Environmental and spontaneous localization

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The dynamical elimination or reduction of macroscopic superpositions has long been of interest, particularly with regard to the quantum theory of measurement. A number of models have demonstrated this for the reduced density matrix of a system interacting with an environment. Alternatively, Ghirardi, Rhimini, and Weber [Phys. Rev. D **34**, 470 (1986)] have proposed a fundamental modification of the Schrödinger equation, quantum mechanics with spontaneous localization (QMSL), which provides a master equation similar in mathematical form. In this paper we consider an isotropic environment that is elastically scattered by the system of interest with negligible momentum transfer, extending a previous result of Joos and Zeh [Z. Phys. B. **59**, 223 (1985)] from small length scales to all length scales. We discuss the physical nature and relevance of the differences between our result and similar open systems calculations. We describe the mathematical similarity between our extended environmental model and the QMSL dynamics determining the QMSL parameters that allow calculations using QMSL to be used as a model for the effect of the environment. That gives us access to a number of interesting results obtained for the QMSL master equation. Finally, we discuss some experimental considerations for the purposes of detecting effective nonunitary evolution of this form.

I. INTRODUCTION

Quantum mechanics, as a fundamental theory, has met tremendous success in its ability to predict physical phenomena. The interpretation of various features of the theory, however, remains fraught with conceptual difficulties when applied to macroscopic systems, as in the quantum theory of measurement.

A significant source of difficulty in the interpretation of quantum mechanics is the superposition principle. Quantum mechanics allows any superposition of states, and yet the superposition of macroscopically distinguishable states has never been observed. Although it begs the question as to how one should go about measuring such superpositions, it seems that macroscopic variables should have well-defined values, in accordance with classical mechanics. The problem is compounded by the collapse postulate of quantum mechanics: when a measurement is completed the system is in an eigenstate corresponding to the result of the measurement. However, when the apparatus is included in the quantum-mechanical description, the apparatus is found to be correlated to the quantum system resulting in a superposition of the apparatus in the various states of outcome.

A number of models have been explored that dynamically reduce, via interaction with an environment, the coherence between macroscopically distinct states for the reduced density operator of the macroscopic system.[1–4] The possibility of the observation of macroscopic superpositions, such as in the Schrödinger cat paradox, is effectively eliminated in these models. This type of decohering mechanism was employed by Zurek to produce a natural selection of the pointer basis in the Everett interpretation of quantum mechanics, and indeed characterizes how quantum-mechanical degrees of freedom can become classical in nature.[5,6] We should point out that Zurek referred to this effect as environment-induced superselection, though it is not a superselection principle in the usual sense. We will generally refer to the dynamical reduction of the coherence terms between spatially separated states as localization.

The effective elimination of macroscopic superpositions by "hiding" the coherence in correlations with an environment has been argued to be an inadequate solution to the measurement problem.[7–9] This has motivated Ghirardi, Rhimini, and Weber (GRW) to propose a fundamental modification of the Schrödinger equation, quantum mechanics with spontaneous localization (QMSL), which produces the desired decoherence between position states of a macroscopic system. Another feature of modified quantum mechanics such QMSL is that the direction of the arrow of time (increasing entropy) is implicit in the fundamental dynamics, independent of the boundary conditions (initial state of the environment). We should emphasize that QMSL does introduce new physics, that is, the modifications to the Schrödinger equation do not in any sense come from within the framework of standard quantum mechanics. Because of the implications of such a mechanism, like those mentioned above, we believe the search for experimental tests of QMSL to be fairly important.

A number of interesting results have been obtained for QMSL, including a free-particle propagator and a nondemolition measurement model.[7,10–12] The analysis of environmental localization in specific measurement models has not been carried out in as great detail. The mathematical similarity between environmental localization and QMSL (which we will display) suggests that

some of the calculations performed for the QMSL model could be employed as environmental models, with the appropriate selection of parameters. One of our objectives in this paper is to obtain a natural selection of these parameters.

We are particularly interested in the effect of a thermal environment on the evolution equations of the density matrix. We will calculate the effect of an isotropic environment which elastically scatters from the system of interest with negligible momentum transfer, extending a previous result of Joos and Zeh.[1] Wigner has written down, on general principles, a master equation for a system interacting with an environment which incorporates dynamical elimination of coherence, which is of the form of the equations which we shall be discussing.[13] Dekker obtained a similar equation by introducing quantal *noise* operators.[14] The mathematical form of localization obtained by Joos and Zeh[1] is given by the modification of the Schrödinger equation,

$$\frac{\partial\rho}{\partial t}=\frac{1}{i\hbar}[H,\rho]-\Lambda[x,[x,\rho]]\ , \tag{1.1}$$

valid at short length scales.

A similar master equation,

$$\frac{\partial\rho}{\partial t}=\frac{1}{i\hbar}[H,\rho]=\frac{\gamma}{i\hbar}[x,\{p,\rho\}]-\frac{2m\gamma k_B T}{\hbar^2}[x,[x,\rho]]\ , \tag{1.2}$$

has been obtained by Caldeira and Leggett (CL) for a system coupled to a distribution of oscillators all at the same temperature T using influence functional techniques.[2] The Hamiltonian in Eq. (1.2) is just the Hamiltonian for the isolated system. The parameter γ is related to the classical dampening constant η by $\gamma=\eta/2m$, which can most easily be understood as the coefficient of velocity for a velocity-dependent frictional force. The second term of the right-hand side of Eq. (1.2) represents the transfer of momentum between the system and the oscillator bath, while the third term corresponds to the environmental localization.

The model composite system used by CL has the total Hamiltonian

$$H_{\rm tot}=\frac{p^2}{2m}+V(x)+x\sum_k C_k R_k+\sum_k\left[\frac{P_k^2}{2M}+\frac{1}{2}M\omega_k^2 R_k^2\right] \tag{1.3}$$

for the system of interest coupled to the oscillator bath. The first two terms are the Hamiltonian of the system of interest, the third is the interaction between the system and the oscillator bath, and the remainder is the self-energy of the oscillators. The propagator for the reduced density operator of the system of interest has been found using path-integration techniques, given the initial conditions of a total density operator which factors into the tensor product of system and environment density operators. The environment initially is in a state of thermal equilibrium at a temperature T. The limit is then taken as the discrete set of oscillators becomes a continuum. CL choose the distribution of oscillators given by

$$\rho_D(\omega)C^2(\omega)=\begin{cases}\dfrac{2m\eta\omega^2}{\pi}, & \omega\le\Omega,\\ 0, & \omega>\Omega,\end{cases} \tag{1.4}$$

where $\rho_D(\omega)$ is the number of oscillators per unit angular frequency interval, $C(\omega)$ is the coupling constant of an oscillator of frequency ω to the system of interest [corresponding to C_k in Eq. (1.3)], and the high frequency cutoff Ω is assumed to be higher than any other relevant frequencies; that is, the relevant time scales are much longer than $1/\Omega$. They obtain the effective master equation under the assumption $2k_bT\ge\hbar\Omega$. The effective master equation for the reduced density operator becomes identical in form to the result of Joos and Zeh when we take the simultaneous limit $T\to\infty$, $\gamma\to\infty$, $\gamma\to 0$, holding γT finite. By taking this limit, we are focusing on localization effects and neglecting the transfer of momentum to the environment. This limit is reasonable if the localization time scale is much shorter than the relaxation time scale of the system of interest (where the system comes to equilibrium with the environment). CL observed that interference effects are destroyed (for their model) on time scales much shorter than the relaxation time scale of the system.[15]

Hakim and Ambegaokar[3] have explored the oscillator bath of CL with nonfactoring initial conditions. They assumed that the composite system was in thermal equilibrium, and found differences from the results of CL in the details of the propagator for the system of interest. These initial conditions are not relevant to the types of systems in which we are interested, particularly where the system of interest is a measurement apparatus assumed to be initially in a preparted state, not in thermal equilibrium with its environment.

