Theory of intracavity atom-field interactions with squeezed inputs

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(Received 7 March 1990)

We give a general theory of the interaction of a large number of two-level atoms in an optical cavity driven by a broadband squeezed input of finite amplitude. No adiabatic elimination of atoms or field is made, and atomic and cavity loss and dispersion are fully incorporated in the theory. We calculate the optical and squeezing spectra that may be observed in transmission and reflection from the cavity; the latter exhibits a novel interference structure superimposed as a result of the squeezed input. Results presented indicate that it is possible to somewhat reduce the vacuum Rabi peaks in transmission, in the strong-coupling regime, with a moderately squeezed input. In the limit of a "bad cavity," the characteristic triplet transmitted under conditions of saturation may have one of its sidebands suppressed and the other enhanced by a squeezed input. These signatures suggest new alternatives for the experimental study of the interaction of atoms with squeezed light.

I. INTRODUCTION

Recent progress in the generation of squeezed states of light[1] by phase-sensitive nonlinear optical processes has led to a reexamination of fundamental atomic radiative processes and the role of the environment. In particular, the presence of a highly correlated bath of squeezed electromagnetic field modes, instead of the usual free-space environment of a thermal bath, or what at optical frequencies amounts to a vacuum bath of field modes. Interesting predictions have been made, such as the inhibited decay of an atomic polarization quadrature,[2] and shown to lead to subnatural spectral linewidths.[3] This reflects the influence of phase-sensitive field correlations on the atomic dipole dynamics, and suggests an experimental test of the role of noise and dissipation in atom-field interactions. To date no experimental confirmation exists. This is in part due to the geometrical problem of squeezing all the electromagnetic field modes to which an atom is coupled in free space, i.e., a 4π solid angle or at least a dipole profile. In practice squeezed light is generated in a beam from a given source, and would thus subtend a small solid angle at the position of an atom. The phase-sensitive features, proportional to the solid angle subtended,[2] would be so small as to be practically unobservable. To circumvent this problem a proposal, where atoms are placed in a microcavity and the squeezed beam is mode matched to the cavity mode, effectively removing the coupling to other free-space modes, has been suggested by Parkins and Gardiner.[4] It is clear in any case that some modification of the isotropic environment of the atom must be made for the effects of squeezing to be manifest. The use of optical cavities is the focus of our attention here.

The radiative properties of atoms inside microwave[5] and optical[6] cavities has received a good deal of theoretical and experimental attention recently. The optical regime has the advantage that direct measurements of the output fields are possible. Apart from cavity and atomic dispersion the intracavity interactions are characterized by three parameters (Ref. 6): g, the electric dipole coupling constant, which determines the frequency of exchange of excitation between an atom and the intracavity field; κ the damping rate of the intracavity field; and γ the atomic spontaneous emission rate into modes other than the resonant cavity mode. For cavities whose mode function subtends only a small fraction of 4π solid angle, γ is approximately equal to the free-space Einstein A coefficient.[7] Weak- and strong-coupling regimes are delineated by the relative magnitudes of these parameters, or combinations of them. Experiments on optical cavities which contain $\mathcal{N}$ two-state atoms have been carried out in the strong-coupling regime $g\sqrt{\mathcal{N}} \gg \gamma \gg \kappa$.[6,7] In this regime oscillatory exchange of excitation between the collective atomic polarization and the field is likely to occur before radiative escape via the cavity mirrors, or spontaneous emission into the continuum.[8] The spectral characteristic is a two-peak structure due to the splitting of the first excited state of the dressed atom-intracavity field system. The linewidths of the doublet, which have been observed experimentally, are an average of the atomic and cavity decay rates, and may thus be smaller than the free-space Einstein A value.[6,7] The splitting is not an atomic saturation effect; it occurs at intracavity photon numbers arbitrarily far below that necessary for atomic saturation, and merely reflects the eigenvalue spacing of the first and second excited dressed states. If the intracavity field is resonantly pumped by an external laser field, this will induce transitions among the dressed eigenstates, possibly exciting higher dressed levels if the field intensity is sufficient. The coherent scattering processes which result have been used to generate squeezed light.[9]

In this paper we investigate the spectral properties of a large number of atoms in a cavity under very general conditions, when the driving field is squeezed in some quadrature.[10] In recent work Courty and Reynaud[11] and Savage[12] have considered the intracavity interaction of single atoms with squeezed cavity fields, in the bad-cavity and strongly coupled regimes, respectively. The presence of the cavity means that the geometrical problem of effectively coupling atoms to a squeezed beam, alluded to earlier, is offset somewhat if the incident field can be mode matched to the resonant cavity mode, as the cavity decay rate may be comparable with the rate of spontaneous decay into the continuum. In addition the dynamics of the atom-cavity system is a good deal richer than that of an atom in free space. From a pragmatic viewpoint this is also perhaps the most experimentally convenient scenario for investigating the response of atoms to squeezed light fields. The essential problem is to find a suitable source at optical frequencies, perhaps a similar cavity stage, and to properly mode match it to the cavity. The other elements of the system are already under exceedingly good experimental control.[6–9]

II. THEORETICAL MODEL

A. Squeezed inputs to a cavity

The input-output formalism of Collett and Gardiner[13] relates freely propagating input and output fields at one port of an optical cavity, to the internal field, through a local boundary condition at the mirror i:

$$a_{\rm in}(t)+a_{\rm out}(t)=\sqrt{2\kappa_i}\,a(t)\ , \tag{2.1}$$

where $a_{\rm in,out}$ are the input and output field annihilation operators and a is the nearest resonant cavity mode annihilation operator. The cavity amplitude decay rate, via mirror i, is κ_i. Since the input and output fields are free, their temporal evolution is simple and they obey (in the optical regime)

$$[a_{\rm in}(t),a_{\rm in}^\dagger(t')]=[a_{\rm out}(t),a_{\rm out}^\dagger(t')]=\delta(t-t')\ . \tag{2.2}$$

We consider a two port cavity, with a broadband field squeezed around the frequency ω_L incident on port A, while the vacuum is incident on port B. The input field (to port A) has a nonzero amplitude at frequency ω_L:

$$\langle a_{\rm in}(t)\rangle=\sqrt{2\kappa_A}\,Ee^{-i\omega_L t}\ , \tag{2.3}$$

where E is complex. This driving field is assumed to be phase coherent with the squeezed field which in practice means they are derived from the same source. The presence of squeezing in the input is indicated by the nonzero correlations

$$\begin{aligned}&\langle a_{\rm in}^\dagger(t),a_{\rm in}(t')\rangle=N\delta(t-t')\ ,\\&\langle a_{\rm in}(t),a_{\rm in}(t')\rangle=Me^{-2i\omega_L t}\delta(t-t')\ ,\end{aligned} \tag{2.4}$$

where $N,M\equiv|M|e^{i2\psi}$ are parameters characterizing the squeezing and $|M|^2\le N(N+1)$. As the squeezed field is incident on port A we will use the convention that the output field from port A will be referred to as the *reflected field* (R), to distinguish it from the output *transmitted field* T from port B.

B. Atoms in a cavity with squeezed input

We consider a large number $\mathcal{N}$ *two-state* atoms suitably prepared, for example, by preoptical pumping, in the cavity configuration discussed above. The intracavity field couples to the collective atomic spin operators $J_\pm$ and J_z defined by

$$\begin{aligned}&J_\pm=\sum_{j=1}^{\mathcal{N}}\sigma_\pm^j e^{\pm i\mathbf{k}\cdot\mathbf{r}_j}\ ,\\&J_z=\sum_{j=1}^{\mathcal{N}}\sigma_z^j\ ,\end{aligned} \tag{2.5}$$

where $\mathbf{k}$ is the cavity mode wave number, $\mathbf{r}_j$ the position, and $\sigma_\pm^j$ and σ_z^j are the Pauli spin operators for atom j. The collective spin operators satisfy the commutation relations

$$\begin{aligned}&[J_z,J_\pm]=\pm J_\pm\ ,\\&[J_+,J_-]=2J_z\ .\end{aligned} \tag{2.6}$$

With coherent field input to a single-ended cavity, Reid[14] has derived c-number stochastic differential equations for this system, in the limit that the number of atoms $\mathcal{N}$ is large enough that the positive P function obeys a Fokker-Planck equation. The derivation associates c-number random variables with the operators of the atom-field system, i.e.,

$$\begin{aligned}&\alpha\leftrightarrow a\ ,\\&\alpha^+\leftrightarrow a^\dagger\ ,\\&v\leftrightarrow J_-\ ,\\&v^+\leftrightarrow J_+\ ,\\&D\leftrightarrow 2J_z\ .\end{aligned} \tag{2.7}$$

