Macroscopic squeezing in three-level laser

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It is shown that a three-level laser can become the first example of an optically active system to generate a squeezed-field state.

I. INTRODUCTION

The three-level atom laser has become quite interesting in connection with subtle optical effects, such as phase and dynamic instabilities (dynamic chaos) and so on.[1,2] However, together with long-term classical properties, it has specific quantum properties that are manifested in amplitude noise and linewidth.

In the present paper the problem of quantum fluctuations in a three-level atom laser (TLL) is discussed. It is shown that TLL with relevant pumping can be used for generating squeezed-field states. In order to create one, we would need an optically active system that directly produces a squeezed state without using a multitude of converting devices. Before discussing the quantum features of three-level generation, we would like to give a brief summary of conventional two-level generation.

II. TWO-LEVEL LASER

The fluctuations in lasers can be quite simply explained in terms of the dipole moment P and population inversion S that are fluctuating. To this end, we write quantum equations for these quantities (for their derivation see Appendix A):

$$\frac{\partial P}{\partial t}=\mathcal{S}-i\frac{\partial}{\partial z^*}\frac{1+S}{2}\ln\rho+i\Lambda P\ln\rho\ , \tag{1}$$

$$\frac{\partial S}{\partial t}=\mathcal{S}+i\Lambda\ln\rho+i\Lambda S\ln\rho,\quad \Lambda\equiv-(\partial/\partial z)P-\text{H.c.} \tag{2}$$

The coupling constant is omitted for the sake of simplicity. $\mathcal{S}$ stands for terms that do not contain derivatives $\partial/\partial z$, $\partial/\partial z^*$. Equations (1) and (2) are derived from the usual set of equations, which is decoupled by the following condition:

$$R\simeq\rho_f\prod_{a=1}^{N}r_a,\quad r_a\equiv\begin{pmatrix}\frac{1+S_a}{2} & P_a\\ P_a^* & \frac{1-S_a}{2}\end{pmatrix}. \tag{3}$$

R is the whole density matrix and ρ_f is the field density matrix. The quantities R, $P=P(z,z^*)$, $S=S(z,z^*)$ are functions of arguments z,z^*, being the relevant coherent-state amplitudes (the Glauber P representation is used). They describe the fluctuating dipole moment and inversion. It should be noted that the decoupling condition (3) differs from the one for undressed atoms that was used in early work.[3,4] In fact, the matrix r_a describes an atom dressed by the field (its matrix elements are functions of field operators).

The procedure we use to consider the function is based on the perturbation theory that is briefly described in Appendix A. Here we dwell upon physical mechanisms determining the amplitude fluctuations. On one hand, the fluctuations arise by spontaneous emission. The probability of spontaneous transitions depends on the time an atom spends in an excited state. On the other hand, fluctuations are defined by the derivatives $\partial/\partial z,\partial/\partial z^*$, in Eqs. (2) and (3). Let us consider contributions from different derivative terms in Eqs. (2) and (3) and their physical meaning.

A. Contribution from purely spontaneous emission

This factor is defined by the second term in the right-hand side (rhs) of Eq. (1). The population of the upper level (depending on pumping only in weak-field limit) $(1+S)/2$ defines the spontaneous emission probability in this case. The contribution to the photon number variance from this factor is given by

$$\langle(\Delta n)^2\rangle_1=\bar{\eta}(\xi-1)+\eta\frac{1+\tilde{\Delta}}{2\tilde{\Delta}}\ . \tag{4}$$

Here $\tilde{\Delta}$ is the threshold inversion, Δ_0 the inversion without field, and $\xi=\Delta_0/\tilde{\Delta}$ the lasing parameter. This result was obtained in early work on laser fluctuations.[3,4] Formula (4) describes fluctuations near the generation threshold fairly well. The "zero" contribution $(\xi-1)\bar{\eta}$, which is determined by "zero" fluctuations in the Glauber P representation, is taken into account.

B. Contribution from stimulated emission

Here the action of the stream of stimulated photons from a single atom should be taken into account:

$$\frac{d\langle n\rangle}{dt}=\frac{d|z|^2}{dt}=z\dot{z}^*+\dot{z}z^*\simeq -izP+\text{c.c.} \tag{5}$$

We obtain exactly this quantity if we substitute into Eq. (1) the inversions calculated with the second term in the rhs of Eq. (2). This second factor provides a negative correction to the factor discussed above, due to the fact that some of the radiated photons are not spontaneous. The contribution from this factor to the variance is

$$\langle(\Delta n)^2\rangle_2=-\tilde{\eta}\frac{\xi-1}{2} . \tag{6}$$

This term in the variance was obtained in a number of papers (see Refs. 5). The contribution of both factors sometimes is referred to as noncorrelation of the atoms. In the following we shall show that this is not so. Moreover, the atoms are statistically dependent even near threshold.