More recently, Unruh and Zurek[4] (UZ) examined environmental localization for a system coupled to a scalar field where the action is given by

$$I=\int\left\{\frac{1}{2}\left[\dot\phi^2-\left[\frac{\partial\phi}{\partial x}\right]^2\right]+\delta(x)\left[\frac{\dot q^2}{2}-\epsilon q\dot\phi-\frac{\Omega_0^2}{2}q^2\right]\right\}dt\,dx\ . \tag{1.5}$$

They examine this action for three cases of the system: a harmonic oscillator (Ω_0 real), a free particle ($\Omega_0=0$), and an "upside-down" harmonic oscillator (Ω_0 imaginary). They solve the equations of motion for the operators ϕ, q, and p (p is the momentum conjugate to q), and assume that the initial density operator of the system is factorable into the tensor product of density operators for the subsystems. The field is assumed to be in thermal equilibrium at a temperature T. They obtain the propagator for the reduced density operator of the system of interest by transforming from the Heisenberg picture to the Schrödinger picture, and then taking a partial trace over the field degrees of freedom. The operator representation for the (Schrödinger picture) master equation is

$$\frac{\partial\rho}{\partial t}=\frac{1}{i\hbar}[H,\rho]+\frac{2\gamma}{i\hbar}[q,\{p,\rho\}]-\frac{4\gamma mh(t,T)}{\hbar^2}[q,[q,\rho]]$$
$$-\frac{4\gamma f(t,T)}{\hbar}[q,[p,\rho]]\ , \tag{1.6}$$

where $\gamma=\epsilon^2/4$, and where the time dependence of the coefficients h and f depend upon the temperature and a cutoff frequency Γ introduced to avoid divergences in the calculations. In general, h and f asymptote to constant values after initial transient behavior. For finite temperatures, the asymptotic value of h is k_BT, and f is inversely proportional to the cutoff frequency Γ. For zero temperature, the behavior of h and f depends on the cutoff frequency and asymptotes to functions of Γ, except for the case of the upside-down harmonic oscillator, where h and f diverge. We have reintroduced the particle mass and Planck's constant to illustrate the similarity between this master equation and others discussed in this section. In the high-temperature limit $k_BT\gg\hbar\Gamma>\hbar\gamma,\hbar\Omega_0$ Eq. (1.6) becomes

$$\frac{\partial\rho}{\partial t}=\frac{1}{i\hbar}[H,\rho]+\frac{2\gamma}{i\hbar}[q,\{p,\rho\}]-\frac{4\gamma mk_BT}{\hbar^2}[q,[q,\rho]]\ , \tag{1.7}$$

which makes a good approximation for times greater than the dynamical time scale γ^{-1} when $\Gamma/\Omega_0\ll\exp(k_bT/\hbar\gamma)$. Again the master equation includes the usual Hamiltonian dynamics plus environment effects [localization is provided by the third term of Eqs. (1.6) and (1.7)]. We expect that only the first and third terms will be relevant for macroscopic systems, for the same reasons that we gave for ignoring the second term in Eq. (1.2): the additional terms in Eqs. (1.6) and (1.7) which correspond to momentum transfer between system and environment are assumed to be negligible for time scales much shorter than the relaxation time scale of the system of interest.

In QMSL,[7] the evolution of the density matrix is given by

$$\frac{\partial\rho(x,x')}{\partial t}=\frac{1}{i\hbar}\langle x|[H,\rho]|x'\rangle$$
$$-\lambda(1-e^{-(\alpha/4)(x-x')^2})\rho(x,x')\ . \tag{1.8}$$

The factor corresponding to the localization has the short length-scale behavior

$$\frac{\partial\rho(x,x')}{\partial t}=-\frac{\lambda\alpha}{4}(x-x')^2\rho(x,x')\ ,$$

and approaches

$$\frac{\partial\rho(x,x')}{\partial t}=-\lambda\rho(x,x')$$

for long length scales.

We contrast this with the environmental localization mechanisms which have the form:

$$\frac{\partial\rho(x,x')}{\partial t}\propto(x-x')^2\rho(x,x')\ .$$

A disturbing feature of this form is that the environment's ability to destroy the off-diagonal coherence in the density matrix can grow without bound. To see why this might not be physical for an environmental model, we imagine a reversible Stern-Gerlach experiment with a two-state system, prepared in a spin-up state and split into two beams with the spin axis orthogonal to the original spin axis. The decoherence which would occur between the two beams depends upon the distance between the beams and results in a mixture of spin-up and spin-down states when the beam is recombined and analyzed. If the localization is indeed simply quadratic, as the models suggest, then the environment can somehow "sense" the separation of the beams, be it microns, meters, or kilometers. But if localization is to be interpreted as being the result of correlations developing between environment degrees of freedom and the system of interest,[5,6] conflict arises between the unbounded increase (with increasing spatial separations) of the decay rate of off-diagonal elements of the density operator and the physical nature of the environment and its coupling to the system of interest. First, the environment is generally correlated with itself only over finite length scales. Second, the interaction between system and environment degrees of freedom is of finite range (indeed, they are local when reduced to fundamental interactions). The unbounded increase in the decay rate is at least suggestive of causality violation. We will return to this characterization of the environment in Sec. IV.

The open systems calculations mentioned above share the quadratic dependence on spatial separations. This turns out to be effectively due to short length-scale approximations inherent in the models and infinite-range interaction between the system of interest and environmental degrees of freedom, and is, in part, our motivation for extending the applicability of Joos and Zeh's calculation. In the Hamiltonian of CL, the coupling term is linear in the position operator of the system, and so the qualitative nature of the oscillator-particle interaction is the same for all oscillators. In a sense, all the oscillators in the bath exist at the same position, with a nonlocal coupling between system and oscillators. If the coupling coefficients were made to depend appropriately upon the position of the particle, the local nature of the interaction between the environment and the system might be recovered. Unfortunately, it is the linear nature of the coupling that allows much of the calculations to proceed. In the model of UZ, the coupling of the particle to the field is at a single point in the field's configurations space. In both models, the interaction potential is linear in the position of the system of interest, resulting in infinite-range interactions, and so the system-environment interactions are not sensitive to spatial separations of different components of the environment. The result is that these models cannot be sensitive to the finite correlation length of the environment.

