Virtual photons and causality in the dynamics of a pair of two-level atoms

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(Received 16 February 1990)

The dynamics of an initially bare pair of two-level atoms at distance R is investigated. The pair is coupled to the vacuum radiation field in the multipolar scheme and in the dipole approximation. The Heisenberg equations of motion for various atomic operators are obtained neglecting terms $O(e^3)$ and for t smaller than the spontaneous relaxation time. A rigorous proof of causality in the atom-atom interaction is given. Interatomic correlations, however, are shown to develop for $t < R/c$. This early development is shown to be compatible with the principle of relativistic causality, and to be related to the peculiar form of the vacuum zero-point fluctuations. The techniques adopted are applied to study dynamics and correlations of a superradiant state.

I. INTRODUCTION

This paper is concerned with the canonical problem of the dynamics of two atoms at distance R coupled to the electromagnetic radiation field. If one of the two atoms is excited and the other is in its ground state at $t=0$, it is often stated that the latter has a nonvanishing probability of being excited only after time R/c,[1] and this statement is reckoned to descend from the general principle of causal propagation of light signals being exchanged between two sources. Another, and perhaps more precise, statement has been proposed according to which the two atoms evolve independently and their mutual influence plays a role only for $t > R/c$.[2] A third statement is that before $t < R/c$ there can be no change in directly observable quantities related to the initially unexcited atom.[3] These three statements are, albeit subtly, different from each other. Moreover, the possibility that QED might exhibit noncausal behavior in the two-atom problem has been considered previously up to very recently.[4-6] Since this problem is vital for the interpretation of quantum theory, and in particular for the theory of measurement, it seems worthwhile to reconsider it in as rigorous terms as possible within the boundaries of nonrelativistic QED. It is appropriate to mention here that the problem of causality in a fully relativistic context is beset with difficulties connected with the definition of particle position and with the process of localization of states.[7] We bypass these difficulties here, since we treat the atomic part of the system in a nonrelativistic fashion.

A preliminary question which arises is that of choosing a model for the bare atoms. A variety of these models has been used in the literature in connection with investigation on causality. In recent years the choice has fallen mainly on harmonic oscillators,[8] multilevel atoms,[9] and two-level systems.[10] Here we discard harmonic oscillators in order to eliminate possible spurious effects due to the homologous structure of the eigenvalues of the bare source with those of any of the field modes. We also discard multilevel atoms in order to keep the mathematics as simple as possible and we choose two-level atoms, which have been widely and successfully adopted in the past.

Another question which we need to consider right at the outset is the meaning to be attributed to bare atomic states, since strictly speaking bare atomic states in QED do not exist. We shall take the point of view that a bare atomic state actually represents a partially dressed atomic state[11] consisting of an atom surrounded by an incomplete cloud of virtual photons. The cloud is incomplete in the sense that it extends only out to a few atomic radii. Even such a radically incomplete cloud, however, is responsible for endowing the atomic electron with most of the physical mass experimentally observed and for preventing the atom from falling apart because of lack of Coulomb forces.[12] We shall not dwell on this question any more here, but we anticipate that the legitimacy of such a point of view is supported by recent results on

spontaneous decay, based on a relativistic quantum-field theoretical approach.[13]

The aim of this paper is to present a set of rigorous results on the dynamics of a pair of initially bare two-level atoms which we hope should illustrate satisfactorily the problems associated with causality and atom-atom correlations.

II. ATOMIC DYNAMICS

We take two identical atoms 1 and 2, localized at points O and $\mathbf{R}$, respectively. Clearly $\mathbf{R}$ is well defined as an atomic space coordinate because we are treating atoms in a nonrelativistic context. The Dicke Hamiltonian for the system is

$$H=\hbar\omega_0(S_z^{(1)}+S_z^{(2)})+\sum_{\mathbf{k},j}\hbar\omega_k a_{\mathbf{k}j}^\dagger a_{\mathbf{k}j}+\sum_{\mathbf{k},j}[\epsilon_{\mathbf{k}j}a_{\mathbf{k}j}(S_+^{(1)}+e^{i\mathbf{k}\cdot\mathbf{R}}S_+^{(2)})+\epsilon_{\mathbf{k}j}^* a_{\mathbf{k}j}^\dagger(S_-^{(1)}+e^{-i\mathbf{k}\cdot\mathbf{R}}S_-^{(2)})]$$
$$-\lambda\sum_{\mathbf{k},j}[\epsilon_{\mathbf{k}j}a_{\mathbf{k}j}^\dagger(S_+^{(1)}+e^{-i\mathbf{k}\cdot\mathbf{R}}S_+^{(2)})+\epsilon_{\mathbf{k}j}^* a_{\mathbf{k}j}(S_-^{(1)}+e^{i\mathbf{k}\cdot\mathbf{R}}S_-^{(2)})]\ , \tag{2.1}$$

where ω_0 is the bare natural frequency of the atoms, ω_k is the frequency of field mode of wave vector $\mathbf{k}$ and polarization index j, ϵ is the atom-photon coupling constant, and λ is a parameter which we introduce for diagnostic purposes [$\lambda=0$ in the rotating-wave approximation (RWA) and $\lambda=1$ otherwise]. We adopt the multipolar coupling scheme in the dipolar approximation, within the Coulomb gauge. Thus[14]

$$\epsilon_{\mathbf{k}j}=i\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}\mathbf{e}_{\mathbf{k}j}\cdot\mathbf{d} \tag{2.2}$$

where V is the quantization volume, $\mathbf{e}$ is the polarization vector, and $\mathbf{d}$ is the matrix element of the electronic dipole operator, which we take as real, between the atomic ground and excited states. Moreover, the S's in (2.1) are the usual pseudospin atomic operators ($i=1,2$) and a is a Bose annihilation operator for photons. Obviously (2.1) is nonrelativistic in the atomic part, so that $\mathbf{R}$ appears as a three-vector in the exponentials.