C. Contribution from mixed states

It is known that in a strong field the atom is in a state that is a mixture of excited and nonexcited states. The mixture decreases with the atom dipole moment P. The presence of the states' interference results in increasing the atom lifetime in the upper level. The quantum dipole appeared to be more coherent than the classical one. This atom dipole coherence gives rise to extracorrelation of atomic quantities (we mean the quantities for the single atom). This correlation leads to a reduction of dipole moment noise caused by the first factor; i.e., it decreases field fluctuations. From a formal point of view, the third factor is taken into account by the last terms in Eqs. (1) and (2). All terms of this type have the same form,

$$i\Lambda F=i\left[-\frac{\partial}{\partial z}P-\text{H.c.}\right]F , \tag{7}$$

where F is the chosen atomic quantities. "Extracoherence" of the quantum atom dipole leads to the following contribution to the variance:

$$\langle(\Delta n)^2\rangle_3=-\tilde{\eta}\frac{m+1}{2m}(\Delta_0-\tilde{\Delta}),\quad m=\frac{\gamma_\perp}{\gamma_\parallel} . \tag{8}$$

A contribution of this type was first obtained by Kazantsev and Surdutovich (for the case $m=1$) as early as 1969 and then was verified by other methods in Refs. 5–8. The relative value of contribution (8) in comparison with the first and second factors is not small for intermediate-range fields. It can be explained as follows. Near threshold there is no state mixing. In the above-threshold range the effect of mixing disappears because of averaging by Rabi oscillations. Therefore, the average value of the overlap goes to zero in this limit. Correlations between atomic quantities are again absent, and the variance is determined by the first-two factors only. Let us return to atom-atom correlation treatment. The contribution (8) to the variance is sometimes called "cooperative" addition. There is a need to elucidate. The fact is, the atoms are not statistically independent in any case, which is caused by the simple reason that all atoms bear fluctuations on the same field. Formally, it follows from the fact that the averages in the atom-atom correlation function are nonfactorized. For instance,

$$\langle P_a S_B\rangle\neq\langle P_a\rangle\langle S_B\rangle .$$

Neglect of the field fluctuations results in a transformation of the inequality to an equality. This statistical dependence of atoms is caused by field fluctuations. Thus the atoms are cooperative even near threshold. The contribution (8) is not related to atom-atom correlation, since it remains even in case of a single atom interacting with the field mode. As for the correlation between atoms dressed by the field, they do not correlate in the given adiabatic limit.[9] Formal consideration of atom-atom N correlations with finite N does not provide the correct result for fluctuations.[10] Thus the decoupling condition (3) can be called a "self-consistent-field" condition.

The above consideration shows that amplitude noise in TLL can be reduced by the following two mechanisms. The first is connected with the mutual influence of the first and second factors discussed previously. The point is that these factors appear to be mutually contradictory. If we try to decrease spontaneous emission by means of decreasing the upper-level population, we would decrease inversion. The latter gives rise to a decrease of the usual contribution from the second factor. One can break the deadlock by means of a third level; namely, a population decrease can be achieved by a simultaneous emission of an atom from both excited and nonexcited acting levels to a third level. This eliminates an unfavorable decrease of the inversion on the acting transition. Another noise-reducing mechanism is extracorrelation of inversion and dipole moment. This correlation is caused by third-level population fluctuations caused by state mixing in a strong field (third factor).

Formally, it is manifested in the introduction of a term such as in Eq. (7) in the equation for the population of the nonacting level.