However, when we extend Joos and Zeh's work, we find the qualitative behavior we expect, and we explore the similarity with the mathematical behavior of the QMSL mechanism. We are then provided with the additional motivation of having various results of calculations performed for QMSL immediately available in our model

of environmental localization, including possible effects at large length scales.

In Sec. II, we calculate the effect of an environment which scatters weakly from the system of interest. The environmental mechanism is compared with the QMSL model so that results obtained for the model can be readily applied to the environmental mechanism with a suitable choice of parameters. We examine in detail the particular case in which the environment is composed of thermal photons. In Sec. III the implications of dissipative evolution of the form of Eq. (1.8) in various experimental tests of quantum mechanics are discussed. Finally, in Sec. IV, we draw our conclusions and discuss additional topics in quantum theory.

II. ENVIRONMENTALLY INDUCED DECORRELATION

It has been suggested that an effective "localizing"[1,5,6] mechanism for macroscopic objects is their interaction with their environment. It has also been observed that one can obtain a strong localization for the position of the center of mass of a macroscopic object if there is a weak localization for the position of the constituent microscopic systems.[1,7,11] We begin this section by examining a scattering model of the interaction between a particle and the environment to obtain an evolution of the density operator in the manner, somewhat elaborated, of Joos and Zeh.[1]

The scattering event will be examined in the context of the $|\mathbf{x}_s\rangle|\chi_l\rangle$ representation, where $|\mathbf{x}_s\rangle$ is the position eigenstate of the system, and $|\chi_l\rangle$ is a member of some convenient complete sets of states for the environment constituent which is being scattered. If the system and environment are initially uncorrelated, then the initial density operator can be factored into the tensor product of the system state and the environment state:

$$\rho=\rho_S\otimes\rho_E \ .$$

The components of the initial environment density operator are given by

$$\rho_{ij}^{(E)}=\langle\chi_i|\rho_E\chi_j\rangle \ .$$

The effect of a single-scattering event on the combined system-environment density matrix can be written

$$\rho\Longrightarrow S\rho S^{\dagger} \ . \tag{2.1}$$

Since

$$\mathbf{K}=\frac{\mathbf{P}}{\hbar}+\mathbf{k}$$

is the generator of translations for the composite system, where $\mathbf{P}$ is the momentum operator of the system of interest, and $\mathbf{k}$ is the wave-number operator for the environment constituent, one can then write

$$S|\mathbf{x}_s\rangle\otimes|\chi_l\rangle=Se^{-i\mathbf{K}\cdot\mathbf{x}_s}(|0\rangle\otimes e^{i\mathbf{k}\cdot\mathbf{x}_s}|\chi_l\rangle) \ , \tag{2.2}$$

where $|0\rangle$ represents the system located at the origin. If we assume the interaction is translationally invariant, then

$$[S,e^{-i\mathbf{K}\cdot\mathbf{x}_s}]=0 \ ,$$

and Eq. (2.2) becomes

$$S|\mathbf{x}_s\rangle\otimes|\chi_l\rangle=e^{-i\mathbf{K}\cdot\mathbf{x}_s}S(|0\rangle\otimes e^{i\mathbf{k}\cdot\mathbf{x}_s}|\chi_l\rangle) \ . \tag{2.3}$$

A critical assumption is now imposed: the system is not significantly disturbed by the scattering of the environment. Physically, one can consider this equivalent to a massive system which is not disturbed when a much lighter particle is scattered by it, and the interaction potential is negligible compared to the kinetic and potential energy of the system of interest. This allows the environment to become correlated with the system without significantly disturbing the system. Zurek characterized the necessary conditions for environment-induced superselection of an observable $\mathbf{O}$ by the commutator $[\mathbf{H}_{ES},\mathbf{O}]=0$, where $\mathbf{H}_{ES}$ is the interaction Hamiltonian. Of course, $\mathbf{H}_{ES}$ must depend upon $\mathbf{O}$, or the necessary correlations between environment and system cannot develop. Since environmental localization is environment-induced superselection of position, the analogous commutator should hold for our model. The conditions of translational invariance and the commutator of a projection operator located at the origin,

$$[\mathbf{H}_{ES},|0\rangle\langle 0|]=0 \ ,$$

form a slightly stronger condition but imply Zurek's condition.

The scattering of the environment constituent by the system located at the origin now can be written

$$S|0\rangle\otimes|\chi_l\rangle=|0\rangle\otimes S_0|\chi_l\rangle \ ,$$

where S_0 is the scattering matrix for the scattering of the environment by the system at the origin. The more general case can then be written

$$\begin{aligned}S|\mathbf{x}_s\rangle\otimes|\chi_l\rangle&=Se^{-i\mathbf{K}\cdot\mathbf{x}_s}(|0\rangle\otimes e^{i\mathbf{k}\cdot\mathbf{x}_s}|\chi_l\rangle)\\&=e^{-i\mathbf{K}\cdot\mathbf{x}_s}(|0\rangle\otimes S_0e^{i\mathbf{k}\cdot\mathbf{x}_s}|\chi_l\rangle)\\&=|\mathbf{x}_s\rangle\otimes e^{-i\mathbf{k}\cdot\mathbf{x}_s}(S_0e^{i\mathbf{k}\cdot\mathbf{x}_s}|\chi_l\rangle) \ .\end{aligned} \tag{2.4}$$

For the reduced density matrix of the system, the scattering event has the effect

$$\rho_s(\mathbf{x},\mathbf{x}')\Longrightarrow\rho_s(\mathbf{x},\mathbf{x}')\sum_{l,J}\rho_{l,J}\langle\chi_J|e^{-i\mathbf{k}\cdot\mathbf{x}'}S_0^{\dagger}e^{i\mathbf{k}\cdot\mathbf{x}'}e^{-i\mathbf{k}\cdot\mathbf{x}}S_0e^{i\mathbf{k}\cdot\mathbf{x}}|\chi_l\rangle \ . \tag{2.5}$$

It is clear that the diagonal terms (those with $\mathbf{x}=\mathbf{x}'$) corresponding to the probability density of position for the system are unaffected by the scattering event. In order to proceed with the calculation, it is assumed that the density matrix for the environment constituent is diagonal in the momentum basis, which in turn requires the use of box normalization for the environment constituent. This assumption is compatible with an isotropic thermal environment, such as a bath

of thermal photons, for which we will use the Planck radiation law later in this chapter. Then the natural choice for the environment basis $|\chi_i\rangle$ is the momentum states $|\mathbf{q}\rangle$. In terms of a single-scattering event (one photon, e.g.), one finds

$$\rho_s(\mathbf{x},\mathbf{x}') \Longrightarrow \rho_s(\mathbf{x},\mathbf{x}')\langle \mathbf{q}|e^{-i\mathbf{k}\cdot\mathbf{x}'}S_0^{\dagger}e^{i\mathbf{k}\cdot\mathbf{x}'}e^{-\imath\mathbf{k}\cdot\mathbf{x}}S_0e^{\imath\mathbf{k}\cdot\mathbf{x}}|\mathbf{q}\rangle \ . \tag{2.6}$$