The only difference in the present case appears in the correlations of the field noise sources, due to the presence of the squeezed input. Details of the derivation which uses techniques developed by Haken[15] and Drummond and Gardiner[16] may be found in Ref. 17. The stochastic differential equations are

$$\begin{aligned}&\dot\alpha=E-\kappa(1+i\phi)\alpha+gv+\Gamma_\alpha(t)\ ,\\&\dot\alpha^+=E^*-\kappa(1-i\phi)\alpha^++gv^++\Gamma_{\alpha^+}(t)\ ,\\&\dot v=-\gamma_\perp(1+i\Delta)v+g\alpha D+\Gamma_v(t)\ ,\\&\dot v^+=-\gamma_\perp(1-i\Delta)v^++g\alpha^+D+\Gamma_{v^+}(t)\ ,\\&\dot D=-\gamma_\parallel(D+\mathcal{N})-2g(v^+\alpha+v\alpha^+)+\Gamma_D(t)\ ,\end{aligned} \tag{2.8}$$

with g the electric dipole coupling coefficient, and the scaled atomic and cavity detunings Δ and ϕ defined by

$$\Delta=\frac{\omega_0-\omega_L}{\gamma_\perp},\quad \phi=\frac{\omega_c-\omega_L}{\kappa}\ , \tag{2.9}$$

where $\kappa=\kappa_A+\kappa_B$ is the total loss rate from the cavity, and the atomic and cavity resonance frequencies are given by ω_0 and ω_c, respectively. Spontaneous decay of atoms into modes other than the cavity occurs at rate $\gamma_\parallel$. For purely radiative damping $\gamma_\perp=\frac{1}{2}\gamma_\parallel$. The zero mean Gaussian noise sources $\Gamma_\lambda(t)$, $\lambda\in\{\alpha,\alpha^+,v,v^+,D\}$ have the nonzero correlations

$$\langle \Gamma_{\alpha^+}(t)\Gamma_{\alpha}(t')\rangle = 2\kappa_A N\delta(t-t') \ , \tag{2.10a}$$

$$\langle \Gamma_{\alpha}(t)\Gamma_{\alpha}(t')\rangle = \langle \Gamma_{\alpha^+}(t)\Gamma_{\alpha^+}(t')\rangle^* = 2\kappa_A M\delta(t-t') \ , \tag{2.10b}$$

$$\langle \Gamma_{v}(t)\Gamma_{v}(t')\rangle = 2g\alpha v\delta(t-t') \ , \tag{2.10c}$$

$$\langle \Gamma_{v^+}(t)\Gamma_{v^+}(t')\rangle = 2g\alpha^+ v^+\delta(t-t') \ , \tag{2.10d}$$

$$\langle \Gamma_{D}(t)\Gamma_{D}(t')\rangle = [\,2\gamma_{\parallel}(D+\mathcal{N}) - 4g(\alpha v^+ + \alpha^+ v)]\delta(t-t') \ . \tag{2.10e}$$

The effect of a squeezed input to port A is contained in the nonzero correlations (2.10a) and (2.10b), which vanish for a coherent input field.

C. Classical steady states and phase relations

Ignoring the noise sources in (2.8), the resultant equations become deterministic with $\alpha^+=\alpha^*$, $v^+=v^*$. The steady-state solutions of these equations are[17]

$$v_0 = \frac{g\alpha_0 D_0}{\gamma_\perp(1+i\Delta)}, \quad D_0 = \frac{-\mathcal{N}(1+\Delta^2)}{1+\Delta^2+I} \ , \tag{2.11}$$

$$Y = I\left[\left[1+\frac{2C}{1+\Delta^2+I}\right]^2 + \left[\phi - \frac{2C\Delta}{1+\Delta^2+I}\right]^2\right] \ , \tag{2.12}$$

where the atomic *cooperativity* parameter C is defined

$$C \equiv \frac{g^2\mathcal{N}}{2\gamma_\perp\kappa} = \frac{g^2\mathcal{N}}{\gamma_\parallel\kappa} \ . \tag{2.13}$$

The intracavity intensity I is expressed in terms of the atomic saturation photon number n_s, where

$$I \equiv \frac{|\alpha_0|^2}{n_s}, \quad n_s = \frac{\gamma_\parallel\gamma_\perp}{4g^2} = \frac{\gamma_\parallel^2}{8g^2} \ . \tag{2.14}$$

The state equation (2.12) relates the intracavity intensity to the input intensity Y defined by

$$Y = \frac{|E|^2}{\kappa^2 n_s} \tag{2.15}$$

and exhibits both absorptive and dispersive bistability in appropriate limits. We make the phase convention that α_0 is real and positive.

The state equation (2.12) contains no phase information. The equation relating the mean input field to the intracavity field is

$$y_{\text{in}} = \sqrt{I}\left[1+i\phi+\frac{2C(1-i\Delta)}{1+\Delta^2+I}\right] \ , \tag{2.16}$$

where

$$y_{\text{in}} = \frac{E}{\kappa\sqrt{n_s}} = \left[\frac{2\kappa_A}{n_s}\right]^{1/2}\frac{\langle a_{\text{in}}\rangle}{\kappa} \tag{2.17}$$

is the mean input field to port A.

The phase ζ_{in} of the input field with respect to the intracavity field is defined by

$$y_{\text{in}} = \sqrt{Y}e^{i\zeta_{\text{in}}} \ , \tag{2.18}$$

where

$$\tan\zeta_{\text{in}} = \frac{\phi - \dfrac{2C\Delta}{1+\Delta^2+I}}{1+\dfrac{2C}{1+\Delta^2+I}} \ . \tag{2.19}$$

This phase angle is helpful in defining the amplitude and phase input quadratures for the field input to port A. Similarly the output quadratures reflected and transmitted by the cavity require the phases of the reflected and transmitted fields. The boundary conditions[13]

$$\langle a_{\text{in}}\rangle + \langle a_{\text{out}}\rangle = \sqrt{2\kappa_A}\langle a\rangle \ , \tag{2.20a}$$

$$\langle b_{\text{in}}\rangle + \langle b_{\text{out}}\rangle = \sqrt{2\kappa_B}\langle a\rangle \ , \tag{2.20b}$$

at mirrors A and B, respectively, where b_{in} and b_{out} are the free field input and output annihilation operators for port B, relate the mean input and output fields to the intracavity field. Noting that $\langle b_{\text{in}}\rangle=0$, $\langle a\rangle=\alpha_0$, Eq. (2.20b) implies that the transmitted field is in phase with the intracavity field and is therefore real and positive. Using (2.20a) we find

$$y_{\text{out}} = \sqrt{I}\left[\kappa_m - i\phi - \frac{2C(1-i\Delta)}{1+\Delta^2+I}\right] \ , \tag{2.21}$$

where

$$y_{\text{out}} = \left[\frac{2\kappa_A}{n_s}\right]^{1/2}\frac{\langle a_{\text{out}}\rangle}{\kappa} \equiv |y_{\text{out}}|e^{i\zeta_{\text{out}}} \ ,$$

$$\tan\zeta_{\text{out}} = -\frac{\phi - \dfrac{2C\Delta}{1+\Delta^2+I}}{\kappa_m - \dfrac{2C}{1+\Delta^2+I}} \ , \tag{2.22}$$

and κ_m is the cavity mismatch factor

$$\kappa_m \equiv \frac{\kappa_A - \kappa_B}{\kappa} \ . \tag{2.23}$$

Equations (2.19) and (2.22) will be used in Sec. IV to define the amplitude and phase quadratures of the input and output fields, for any set of atomic and cavity parameters.

III. LINEARIZED ANALYSIS OF QUANTUM FLUCTUATIONS

The nonlinear stochastic equations may be solved approximately by linearization about a stable steady-state solution. Define $\underline{\alpha}(t)=(\alpha,\alpha^+,v,D,v^+)^T$, $\underline{\alpha}_0=(\alpha_0,\alpha_0,v_0,D_0,v_0^*)^T$, and $\delta\underline{\alpha}(t)=\underline{\alpha}(t)-\underline{\alpha}_0$, one finds to first order the Ornstein-Uhlenbeck equations

$$\frac{d}{dt}\delta\underline{\alpha}(t) = -\underline{A}\,\delta\underline{\alpha}(t)+\underline{\Gamma}(t) \ , \tag{3.1}$$

where the drift matrix $\underline{A}$ is defined

$$\underline{A}=\begin{bmatrix} \kappa(1+i\phi) & 0 & -g & 0 & 0 \\ 0 & \kappa(1-i\phi) & 0 & 0 & -g \\ -gD_0 & 0 & \gamma_\perp(1+i\Delta) & -g\alpha_0 & 0 \\ 2gv_0^* & 2gv_0 & 2g\alpha_0 & \gamma_\parallel & 2g\alpha_0 \\ 0 & -gD_0 & 0 & -g\alpha_0 & \gamma_\perp(1-i\Delta) \end{bmatrix}, \tag{3.2}$$

and the linearized noise source $\underline{\Gamma}(t)$ satisfies

$$\langle \underline{\Gamma}(t)\rangle=\underline{0};\langle \underline{\Gamma}(t)\underline{\Gamma}^T(t')\rangle=\underline{D}\delta(t-t') . \tag{3.3}$$

The diffusion matrix $\underline{D}$ is partitioned onto orthogonal subspaces spanned by field and atomic variables, respectively, i.e.,

$$\underline{D}=\begin{bmatrix} \underline{D}_F & \underline{0} \\ \underline{0} & \underline{D}_A \end{bmatrix}, \tag{3.4}$$

where

$$\underline{D}_F=2\kappa_A\begin{bmatrix} M & N \\ N & M^* \end{bmatrix}, \tag{3.5}$$

$$\underline{D}_A=\mathrm{diag}(d_v,d_D,d_v^*) , \tag{3.6}$$

and

$$\begin{aligned} d_v&=2g\alpha_0 v_0 , \\ d_D&=2\gamma_\parallel(D_0+\mathcal{N})-4g(v_0^*\alpha_0+v_0\alpha_0^*) . \end{aligned} \tag{3.7}$$

For a coherent input field the matrix $\underline{D}_F$ would be zero, $N=M=0$.