The equation of motion for the population operator of atom 1 at the origin is

$$\dot{S}_z^{(1)}=-\frac{i}{\hbar}[S_z^{(1)},H]=-\frac{i}{\hbar}\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}(a_{\mathbf{k}j}+a_{\mathbf{k}j}^\dagger)S_+^{(1)}+\text{H.c.} \tag{2.3}$$

Moreover

$$(a_{\mathbf{k}j}\dot{S}_+^{(1)})=i(\omega_0-\omega_k)a_{\mathbf{k}j}S_+^{(1)}-\frac{i}{\hbar}\epsilon_{\mathbf{k}j}^*(S_z^{(1)}+\tfrac{1}{2}+S_+^{(1)}S_-^{(2)}e^{-i\mathbf{k}\cdot\mathbf{R}})+\frac{i}{\hbar}\lambda\epsilon_{\mathbf{k}j}S_+^{(1)}S_+^{(2)}e^{-i\mathbf{k}\cdot\mathbf{R}}$$
$$-\frac{i}{\hbar}\sum_{\mathbf{k}',j'}2\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}'j'}^\dagger a_{\mathbf{k}j}-\lambda a_{\mathbf{k}'j'}a_{\mathbf{k}j})S_z^{(1)}\ ,$$
$$(a_{\mathbf{k}j}^\dagger\dot{S}_+^{(1)})=i(\omega_0+\omega_k)a_{\mathbf{k}j}^\dagger S_+^{(1)}+\frac{i}{\hbar}\epsilon_{\mathbf{k}j}S_+^{(1)}S_+^{(2)}e^{i\mathbf{k}\cdot\mathbf{R}}+\frac{i}{\hbar}\lambda\epsilon_{\mathbf{k}j}^*(S_z^{(1)}-\tfrac{1}{2}-S_+^{(1)}S_-^{(2)}e^{i\mathbf{k}\cdot\mathbf{R}}) \tag{2.4}$$
$$-\frac{i}{\hbar}\sum_{\mathbf{k}',j'}2\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'}^\dagger-\lambda a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'})S_z^{(1)}\ .$$

Equations (2.4) can be integrated formally, yielding

$$(a_{\mathbf{k}j}S_+^{(1)})_t=-\frac{i}{\hbar}\epsilon_{\mathbf{k}j}^*e^{i(\omega_0-\omega_k)t}\int_0^t e^{-i(\omega_0-\omega_k)t'}[S_z^{(1)}(t')+\tfrac{1}{2}]dt'-\frac{i}{\hbar}\epsilon_{\mathbf{k}j}^*e^{-i\mathbf{k}\cdot\mathbf{R}}e^{i(\omega_0-\omega_k)t}\int_0^t e^{-i(\omega_0-\omega_k)t'}(S_+^{(1)}S_-^{(2)})_{t'}dt'$$
$$+\frac{i}{\hbar}\lambda\epsilon_{\mathbf{k}j}e^{-i\mathbf{k}\cdot\mathbf{R}}e^{i(\omega_0-\omega_k)t}\int_0^t e^{-i(\omega_0-\omega_k)t'}(S_+^{(1)}S_+^{(2)})_{t'}dt'+(a_{\mathbf{k}j}S_+^{(1)})_0e^{i(\omega_0-\omega_k)t}$$
$$-\frac{2i}{\hbar}\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*e^{i(\omega_0-\omega_k)t}\int_0^t e^{-i(\omega_0-\omega_k)t'}(a_{\mathbf{k}'j'}^\dagger a_{\mathbf{k}j}S_z^{(1)})_{t'}dt'$$
$$+\frac{2i}{\hbar}\lambda\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*e^{i(\omega_0-\omega_k)t}\int_0^t e^{-i(\omega_0-\omega_k)t'}(a_{\mathbf{k}'j'}a_{\mathbf{k}j}S_z^{(1)})_{t'}dt'\ ,$$
$$(a_{\mathbf{k}j}^\dagger S_+^{(1)})_t=\frac{i}{\hbar}\lambda\epsilon_{\mathbf{k}j}^*e^{i(\omega_0+\omega_k)t}\int_0^t e^{-i(\omega_0+\omega_k)t'}[S_z^{(1)}(t')-\tfrac{1}{2}]dt'+\frac{i}{\hbar}\epsilon_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{R}}e^{i(\omega_0+\omega_k)t}\int_0^t e^{-i(\omega_0+\omega_k)t'}(S_+^{(1)}S_+^{(2)})_{t'}dt' \tag{2.5}$$
$$-\frac{i}{\hbar}\lambda\epsilon_{\mathbf{k}j}^*e^{i\mathbf{k}\cdot\mathbf{R}}e^{i(\omega_0+\omega_k)t}\int_0^t e^{-i(\omega_0+\omega_k)t'}(S_+^{(1)}S_-^{(2)})_{t'}dt'+(a_{\mathbf{k}j}^\dagger S_+^{(1)})_0e^{i(\omega_0+\omega_k)t}$$
$$-\frac{2i}{\hbar}\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*e^{i(\omega_0+\omega_k)t}\int_0^t e^{-i(\omega_0+\omega_k)t'}(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'}^\dagger S_z^{(1)})_{t'}dt'$$
$$+\frac{2i}{\hbar}\lambda\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*e^{i(\omega_0+\omega_k)t}\int_0^t e^{-i(\omega_0+\omega_k)t'}(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'}S_z^{(1)})_{t'}dt'\ .$$

The procedure is exact up to this point. We now evaluate (2.5) for small times such that the various operators appearing in the integrals on the rhs of (2.5) can be approximated by their free-field expressions. This means that time t must be so small that the changes in atomic as well as in field populations due to the atom-photon coupling are negligible. Thus $t \lesssim \gamma^{-1}$, where γ^{-1} is the single atom spontaneous emission time. This is equivalent to the first Born approximation, which is obtained by neglecting terms $O(\epsilon^2)$ in (2.5) since these would contribute terms $O(\epsilon^3)$ in (2.3). In this way we obtain

$$
\begin{aligned}
(a_{\mathbf{k}j}S_+^{(1)})_t = &\frac{1}{\hbar}\epsilon_{\mathbf{k}j}^*[S_z^{(1)}(0)+\tfrac{1}{2}]F(k_0-k)+\frac{1}{\hbar}\epsilon_{\mathbf{k}j}^* e^{-i\mathbf{k}\cdot\mathbf{R}}(S_+^{(1)}S_-^{(2)})_0 F(k_0-k) \\
&+\frac{1}{\hbar}\lambda\epsilon_{\mathbf{k}j}e^{-i\mathbf{k}\cdot\mathbf{R}}(S_+^{(1)}S_+^{(2)})_0 e^{2i\omega_0 t}F^*(k_0+k)+(a_{\mathbf{k}j}^\dagger S_+^{(1)})_0 e^{i(\omega_0-\omega_k)t} \\
&-\frac{2}{\hbar}\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}'j'}^\dagger a_{\mathbf{k}j}S_z^{(1)})_0 e^{i(\omega_0-\omega_k)t}F^*(k_0-k')+\frac{2}{\hbar}\lambda\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}'j'}a_{\mathbf{k}j}S_z^{(1)})_0 e^{i(\omega_0-\omega_k)t}F^*(k_0+k')\,, \\
(a_{\mathbf{k}j}^\dagger S_+^{(1)})_t = &-\frac{1}{\hbar}\lambda\epsilon_{\mathbf{k}j}^*[S_z^{(1)}(0)-\tfrac{1}{2}]F(k_0+k)+\frac{1}{\hbar}\epsilon_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{R}}(S_+^{(1)}S_+^{(2)})_0 e^{2i\omega_0 t}F^*(k_0-k) \\
&+\frac{1}{\hbar}\lambda\epsilon_{\mathbf{k}j}^* e^{i\mathbf{k}\cdot\mathbf{R}}(S_+^{(1)}S_-^{(2)})_0 F(k_0+k)+(a_{\mathbf{k}j}^\dagger S_+^{(1)})_0 e^{i(\omega_0+\omega_k)t} \\
&-\frac{2}{\hbar}\sum_{\mathbf{k}',j'}\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'}^\dagger S_z^{(1)})_0 e^{i(\omega_0+\omega_k)t}F^*(k_0-k')+\frac{2}{\hbar}\lambda\sum_{\mathbf{k},'j'}\epsilon_{\mathbf{k}'j'}^*(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'}S_z^{(1)})_0 e^{i(\omega_0+\omega_k)t}F^*(k_0+k')\,,
\end{aligned}
\tag{2.6}
$$