We now address the perturbative nature of the interaction. To second order, one can write

$$S=S^{(0)}+S^{(1)}+S^{(2)} \ .$$

By convention, $S^{(0)}=1$. Formally, unitarity is required to every order in the perturbation, so that

$$SS^{\dagger}=I \ .$$

This yields the following set of conditions:

$$\begin{aligned} &S^{(0)}(S^{(0)})^{\dagger}=I \ , \\ &S^{(1)}+(S^{(1)})^{\dagger}=0 \ , \\ &S^{(1)}(S^{(1)})^{\dagger}+S^{(2)}+(S^{(2)})^{\dagger}=0 \ . \end{aligned} \tag{2.7}$$

Using these conditions, Eq. (2.1) becomes

$$\begin{aligned} \rho \Longrightarrow S\rho S^{\dagger} &\cong (S^{(0)}+S^{(1)}+S^{(2)})\rho[(S^{(0)})^{\dagger}+(S^{(1)})^{\dagger}+(S^{(2)})^{\dagger}] \\ &=\rho+S^{(1)}\rho+\rho(S^{(1)})^{\dagger}+S^{(1)}\rho(S^{(1)})^{\dagger}+S^{(2)}\rho+\rho(S^{(2)})^{\dagger} \ , \end{aligned} \tag{2.8}$$

where terms to second order have been kept.

The perturbation expansion is used to evaluate the matrix element in Eq. (2.6) to obtain

$$\langle \mathbf{q}|e^{-i\mathbf{k}\cdot\mathbf{x}'}S_0^{\dagger}e^{i\mathbf{k}\cdot\mathbf{x}'}e^{-i\mathbf{k}\cdot\mathbf{x}}S_0e^{i\mathbf{k}\cdot\mathbf{x}}|\mathbf{q}\rangle = \langle \mathbf{q}|1+e^{-i\mathbf{k}\cdot\mathbf{x}'}(S_0^{(1)})^{\dagger}e^{i\mathbf{k}\cdot\mathbf{x}'}e^{-\imath\mathbf{k}\cdot\mathbf{x}}S_0^{(1)}e^{i\mathbf{k}\cdot\mathbf{x}}-(S_0^{(1)})^{\dagger}S_0^{(1)}|\mathbf{q}\rangle \ , \tag{2.9}$$

where the consistency conditions of perturbation theory as well as the c-number evaluation of momentum in the exponentials were used to simplify the matrix element. It is this c-number evaluation and the consistency requirements of the perturbation calculation which allow us to proceed with the calculation, explicitly using only the first-order terms in the scattering matrix in a calculation which is implicitly second order. However, for an environment that is only weakly coupled to the system of interest, the higher-order effects should not have a significant effect. Inserting a complete set of states, one obtains for the change in the density operator

$$\Delta\rho_s(\mathbf{x},\mathbf{x}')=\rho_s(\mathbf{x},\mathbf{x}')\sum_{\mathbf{q}'}(e^{\imath(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')}-1)\langle \mathbf{q}|(S_0^{(1)})^{\dagger}|\mathbf{q}'\rangle\langle \mathbf{q}'|S_0^{(1)}|\mathbf{q}\rangle \ . \tag{2.10}$$

Converting the sum to an integral using box normalization in a volume V yields

$$\begin{aligned} \Delta\rho_s(\mathbf{x},\mathbf{x}') &= \rho_s(\mathbf{x},\mathbf{x}')\int d^3q'\frac{(2\pi)^3}{V}(e^{\imath(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')}-1)|\langle \mathbf{q}'|S_0^{(1)}|\mathbf{q}\rangle|^2 \\ &= \rho_s(\mathbf{x},\mathbf{x}')\int d^3q'\frac{(2\pi)^3}{V}(e^{\imath(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')}-1)(2\pi q)^{-2}|f(\mathbf{q},\mathbf{q}')|^2\delta(q-q')V^{1/3} \ , \end{aligned} \tag{2.11}$$

where the standard representation for the scattering matrix

$$\langle \mathbf{q}'|S_0^{(1)}|\mathbf{q}\rangle = \frac{i}{2\pi q}f(\mathbf{q},\mathbf{q}')\delta(q-q')$$

was used, and $\delta^2(q-q')=\delta(q-q')V^{1/3}$ with the box normalization. We now average over a uniform distribution in the direction of the incident momentum, and evaluate the integral of the δ function to obtain

$$\Delta\rho_s(\mathbf{x},\mathbf{x}')=\rho_s(\mathbf{x},\mathbf{x}')\int\frac{d\Omega d\Omega'}{2V^{2/3}}(e^{\imath(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')}-1)|f(\mathbf{q},\mathbf{q}')|^2 \ , \tag{2.12}$$

with $q=q'$, which is assumed in the remainder of the paper. The flux of particles with wave number q passing through the volume V in an interval Δt is $V^{2/3}v(q)n(q)\Delta t$, where $n(q)$ is the number density, and $v(q)$ is the speed. Adding up the contribution over all wave numbers, the change in the density matrix becomes

$$\Delta\rho_s(\mathbf{x},\mathbf{x}')=-\rho_s(\mathbf{x},\mathbf{x}')\int dq\ n(q)v(q)\int\frac{d\Omega d\Omega'}{2}(1-e^{\imath(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')})|f(\mathbf{q},\mathbf{q}')|^2\Delta t \tag{2.13a}$$

or

$$\frac{\Delta\rho_s(\mathbf{x},\mathbf{x}')}{\Delta t}=-\rho_s(\mathbf{x},\mathbf{x}')\int dq\ n(q)v(q)\int\frac{d\Omega d\Omega'}{2}(1-e^{i(\mathbf{q}-\mathbf{q}')\cdot(\mathbf{x}-\mathbf{x}')})|f(\mathbf{q},\mathbf{q})|^2 \ . \tag{2.13b}$$

We have found the contribution of the environment to the equations of motion of the reduced density matrix. We write this then as

$$\frac{\partial \rho_s(\mathbf{x},\mathbf{x}')}{\partial t} = \frac{1}{i\hbar}\langle \mathbf{x}|[H,\rho]|\mathbf{x}'\rangle - F(\mathbf{x}-\mathbf{x}')\rho_s(\mathbf{x},\mathbf{x}') , \tag{2.14a}$$

where

$$F(\mathbf{r}) = \int dq\, n(q)v(q) \times \int \frac{d\Omega d\Omega'}{2}(1-e^{[i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{r}]})|f(\mathbf{q},\mathbf{q}')|^2 . \tag{2.14b}$$

We take the scattering process to be independent of the orientation of the scattering center. and hence, of the coordinate system. This is a reasonable assumption since the scattering center can be taken to be symmetric by nature or by averaging over random orientations. By taking the direction of $\mathbf{r}$ as the z-coordinate axis, we have

$$F(\mathbf{r}) = F(r) = \int dq\, n(q)v(q) \times \int \frac{d\Omega d\Omega'}{2}\{1-\cos[(q_z-q_z')r]\} \times |f(\mathbf{q},\mathbf{q}')|^2 . \tag{2.15}$$