In Fourier space the linear equation (3.1) becomes algebraic:

$$(\underline{A}-i\omega\underline{1})\delta\underline{\alpha}(\omega)=\underline{\Gamma}(\omega) , \tag{3.8}$$

where the Fourier transform is defined

$$f(\omega)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt . \tag{3.9}$$

The atomic variables may be exactly eliminated from the set of 5×5 linear equations in (3.8) to leave a set of 2×2 equations for the field variables alone,[14] i.e.,

$$[\underline{A}_f(\varpi)-i\omega\underline{1}]\delta\underline{\alpha}_f(\omega)=\underline{\Gamma}_f(\omega) , \tag{3.10}$$

where $\delta\underline{\alpha}_f(\omega)=(\delta\alpha(\omega),\delta\alpha^+(\omega))^T$, and the field drift matrix is

$$\underline{A}_f(\varpi)\equiv\kappa\begin{bmatrix} a(\varpi) & b(\varpi) \\ b^*(-\varpi) & a^*(-\varpi) \end{bmatrix} \tag{3.11}$$

with $\varpi\equiv\omega/\gamma_\perp$. The noise correlations define the field diffusion matrix $\underline{D}_f(\varpi)$

$$\langle \underline{\Gamma}_f(\omega)\underline{\Gamma}_f(\omega')\rangle=\underline{D}_f(\varpi)\delta(\omega+\omega') , \tag{3.12}$$

where

$$\underline{D}_f(\varpi)\equiv\kappa\begin{bmatrix} \bar{d}(\varpi) & \bar{\Lambda}(\varpi) \\ \bar{\Lambda}(-\varpi) & \bar{d}^*(-\varpi) \end{bmatrix} \tag{3.13}$$

with

$$\begin{aligned} \bar{d}(\varpi)&=\bar{d}(-\varpi)=d(\varpi)+2\frac{\kappa_A}{\kappa}M , \\ \bar{\Lambda}(\varpi)&=\bar{\Lambda}(-\varpi)=\Lambda(\varpi)+2\frac{\kappa_A}{\kappa}N . \end{aligned} \tag{3.14}$$

The drift matrix elements are[14]

$$\begin{aligned} a(\varpi)&=1+i\phi+\gamma(\varpi) , \\ b(\varpi)&=-\frac{CI}{\Pi(0)\Pi(\varpi)}\frac{1}{[1+i\Delta(\varpi)]}\times\left[\frac{1}{1+i\Delta}+\frac{1}{1-i\Delta(-\varpi)}\right] \end{aligned} \tag{3.15}$$

with

$$\gamma(\varpi)=\frac{2C}{\Pi(0)[1+i\Delta(\varpi)]}\left[1-\frac{I}{2(1-i\Delta)\Pi(\varpi)}-\frac{I}{2[1+i\Delta(\varpi)]\Pi(\varpi)}\right],$$

$$\Pi(\varpi)=1-\frac{i\varpi}{2}+\frac{I}{2}\left[\frac{1}{1+i\Delta(\varpi)}+\frac{1}{1-i\Delta(-\varpi)}\right], \tag{3.16}$$

$$\Delta(\varpi)\equiv\Delta-\varpi .$$

The real and imaginary parts of $a(\varpi)$ represent atomic and cavity absorption or loss, and dispersion, respectively. The diffusion matrix elements are given by

$$\begin{aligned} d(\varpi)=&-\frac{2CI}{\Pi(0)(1+i\Delta)}\frac{1}{[1+i\Delta(\varpi)][1+i\Delta(-\varpi)]}\left[1-\frac{I}{2\Pi(\varpi)[1+i\Delta(\varpi)]}\right]\left[1-\frac{I}{2\Pi^*(\varpi)[1+i\Delta(-\varpi)]}\right] \\ &+\frac{2CI^2}{\Pi(0)(1+\Delta^2)}\frac{1}{[1+i\Delta(\varpi)][1+i\Delta(-\varpi)]|\Pi(\varpi)|^2}-\frac{CI^3}{2\Pi(0)(1-i\Delta)}\frac{1}{|[1+i\Delta(\varpi)][1-i\Delta(-\varpi)]\Pi(\varpi)|^2} \end{aligned} \tag{3.17}$$

and

$$\Lambda(\varpi)=\frac{2CI^2}{\Pi(0)(1+\Delta^2)}\frac{1}{|[1+i\Delta(\varpi)]\Pi(\varpi)|^2}\left\{1+\mathrm{Re}\left[\frac{\Pi(\varpi)(1-i\Delta)}{[1+i\Delta(-\varpi)]}\left[1-\frac{I}{2\Pi(\varpi)[1+i\Delta(\varpi)]}\right]\right]\right\}. \tag{3.18}$$

A thorough discussion of the physical interpretation of $d(\varpi)$ and $\Lambda(\varpi)$ was given by Reid.[14] To summarize, $d(\varpi)$ is a phase-sensitive atomic noise source, and is responsible for squeezing, while $\Lambda(\varpi)$ is the phase-insensitive source term which tends to destroy phase sensitive features in the spectral region around the pump frequency ($\varpi=0$ in the rotating frame). These terms represent the atomic noise induced by the mean intracavity intensity I. With a squeezed input the diffusion matrix elements $\bar{d}(\varpi)$ and $\bar{\Lambda}(\varpi)$ contain contributions due to the squeezed light which is coupled into the cavity through port A, the coupling efficiency being determined by the ratio κ_A/κ of Eq. (3.14).

IV. CAVITY REFLECTION AND TRANSMISSION SPECTRA

A. Input and output quadratures

Define the input quadrature operator for port A, in the interaction picture as

$$X_\theta^{\mathrm{in}}(t)=e^{-i(\zeta_{\mathrm{in}}+\theta)}a_{\mathrm{in}}(t)+e^{i(\zeta_{\mathrm{in}}+\theta)}a_{\mathrm{in}}^\dagger(t) . \tag{4.1}$$

The output quadrature operator for port A (reflected field) is similarly defined

$$X_\theta^{\mathrm{out}}(t)=e^{-i(\zeta_{\mathrm{out}}+\theta)}a_{\mathrm{out}}(t)+e^{i(\zeta_{\mathrm{out}}+\theta)}a_{\mathrm{out}}^\dagger(t) . \tag{4.2}$$

The phases ζ_{in} and ζ_{out} are given by Eqs. (2.19) and (2.22), respectively. The output quadrature operator for port B (transmitted field) is defined by an expression similar to (4.2) with $\zeta_{\mathrm{out}}=0$. The advantage of these definitions is that the amplitude and phase quadratures of the input and output fields corresponds to $\theta=0$ and $\pi/2$, respectively, for any set of atomic and cavity parameters.

B. Optical and squeezing spectra

The squeezing spectrum of quadrature fluctuations is defined

$$V(\omega,\theta)\equiv\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle X_\theta(t+\tau),X_\theta(t)\rangle , \tag{4.3}$$

where X_θ is the appropriate input or output quadrature of interest. The time t is a time at which steady state has been reached. Using the commutation relations (2.2) enables (4.3) to be written

$$V(\omega,\theta)=1+\phi(\omega)+\phi(-\omega)+\chi(\omega;\theta) , \tag{4.4}$$

where the constant 1 is the vacuum or short noise level, $\phi(\omega)$ is the *optical spectrum* which is independent of the quadrature phase angle θ, and $\chi(\omega;\theta)$ is a phase-sensitive contribution. These are defined for the transmitted output fields by

$$\phi(\omega)\equiv\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle a_{\mathrm{out}}^\dagger(t),a_{\mathrm{out}}(t+\tau)\rangle ,$$
$$\chi(\omega;\theta)\equiv 2\mathrm{Re}\left[e^{-i2(\theta+\zeta_{\mathrm{out}})}\times\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle a_{\mathrm{out}}(t+\tau),a_{\mathrm{out}}(t)\rangle\right] , \tag{4.5}$$

where ζ_{out} is given by (2.22) if we are considering the reflected field, and $\zeta_{\mathrm{out}}=0$ for the transmitted field.

Similar definitions apply to the input fields. Consider the squeezed input to port A (Ref. 13):

$$\langle a_{\mathrm{in}}^\dagger(\omega),a_{\mathrm{in}}(\omega')\rangle=N\delta(\omega+\omega') ,$$
$$\langle a_{\mathrm{in}}(\omega),a_{\mathrm{in}}(\omega')\rangle=M\delta(\omega+\omega') \tag{4.6}$$

with $M=|M|e^{i2\psi}$ imply that

$$\phi_{\mathrm{in}}(\omega)=N , \tag{4.7}$$

$$V_{\mathrm{in}}(\omega,\theta)=1+2\{N+|M|\cos[2(\theta+\zeta_{\mathrm{in}}-\psi)]\} . \tag{4.8}$$

The input is said to be *amplitude squeezed* when

$$\psi=\zeta_{\mathrm{in}}+(n+\tfrac{1}{2})\pi,\quad n \text{ integer} \tag{4.9}$$

and *phase squeezed* when

$$\psi=\zeta_{\mathrm{in}}+n\pi,\quad n \text{ integer} . \tag{4.10}$$

The input to port B is just vacuum, $V_{\mathrm{in}}=1$.