where

$$
k_0=\omega_0/c,\quad F(k_a+k_b)=-i\int_0^t \exp[i(\omega_{k_a}+\omega_{k_b})t']dt'=\frac{1-\exp[i(\omega_{k_a}+\omega_{k_b})t]}{\omega_{k_a}+\omega_{k_b}}\,. \tag{2.7}
$$

Substitution of (2.6) in (2.3) yields

$$
\begin{aligned}
\dot S_z^{(1)} = &-\frac{i}{\hbar^2}\Bigg[\sum_{k,j}|\epsilon_{\mathbf{k}j}|^2\{[S_z^{(1)}(0)+\tfrac{1}{2}]F(k_0-k)+\lambda^2[S_z^{(1)}(0)-\tfrac{1}{2}]F(k_0+k)\} \\
&+(S_+^{(1)}S_-^{(2)})_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-\lambda^2 e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)] \\
&-\lambda(S_+^{(1)}S_+^{(2)})_0 e^{2i\omega_0 t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)-e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)]\Bigg] \\
&-\frac{1}{\hbar}\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}[(a_{\mathbf{k}j}S_+^{(1)})_0 e^{i(\omega_0-\omega_k)t}-\lambda(a_{\mathbf{k}j}^\dagger S_+^{(1)})_0 e^{i(\omega_0+\omega_k)t}] \\
&+\frac{2i}{\hbar^2}\sum_{\mathbf{k},\mathbf{k}',j,j'}\epsilon_{\mathbf{k}j}\epsilon_{\mathbf{k}'j'}^*\{[(a_{\mathbf{k}'j'}^\dagger a_{\mathbf{k}j})_0 e^{i(\omega_0-\omega_k)t}-\lambda(a_{\mathbf{k}'j'}^\dagger a_{\mathbf{k}j}^\dagger)_0 e^{i(\omega_0+\omega_k)t}]S_z^{(1)}(0)F^*(k_0-k') \\
&-[\lambda(a_{\mathbf{k}j}a_{\mathbf{k}'j'})_0 e^{i(\omega_0-\omega_k)t}-\lambda^2(a_{\mathbf{k}j}^\dagger a_{\mathbf{k}'j'})_0 e^{i(\omega_0+\omega_k)t}]S_z^{(1)}(0)F^*(k_0+k')\}+\text{H.c.}\,,
\end{aligned}
\tag{2.8}
$$

where $\epsilon^2=-|\epsilon|^2$ has been used.

We now take quantum averages of (2.8) on the set of states

$$
|\phi(S^{(1)},S^{(2)}),\{O_{\mathbf{k}j}\}\rangle \tag{2.9}
$$

These states are factorizable in an atomic and a field part which is taken to be the vacuum. The atomic part is quite general and no assumption is made at this point about its structure. Clearly only the terms within the first pair of curly brackets on the rhs of (2.8) contribute to

$$
\langle \dot S_z^{(1)}\rangle_t \equiv \langle \phi(S^{(1)},S^{(2)}),\{O_{\mathbf{k}j}\}|\dot S_z^{(1)}(t)|\phi(S^{(1)},S^{(2)}),\{O_{\mathbf{k}j}\}\rangle
$$

and one gets

$$
\begin{aligned}
\langle \dot S_z^{(1)}\rangle_t = &-\frac{i}{\hbar^2}\Bigg[\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[(\langle S_z^{(1)}\rangle_0+\tfrac{1}{2})F(k_0-k)+\lambda^2(\langle S_z^{(1)}\rangle_0-\tfrac{1}{2})F(k_0+k)] \\
&+\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-\lambda^2 e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)] \\
&-\lambda\langle S_+^{(1)}S_+^{(2)}\rangle_0 e^{2i\omega_0 t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)-e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)]\Bigg]+\text{c.c.}
\end{aligned}
\tag{2.10}
$$

The sums on the rhs of (2.10) can be readily transformed into integrals with the following results:

$$
\begin{aligned}
&-\frac{i}{\hbar^2}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0-k)+\text{c.c.}=-\frac{\gamma}{\pi k_0^3}\left[D_1^t\{\mathrm{Si}(k_0ct)+\mathrm{Si}[(k_M-k_0)ct]\}+D_2^t\left[\frac{1}{ct}[\cos k_0ct-\cos(k_M-k_0)ct]\right]\right],\\
&-\frac{i}{\hbar^2}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0+k)+\text{c.c.}=\frac{\gamma}{\pi k_0^3}\left[D_1^t\{\mathrm{Si}[(k_M+k_0)ct]-\mathrm{Si}(k_0ct)\}-D_2^t\left[\frac{1}{ct}[\cos k_0ct-\cos(k_M+k_0)ct]\right]\right],\\
&-\frac{i}{\hbar^2}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-\lambda^2e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)]=\frac{i}{\pi\hbar}d_md_nD_{mn}^R\left[\lambda^2\int_{-\infty}^{0}\frac{1-e^{i(k_0-k)ct}}{k_0-k}\sin kR\,dk\right.\\
&\qquad\left.+\int_0^{\infty}\frac{1-e^{i(k_0-k)ct}}{k_0-k}\sin kR\,dk\right],\\
&-\frac{i}{\hbar^2}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)-e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)]=-\frac{i}{\pi\hbar}d_md_nD_{mn}^R\int_{-\infty}^{\infty}\frac{1-e^{i(k_0-k)ct}}{k_0-k}\sin kR\,dk,
\end{aligned}
\tag{2.11}
$$

where

$$
\gamma=\frac{2\pi}{\hbar^2}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2\delta(\omega_0-\omega_k)=\frac{4|\mathbf{d}|^2\omega_0^3}{3\hbar c^3}
$$

is the single-atom spontaneous relaxation rate,

$$
\begin{aligned}
&D_1^t=k_0^3-3\frac{k_0}{c^2}\frac{\partial^2}{\partial t^2},\quad D_2^t=3k_0^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2},\\
&D_{mn}^R=\frac{1}{R}\left[(\delta_{mn}-\hat{R}_m\hat{R}_n)\frac{\partial^2}{\partial R^2}\right.\\
&\qquad\left.+(\delta_{mn}-3\hat{R}_m\hat{R}_n)\left[\frac{1}{R^2}-\frac{1}{R}\frac{\partial}{\partial R}\right]\right]
\end{aligned}
\tag{2.12}
$$

and k_M is a cutoff wave vector typical of nonrelativistic QED which we take here to be of the same order of magnitude as the inverse Bohr radius $1/a_0$.

