langle a_2^\dagger a_2\rangle^2}\ , \tag{42}$$

or by the interbeam coherence function

$$g_{12}^{(2)}=\frac{\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle-\langle a_1^\dagger a_2\rangle\langle a_2^\dagger a_2\rangle}{\langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle}\ . \tag{43}$$

For simplicity we assume that $q=0$ for our analysis, and thus $g_{11}^{(2)}=g_{22}^{(2)}=g^{(2)}=g_{12}^{(2)}-1/\langle a_1 a_1\rangle$, because the modes are degenerate. We find[14]

$$\langle a_i^\dagger a_i\rangle=\tfrac{1}{2}+\sum_n n_i P_n-\tfrac{1}{2}\sum_n P_n\cos[2gt(n+1)]\ , \tag{44}$$

$$\langle (a_i^\dagger)^2 a_i^2\rangle=\sum_n n_i^2 P_n-\sum_n n_i P_n\cos[2gt(n+1)],\quad i=1,2 \tag{45}$$

where P_n is the same as in Eq. (14) for $q=0$.

The expression $g^{(2)}(t)$ can be simplified for $|\zeta|\gg 1$, and we can write

$$\frac{\langle (a^\dagger)^2 a^2\rangle}{\langle a^\dagger a\rangle^2}\cong\frac{\langle (a^\dagger)^2 a^2\rangle}{\langle a^\dagger a\rangle^2}\left[1+\frac{1}{\langle n\rangle}+2\frac{\langle S_z\rangle}{\langle n\rangle}+\cdots\right]. \tag{46}$$

Retaining the terms of leading order in Eq. (46), we obtain

$$g^{(2)}(t)\cong\frac{\langle n^2\rangle}{\langle n\rangle^2}\left[1-\frac{1}{\langle n\rangle}+\frac{3}{\langle n\rangle^2}\right]+\left[\frac{\langle n^2\rangle}{\langle n\rangle^3}\frac{1}{\sqrt{2\pi|\zeta|}}e^{-2|\zeta|}\sin[2|\zeta|\sin(gt)+2gt]\sin(gt)\cosh[2|\zeta|\cos(gt)]\right]$$
$$+\frac{\langle n^2\rangle}{\langle n\rangle^4}\left[\cdots\right]-1\ . \tag{47}$$

The envelope function of the dominating term (the term in the second square brackets) has a minimum at $t=t_r=\pi/g$ and extrema at $t_r\pm t_r/\pi\sqrt{2|\zeta|}$, thus showing "doublet" structure in the revivals. The expression of $g^{(2)}(t)$ for an ordinary coherent state has been given in Ref. 9. However, in the case of the coherent state the minimum in the revival occurs at $t=t_r$, but extrema occur at $t=t_r\pm t_r/2\pi|\zeta|$.

IV. CONCLUSION

In conclusion, we find that quantum effects due to any nonclassical field can be easily monitored by allowing the field to interact with an atom in a lossless cavity, and that the observed collapses and revivals phenomenon will reflect the nature of the field. Thus we find that, due to remarkable quantum features of the pair-coherent-state field (e.g., sub-Poissonian statistics, correlations in number fluctuations, and violation of Cauchy-Schwarz inequalities), its signature is apparent in the collapses and revivals phenomenon of Rabi oscillations, making it vastly different from an ordinary coherent-state field.

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