If the optical spectrum is an even function $\phi(\omega)=\phi(-\omega)$, then (4.4) implies

$$\phi(\omega)+\tfrac{1}{2}=\frac{1}{4}\left[V(\omega,\theta)+V\left[\omega,\theta+\frac{\pi}{2}\right]\right] , \tag{4.11}$$

a result whose significance has been pointed out by Rice and Carmichael.[18] For example, if there exists a quadrature θ_0 such that $V(\omega,\theta_0)\sim 1$, the optical fluctuation spectrum is due only to fluctuations of the conjugate quadrature $\theta_0+\pi/2$. This sort of possibility arises if the optical spectrum is symmetric, and in general this is not so.

The reflected field is a superposition of light reflected by mirror A, and light which exits the cavity through mirror A. Both contributions are influenced by the squeezed input since some is reflected off A without entering the cavity and the residual squeezes the intracavity field and therefore modifies the atomic response. A detector of R will register interference as a result of the superposition. The transmitted field T, contains no such interference, as it consists of a superposition of the field exiting port B (including squeezed light that has passed through the cavity) and vacuum field reflected from port B. The interference on the reflected signal can thus be isolated by comparing the reflected and transmitted field spectra, suitably scaled by the exit port bandwidths. Explicitly the reflected $\phi_R(\omega)$ and transmitted $\phi_T(\omega)$ spectra are related by

$$\phi_R(\omega)=\frac{\kappa_B}{\kappa_A}\phi_T(\omega)+N-\frac{4\kappa_A}{\kappa}\mathrm{Re}\left[\frac{N[a(-\varpi)+iq\varpi]-Mb^*(\varpi)}{P^*(-i\omega)}\right], \tag{4.12}$$

where $q\equiv\gamma_\perp/\kappa$ and

$$P(-i\omega)\equiv[a(\varpi)-iq\varpi][a^*(-\varpi)-iq\varpi]-b(\varpi)b^*(-\varpi)\;. \tag{4.13}$$

The details of the derivation are given in Sec. V. The interference contribution is seen to vanish when $N=M=0$. The interference feature is not itself a nonclassical effect, as is easily observed by setting $M=0$, indicating that a classical thermal noise input produces interference; at optical frequencies however, this would be completely negligible. Moreover the phase dependence of the optical spectrum as ψ is varied can be produced by a phase-sensitive input with noise in excess of the vacuum $N>|M|$. A phase-sensitive nonclassical feature, suppression of the vacuum noise spectra, will be discussed in Sec. VI.

V. COMPUTATION OF SPECTRA

We first compute the reflected optical spectrum in detail. By definition

$$\phi_R(-\omega)\equiv\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle a_{\rm out}^\dagger(t+\tau),a_{\rm out}(t)\rangle\;. \tag{5.1}$$

For squeezed inputs, the correlation function in (5.1) may be expressed in terms of input and internal field correlations as[19]

$$\langle a_{\rm out}^\dagger(t+\tau),a_{\rm out}(t)\rangle=\langle a_{\rm in}^\dagger(t+\tau),a_{\rm in}(t)\rangle+2\kappa_A(\langle a^\dagger(t+\tau),a(t)\rangle+N\langle[a^\dagger(t+\tau),a(t)]\rangle-\{M\Theta(\tau)\langle[a^\dagger(t+\tau),a^\dagger(t)]\rangle+M^*\Theta(-\tau)\langle[a(t+\tau),a(t)]\rangle\})\;, \tag{5.2}$$

where $\Theta(\tau)$ is Heaviside's step function. For a coherent input the input correlation and the terms proportional to N and M vanish leaving the output correlation function

$$\langle a_{\rm out}^\dagger(t+\tau),a_{\rm out}(t)\rangle=2\kappa_A\langle a^\dagger(t+\tau),a(t)\rangle\;. \tag{5.3}$$

The optical spectrum may then be evaluated alone in terms of normal and time-ordered correlations of the internal field. These are precisely the orderings which are evaluated in a normally ordered phase-space representation such as the Glauber-Sudarshan P or positive-P distributions. In a linearized fluctuations analysis the spectrum for coherent input could then be written

$$\phi_R(-\omega)=2\kappa_A\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t+\tau)\delta\alpha(t)\rangle\;. \tag{5.4}$$

The presence of a squeezed input means we must consider the extra contributions in (5.2), which lead to non-normal and non-time-ordered correlations of the internal field. These correlations may however be calculated straightforwardly with a normal and time-ordered representation in a linearized analysis, with the aid of the regression theorem,[20] as we now show. Substituting (5.2) into (5.1) gives

$$\phi_R(-\omega)=\phi_{\rm in}(-\omega)+2\kappa_A\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t+\tau)\delta\alpha(t)\rangle+2\kappa_A\left[\int_0^\infty d\tau\, e^{i\omega\tau}\{N\langle[a^\dagger(t+\tau),a(t)]\rangle-M\langle[a^\dagger(t+\tau),a^\dagger(t)]\rangle\}+\int_0^\infty d\tau\, e^{-i\omega\tau}\{N\langle[a^\dagger(t),a(t+\tau)]\rangle-M^*\langle[a(t),a(t+\tau)]\rangle\}\right], \tag{5.5}$$

where $\phi_{\rm in}(-\omega)=\phi_{\rm in}(\omega)=N$. The correlation functions $\langle a(t),a^\dagger(t+\tau)\rangle$ and $\langle a(t+\tau),a^\dagger(t)\rangle$ are non-normal-ordered, and $\langle a^\dagger(t+\tau),a^\dagger(t)\rangle$, $\langle a(t),a(t+\tau)\rangle$ $(\tau>0)$ are non-time-ordered. These are computed in Appendix A. Using the results derived there, one finds (5.5) may be written

$$\phi_R(-\omega)=\phi_{\rm in}(-\omega)+2\kappa_A\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t+\tau)\delta\alpha(t)\rangle-2\kappa_A\left[\int_0^\infty d\tau\, e^{i\omega\tau}[N(e^{-\underline{A}\tau})_{22}+M(e^{-\underline{A}\tau})_{21}]+\int_0^\infty d\tau\, e^{-i\omega\tau}[N(e^{-\underline{A}\tau})_{11}+M^*(e^{-\underline{A}\tau})_{12}]\right], \tag{5.6}$$

where $\underline{A}$ is the 5×5 drift matrix of Eq. (3.2).

Using the result (see Appendix B)

$$\int_0^\infty d\tau\, e^{\mp i\omega\tau}(e^{-\underline{A}\tau})_{ij}=(\underline{A}\pm i\omega\underline{1})_{ij}^{-1}=[\underline{A}_f(\varpi)\pm i\omega\underline{1}]_{ij}^{-1}\quad(i,j=1,2) \tag{5.7}$$

we have

$$\phi_R(\omega)=\phi_{\rm in}(\omega)+2\kappa_A\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t)\delta\alpha(t+\tau)\rangle-2\kappa_A\{N[(\underline{A}+i\omega\underline{1})_{22}^{-1}+(\underline{A}-i\omega\underline{1})_{11}^{-1}]+M(\underline{A}+i\omega\underline{1})_{21}^{-1}+M^*(\underline{A}-i\omega\underline{1})_{12}^{-1}\}\;, \tag{5.8}$$

which gives

$$\phi_R(\omega)=2\kappa_A\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t)\delta\alpha(t+\tau)\rangle+N-\frac{4\kappa_A}{\kappa}\mathrm{Re}\left[\frac{N[a(-\varpi)+iq\varpi]-Mb^*(\varpi)}{P^*(-i\omega)}\right]. \quad (5.9)$$

The evaluation of the usual integral term in (5.9) is standard spectral matrix theory. The spectral matrix is defined as

$$\underline{S}(\omega)\equiv\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\underline{\alpha}_f(t)\delta\underline{\alpha}_f^T(t+\tau)\rangle=[\underline{A}_f(\varpi)-i\omega\underline{1}]^{-1}\underline{D}_f(\varpi)[\underline{A}_f^T(-\varpi)+i\omega\underline{1}]^{-1}. \quad (5.10)$$

Thus

$$S_{12}(\omega)\equiv\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\delta\alpha^+(t)\delta\alpha(t+\tau)\rangle=\frac{1}{\kappa|P(-i\omega)|^2}(\,[|a(-\varpi)+iq\varpi|^2+|b(\varpi)|^2]\bar{\Lambda}(\varpi)-2\,\mathrm{Re}\{b(\varpi)[a(-\varpi)+iq\varpi]\bar{d}^*(\varpi)\}). \quad (5.11)$$