It is easy to see that the first sum in (2.10) yields $-\gamma(\langle S_z^{(1)}\rangle_0+\frac{1}{2})$, which represents the effect of exponential decay of atom 1, plus nonphysical terms oscillating at the very high frequency ω_M. The latter terms represent spurious nonexponential corrections to this exponential decay, as we shall discuss later on in the course of this paper, and they are related to the non-Markovian nature of our treatment. In any case, all the terms in the first sum are the same as in absence of atom 2. On the contrary, the second and third sums in (2.10) represent the effect of atom 2. Since

$$
\int_{-\infty}^{\infty}\frac{1-e^{\pm i(k_0-k)ct}}{k_0-k}\sin kR\,dk=-\pi e^{\pm ik_0R}\theta(ct-R)
\tag{2.13}
$$

for $\lambda=1$ these two sums do not contribute to the time evolution of $\langle S_z^{(1)}\rangle$ until $t=R/c$. For $\lambda=0$, however, causality in interatomic forces is obviously lost. We remark that in previous treatments[1,2] causality in the two-level atom pair was obtained as an approximation, which is clearly unsatisfactory. The present treatment shows that causality can be obtained exactly at order e^2.

Starting from the Heisenberg equation for S_+ and following the same procedure above leads to

$$
\begin{aligned}
\langle\dot{S}_+^{(1)}\rangle_t=&\,i\omega_0\langle S_+^{(1)}\rangle_t+\frac{i}{\hbar^2}\left[\langle S_+^{(1)}\rangle_0e^{i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F^*(k_0-k)+\lambda^2F^*(k_0+k)]\right.\\
&+\lambda\langle S_-^{(1)}\rangle_0e^{-i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F(k_0-k)+F(k_0+k)]\\
&-2\langle S_z^{(1)}S_+^{(2)}\rangle_0e^{i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)-\lambda^2e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)]\\
&\left.+2\lambda\langle S_z^{(1)}S_-^{(2)}\rangle_0e^{-i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)-e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)]\right]
\end{aligned}
\tag{2.14}
$$

within the same approximations as in (2.10). Altogether for $\gamma t\ll1$ and $\lambda=1$ one obtains from (2.10) and (2.14)

$$
\langle\dot{S}_z^{(1)}\rangle_t=\langle\dot{S}_z^{(1)}\rangle_t(A_1)+\left[-\frac{i}{\hbar}d_md_nD_{mn}^R[(\langle S_+^{(1)}S_-^{(2)}\rangle_0e^{ik_0R}+\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}e^{-ik_0R})\theta(ct-R)]+\text{c.c.}\right]
\tag{2.15a}
$$

$$
\langle\dot{S}_+^{(1)}\rangle_t=\langle\dot{S}_+^{(1)}\rangle_t(A_1)-\frac{2i}{\hbar}d_md_nD_{mn}^R[(\langle S_z^{(1)}S_+^{(2)}\rangle_0e^{i\omega_0t}e^{-ik_0R}+\langle S_z^{(1)}S_-^{(2)}\rangle_0e^{-i\omega_0t}e^{ik_0R})\theta(ct-R)],
\tag{2.15b}
$$

where A_1 refers to atom 1 in the absence of atom 2. From Eqs. (2.15) it is evident that an observer watching atom 1 has no way of detecting the existence of atom 2 until $t=R/c$. The conclusion would have been different for $\lambda=0$ in the RWA, since in that case the influence of atom 2 would have changed the equations of motion of atom 1 immediately after $t=0$.

III. CAUSALITY AND CORRELATIONS

It has been suggested[5] that correlations in the two-atom system under scrutiny should display noncausal behavior, even when the counterrotating terms are taken into account. In particular, apparently noncausal behavior has been obtained in the transition amplitude for the exchange of one atomic excitation between atoms 1 and 2. Here we shall take a more general point of view and we shall show that this apparently noncausal behavior is related to the properties of the interatomic correlation function. As discussed by Barnett and Phoenix,[15] two systems 1 and 2 are correlated if the measurement of an observable of 1 projects 2 into a new state. In fact, if A is an operator pertaining to 1 and B is an operator pertaining to 2, it is well known that the two systems are uncorrelated if $\langle AB\rangle=\langle A\rangle\langle B\rangle$, and $\langle AB\rangle-\langle A\rangle\langle B\rangle$ may be taken as a criterion for the degree of correlations between the two systems induced by the structure of the state on which the quantum averages are taken.

Here we consider in particular the time development of $\langle S_z^{(1)}S_z^{(2)}\rangle-\langle S_z^{(1)}\rangle\langle S_z^{(2)}\rangle$ where the operators are in the Heisenberg representation and the quantum averages are taken on a state for the form (2.9). One obtains first

$$\dot{(S_z^{(1)}S_z^{(2)})}=-\frac{i}{\hbar}[S_z^{(1)}S_z^{(2)},H]=-\frac{i}{\hbar}\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}[a_{\mathbf{k}j}(S_+^{(1)}S_z^{(2)}+e^{i\mathbf{k}\cdot\mathbf{R}}S_z^{(1)}S_+^{(2)})-\lambda a_{\mathbf{k}j}^{\dagger}(S_+^{(1)}S_z^{(2)}+e^{-i\mathbf{k}\cdot\mathbf{R}}S_z^{(1)}S_+^{(2)})]+\text{H.c.} \tag{3.1}$$

A long and tedious but straightforward procedure, analogous to that developed in Sec. II, leads to the following expression, valid up to terms of order ϵ^2 and for $t\ll\gamma^{-1}$:

$$\begin{aligned}
-\frac{i}{\hbar}\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}\langle a_{\mathbf{k}j}S_+^{(1)}S_z^{(2)}\rangle_t&=-\frac{i}{\hbar^2}(\langle S_z^{(1)}S_z^{(2)}\rangle_0+\langle S_z^{(2)}\rangle_0/2)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0-k)\\
&+\frac{i}{\hbar^2}\frac{1}{2}\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)\\
&+\frac{i}{\hbar^2}\lambda\tfrac{1}{2}\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)\,,\\
-\frac{i}{\hbar}\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{R}}\langle a_{\mathbf{k}j}S_z^{(1)}S_+^{(2)}\rangle_t&=-\frac{i}{\hbar^2}(\langle S_z^{(1)}S_z^{(2)}\rangle_0+\langle S_z^{(1)}\rangle_0/2)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0-k)\\
&+\frac{i}{\hbar^2}\frac{1}{2}\langle S_-^{(1)}S_+^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)\\
&+\frac{i}{\hbar^2}\lambda\tfrac{1}{2}\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0+k)\,,\\
\frac{i}{\hbar}\lambda\sum_{\mathbf{k},j}\epsilon_{\mathbf{k}j}\langle a_{\mathbf{k}j}^{\dagger}S_+^{(1)}S_z^{(2)}\rangle_t&=-\frac{i}{\hbar^2}(\langle S_z^{(1)}S_z^{(2)}\rangle_0-\langle S_z^{(2)}\rangle_0/2)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0+k)\\
&+\frac{i}{\hbar^2}\lambda^2\tfrac{1}{2}\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)\\
&+\frac{i}{\hbar^2}\lambda\tfrac{1}{2}\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)\,,\\
\frac{i}{\hbar}\lambda\sum_{\mathbf{k},j}e^{-i\mathbf{k}\cdot\mathbf{R}}\langle a_{\mathbf{k}j}^{\dagger}S_z^{(1)}S_+^{(2)}\rangle_t&=-\frac{i}{\hbar^2}\lambda^2(\langle S_z^{(1)}S_z^{(2)}\rangle_0-\langle S_z^{(1)}\rangle_0/2)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2F(k_0+k)\\
&+\frac{i}{\hbar^2}\lambda^2\tfrac{1}{2}\langle S_-^{(1)}S_+^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)\\
&+\frac{i}{\hbar^2}\lambda\tfrac{1}{2}\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)\,.
\end{aligned}\tag{3.2}$$

Taking the quantum average of (3.1) on the state (2.9) and using the result (3.2) one obtains

$$\langle S_z^{(1)}\dot{S}_z^{(2)}\rangle_t = -\frac{i}{\hbar}\Big[2\langle S_z^{(1)}S_z^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F(k_0-k)+\lambda^2F(k_0+k)]+\tfrac{1}{2}(\langle S_z^{(1)}\rangle_0+\langle S_z^{(2)}\rangle_0)\sum_{\mathbf{k},j}[F(k_0-k)-\lambda^2F(k_0+k)]$$
$$-\tfrac{1}{2}\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)+e^{i\mathbf{k}\cdot\mathbf{R}}\lambda^2F(k_0+k)]$$
$$-\tfrac{1}{2}\langle S_-^{(1)}S_+^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-e^{-i\mathbf{k}\cdot\mathbf{R}}\lambda^2F(k_0+k)]$$
$$-\lambda\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2\cos\mathbf{k}\cdot\mathbf{R}[F^*(k_0-k)+F^*(k_0+k)]\Big]+\text{c.c.} \tag{3.3}$$

It is also easy to show that for $t\ll\gamma^{-1}$ and neglecting $O(\epsilon^3)$

$$\langle\dot{S}_z^{(1)}\rangle_t\langle S_z^{(2)}\rangle_t+\langle S_z^{(1)}\rangle_t\langle\dot{S}_z^{(2)}\rangle_t$$
$$=-\frac{i}{\hbar^2}\Big[2\langle S_z^{(1)}\rangle_0\langle S_z^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F(k_0-k)+\lambda^2F(k_0+k)]$$
$$+\tfrac{1}{2}(\langle S_z^{(1)}\rangle_0+\langle S_z^{(2)}\rangle_0)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F(k_0-k)-\lambda^2F(k_0+k)]$$
$$+\langle S_z^{(2)}\rangle_0\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-e^{i\mathbf{k}\cdot\mathbf{R}}\lambda^2F(k_0+k)]$$
$$+\langle S_z^{(1)}\rangle_0\langle S_-^{(1)}S_+^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)-e^{-i\mathbf{k}\cdot\mathbf{R}}\lambda^2F(k_0+k)]$$
$$-\lambda\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[(\langle S_z^{(1)}\rangle_0e^{i\mathbf{k}\cdot\mathbf{R}}+\langle S_z^{(2)}\rangle_0e^{-i\mathbf{k}\cdot\mathbf{R}})F^*(k_0+k)$$
$$-(\langle S_z^{(1)}\rangle_0e^{-i\mathbf{k}\cdot\mathbf{R}}+\langle S_z^{(2)}\rangle_0e^{i\mathbf{k}\cdot\mathbf{R}})F^*(k_0-k)]\Big]+\text{c.c.} \tag{3.4}$$

From (3.3) and (3.4) one obtains

$$\frac{d}{dt}(\langle S_z^{(1)}S_z^{(2)}\rangle_t-\langle S_z^{(1)}\rangle_t\langle S_z^{(2)}\rangle_t)$$
$$=-\frac{2i}{\hbar^2}(\langle S_z^{(1)}S_z^{(2)}\rangle_0-\langle S_z^{(1)}\rangle_0\langle S_z^{(2)}\rangle_0)\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[F(k_0-k)+\lambda^2F(k_0+k)]$$
$$+\frac{i}{\hbar^2}\Big[\langle S_+^{(1)}S_-^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[(\tfrac{1}{2}+\langle S_z^{(2)}\rangle_0)e^{-i\mathbf{k}\cdot\mathbf{R}}F^*(k_0-k)+\lambda^2(\tfrac{1}{2}-\langle S_z^{(2)}\rangle_0)e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)]$$
$$+\langle S_-^{(1)}S_+^{(2)}\rangle_0\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[(\tfrac{1}{2}+\langle S_z^{(1)}\rangle_0)e^{i\mathbf{k}\cdot\mathbf{R}}F(k_0-k)+\lambda^2(\tfrac{1}{2}-\langle S_z^{(1)}\rangle_0)e^{-i\mathbf{k}\cdot\mathbf{R}}F(k_0+k)]$$
$$+\lambda\langle S_+^{(1)}S_+^{(2)}\rangle_0e^{2i\omega_0t}\sum_{\mathbf{k},j}|\epsilon_{\mathbf{k}j}|^2[(\cos\mathbf{k}\cdot\mathbf{R}+\langle S_z^{(1)}\rangle_0e^{-i\mathbf{k}\cdot\mathbf{R}}+\langle S_z^{(2)}\rangle_0e^{i\mathbf{k}\cdot\mathbf{R}})F^*(k_0-k)$$
$$+(\cos\mathbf{k}\cdot\mathbf{R}-\langle S_z^{(2)}\rangle_0e^{-i\mathbf{k}\cdot\mathbf{R}}-\langle S_z^{(1)}\rangle_0e^{i\mathbf{k}\cdot\mathbf{R}})F^*(k_0+k)]\Big]+\text{c.c.} \tag{3.5}$$

The first term on the rhs of (3.5) does not depend on $\mathbf{R}$ and consequently it corresponds to a local effect of the interaction with the field, which induces spontaneous decay of each atom independently of the other and which tends to decrease the degree of correlation initially present in the system. The other term is $\mathbf{R}$ dependent, and it is easy to convince oneself that it cannot be expressed entirely in terms of θ functions like in (2.15). Indeed, one has to face the fact that atomic correlations develop "noncausally" due to this term. In particular, if the initial $\Phi(S^{(1)},S^{(2)})$ state is taken to be completely uncorrelated, some correlations develop before $t=R/c$. This, however, is not by itself in contradiction with the principle of relativistic causality. In fact, in order to measure these correlations one needs two observers who take simultaneous measurements on atoms 1 and 2 separately. The existence of these correlations can be established only after comparison of the results obtained by the two observers, which implies some form of communicating information between them; it is this necessity of transmitting information which ensures overall causality in any process of measurement of interatomic correlations. Consequently the real question here is not that the presence of these early correlations itself is paradoxical, but rather what is

their physical origin.