III. FORMULATION OF PROBLEM FOR THREE-LEVEL LASER

Active atoms are considered to have three levels with states $|0\rangle,|1\rangle,|2\rangle$. Radiative transitions take place between states $|0\rangle$ (ground) and $|1\rangle$ (excited). The interaction is chosen to be of the form $\alpha\sigma_{01}b+\text{H.c.}$, where α is the coupling constant and b is the annihilation operator. $\sigma_{ij}\equiv|i\times j|$. The field is described in the Glauber-Sudarshan representation. The total density matrix obeys the Liouville equation

$$\left[\frac{\partial}{\partial t}+\Lambda_A+\Lambda_f+iL_{Af}\right]R(t)=0 , \tag{9}$$

$$\Lambda_f\Psi=\kappa\left[\frac{\partial}{\partial z}z+\text{c.c.}\right]\Psi , \tag{10}$$

$$\Lambda_A\Psi=-\sum_{i,j=0}^{2}\gamma_{ij}([\sigma_{ji}\Psi,\sigma_{ij}]+[\sigma_{ji},\sigma_{ij}\Psi]) . \tag{11}$$

Here κ is the cavity damping constant, γ_{ij} are the gen-

eralized rates of radiationless transitions between atom states; Ψ is an arbitrary operator. Relaxation of the form of Eq. (11) does not consider phase destruction processes. The Liouville equation (9) introduces a series of equations for reduced atom-field density matrices. All calculations can be performed by the same methods as for the two-level lasers.[5,7,9] The set of equations for the polarization and populations is written in Appendix C. The results to follow on the field statistics are obtained using the approach described in Appendix B.

IV. FIELD STATISTICS

As was indicated in Sec. II, the amplitude fluctuations are the sum of three terms.

A. Dipole momentum fluctuations

These fluctuations are proportional to the upper operating level population $(1+S)/2$ (probability of spontaneous emission). The contribution from this factor to the photon-number variance is positive and equals

$$\langle(\Delta n)^2\rangle_1=\tilde{\eta}\left[\frac{1+\tilde{\Delta}}{2\tilde{\Delta}}-\frac{K}{2\tilde{\Delta}}\right], \tag{12}$$

where the population of the third level K is introduced:

$$K\equiv[(\gamma_{12}-\gamma_{02})\Delta_0 f+\gamma_{12}+\gamma_{02}]/\gamma^*, \tag{13}$$

$$\Delta_0\equiv\frac{1}{D}[(\gamma_{01}-\gamma_{10})(\gamma_{21}+\gamma_{20})+\gamma_{21}\gamma_{02}-\gamma_{20}\gamma_{12}], \tag{14}$$

$$D=(\gamma_{01}+\gamma_{10})(\gamma_{21}+\gamma_{20})+\gamma_{02}(\gamma_{10}+\gamma_{12}+\gamma_{21})+\gamma_{12}(\gamma_{20}+\gamma_{01}), \tag{15}$$

$$\gamma^*\equiv\gamma_{12}+\gamma_{02}+2(\gamma_{20}+\gamma_{21}), \tag{16}$$

$$\tilde{\Delta}\equiv\frac{\kappa\gamma_\perp}{N\alpha^2},\quad f\equiv(1+x)^{-1},\quad x\equiv\frac{|z|^2}{\tilde{\eta}},\quad \tilde{\eta}\equiv\frac{\gamma_\perp\gamma_\parallel}{4\alpha^2},$$

$$\gamma_\parallel=\frac{4D}{\gamma^*},\quad \gamma_\perp\equiv\gamma_{10}+\gamma_{01}+\gamma_{12}+\gamma_{02}.$$

The first term in (12) coincides with the analogous term for a two-level laser. The second term adds negatively, giving a reduction of the spontaneous emission in the presence of a third level [according to the formula $\rho_{11}=(1+S-K)/2$].

B. Contribution from stimulated emission

The contribution in the case of three-level atoms has the same appearance as for the two-level system (see Sec. II),

$$\langle(\Delta n)^2\rangle_2=-\tilde{\eta}\frac{\xi-1}{2}. \tag{17}$$

C. Reducing amplitude noise caused by mixing of the atom states in strong fields

We give the result of a negative contribution from this factor,

$$\langle(\Delta n)^2\rangle_3=-\tilde{\eta}\frac{m+1}{2m}(\Delta_0-\tilde{\Delta})-\tilde{\eta}\frac{\gamma_1^*}{2\gamma^*}K(\xi-1), \tag{18}$$

$$\gamma_1^*\equiv 2(\gamma_{21}-\gamma_{20}+\gamma_{10}-\gamma_{01})+\gamma_{12}-\gamma_{02}.$$

The second term in Eq. (18) determines the contribution from correlations between the dipole moment and the population of third level (as a result of mixing of atomic states).