This completes the computation of the optical spectrum of reflected light. The transmitted optical spectrum $\phi_T(\omega)$ is simply given by

$$\phi_T(\omega)=2\kappa_B S_{12}(\omega). \quad (5.12)$$

Calculation of the reflected field squeezing spectrum requires the computation of $\chi(\omega;\theta)$ of Eq. (4.4). To evaluate this we make use of the result[19]

$$\langle a_{\text{out}}(t+\tau),a_{\text{out}}(t)\rangle=\langle a_{\text{in}}(t+\tau),a_{\text{in}}(t)\rangle+2\kappa_A[\,(N+1)\langle T(a(t+\tau),a(t))\rangle-N\langle\tilde{T}(a(t+\tau),a(t))\rangle+M\{\Theta(-\tau)\langle[a^\dagger(t+\tau),a(t)]\rangle+\Theta(\tau)\langle[a^\dagger(t),a(t+\tau)]\rangle\}], \quad (5.13)$$

where $T\,(\tilde{T})$ is the time (antitime) ordering operator, i.e., earlier times to the right (left). By an analysis similar to that used for the optical spectrum one finds that

$$\chi_R(\omega;\theta)=e^{-i2(\theta+\zeta_{\text{out}})}\{\,2\kappa_A S_{11}(\omega)+M-2\kappa_A N[(\underline{A}+i\omega\underline{1})_{12}^{-1}+(\underline{A}-i\omega\underline{1})_{12}^{-1}]-2\kappa_A M[(\underline{A}+i\omega\underline{1})_{11}^{-1}+(\underline{A}-i\omega\underline{1})_{11}^{-1}]\}+\text{c.c.}, \quad (5.14)$$

which yields

$$\chi_R(\omega;\theta)=e^{-i2(\theta+\zeta_{\text{out}})}\left[2\kappa_A S_{11}(\omega)+M+\frac{2\kappa_A}{\kappa}\left[\frac{Nb(\varpi)-M[a^*(-\varpi)-iq\varpi]}{P(-i\omega)}+\frac{Nb(-\varpi)-M[a^*(\varpi)+iq\varpi]}{P^*(-i\omega)}\right]\right]+\text{c.c.}, \quad (5.15)$$

where

$$S_{11}(\omega)=\frac{1}{\kappa|P(-i\omega)|^2}(\,[a^*(-\varpi)-iq\varpi][a^*(\varpi)+iq\varpi]\bar{d}(\varpi)+b(\varpi)b(-\varpi)\bar{d}^*(\varpi)-\{[a^*(\varpi)+iq\varpi]b(\varpi)+[a^*(-\varpi)-iq\varpi]b(-\varpi)\}\bar{\Lambda}(\varpi)). \quad (5.16)$$

The transmitted phase function is given by

$$\chi_T(\omega;\theta)=2\kappa_B S_{11}(\omega)e^{-i2\theta}+\text{c.c.} \quad (5.17)$$

Equations (5.9), (5.12), (5.15), and (5.17) give the reflected and transmitted spectra of squeezing for arbitrary output quadratures, for any quadrature of broadband squeezed input to port A, within a linearized approximation.

VI. RESULTS

In this section we illustrate some of the squeezed input induced modifications to the optical spectra seen in transmission and reflection. We denote $\Delta\phi_T(\varpi)$ the contribution to the transmitted optical spectrum due to the squeezed input. For a coherent input this is identically zero. This may be written

$$\Delta\phi_T(\varpi)=\frac{(1-\kappa_m^2)}{|P(-i\omega)|^2}\{\,N[\,|a(-\varpi)+iq\varpi|-|b(\varpi)|\,]^2-2[\,|M|\eta-N]|[\,a(-\varpi)+iq\varpi]b(\varpi)|\}, \quad (6.1)$$

where η is a phase factor which varies between $[-1,1]$ as the phase ψ is varied. It is therefore possible to achieve $\Delta\phi_T(\omega)<0$, an effect which depends on the quantum-mechanical nature of the input, for at least some regions of the optical spectrum with a suitably phased, squeezed input and provided that the degree of squeezing is not too large. The latter is a somewhat surprising condition and derives from the fact that the intensity of squeezed input N should not be too large otherwise the positive definite term in (6.1) will dominate; the minimum quadrature noise of input for a given N is $V_{\min}=1-2[\sqrt{N(N+1)}-N]$.

In Figs. 1–3 we illustrate the reflected and transmitted spectra in the strong-coupling regime, where for a coherent input and intracavity intensity well below atomic saturation $I \ll 1$, the spectra consist of a doublet positioned at the collective Rabi frequencies $\omega=\pm g\sqrt{\mathcal{N}}$, in the rotating frame.[5–9] The doublet is due to single quantum splitting of the first pair of excited levels of the interacting atom-field system, and is also sometimes re-

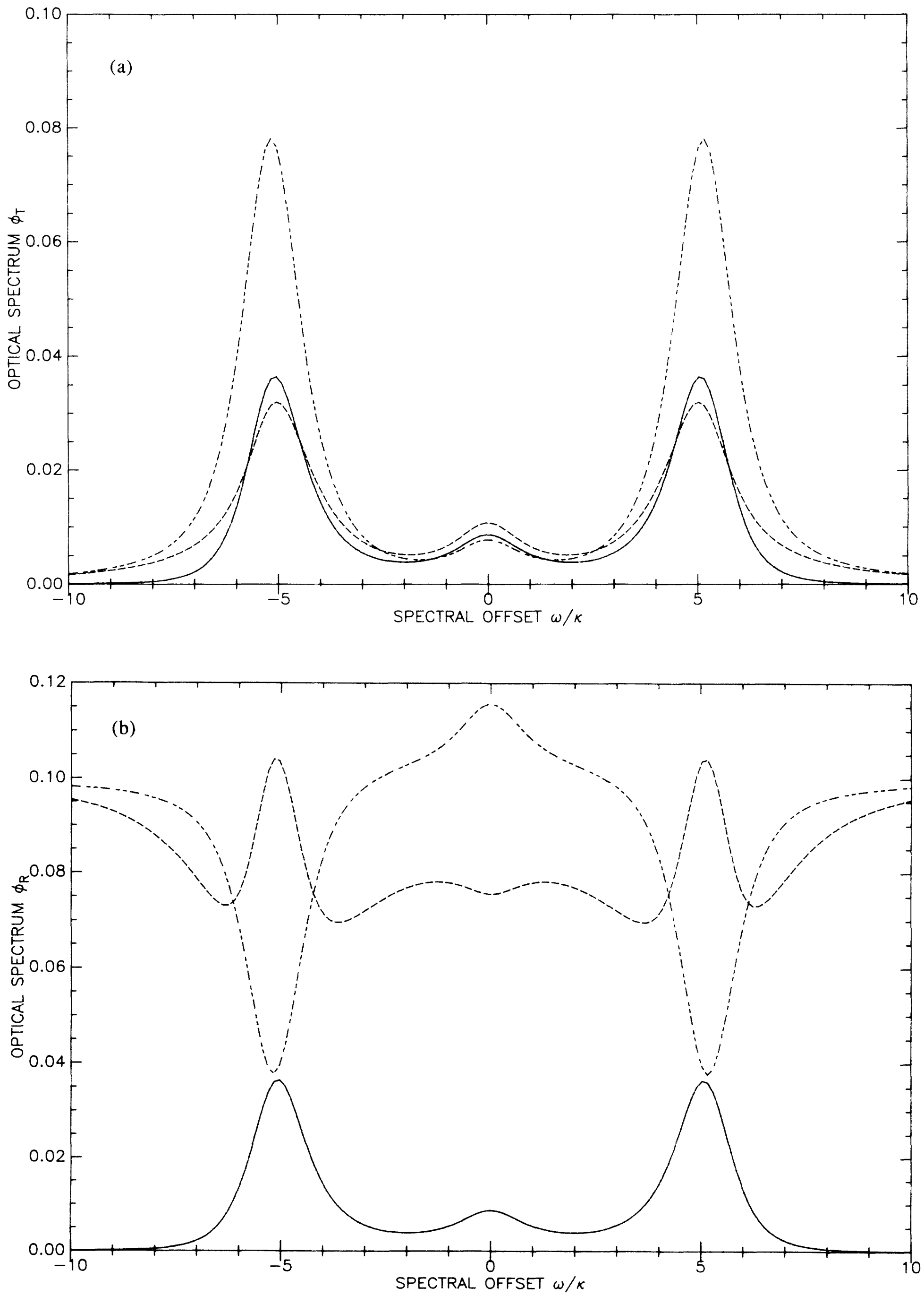

FIG. 1. Shows the (a) transmitted and (b) reflected spectra, for the strong-coupling regime with $I=0.5$, $C=20$, $\Delta=\phi=0$, $\kappa_m=0$, and $q=1$. The solid line shows the spectra with a coherent input, the dashed line shows that for a phase squeezed input, $N=0.1$, $|M|=\sqrt{0.11}$, and the chain-dashed line shows that for an amplitude squeezed input with the same values of N and $|M|$.

ferred to as vacuum Rabi splitting. The input is chosen to be a moderate 46% squeezed ($V_{\min}=0.54$, with $N=0.1$ and $|M|=\sqrt{0.11}$). For the transmitted spectra a phase squeezed input suppresses the vacuum Rabi peaks somewhat, while the amplitude squeezed input with its enhanced phase fluctuations enhances the doublet. Conversely, the reflected spectrum has fluctuations below the level of the input N, for near-resonant normal-mode frequencies with amplitude squeezed input.