The limiting behavior of the localization mechanism can now be readily evaluated. We obtain for small r, by expanding the cosine in powers of r and keeping the lowest contributing order, the familiar result of Joos and Zeh:

$$F(r) \approx r^2 \int dq\, n(q))v(q) \int \frac{d\Omega d\Omega'}{4}(q_z-q_z')^2|f(\mathbf{q},\mathbf{q}')|^2 . \tag{2.16}$$

The small length-scale approximation actually takes place earlier in the calculation of Joos and Zeh, where it is assumed that $rq \ll 1$, for typical wave numbers of the environment. For large r, the cosine in the integral undergoes rapid oscillations, and hence does not contribute to the integral, as a consequence of the smooth behavior of the other functions in the integrand. In this region, then,

$$F(r) = \int dq\, n(q)v(q) \int \frac{d\Omega d\Omega'}{2}|f(\mathbf{q},\mathbf{q}')|^2 = \int dq\, n(q)v(q)2\pi\sigma(q) \equiv F(\infty) , \tag{2.17}$$

where $\sigma(q)$ is the total cross section. Thus the localization rate does not increase without bound for arbitrarily large separations, but approaches a constant for large r. The appropriate length scale for determining what constitutes large r is determined physically by the coherence length of the environment, as discussed in Sec. I. Since wave number and position are Fourier transform pairs, the width of the autocorrelation function is the inverse of the width of the wave-number distribution. It is the width of the autocorrelation function that characterizes the coherence length of the environment. For a thermal bath, the wave-number distribution has a width on the order of the inverse of the thermal wavelength, so that we can easily determine the appropriate length scales of thermal environments.

To model the environment with a master equation of the form of the QMSL master equation, we wish to replace $F(r)$ with $\lambda(1-e^{-(\alpha/4)r^2})$. The choice of parameters is by now obvious. To match the short length-scale behavior with Eq. (2.16), we take

$$\frac{\lambda\alpha}{4} = \int dq\, n(q)v(q) \int d\Omega d\Omega' \tfrac{1}{4}(q_z-q_z')^2|f(\mathbf{q},\mathbf{q}')|^2 ,$$

and to match the asymptotic behavior at large r, $\lambda = F(\infty)$.

We will now consider some specific examples, using a background of thermal photons (with the Planck distribution), and take the specific form of the differential cross section to be

$$|f(\mathbf{q},\mathbf{q}')|^2 = gq^m \frac{1}{2}\left[1+\left|\frac{\mathbf{q}\cdot\mathbf{q}'}{q^2}\right|^2\right] , \tag{2.18}$$

with the exponent m varying in the examples. This form occurs often enough in relevant situations so as to be of interest. For Thompson scattering we have $m=0$, and g is the square of the classical electron radius, while for Rayleigh scattering $m=4$, $g = a^6|(\epsilon-1)/(\epsilon+1)|^2$, where a is the scatterer's radius and ϵ is the dielectric constant. The angular integrals of Eq. (2.15) can now be evaluated to obtain

$$F(r) = \int_0^\infty dq \frac{q^2 c}{e^{\beta\hbar cq}-1}\frac{gq^m}{2}(\tfrac{32}{3} - 3\{[I_1(qr)]^2+[I_2(qr)]^2\} + 2I_1(qr)I_2(qr)) , \tag{2.19a}$$

where

$$I_1(s) = 2\frac{\sin(s)}{s} , \qquad I_2(s) = s^{-3}[4s\cos(s)+2(s^2-2)\sin(s)] . \tag{2.19b}$$

We factor the asymptotic behavior using Eq. (2.17) and write this as

$$\frac{F(r)}{F(\infty)} = \frac{3}{32\Gamma(m+3)\zeta(m+3)}\int_0^\infty dt \frac{t^{2+m}}{e^t-1} I\left[t\frac{r}{\beta\hbar c}\right] , \tag{2.20a}$$

where

$$I(s) = \tfrac{32}{3} - 3[(I_1)^2+(I_2)^2]+2I_1I_2 . \tag{2.20b}$$

It is convenient to note that for $s \gg 1$,

$$I(s) \approx \tfrac{32}{3} ,$$

and that for $s \ll 1$, when we expand $I(s)$ in a Taylor series, we obtain

$$I(s) \approx \tfrac{32}{9}s^2 - \tfrac{112}{225}s^4 + \tfrac{176}{4725}s^6 + O(s^8) .$$

By making these approximations the limiting behavior, both long range and short range, can be readily evalu-

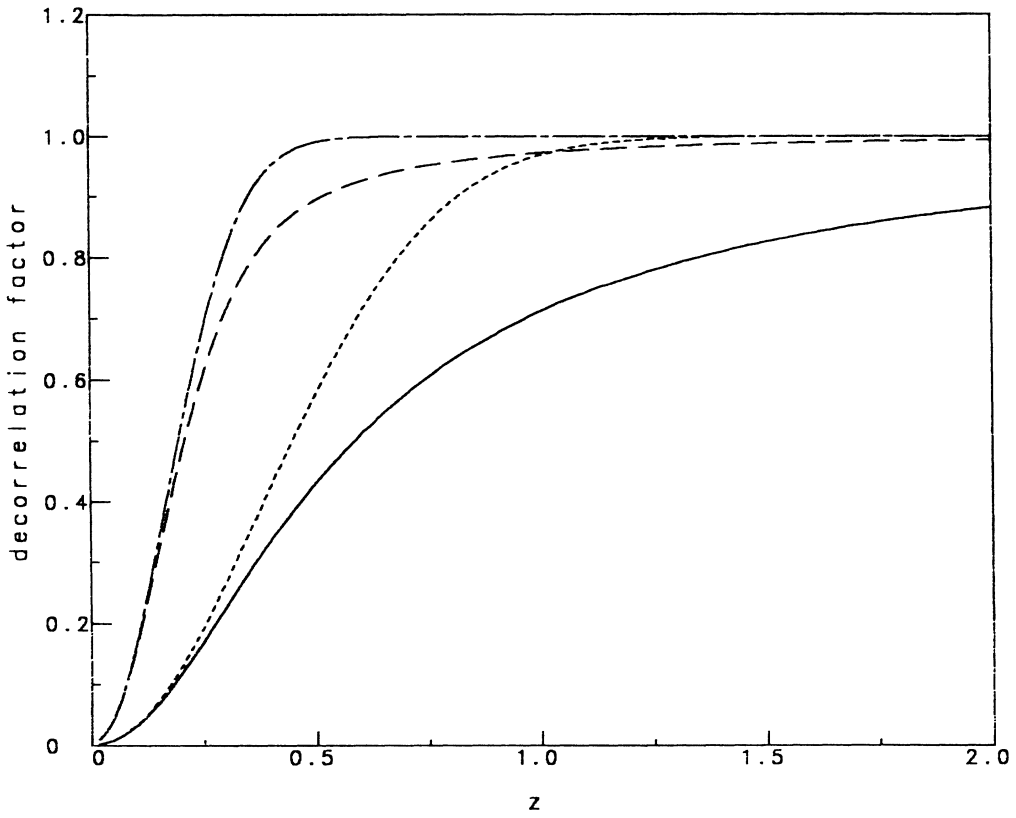

FIG. 1. The fraction of the asymptotic value of the decorrelation factor is plotted vs $z = r/(\beta\hbar c)$. The solid line is for Thompson scattering, and the short-dashed line is the closest fitting QMSL-like curve. The long-dashed line is for Rayleigh scattering and the alternating dashed line is the closest fitting QMSL-like curve. The closest fit was defined by matching the small-z behavior and the asymptotic behavior at large z.

ated. The departure from the quadratic behavior of the localization factor at short length scales can be examined by keeping any terms of higher order than the quadratic in the expansion of $I(s)$.