The above formulas show that it is impossible to confine consideration of TLL by renormalizing only the constants. While doing so, we miss a significant contribution to the variance. Finally, the amplitude noise (taking into account "zero" fluctuations in the Glauber P representation) can be written as

$$\langle(\Delta n)^2\rangle=\tilde{\eta}(\xi-1)+\langle(\Delta n)^2\rangle_1+\langle(\Delta n)^2\rangle_2+\langle(\Delta n)^2\rangle_3. \tag{19}$$

This formula was fist obtained in Ref. 11.

V. DISCUSSION OF RESULTS

Formulas (12), (17), and (18) allow us to calculate the variance for different pumping schemes. In the examples given pumping was chosen to obtain the best amplitude squeezing, which is determined as $[1-\langle(\Delta n)^2\rangle/\langle\Delta n\rangle]$ (100%). We are striving for a reduction of spontaneous emission, not at the expense of a drastic reduction in inversion, but at the expense of an increase in population of the third level in steady state. When the field is switched off, the population of the third level is small, but in the steady state it is considerable. The smallness of K_0 creates a semblance of two-level behavior in the atom. Meanwhile the fluctuation properties of such a system differs to a great extent from two-level one. In reality, the maximum squeezing that can be reached in the two-level laser in the absence of phase distruction processes does not surpass 5%, whereas in a three-level laser it can be significantly increased by means of increasing the distance from equilibrium of the pumping. In most nonequilibrium cases with only γ_{02} and $\gamma_{21}\neq 0$, the following expression for relative variance yields:

$$\frac{\langle(\Delta n)^2\rangle}{\langle n\rangle}=\frac{\xi}{\xi-1}-\frac{m+1}{2m\xi}-\frac{\gamma_{02}(1-f)}{2(\gamma_{02}+2\gamma_{21})}\left[\frac{\xi}{\xi-1}+\frac{2\gamma_2-\gamma_{02}}{2\gamma_2+\gamma_{02}}\right]. \tag{20}$$

We see that the maximum squeezing is reached in the limit of strong fields $\xi\gg 1$ and amounts to 25% with $\gamma_{02}=2\gamma_{21}$. Thus, we obtain the "macroscopic quantum state of the field" in an active system. Moreover, the squeezing increases with the field. In Fig. 1 are given for comparison the degree of squeezing in both a two-level and a three-level laser, as functions of the lasing parameter. In case of a two-level laser in the strong-field limit we have Poisson statistics, whereas in the TLL squeezed

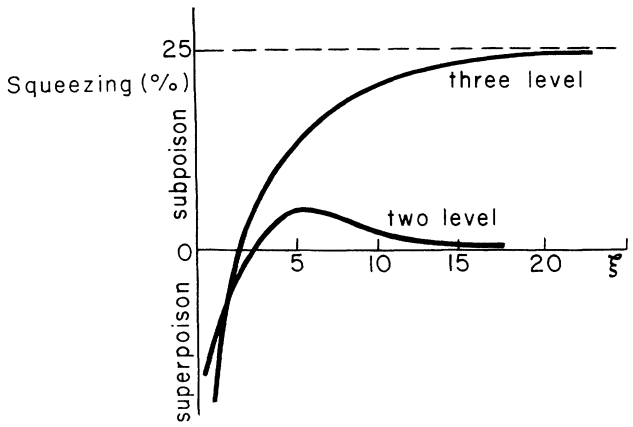

FIG. 1. The principal difference between field statistics of a two-level laser and TLL.

state 25% squeezing appears in the above-mentioned limit. This result allows us to hope for experimental realization of a macroscopic squeezed light generator.

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APPENDIX A: DERIVATION OF CLOSED SET OF EQUATIONS

Let us consider a derivation of the closed set of quantum equations for the population inversion S and the dipole moment P in the Glauber P representation. We proceed from the conventional single-mode laser problem. The whole "N atoms plus field" system density matrix $R(t)$ obeys the generalized Liouville equation

$$\left[\frac{\partial}{\partial t}+\Lambda_H+\Lambda_f+i\mathcal{L}_{Hf}\right]R(t)=0 , \tag{A1}$$