Increasing the intracavity intensity to 75% of saturation ($I=0.75$), the central component of the spectrum grows, since real excitation of higher dressed states of the atom-field system leads to spontaneous transitions at this frequency. The central component is also modified by the squeezed input, but in an opposite sense to the doublet, and this may be observed in transmission and reflection.

Figure 3 illustrates the transmitted spectrum for the parameters of Fig. 2, as the phase of squeezing is continu-

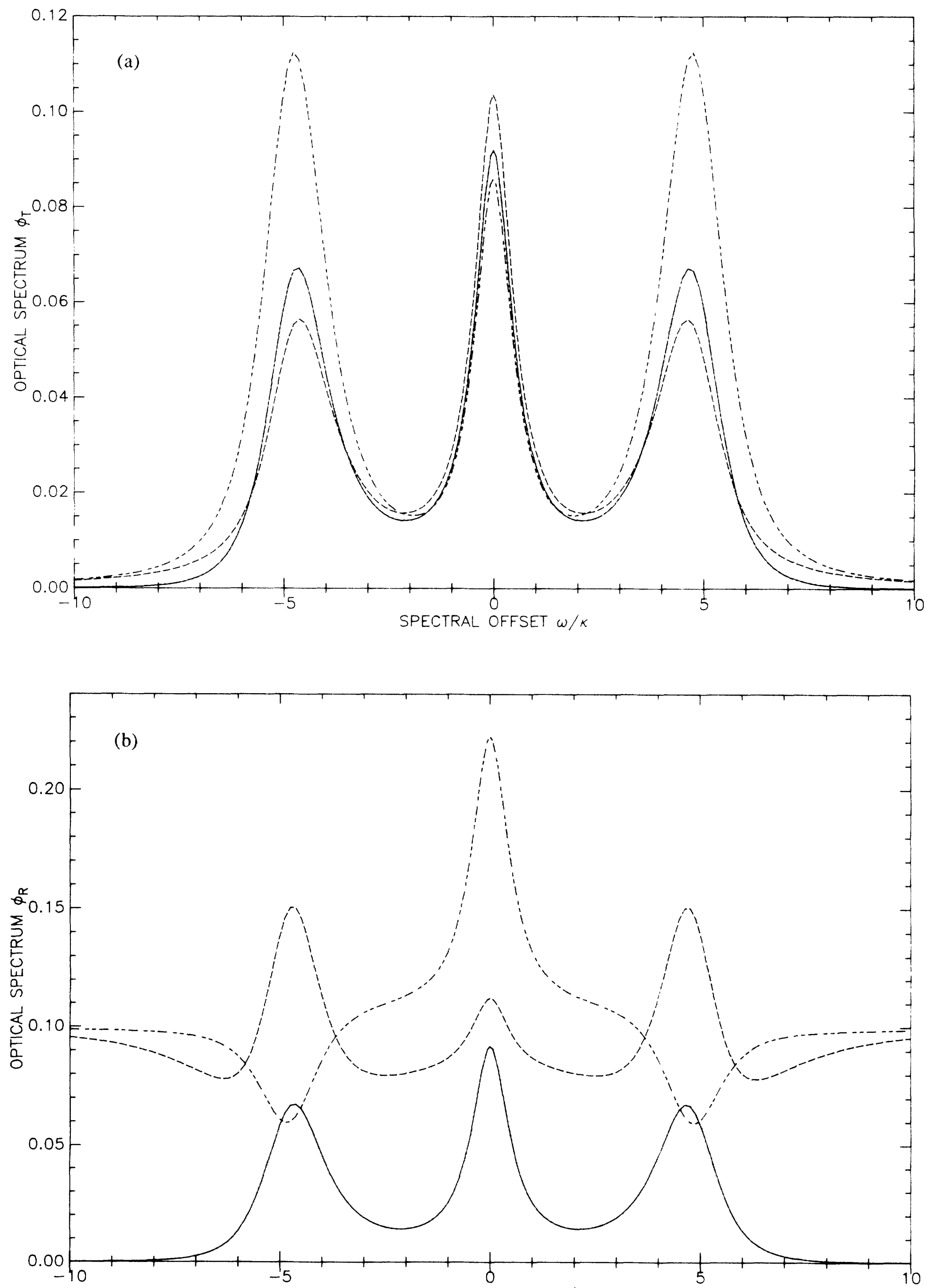

FIG. 2. Same as Fig. 1 except that $I=0.75$.

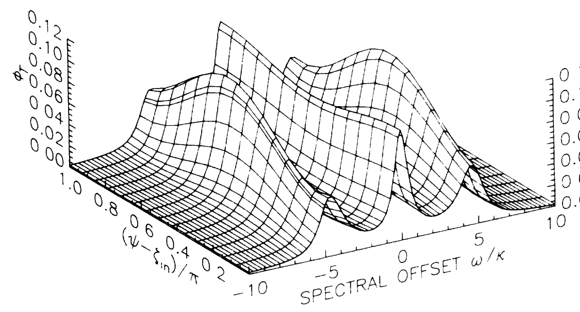

FIG. 3. This shows the transmitted optical spectrum of Fig. 2(a), without the coherent reference, as the phase of squeezing input is continuously varied from phase to amplitude and back to phase squeezing.

ously varied from phase squeezed input to amplitude squeezed input and back to phase squeezed input. Although not clearly apparent from the figure, the spectrum is only symmetric for the phase and amplitude squeezed inputs $\psi-\zeta_{\rm in}=0(\bmod \pi)$ and 0.5π, respectively. Even this symmetry is a consequence of assuming atomic resonance $\Delta=0$.

The generality of our theory allows us to investigate many different parameter regimes. So far we have considered the strong-coupling regime when the timescale for atomic decay via spontaneous emission is comparable with that for loss via the cavity mirrors, and both are longer than the time in which the collective atomic polarization may exchange quanta with the resonant cavity mode; hence the normal mode splitting. Other interesting features may be expected in the so-called bad-cavity limit where the cavity decay time is much shorter than the atomic decay time, i.e., $q \ll 1$. Intuitively one expects that since cavity loss is the dominant source of dissipation, the squeezed input may most strongly influence the system dynamics, as it is precisely this decay route which it modifies. Collective effects in the transmission spectra, for coherent inputs, have already been considered in detail in this regime.[14,21] Moreover it has been pointed out that in the case of a single atom in a cavity the bad-cavity limit is isomorphic with single-atom resonance fluorescence in a vacuum except that $\gamma_{\parallel}$, the longitudinal atomic decay rate, is replaced by $\gamma(1+2C)$, where the first and second factors represent loss other than via the cavity mode and via the cavity mode, respectively, and C is the single-atom cooperativity.[22] For large enough intensities I, when atomic saturation becomes dominant, the spectrum becomes a triplet with $P(-i\omega)$ proportional to the Torrey polynomial which determines the Mollow triplet positions in free-space (single-atom) resonance fluorescence.[23] This can be seen by taking the limit $q\to 0$, $C/I \to 0$ with $I \gg 1$, in Eq. (34) of Reid.[14] Hence in this limit the sidebands are located at the Rabi frequency $\bar{\omega}=\pm(\Delta^2+2I)^{1/2}$.

In Fig. 4(a) we show that in the bad-cavity limit it is possible to significantly suppress one of the sidebands in transmission by use of a suitably squeezed input when atomic dispersion is included. The spectrum for a coherent input is shown for comparison. Note that the intensity chosen is quite moderate, and in particular the sidebands are separated by less than the Rabi frequency given above. For phase squeezed input the left peak is significantly suppressed in transmission, while the right peak is greatly enhanced. The spectrum is nearly symmetric for the amplitude squeezed input, with the sidebands much larger than the central peak. The reflected spectra shown in Fig. 4(b) are rather complex, but we note that for the amplitude squeezed input the spectrum is very asymmetric, in contrast to transmission. Figure 5 shows the transmitted spectrum as a continuous function of the phase of squeezing, for the same parameters, and may be compared with Fig. 4(a). In Fig. 6 the effect of increasing the cavity mismatch on the transmitted spectrum is considered. This effectively increases the relative loss through port A which is squeezed, and the suppression of the left peak appears more pronounced. Note that in Figs. 4–6 the degree of squeezing is less than 83% ($V_{\rm min}=0.17$), and no attempt has been made to optimize the effect by fine tuning the parameters.

Recently Courty and Reynaud[11] reported the suppression of one of the Mollow sidebands for single atom resonance fluorescence, from the sides of a cavity driven by a squeezed input, in what amounts to the bad-cavity limit. This they interpreted as being due to dressed state population trapping, which effectively stops spontaneous emission from one of the dressed states and thus at one of the sideband frequencies, and required that the atom be excited off resonance. It appears likely that the phenomena we observe in the transmission spectra for the many-atom case are related, though we will not discuss this further here.