In order to investigate this origin we consider as an example two initially completely uncorrelated atoms, with

$$|\phi(S^{(1)},S^{(2)})\rangle=\tfrac{1}{2}(|\uparrow_1\rangle+|\downarrow_1\rangle)(|\uparrow_2\rangle+|\downarrow_2\rangle) \tag{3.6}$$

in such a way that

$$\langle S_z^{(1)}\rangle_0=\langle S_z^{(2)}\rangle_0=\langle S_z^{(1)}S_z^{(2)}\rangle_0=0\ ,$$

$$\langle S_-^{(1)}S_+^{(2)}\rangle_0=\langle S_+^{(1)}S_+^{(2)}\rangle_0=\tfrac{1}{4}\ .$$

Thus the decorrelation due to local effects of the vacuum fluctuation vanishes, and for $\lambda=1$ one obtains after some algebra

$$\frac{d}{dt}(\langle S_z^{(1)}S_z^{(2)}\rangle_t-\langle S_z^{(1)}\rangle_t\langle S_z^{(2)}\rangle_t)=\frac{d_m d_n}{2\pi\hbar}\,\mathrm{Im}\left[(1-e^{-2i\omega_0 t})D_{mn}^R\left[\int_0^\infty\frac{1-e^{i(k_0-k)ct}}{k_0-k}\sin kR\,dk\right.\right.$$

$$\left.\left.+\int_0^\infty\frac{1-e^{i(k_0+k)ct}}{k_0+k}\sin kR\,dk\right]\right]\ , \tag{3.7}$$

which is clearly "noncausal." For $R\gg ct$ one can approximate

$$\int_0^\infty\frac{1-\exp[i(k_0\pm k)ct]}{k_0\pm k}\sin kR\,dk=-\frac{ic}{2}\left[\int_0^t\frac{\exp(ik_0ct')}{R+ct'}dt'+\int_0^t\frac{\exp(ik_0ct')}{R-ct'}dt'\right]\approx-\frac{2}{k_0R}[\exp(i\omega_0t)-1]\ . \tag{3.8}$$

Substituting (3.8) into (3.7), performing the t integration and the R derivatives one gets

$$\langle S_z^{(1)}S_z^{(2)}\rangle_t-\langle S_z^{(1)}\rangle_t\langle S_z^{(2)}\rangle_t$$

$$\equiv\langle S_z^{(1)}S_z^{(2)}\rangle_t=-\frac{8d_m d_n}{\pi\hbar k_0}F(t)\frac{1}{R^4}(\delta_{mn}-2\hat{R}_m\hat{R}_n)\ , \tag{3.9}$$

$$F(t)=\frac{1-\cos\omega_0t}{\omega_0}-\frac{1-\cos2\omega_0t}{4\omega_0}\ .$$

It is interesting to compare (3.9) with the vacuum field correlations at atomic location 0 and **R**. One obtains

$$\langle E_m(0)E_n(\mathbf{R})\rangle=-\frac{4\hbar c}{\pi R^4}(\delta_{mn}-2\hat{R}_m\hat{R}_n)\ . \tag{3.10}$$

We consider the similarity between (3.9) and (3.10) as strong evidence in support of the hypothesis that the apparently noncausal atomic correlations are indeed induced by the correlations existing in the vacuum electromagnetic field.[6,16] In fact, it would be appropriate to state that atom-atom correlations (3.9), if experimentally confirmed, would provide evidence for correlations in the vacuum fluctuations of the electromagnetic field of the form (3.10). One can also express such a statement in the form

$$\langle S_z^{(1)}S_z^{(2)}\rangle_t=\frac{\alpha_{mn}}{\hbar\omega_0}F(t)\langle E_m(0)E_n(\mathbf{R})\rangle$$

$$(\alpha_{mn}=\frac{2d_m d_n}{\hbar\omega_0},\quad R\gg ct) \tag{3.11}$$

where α is the ground-state polarizability of the (identical) two-level atoms.

IV. THE DICKE SUPERRADIANT STATE

As a further illustrative example we consider a strongly correlated initial state. We choose the so-called Dicke superradiant state,[17] which is of interest on its own, apart from being of the same form of the state normally used in the discussion of the Einstein-Podolsky-Rosen (EPR) paradox. This state is

$$|\phi(S^{(1)},S^{(2)})\rangle=\frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle+|\downarrow_1\uparrow_2\rangle) \tag{4.1}$$

For this state we have

$$\langle S_z^{(1)}\rangle_0=\langle S_z^{(2)}\rangle_0=0,\quad\langle S_+^{(1)}\rangle_0=\langle S_-^{(1)}\rangle_0=0\ ,$$

$$\langle S_z^{(1)}S_z^{(2)}\rangle_0=-\tfrac{1}{4},\quad\langle S_+^{(1)}S_+^{(2)}\rangle_0=0,\quad\langle S_+^{(1)}S_-^{(2)}\rangle_0=\tfrac{1}{2}\ .$$

Thus using (2.11) we obtain from (2.10)

$$\langle \dot{S}_z^{(1)} \rangle_t(A_1) = -\frac{\gamma}{2\pi k_0^3}\left[D_1^t\{(1-\lambda^2)\mathrm{Si}(k_0 ct)+\mathrm{Si}[(k_M-k_0)ct]+\lambda^2\mathrm{Si}[(k_M+k_0)ct]\} \right.$$
$$\left. +D_2^t\left[\frac{1}{ct}[(1-\lambda^2)\cos k_0 ct-\cos(k_M-k_0)ct+\lambda^2\cos(k_M+k_0)ct]\right]\right]$$
$$= -\frac{\gamma}{2\pi}\left[(1-\lambda^2)\mathrm{Si}(\omega_0 t)+\mathrm{Si}[(\omega_M-\omega_0)t]+\lambda^2\mathrm{Si}[(\omega_M+\omega_0)t]\right.$$
$$-\left[1+\frac{\omega_M}{\omega_0}+\frac{\omega_M^2}{\omega_0^2}\right]\frac{\cos(\omega_M-\omega_0)t}{\omega_0 t}+\lambda^2\left[1-\frac{\omega_m}{\omega_0}+\frac{\omega_M^2}{\omega_0^2}\right]\frac{\cos(\omega_M+\omega_0)t}{\omega_0 t}+(1-\lambda^2)\frac{\cos\omega_0 t}{\omega_0 t}$$
$$+\left[1+\frac{2\omega_M}{\omega_0}\right]\frac{\sin(\omega_M-\omega_0)t}{(\omega_0 t)^2}+\lambda^2\left[1-\frac{2\omega_M}{\omega_0}\right]\frac{\sin(\omega_M+\omega_0)t}{(\omega_0 t)^2}+(1-\lambda^2)\frac{\sin\omega_0 t}{(\omega_0 t)^2}$$
$$\left.+2\frac{\cos(\omega_M-\omega_0)t}{(\omega_0 t)^3}-2\lambda^2\frac{\cos(\omega_M+\omega_0)t}{(\omega_0 t)^3}-2(1-\lambda^2)\frac{\cos\omega_0 t}{(\omega_0 t)^3}\right]. \tag{4.2}$$