where

$$\Lambda_f\Psi=-\kappa([B\Psi,B^\dagger]+[B,\Psi B^\dagger]) , \tag{A2}$$

$$\Lambda_H=\sum_{a=1}^{N}\Lambda_a , \tag{A3}$$

$$\mathcal{L}_{Hf}=\sum_{a=1}^{N}\mathcal{L}_{af} , \tag{A4}$$

$$\Lambda_a\begin{pmatrix}\Psi_{\parallel} & \Psi_{12}\\ \Psi_{21} & \Psi_{22}\end{pmatrix}=\begin{pmatrix}\gamma_\uparrow\Psi_{\parallel}-\gamma_\downarrow\Psi_{22} & \gamma_\perp\Psi_{12}\\ \gamma_\perp\Psi_{21} & -\gamma_\uparrow\Psi_{\parallel}+\gamma_\downarrow\Psi_{22}\end{pmatrix} \tag{A5}$$

$$\mathcal{L}_{af}\Psi=[\sigma_a^\dagger B+\text{H.c.},\Psi] . \tag{A6}$$

The operators Λ_f,Λ_q describe linear relaxations of the field and the single atom, respectively. b ($b^\dagger$) is the cavity mode annihilation (creation) operator. σ ($\sigma^\dagger$) is the lowering (rasing) atomic operator. κ is the cavity damping constant. $\gamma_\uparrow,\gamma_\downarrow,\gamma_\perp$ are the atomic relaxation constants describing pumping and the decay. The coupling constant in the interaction operator $\mathcal{L}_{af}$ is omitted for simplicity.

The problem to be handled is a many-particle problem. Thus, to obtain a closed set of equations it is necessary to use some decoupling procedure. It is known[5] that the usage of "pure" atom-atom n-particle correlations does not allow one to confine with any finite number n. To avoid this difficulty, we consider correlations between dressed atoms. To do this, the following correlation forms are to be introduced:

$$Sp_{1,2,\ldots,N}R(t,z)\equiv\rho_f(t,z) , \tag{A7}$$

$$Sp_{2,3,\ldots,N}R(t,z)\equiv\rho_f(t,z)r_1(t,z) , \tag{A8}$$

$$Sp_{3,4,\ldots,N}R(t,z)\equiv\rho_f(t,z)r_{12}(t,z) , \tag{A9}$$

and so on. z is the argument of the Glauber P representation. Here, ρ_f is the field density matrix. r_1 and r_2 are one- and two-field density matrices describing dressed atoms. It can be seen[9] that the correlation form $\delta r_{12}=r_{12}-r_1r_2$ is very small and can be neglected in the adiabatic limit $\kappa\ll\gamma_\perp,\gamma_\uparrow,\gamma_\downarrow$ in the calculations of quantum fluctuations. r_1 represents the atomic density matrix, whose elements depend on the field variables t,z. In terms of random processes, they are random quantities that fluctuate with fluctuating field. An explicit expression for r_a in terms of the fluctuating dipole moment $P_a=P_a(t,z)$ and population inversion $S_a=S_a(t,z)$ is

$$r_a=\begin{pmatrix}\frac{1+S_a}{2} & P_a\\ P_a^* & \frac{1-S_a}{2}\end{pmatrix} . \tag{A10}$$

To obtain equations for ρ_f,r_1, traces in Eq. (A1) should be taken with respect to $N,N-1,\ldots,2$ atom variables, respectively.

Using the decoupling condition $\delta r_{12}=0$, one can obtain from Eq. (A1) the one-particle equation

$$\frac{1}{i}\frac{\partial r_a}{\partial t}+\frac{1}{i}\Lambda_a r_a+[h_a,r_a]=\tilde{\sigma}_a r_a\frac{\partial}{\partial z}\ln\rho_f-\text{H.c.} , \tag{A11}$$

where $\tilde{\sigma}_a=\sigma_a-Sp_a(\sigma_a r_a)$, and h_a is the "classical" part of the interaction Hamiltonian, which does not contain quantum derivatives $\partial/\partial z,\partial/\partial z^*$:

$$h_a\equiv z^*\sigma_a+z\sigma_a^\dagger . \tag{A12}$$

Equation (A11) is the basic closed master equation (ME).[9] It differs from this in simple Scully-Lamb theory[3] due to differences between $\tilde{\sigma}$ and σ. Equation (A11) takes into account all orders of pure atom-atom correlation. On the other hand, in terms of the dressed atoms, Eq. (A11) is a one-particle equation. Indeed, it does not contain any quantities but those belonging to the same

atom a. The left-hand side of Eq. (A11) defines the nonlinear (with respect to field) classical (without quantum fluctuations) evolution of the total density matrix, while its rhs has a pure quantum origin. It contains only terms with derivatives $\partial/\partial z, \partial/\partial z^*$.