VII. SUMMARY

In this paper we have given a general theory of the interaction of a broadband squeezed light input with a large number of atoms in an optical cavity under general conditions of atomic and cavity loss, and dispersion. We have concentrated on the optical spectra which may be observed in transmission and reflection. The latter is quite novel in that interference between the squeezed light reflected from the input mirror and the cavity leakage field leads to distinctive structure, which changes qualitatively with both the intensity and phase of squeezing at fixed coherent driving amplitude. The structure of the transmitted spectra is more closely related to that which is found in the absence of squeezing. With a squeezed input of appropriate phase it is possible to somewhat suppress the vacuum Rabi peaks in the strong coupling regime, provided the intensity of squeezing, and hence the degree of squeezing, it not too large; this is a nonclassical effect. In the "bad-cavity" limit in which the cavity mode is the dominant decay rate, it is also possible to suppress one of the sidebands of the triplet found for saturated atoms excited off resonance. Recent studies of single-atom resonance fluorescence with squeezed inputs in the strong coupling and bad cavity limits, by Savage[11] and Courty and Reynaud,[10] respectively, have admitted similar qualitative features.

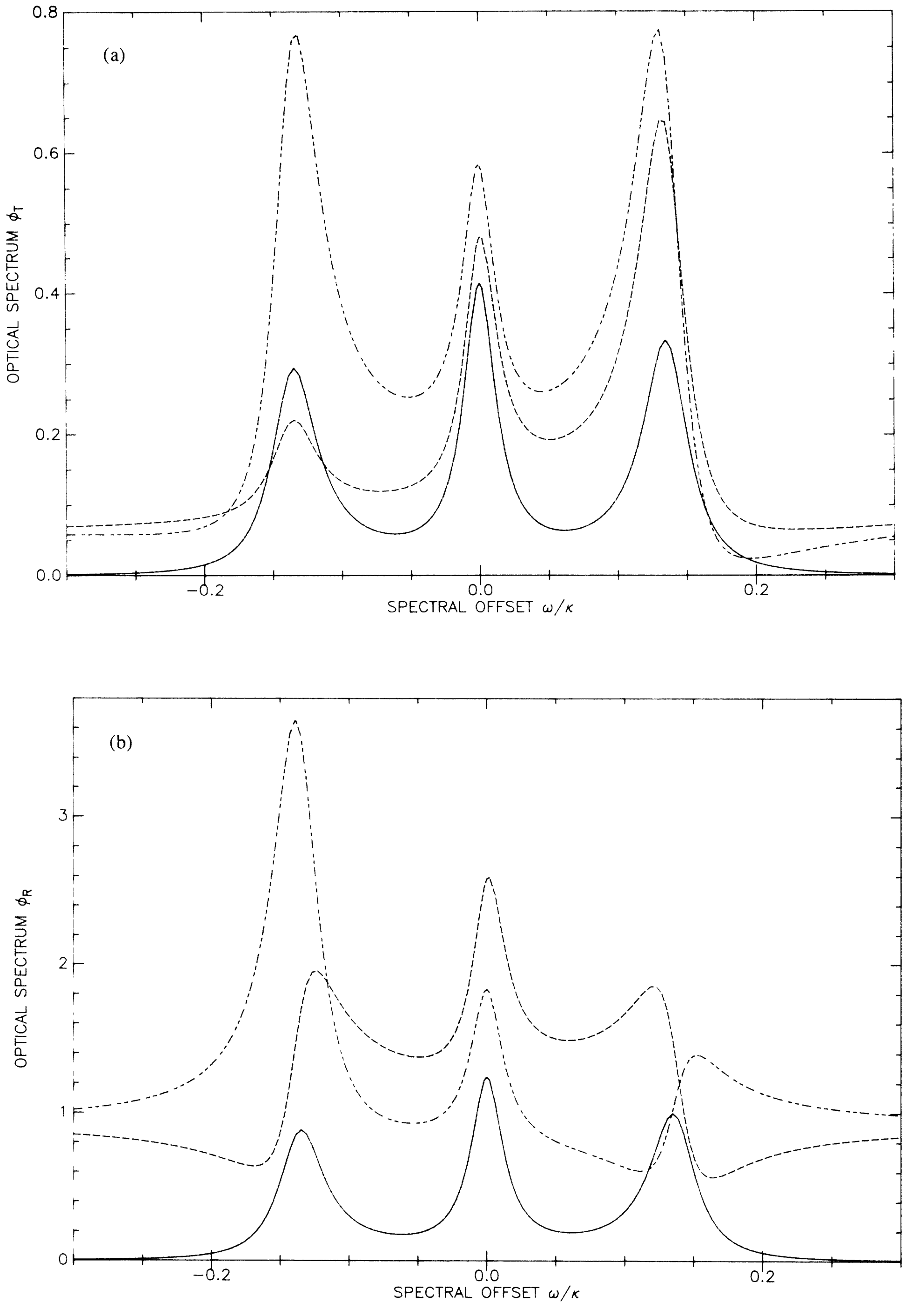

FIG. 4. Illustration of the "bad-cavity" limit (a) transmitted and (b) reflected spectra, for $I=144$, $C=20$, $\Delta=5$, $\phi=3$, $\kappa_m=0.5$, and $q=0.01$. The solid line indicates the spectra for a coherent input, the dashed line for a phase squeezed input with $N=1$ and $M=\sqrt{2}$, and the chain-dashed line for an amplitude squeezed input with the same N and $|M|$.

The results presented here may also serve as benchmarks for calculations in which a linearized analysis is not sufficient, particularly if the number of atoms or the saturation photon number is not large enough.

ACKNOWLEDGMENTS

This work was supported by the New Zealand Universities Grants Committee.

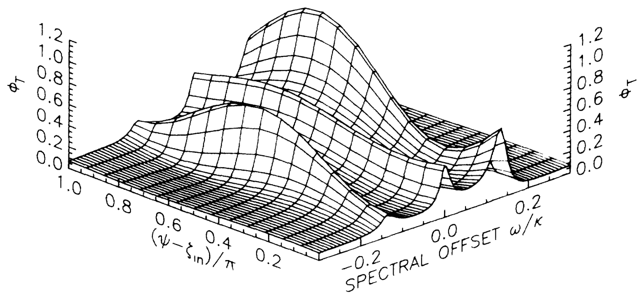

FIG. 5. The transmitted of Fig. 4(a), without the coherent reference, as the phase of squeezing input is continuously varied from phase to amplitude to phase squeezing.

APPENDIX A: NON-NORMAL AND NON-TIME-ORDERED CORRELATION FUNCTIONS

Due to the presence of a squeezed input, we require non-normal and non-time-ordered correlation functions of the intracavity field. To compute these we first introduce some formalism which defines the P distribution. We define the normally ordered characteristic function

$$\chi(\underline{\lambda},t) \equiv \mathrm{Tr}[\rho(t)O_A O_F] \equiv \int d\mu(\underline{\alpha}) P(\underline{\alpha},t) e^{i\underline{\lambda}\cdot\underline{\alpha}} , \qquad \text{(A1)}$$