We have evaluated Eq. (2.20) numerically for the two cases described above, the results of which are shown in Fig. 1. The environmental calculations have been plotted on a dimensionless length scale in terms of the thermal wavelength. The localization rate does asymptote above this length scale, as expected. At small enough length scales, the quadratic behavior does hold. What determines small enough depends upon the relative sizes of the terms in an expansion of the integrand in Eq. (2.15) above, which in turn would depend upon the distribution of particles (the Planck distribution, in our examples) and the nature of the coupling, which is illustrated by the different behavior in our two examples as seen in Fig. 1.

III. EXPERIMENTAL CONSIDERATIONS

In this section, we explore possible experimental implications of modifications to the Schrödinger equation such as that given by Eq. (1.4). Because the effect of interest provided by these mechanisms is the destruction of quantum interference, the obvious place to begin is with a simple interference experiment. We note that, for the values of the parameters selected by GRW for QMSL, the neutron interference experiments we will be initially discussing will be insensitive to the presence of spontaneous localization. However, since we are interested in *any* deviations from the Schrödinger equation, we will pursue this avenue for a short while.

Neutron diffraction experiments have already been examined in the context of violations of unitary evolution by Ellis *et al.*[16] Their analysis of the violation is formulated in terms of a super Hamiltonian, which generates Hawking's superscattering matrix.[17] For dynamics of the form of Eq. (1.4), the nonunitary part of the super-Hamiltonian is given by

$$|\delta H| = \hbar\lambda(1 - e^{-[(\alpha/4)(x-x')^2]}) \leq \hbar\lambda \ . \tag{3.1}$$

The limit set by neutron interferometry by Ellis is $\delta H \leq 2\times 10^{-12}$ eV. This limit is obtained by observing that double-slit interference of neutrons can be observed for a time of flight of 1/3000 s. It is possible to improve upon this limit a couple of orders of magnitude with more recent experiments using cold neutrons.[18] For these experiments, the double-slit interference pattern of 20-Å neutrons with a slit to observation plane distance of 5 m was performed. The limit on the nonunitary portion of the super-Hamiltonian becomes $\delta H \leq 2.6\times 10^{-14}$ eV. We note that, for GRW's choice of parameters $\delta H \leq 7\times 10^{-32}$ eV, so that it becomes evident that some-

thing other than an improved neutron experiment would be required to experimentally test QMSL.

In order to extract better limits from the neutron experiments, we now examine the double-slit experiment in increasing detail. By taking the simultaneous limit

$$\gamma \to 0, \quad T \to \infty, \quad \frac{2m\gamma k_B T}{\hbar^2} = \frac{\lambda\alpha}{4} , \tag{3.2}$$

Eq. (1.2a), the effective master equation of Caldeira and Leggett becomes the short length-scale limit of (1.4). We can now immediately employ the results of Savage and Walls[19] to analyze the effect of localization in an ideal double-slit experiment. The initial state of the system is given by a superposition of two plane waves, which correspond to the momenta directed from two point sources in the double slit experiment. The magnitudes of the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ are the same, the directions determined by the relative geometry of the respective slit to the relevant point on the observation plane. The intensity at the observation plane is then given by

$$P(\mathbf{r}) = 1 + e^{-\eta}\cos[\mathbf{r}\cdot(\mathbf{k}_2 - \mathbf{k}_1)] , \tag{3.3a}$$

where

$$\eta = \frac{\lambda\alpha}{4}\frac{\hbar^2}{3m^2}t^3|\mathbf{k}_2 - \mathbf{k}_1|^2 . \tag{3.3b}$$

The origin has been chosen to be exactly between the two slits. The geometry is essentially two dimensional: we let R be the distance from the slits to the observation plane, and z be the transverse distance along the observation plane (parallel to the slit separation). The value of t is determined by the time of flight from the slits to the observation plane. With a slit separation of d, and small-angle approximations, the interference pattern becomes

$$P(z) = 1 + e^{-\eta}\cos\left[\frac{dkz}{R}\right] , \tag{3.4a}$$

where

$$\eta = \frac{\lambda\alpha d^2 R m}{12\hbar k} . \tag{3.4b}$$

If, on the other hand, the transverse length scales are large, then the interference pattern will be given by

$$P(z) = 1 + e^{-\lambda(Rm/\hbar k)}\cos\left[\frac{dkz}{R}\right] . \tag{3.5}$$

We now show how to extend these concepts to a more detailed analysis of the neutron experiments for finite slit widths. To this end, we turn to the numerical analysis of Zeilinger *et al.*,[18] and examine the departure from their analysis that the dissipative evolution would introduce. We do this for an elementary system composed of a point source and a double-slit object. The problem is treated as strictly two dimensional. The contribution to the amplitude of a ray passing through a portion of width dS_0 of the aperture in the object plane S_0 is given schematically by

$$du(z) = e^{ik(r+s)}dS_0 , \tag{3.6}$$

where r is the distance from the point source to the point in the object plane, and s is the distance from the point in the object plane to the observation point. To find the interference pattern, one would then add up the contributions over the entire object plane, then square the resulting amplitude to determine the observed intensity. Now, the mechanisms we have been discussing have the effect of destroying the interference between different paths. We then write the incremental contribution to the intensity due to two paths (not necessarily different) as

$$dI(z) = du(z)[du'(z)]^* \propto e^{ik(r+s)}dS_0 e^{-ik(r'+s')}dS_0' e^{-D(y,y')} . \tag{3.7}$$

The loss of coherence between the two interfering paths is given by the last factor. For our simple situation, the different paths can be denoted simply by the point by which they pass through the object (y and y'). $D(y,y')$ represents the integrated loss of coherence as the paths are simultaneously traversed.