The oscillations at frequency ω_0 are seen to disappear for $\lambda=1$. A similar result was obtained for the isolated two-level atom with $\langle S_z\rangle_0=0$ using the minimal coupling scheme.[18] Using the RWA, however, these ω_0 oscillations would influence the dynamics of the superradiant state, by delaying superradiance. In fact, as we shall discuss below, one should have $k_0R \ll 1$ for superradiance; hence at $t=R/c$ one would have $t \ll \omega_0^{-1}$ and the two atoms could not cooperate efficiently until $t \gg \omega_0^{-1}$. The fact that Arecchi and Courtens[2] do not find such a delay using the RWA is due to the approximations performed in their calculations. From (4.2) with $\lambda=1$ we see that only oscillations around the unphysically high frequency ω_M remain. These ω_M oscillations could not be detected by any realistic photodetection process and moreover they are strongly model dependent, in the sense that they disappear when an atomic form factor is used rather than introducing the dipole approximation and the cutoff at frequency $\omega_M=ck_M$ as we have done. Thus we feel entitled to neglect these oscillations completely, and we obtain from (4.2)

$$\langle \dot{S}_z^{(1)} \rangle_t(A_1) = -\gamma/2 \quad (\lambda=1) \tag{4.3}$$

as a very good approximation for $\langle \dot{S}_z^{(1)} \rangle_t$ up to times $t<R/c$. For $t>R/c$, the second term on the rhs of (2.15a) takes the form

$$-\frac{i}{2\hbar}d_m d_n D_{mn}^R e^{ik_0R}+\text{c.c.} = \frac{d_m d_n}{\hbar}D_{mn}^R \sin k_0 R$$
$$= -\frac{d_m d_n}{\hbar}k_0^3\left[(\delta_{mn}-\hat{R}_m\hat{R}_n)\frac{\sin k_0R}{k_0R}+(\delta_{mn}-3\hat{R}_m\hat{R}_n)\left[\frac{\cos k_0R}{k_0^2R^2}-\frac{\sin k_0R}{k_0^3R^3}\right]\right]. \tag{4.4}$$

For $k_0R \ll 1$, (4.4) tends to

$$-\frac{2}{3}\frac{|\mathbf{d}|^2\omega_0^2}{\hbar c^3} = -\gamma/2 \quad (k_0R \ll 1)$$

which adds to the $-\gamma/2$ obtained from (4.3) in order to yield the superradiant result in (2.15a)

$$\langle \dot{S}_z^{(1)} \rangle_t = -\gamma \quad (t>R/c). \tag{4.5}$$

We emphasize that this causal result is strictly related to use of $\lambda=1$, and cannot be obtained in the RWA. For $k_0R \gg 1$, (4.4) tends to[19]

$$-\frac{d_m d_n}{\hbar c^3}\omega_0^3(\delta_{mn}-\hat{R}_m\hat{R}_n)\frac{\sin k_0R}{k_0R} \quad (k_0R \gg 1)$$

which is too small to induce superradiant behavior.

We now turn to correlations in state (4.1), and from (3.5) and (2.11) we have

$$\frac{d}{dt}(\langle S_z^{(1)}S_z^{(2)}\rangle_t-\langle S_z^{(1)}\rangle_t\langle S_z^{(2)}\rangle_t)$$
$$=\frac{\gamma}{2\pi k_0^3}\left[D_1^t\{(1+\lambda^2)\mathrm{Si}(k_0ct)+\mathrm{Si}[(k_M-k_0)ct]-\lambda^2\mathrm{Si}[(k_M+k_0)ct]\}\right.$$
$$\left.+D_2^t\left[\frac{1}{ct}[(1+\lambda^2)\cos k_0ct-\cos(k_M-k_0)ct-\lambda^2\cos(k_M+k_0)ct]\right]\right]$$
$$+\frac{d_md_n}{\pi\hbar}\mathrm{Im}\left[D_{mn}^R\left[\int_0^\infty\frac{1-e^{i(k_0-k)ct}}{k_0-k}\sin kR\,dk+\lambda^2\int_0^\infty\frac{1-e^{i(k_0+k)ct}}{k_0+k}\sin kR\,dk\right]\right]. \quad (4.6)$$

The first term on the rhs represents the local effects of spontaneous decay on interatomic correlations. It contains oscillations at frequency ω_0 due to the dressing of the atoms by virtual photons.[18] These oscillations decay for $\omega_0 t \gg 1$, leaving for $\lambda=1$ a positive asymptotic term $\gamma/2$. Since the initial correlation $-\frac{1}{4}$ is negative and a minimum of $\langle S_z^{(1)}S_z^{(2)}\rangle-\langle S_z^{(1)}\rangle\langle S_z^{(2)}\rangle$, this term decreases the correlations. The second nonlocal term looks "noncausal" and it can be evaluated on the basis of (3.8) for $R \gg ct$. One easily obtains

$$-(1+\lambda^2)\frac{4d_md_n}{\pi\hbar k_0}\sin\omega_0t\frac{1}{R^4}(\delta_{mn}-2\hat{R}_m\hat{R}_n). \quad (4.7)$$