where

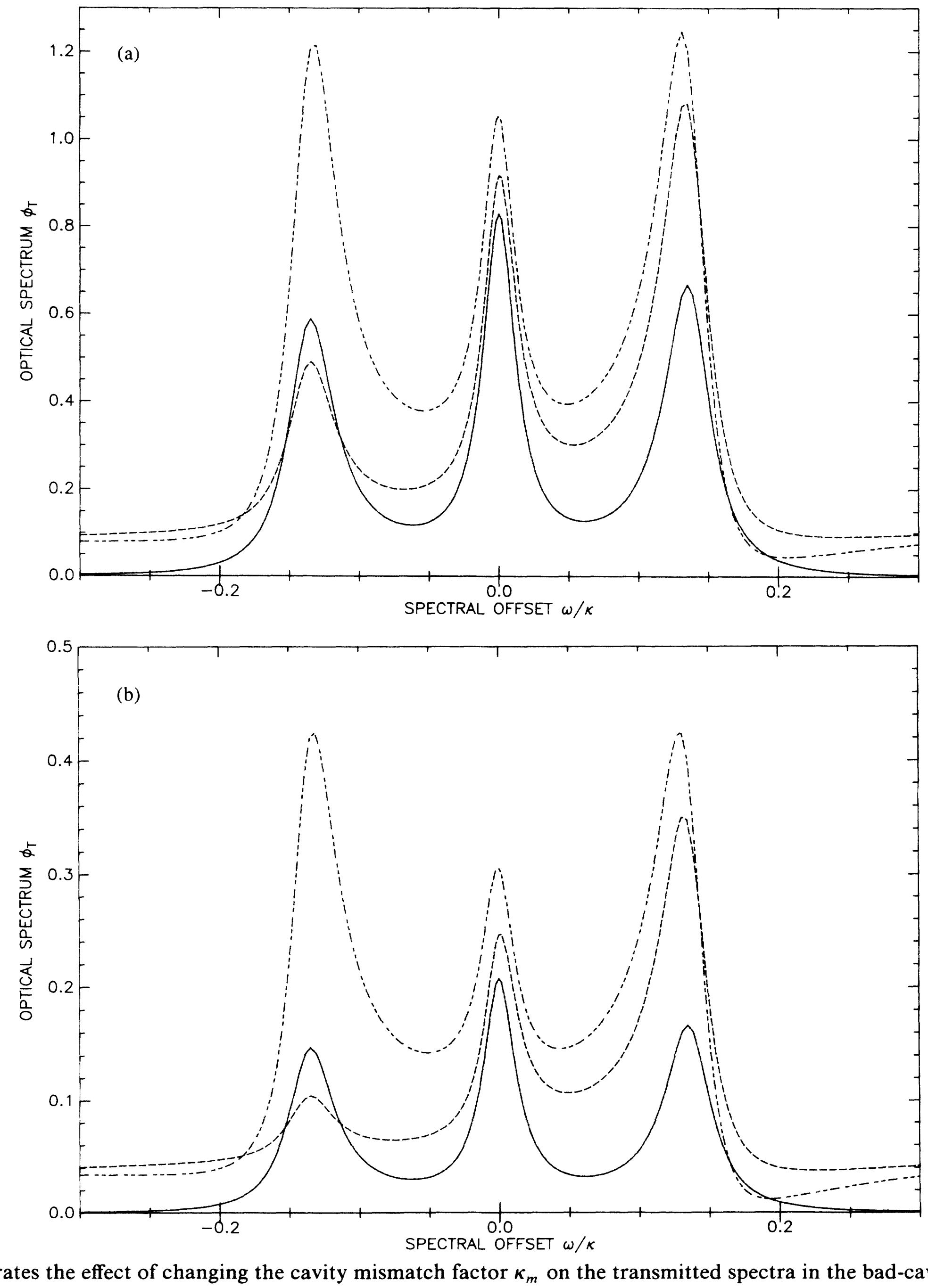

FIG. 6. Illustrates the effect of changing the cavity mismatch factor κ_m on the transmitted spectra in the bad-cavity case, using the parameters of Fig. 4, except (a) $\kappa_m = 0$, and (b) $\kappa_m = 0.75$. These may also be compared with the intermediate case shown in Fig. 4(a).

$$\underline{\lambda}=(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)^T ,$$
$$\underline{\alpha}=(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5)^T=(\alpha,\alpha^+,v,D,v^+)^T \quad \text{(A2)}$$

and

$$O_A=e^{i\lambda_5 J_+}e^{i\lambda_4 2J_z}e^{i\lambda_3 J_-} ,$$
$$O_F=e^{i\lambda_2 a^\dagger}e^{i\lambda_1 a} \quad \text{(A3)}$$

are atomic and field operators, respectively. The measure $d\mu(\underline{\alpha})$ defines the distribution $P(\underline{\alpha},t)$. Here we will consider the Glauber-Sudarshan-Haken P distribution for which

$$\chi(\underline{\lambda},t)=\int d\mu_H(\underline{\alpha})e^{i\underline{\alpha}\cdot\underline{\lambda}}P(\underline{\alpha},t) \quad \text{(A4)}$$

with $\underline{\lambda}=(\lambda^*,\lambda,\xi^*,\zeta,\xi)^T$, $\underline{\alpha}=(\alpha,\alpha^*,v,D,v^*)^T$, and $d\mu_H(\underline{\alpha})=d^2\alpha\, d^2v\, dD$. We write the master equation in the form

$$\dot\rho(t)=\mathcal{L}\rho(t) , \quad \text{(A5)}$$

which defines the Liouvillian $\mathcal{L}$.

Consider, for example, the non-normally ordered correlation function

$$\langle a(t)a^\dagger(t+\tau)\rangle=\mathrm{Tr}\{a^\dagger e^{\mathcal{L}\tau}[\rho(t)a]\} . \quad \text{(A6)}$$

From (A1) and (A4) we find

$$\mathrm{Tr}[\rho(t)aO_AO_F]=\int\left[\alpha-\frac{\partial}{\partial\alpha^*}\right]P(\underline{\alpha},t)e^{i\underline{\lambda}\cdot\underline{\alpha}}d\mu_H(\underline{\alpha}) , \quad \text{(A7)}$$

and define

$$G(\underline{\lambda})\equiv\mathrm{Tr}\{e^{\mathcal{L}\tau}[\rho(t)a]O_AO_F\} \quad \text{(A8)}$$

in terms of which

$$\langle a(t)a^\dagger(t+\tau)\rangle=\frac{\partial}{\partial i\lambda}G(\underline{\lambda})|_{\underline{\lambda}=\underline{0}} . \quad \text{(A9)}$$

Using (A1), (A7), and (A8) we find

$$G(\underline{\lambda})=\int d\mu_H(\underline{\alpha})d\mu_H(\underline{\alpha}')e^{i\underline{\lambda}\cdot\underline{\alpha}}P(\underline{\alpha},t+\tau|\underline{\alpha}',t)\times\left[\alpha'-\frac{\partial}{\partial\alpha'^*}\right]P(\underline{\alpha}',t) , \quad \text{(A10)}$$

hence

$$\langle a(t)a^\dagger(t+\tau)\rangle=\langle\alpha^*(t+\tau)\alpha(t)\rangle+\left\langle\frac{\partial}{\partial\alpha'^*}\langle\alpha^*(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle , \quad \text{(A11)}$$

where

$$\langle\alpha^*(t+\tau)\alpha(t)\rangle=\int d\mu_H(\underline{\alpha})d\mu_H(\underline{\alpha}')\alpha^*\alpha'\times P(\underline{\alpha},t+\tau|\underline{\alpha}',t)P(\underline{\alpha}',t) ,$$
$$\langle\alpha^*(t+\tau)|[\underline{\alpha}',t]\rangle=\int d\mu_H(\underline{\alpha})\alpha^*P(\underline{\alpha},t+\tau|\underline{\alpha}',t) . \quad \text{(A12)}$$

Using the normally ordered correspondence

$$\langle a^\dagger(t+\tau)a(t)\rangle=\langle\alpha^*(t+\tau)\alpha(t)\rangle ,$$

(A11) may be written

$$\langle[a(t),a^\dagger(t+\tau)]\rangle=\left\langle\frac{\partial}{\partial\alpha'^*}\langle\alpha^*(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle . \quad \text{(A13)}$$

Similarly one may derive the following results:

$$\langle[a^\dagger(t+\tau),a^\dagger(t)]\rangle=\left\langle\frac{\partial}{\partial\alpha'}\langle\alpha^*(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle ,$$
$$\langle[a(t),a(t+\tau)]\rangle=\left\langle\frac{\partial}{\partial\alpha'^*}\langle\alpha(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle , \quad \text{(A14)}$$
$$\langle[a(t+\tau),a^\dagger(t)]\rangle=\left\langle\frac{\partial}{\partial\alpha'}\langle\alpha(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle \quad (\tau\geq 0) .$$

The conditional averages in Eqs. (A13) and (A14) may be computed in a linearized approximation by means of the regression theorem. The regression theorem states that

$$\frac{d}{d\tau}\langle\underline{\alpha}(t+\tau)|[\underline{\alpha}',t]\rangle=-\underline{A}\langle\underline{\alpha}(t+\tau)|[\underline{\alpha}',t]\rangle \quad (\tau\geq 0) , \quad \text{(A15)}$$

where $\underline{A}$ is the linearized drift matrix (3.2). Hence

$$\left\langle\frac{\partial}{\partial\alpha'_j}\langle\alpha_i(t+\tau)|[\underline{\alpha}',t]\rangle\right\rangle=(e^{-\underline{A}\tau})_{ij} \quad (\tau\geq 0;i,j=1,\ldots,5) . \quad \text{(A16)}$$

This result is used in Eq. (5.6).

APPENDIX B: DRIFT MATRIX INVERSION

Comparison of Eqs. (3.8) and (3.10) implies that

$$\delta\alpha_\mu(\omega)=\sum_{j=1}^{5}(\underline{A}-i\omega\underline{1})^{-1}_{\mu j}\Gamma_j(\omega)=\sum_{\nu=1}^{2}[\underline{A}_f(\bar\omega)-i\omega\underline{1}]^{-1}_{\mu\nu}\Gamma_{f,\nu}(\omega) \quad (\mu=1,2) , \quad \text{(B1)}$$

where for clarity we label field variables with Greek letters, and use Latin letters for a general (atom or field) index.

1. We multiply (B1) by $\Gamma_\sigma(\omega')$ ($\sigma=1,2$), and take stochastic averages to give

$$\sum_{j=1}^{5}(\underline{A}-i\omega\underline{1})^{-1}_{\mu j}D_{j\sigma}\delta(\omega+\omega')=\sum_{\nu=1}^{2}[\underline{A}_f(\bar\omega)-i\omega\underline{1}]^{-1}_{\mu\nu}\langle\Gamma_{f,\nu}(\omega)\Gamma_\sigma(\omega')\rangle . \quad \text{(B2)}$$

2. We use the results

$$\langle\Gamma_{f,\nu}(\omega)\Gamma_\sigma(\omega')\rangle=D_{\nu\sigma}\delta(\omega+\omega') \quad (\nu,\sigma=1,2) , \quad \text{(B3)}$$
$$D_{j\sigma}\neq 0\Longrightarrow j=1,2 \quad \text{(B4)}$$

in (B2) to give

$$(\underline{A}-i\omega\underline{1})^{-1}_{\mu\nu}=[\underline{A}_f(\bar\omega)-i\omega\underline{1}]^{-1}_{\mu\nu} \quad (\mu,\nu=1,2) . \quad \text{(B5)}$$

As a result of (B5) only inversion of the 2×2 reduced drift matrix is necessary in Eq. (5.8).

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