We know that the dissipative part of the evolution of the density matrix is given by

$$\frac{\partial\rho}{\partial t} = \lambda(1 - e^{-(\alpha/4)(x-x')^2})\rho . \tag{3.8}$$

If we let $\Delta x(r)$ be the transverse distance between the paths, as a function of the longitudinal displacement from the point source, and note that $dr = vdt = (\hbar k/m)dt$, we find that

$$D(y,y') = \int_0^R dr \frac{m}{\hbar k}\lambda(1 - e^{-(\alpha/4)[\Delta x(r)]^2}) . \tag{3.9}$$

Since the straight-line paths are determined by their common origin, their respective points of contact with the object plane and the observation point, the transverse distance between the paths is essentially only a function of y and y', so that we are justified in our simple parametrization of D. For identical paths ($y = y'$), it should be obvious that $D(y,y) = 0$. For those paths whose transverse distance is large through most of their time of flight, we find

$$D(y,y') \approx \lambda\frac{mR}{\hbar k} . \tag{3.10}$$

For those paths whose transverse distance remains small throughout the time of flight, we could again make the quadratic approximation for the integrand of Eq. (3.9).

The total intensity for our system is then

$$I(z) \propto \int_{S_0}\int dy\, dy' e^{ik(r+s)}e^{-ik(r'+s')}e^{-D(y,y')} . \tag{3.11}$$

We could then perform a statistical fit for possible values of the parameters α and λ with this more sophisticated model of the interference pattern. Bounds on nonunitary evolution could be improved by perhaps a few orders of magnitude.

We now turn to a more speculative experiment, and examine the possible ramifications of QMSL in a macroscopic tunneling experiment with a superconducting quantum interference device (SQUID).[20,21] For simplicity, we ignore the possible presence of dissipation in any

conventional sense, so that the Hamiltonian for a SQUID is given by

$$H=\frac{1}{2}C\dot{\Phi}^2+\frac{1}{2L}(\Phi-\Phi_{\text{ext}})^2-\frac{J_c\Phi_0}{2\pi}\cos(2\pi\Phi/\Phi_0)\ , \tag{3.12}$$

where Φ is the flux through the loop, C is the capacitance of the junction, L is the self-inductance of the loop, Φ_{ext} is the external flux, and Φ_0 is the flux quanta $h/2e$. We wish to recast this in terms of the supercurrent, where $\Phi=LJ_s+\Phi_{\text{ext}}$, and hold the external flux constant so that the Hamiltonian becomes

$$H=\frac{1}{2}CL^2\dot{J}_s^2+\frac{L}{2}J_s^2-\frac{J_c\Phi_0}{2\pi}\cos\left[2\pi\frac{LJ_s-\Phi_{\text{ext}}}{\Phi_0}\right]\ . \tag{3.13}$$

We see the familiar washboard potential in terms of the supercurrent. Tunneling between wells in the washboard corresponds to tunneling between different current states. The connection between the supercurrent and the microscopic degrees of freedom is given by

$$J_s=\sigma 2en_c v_s\ ,$$

where σ is the area of the junction, n_s is the number density of superconducting pairs, and v_s is the velocity of the charge carriers. Since σ and n_s are usually taken to be constant, the different current states (on the macroscopic level) correspond to different velocity states on the microscopic level.

We are now going to discuss how the localization mechanism comes into play in a tunneling experiment using a SQUID. We imagine that the system is initially in a state corresponding to one of the wells in the washboard potential. After a finite amount of time, the system has evolved, via the tunneling mechanism, to a superposition of two current states, corresponding to adjacent wells. Then, on the microscopic level, we have a pair state which is in a superposition of two velocity states. If we take the separation between the states to be $\Delta x=\Delta v_s t$, then the rate of "localization," or loss of coherence, will be

$$\frac{\partial\rho_{J,J'}}{\partial t}=2N_s\lambda(1-e^{-[(\alpha/4)(\Delta x)^2]})\rho_{J,J'}\ , \tag{3.14}$$

where Ns is the total number of superconducting pairs. We have taken advantage of the additive property for localization of macrosystems composed of weakly localized microsystems discussed by GRW in Ref. 7. With the proper choice of system parameters, one should expect a demonstrable effect on the tunneling time of the junction.

To illustrate the effect on tunneling time, we examine the implications for quantum tunneling when the two-state system of Leggett *et al.*[22] is appropriate. For simplicity, we ignore normal dissipative effects (due to the environment) and take the value of the detuning parameter to be zero. We take Δx to become large (in the localization mechanism's length scale) very quickly, so that the dissipative part of QMSL can be approximated by

$$\frac{\partial\rho_{J,J'}}{\partial t}\approx 2N_s\lambda\rho_{J,J'}\ . \tag{3.15}$$

The evolution of our simple system, in terms of the Pauli matrices, becomes

$$i\hbar\frac{\partial\rho}{\partial t}=\hbar\Delta_0[\sigma_y,\rho]-i\hbar\frac{N_s\lambda}{4}[\sigma_z,[\sigma_z,\rho]]\ . \tag{3.16}$$

The evolution equations for the "polarization" of the density matrix is essentially that for harmonic oscillation with dampening. If the state is initially polarized, that is, if initially it will be found in the $+z$ state with certainty, then the polarization [probability of $(+z)$ minus the probability of $(-z)$] becomes

$$P_z(t)=\cos(\omega t)e^{-\Lambda t}+\frac{\Lambda}{\omega}\sin(\omega t) \tag{3.17a}$$

for the underdamped case, and

$$P_z(t)=\frac{\nu_+}{\nu_+-\nu_-}e^{-\nu_+ t}+\frac{-\nu_-}{\nu_+-\nu_-}e^{-\nu_- t} \tag{3.17b}$$

for the overdamped case, where

$$\Lambda\equiv\frac{N_s\lambda}{2}\ ,$$
$$\omega\equiv(\Delta_0^2-\Lambda^2)^{1/2}\ , \tag{3.17c}$$
$$\nu_\pm\equiv\Lambda\pm(\Lambda^2-\Delta_0^2)^{1/2}\ .$$

The effect of QMSL would then be characterized by a decrease in the tunneling frequency ω.

IV. CONCLUSIONS AND COMMENTS

In Sec. II, we obtained an extension of Joos and Zeh's localization calculation. One appealing result is the saturation of the localization rate at large length scales. Intuitively, one would not expect an environment to be able to "distinguish" different separations with increasing sensitivity when those separations are already larger than the coherence length of the environment ($\cong\hbar c\beta$ for thermal photons) when the interaction between the system of interest and the environment is local. The effect of finite-range interactions complicates the interpretation, but one can generally determine an effective correlation length to determine at what length scales the localization will saturate.

To obtain the appropriate characteristic length, we first consider two extreme cases. First, if the correlation length of the environment is much longer than the range of the interaction, then the interaction is effectively a local one on the scale of the environment correlation length. In this case we have the situation encountered in Sec. II, and the characteristic length is simply the correlation length of the environment. Second, if the correlation length of the environment is much shorter than the range of the interaction, then the environment degrees of freedom are completely uncorrelated on the length scale of the interaction. In this case the appropriate characteristic length is the range of the interaction. In the regime between the two extremes, the shape of the interac-

tion, as a function of separation between the environment degree of freedom and the system of interest's coordinate, serves as an instrument function. To obtain the appropriate characterization of the environment-system interaction, one could convolute a function with the spatial form of the interaction with the (spatial) autocorrelation function of the environment. The width of the resulting function is the characteristic length we seek. This characteristic length might be interpreted as the correlation length of the effective forces to which the environment is subjected; however, caution is in order: it has already been observed that the localization effect of the environment can be quite strong even when the dissipative effect of the random forces are negligible.