To derive the equation for the reduced field density matrix, the trace over atomic variables in Eq. (A1) should be calculated. Rewriting Eq. (A11) for matrix elements and taking into account Eq. (A10), we derive Eqs. (1) and (2), where $\mathcal{S}$ stands for classical (nonquantum) terms. The explicit form of the equations is the following:

$$\frac{\partial \rho_f}{\partial t} = \left[\frac{\partial}{\partial z} \left[\kappa z + i \sum_a P_a \right] + \text{H.c.} \right] \rho_f \ , \tag{A13}$$

$$\frac{\partial P}{\partial t} + \gamma_\perp P - izS = i \left[\frac{\partial}{\partial z^*} \frac{1+S}{2} - P\Lambda \right] \ln \rho_f \ , \tag{A14}$$

$$\frac{\partial S}{\partial t} + \gamma_\parallel S + (2izP^* + \text{c.c.}) - \gamma_\parallel \Delta_0 = i\Lambda(1+S)\ln\rho_f \ ,$$

$$\gamma_\parallel \equiv 2(\gamma_\downarrow + \gamma_\uparrow), \quad \Delta_0 \equiv \frac{\gamma_\uparrow - \gamma_\downarrow}{\gamma_\uparrow + \gamma_\downarrow}, \quad \Lambda \equiv -\frac{\partial}{\partial z} P - \text{H.c.} \tag{A15}$$

APPENDIX B: CALCULATION OF FLUCTUATIONS

We briefly describe the method to derive the quantum Fokker-Planck equation (FPE) for the field density matrix ρ_f from the set of Eqs. (A13) and (A15), here using the usual perturbation theory with respect to quantum fluctuations. In the process of lasing, the photon number distribution is very narrow (near Gaussian). Therefore, quantum fluctuations represented by terms with derivatives in Eqs. (A13)–(A15) are small. It takes two steps of perturbation theory in order to the obtain quantum FPE for the field density matrix ρ_f.

The first step is to neglect quantum terms in Eqs. (A14) and (A15) (i.e., rhs terms) and to solve an algebraic set of equations. The zero-order solutions $S^{(0)}(z)$, $P^{(0)}(z)$, $P^{(0)*}(z)$ result.

The second step is performed by inserting the zero-order solutions into the rhs of Eqs. (A14) and (A15) and solving the equations again. The first-order solutions $S^{(i)}(z)$, $P^{(1)}(z)$, $P^{(1)*}(z)$ are obtained. These functions now contain first-order derivatives $\partial/\partial z, \partial/\partial z^*$. Inserting $P^{(1)}(z)$, $P^{(1)*}(z)$ into Eq. (A13), one obtains the FPE in question. Its diffusion coefficients allow us to calculate various correlation functions necessary[5] for discussions of the quantum properties of the field.

APPENDIX C: THREE-LEVEL CASE

Equation (A11) is valid for the three-level atom density matrix as well as if we substitute $\sigma = |1\rangle\langle 0|$ into it ($|1\rangle\langle 0|$ is the projection transition operator for acting levels). Projection of Eq. (A11) on proper matrix elements results in the following set of equations:

$$\frac{\partial P}{\partial t} + \gamma_\perp P - izS = -i \left[\frac{\partial}{\partial z^*} \frac{1+S-K}{2} - \Lambda \right] \ln \rho_f \ , \tag{C1}$$

$$\frac{\partial S}{\partial t} + \gamma_1 S - \gamma_1^* K + (2izP^* + \text{c.c.}) - \gamma_2 = -i\Lambda(1+S)\ln\rho \ , \tag{C2}$$

$$\frac{\partial K}{\partial t} + \gamma^* K + (\gamma_{02} - \gamma_{12})S - (\gamma_{12} - \gamma_{02}) = -i\Lambda K \ , \tag{C3}$$

where

$$\Lambda \equiv -\frac{\partial}{\partial z} P - \text{h.c.} \ ,$$

$$\gamma_1 \equiv \gamma_{12} + \gamma_{02} + 2(\gamma_{01} + \gamma_{10}) \ ,$$

$$\gamma_2 \equiv \gamma_{12} - \gamma_{02} + 2(\gamma_{10} - \gamma_{01}) \ ,$$

P and S are the polarization and population inversion for acting transition, and K is the population of the third level ($P = r_{10}$, $S = r_\parallel - r_\infty$, $K = r_{22}$). The other designations are given in the text. Equations (C1)–(C3) can be treated as described in Appendix A.

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