This nonlocal terms is of the same form as in the previously considered case of decorrelated atoms. Like in that case it can be interpreted in terms of correlations induced by correlated zero-point fluctuations of the field. The form of this term, however, becomes complicated for $R \approx ct$. It should be noted that in superradiant conditions ($k_0R \ll 1$) for $t \approx R/c$ one has $\omega_0 t \ll 1$ and the first term on the rhs of (4.6) has not yet settled to its asymptotic value $\gamma/2$. Thus the time dependence of the interatomic correlations for $t > R/c$ is rather complicated, since it results from interference of oscillations coming from both terms on the rhs of (4.6). In summary, for $t < R/c$ the action of the vacuum field on the pair of atoms can be described as twofold. On the one hand it tends to destroy by spontaneous emission [first term on the rhs of (3.5)] the correlations initially present in the pair, and on the other it tries to impose its own correlations on the atoms [second term on the rhs of (3.5)]. This point of view leads us to ask what is the result of these conflicting actions at relatively long time, that is for $t > R/c$ and for $\gamma^{-1} > t > \omega_0^{-1}$ in superradiant conditions $k_0R \ll 1$. The nonlocal term on the rhs of (4.6) for $t \gg \omega_0^{-1}$ can be expressed as

$$\frac{d_md_n}{\pi\hbar}D_{mn}^R\mathrm{Im}\left[\int_0^\infty\zeta(k-k_0)\sin kR\,dk+\lambda^2\int_0^\infty\zeta(k+k_0)\sin kR\,dk\right]$$
$$=-\frac{d_md_n}{\hbar}D_{mn}^R\sin k_0R\approx-\frac{2}{3}\frac{d_md_n}{\hbar}k_0^3\delta_{mn}=-\gamma/2 \quad (\omega_0t\gg1,\ R\ll ct,\ k_0R\ll1) \quad (4.8)$$

where $\zeta(x)$ is the Heitler function. Contribution (4.8) cancels the decorrelating contribution from the first term on the rhs of (4.6). This is an interesting result, because it shows that the effects coming from local and nonlocal interactions of the superradiant atomic pair with the field tend to cancel for $\lambda=1$ and $\gamma^{-1} > t > \omega_0^{-1}$. It is this cancellation for $\lambda=1$ which makes cooperative spontaneous decay possible at least until $t \approx \gamma^{-1}$.

We are induced to conclude that in a pair of initially bare and correlated two-level atoms, and for $k_0R \ll 1$, the superradiant decay rate sets in after $t=R/c$, while correlations start changing immediately after $t=0$. For $t \gg R/c$ the correlations tend to become stationary and the superradiant decay rate can be maintained throughout the complete dynamical evolution of the pair. The virtual photons related to the presence of the counterrotating terms in (2.1) are seen to be essential both in turning on superradiance in a causal way and in ensuring asymptotic cancellation of decorrelating local effects with correlating nonlocal effects of the vacuum fluctuations.

V. SUMMARY AND CONCLUSIONS

We have considered atomic dynamics of a pair of two-level atoms coupled with the vacuum fluctuations of the electromagnetic field, using the multipolar coupling scheme in the dipole approximation. The two atoms are taken to be initially bare. The Heisenberg equations of motion have been solved neglecting terms $O(e^3)$ and for t smaller than the spontaneous relaxation time γ^{-1}. We have shown that an observer performing measurements locally on either of the two atoms cannot detect the presence of the other atom before $t=R/c$, quite independently of the initial correlations existing in the pair. We have also confirmed that starting from $\langle S_z^{(i)}\rangle_0=0$ ($i=1,2$), oscillations at frequency ω_0 do not contribute for $\lambda=1$ to

the nonexponential decay of $\langle s_z \rangle_t$ for $t < R/c$, a result which had been previously obtained using the minimal coupling scheme.[18] It should be noted, however, that this lack of ω_0 oscillations may be due to the peculiarities of the two-level model used for the atoms. After $t = R/c$ the dynamics of each atom is influenced by the presence of the other in a way which is observable by local means. This effect of strictly causality in atomic dynamics has been shown to exist only if one takes into account the virtual photons coming from the counterrotating terms in (2.1), and to be lost in the RWA. Previous calculations yielding causality in a two-level atom pair[1,2] exploited deviation from the exact treatment.

We have also considered the dynamics of interatomic correlations of the pair of atoms. We have been able to separate two contributions to this dynamics. The first comes from local effects of the vacuum fluctuations on each atom of the pair, which tend to decrease the initial correlations. The second contribution is R dependent, and we have been able to attribute it to the field-field correlations peculiar to the vacuum fluctuations. Both contributions are active immediately after $t = 0$ and well before $t = R/c$. This however does not involve any violation of relativistic causality, since in order to measure them one needs two observers, each performing local measurements on atoms 1 and 2 separately, and subsequently transmitting the results to each other or to a common "supervisor;" it is the last step which ensures overall relativistic causality. The above results are related to previously reported cases which one might be tempted to interpret as violations of causality in the one-photon exchange between two atoms of a pair.[5,6]

Finally, we have concentrated on the dynamics of the highly correlated Dicke superradiant state of the form (4.1) for $k_0 R \ll 1$. We have shown that superradiant dynamics of the atomic pair can only be obtained for $t > R/c$ and if the counterrotating terms are taken into account. As for interatomic correlations in the superradiant state, they have been shown to change at frequency ω_0 for $t \ll R/c$, to have a rather complicated behavior for $t \approx R/c$ and to settle down to a constant value for $t \gg R/c$ and $\omega_0 t \gg 1$. This balance is possible only because of the presence of the virtual photons created by the counterrotating terms, and it is essential in order to ensure superradiant dynamics at times of the order of γ^{-1}.

In conclusion, we remark that none of the statements referring to relativistic causality reported in the Introduction is really adequate to cover all possible situations in the two-atom system with one atom excited and the other unexcited. In fact, it is not strictly true that the ground state atom can be excited only after time R/c, if one takes into account the counterrotating terms in the Hamiltonian (2.1). Moreover, it is not strictly true that the two atoms evolve independetly for $t < R/c$, because interatomic correlations evolve for $t < R/c$ even for an initial completely uncorrelated atomic state: what is actually true is that these correlations do not show up in local measurements performed on each atom. For the same reasons, it is also not true that before $t = R/c$ there can be no change in directly observable quantities related to the initially unexcited atom, both because this statement does not account for excitation during emission of a virtual photon and because the early interatomic correlations at $t < R/c$, which are induced by the correlated vacuum field fluctuations, are observable. On the other hand, as shown in this paper, one gets precise statements in terms of two observers localized near atoms 1 and 2. From this point of view one can say that the observer near the initially unexcited atom will occasionally find the atom excited before $t = R/c$, but there is no way for him to tell by performing measurement on this atom alone that there is another atom in the neighborhood. If both observers, however, send information of their measurements to a "supervisor," he will be able to discover interatomic correlations at $t < R/c$. This interpretation seems to coincide qualitatively with the conclusions of Valentini's analysis of correlations at $t < R/c$.[6]

ACKNOWLEDGMENTS

The authors are indebted to E. A. Power and to I. Bialynicki-Birula for interesting conversations, and to A. Valentini for communication of unpublished results relevant to the subject of this paper. They also acknowledge partial financial support by Comitato Regionale Ricerche Nucleari e Struttura della Materia and by Istituto Nazionale di Fisica della Materia. Finally, two of the authors (A.K.B. and F.P.) are grateful to the International Centre for Theoretical Physics for hospitality during part of the completion of this work.

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