Previous calculations have not been sensitive to these aspects of real environments, due to limitations of environment models. In the model of CL,[2] which is also used by Hakim and Ambegoakar,[3] the environment consists of oscillators, each with linear coupling to the system of interest. The model does not contain any explicit information about the spatial distribution of the oscillators, and the identical nature of the coupling suggests that the oscillators all exist at the same position, in terms of the coordinates of the system of interest. In the model of UZ, the system of interest is coupled to a massive scalar field which is spatially extended, but only at a point in the field's configuration space, which prohibits any dependence on the effective equations of motion of the coupled system on the spatial correlations of the environment.[4] If $\delta(x)$ in Eq. (1.3) is with $\delta(x-q)$, and if the coupling constants in Eq. (1.2b) were made to depend upon x, the spatial character of the environment and the local coupling between system and environment might be recovered. Unfortunately, this destroys the linear nature of the coupling which allows much of the calculations to proceed.

All these models share an environment-system interaction which is linear in the coordinate of the system of interest. The range of the interaction is therefore infinite, which will not allow any reflection of the spatial correlation functions of the environment in the effective dynamic of the system of interest. To the extent that this is an appropriate approximation of the physical system of interest, we would not expect to see the saturating effect in the localization of the system of interest, but the quadratic result for these models should not be construed as universal.

Another major difference between our results and the models interactions of UZ and CL is that we have explicitly assumed momentum transfer to the system of interest is negligible. The additional terms in the master equations of UZ and CL correspond to the effects of momentum transfer. It is relatively easy to determine the legitimacy of these approximations. When considering localization by photons, negligible momentum transfer can be reduced to the assumption that the energy of the photons is much less than the rest mass energy of the system of interest. When considering localization by dust particles, as discussed by Joos and Zeh, the assumption is that the mass of the dust particles is much less than the mass of the system of interest. If the approximations are to be reasonable and localization is to be effective, the time scales of interest should be much less than the relaxation time scale of the system of interest, but longer than the mean time between scattering events with the environment degrees of freedom.

One final difference between our calculation and the models of CL and UZ is that they consider coupling to a nontrivial vacuum. By this we mean that they consider effects at zero temperature, when the environment is in its ground state. When the localization is by photons or by dust particles, the zero-temperature or vacuum state would not affect the evolution of the system of interest, and unitary evolution would be recovered, so that the models of CL and UZ are not an appropriate description of these systems. The zero-temperature effects, including strong localization effects, may be appropriate in the context of electrons in a solid, but not universally so: the experimental observation of interference effects, such as Aharonov-Bohm oscillations in magnetoresistance measurements,[23] suggests that the localization of electrons by electron-phonon interaction is at least significantly reduced at low temperatures. Persistent currents in superconductors provide an example of the disappearance of dissipative effects at low temperatures.

Of course, this flattening of the localization rate may not always be significant. In order for the asymptotic behavior to be important, the system must be driven to superpositions of positions on length scales comparable to the characteristic length of the environment on time scales shorter than the lifetimes of the off-diagonal elements of the system of interest. All of the environmental models discussed suggest that these lifetimes should be very short for macroscopic systems, and so the system would be likely to be localized before reaching the characteristic length scale of the environment. On the other hand, if microscopic systems are tested for the presence of these localization mechanisms by the type of interference experiment discussed in Sec. III, for example, then a deep understanding of the environmental localization mechanism on all length scales would be essential to interpret the results. It is possible that by taking advantage of the saturation of the localization rate, one can overcome a difficulty in experimental observation of environmental localization discussed by UZ and CL.

For the dissipative two-state system, as applied to a SQUID, the interference is between two current states. The picture is of spatially extended supercurrent states, all occupying the same physical volume, and interacting with the same portions of the environment. In this case the spatial characterization of the environment is probably not relevant. We will return to this characterization of the pair states in the SQUID and its relevance to the proposed QMSL experiment shortly.

Having obtained the mathematical form for environmental localization, we see that it is qualitatively the same as that for QMSL, so that with the appropriate selection of parameters described in Sec. II, we have a variety of results obtained for QMSL which can be immediately applied to an environmental model. Two important results include a free particle propagator,[7] a localized harmonic oscillator,[12] and a simple measurement model.[10]

In Sec. III, we discussed two experiments for the detection of a dissipative term in the evolution of the density matrix of the form employed by QMSL. The first experiment uses neutron interference experiments as a precision test of quantum coherence. We determined the effect of the nonunitary evolution on the neutron interference pattern. A statistical fit on existing and future data could be used to set bounds upon the parameters α and λ. Although the neutron experiments are not and will not likely become sensitive to the choice of parameters made for QMSL,[7] the form of the dissipation is fairly general, leaving neutron experiments as a viable test of unitarity in quantum mechanics.

Because QMSL has received a great deal of recent attention, there would be great interest in an experimental test. To this end, we have introduced the decorrelating effect of QMSL on a SQUID, by examining the implications of a superposition of quantum states with different superconducting currents. Distinct values for the current on the macroscopic level corresponds to different velocity states for the electron pair state on the microscopic level. The superposition of different currents then leads to a superposition of spatially separated wave packets for the pair state when the system is given time to evolve. The presence of QMSL in a SQUID would manifest itself as a decrease in the tunneling frequency.

There is a potential inconsistency within our picture of the tunneling experiment, which brings to light a difficulty with the "universal" nature of QMSL. In determining how localization would effect the superconducting pair states, we assumed that the wave packets were of (relatively) small extent. It has already been noted that localization is not effective in destroying coherence between momentum states,[11] so that an assumption such as this must be made if QMSL is to be effective in destroying the macroscopic coherence. The conflict arises between this picture of pair states, and the usual description of the wave function in a SQUID, which is considered to be of constant amplitude and varying phase throughout the device. If localization does not appear in a manner at least similar to our description, it seems unlikely that there be another possibility for the spatially separated wave functions required by QMSL to manifest themselves. It is then quite possible that even if QMSL exits, there may be macroscopic systems for which the quantum coherence between macroscopically distinct states is not affected by QMSL. The difficulty with QMSL is that it performs its desired function only if macroscopic superpositions are invariably superpositions of spatially separated states. It might be argued that in the context of measurement, the superposition of spatially separated states is indeed always a part of the process at some part of the transfer of information to the observer. The debate as to the satisfactory nature of this solution is certain to persist.

The existence of effective nonunitary evolution of a reduced density operator due to interaction with an environment has long been known. The possibility of fundamental nonunitary evolution[7–12,14,16,17,24–26] brings with it a multitude of interesting considerations. Implementation of second quantization,[26] special relativity,[27,28] superselection principles, and conservation law are just a few difficulties facing any generalization of a fundamental modification of quantum mechanics such as QMSL.

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