Temporal correlations of sidebands of the fluorescent spectra from a three-level atom

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An analytical study of the sideband correlations in a strongly driven three-level atom in the cascade configuration is presented. The quantum nature of the sidebands is discussed in terms of intensity-intensity correlations and photon statistics.

I. INTRODUCTION

The study of temporal correlations between fluorescent photons has been a subject of much theoretical[1-4] and experimental[5-8] interest. Measurements involving the correlation of the intensity of the light or the degree of second-order coherence can distinguish between classical and quantum predictions.[1] A number of experiments relating to violation of classical inequalities have focused attention on single-beam resonance fluorescence measurements and on double-beam intensity correlations in two-photon cascade emission. Various methods[5] for observing violations of the Cauchy inequality rely on processes in which two photons of different frequency are emitted more or less simultaneously. Another important application[6] of the measurement of photon correlation is found in tests of Bell's inequalities in studies of polarization correlations for photons emitted in an atomic cascade. Kimble, Mezzacappa, and Milonni[4] investigate the photon statistics of the field arising from the atomic cascade with photodetectors placed in the far field of the atomic emission. Also reported[8] is the experimental observation of strong Zeeman beats in the temporal correlations between the two photons emitted in an excited atomic cascade. Further, the measurement of the correlation of photons emitted in an atomic cascade is an accepted technique for determining lifetimes of excited atomic levels.[9] Time correlation of photon detections at given frequency is of interest for radiation fields generated by multiphoton processes such as emission of photons in many-level atoms, resonance fluorescence, and Raman scattering under strong irradiation. The intensity correlations between fluorescent photons have also been analyzed in the literature revealing some quantum features of the fluorescent light such as antibunching, quantum jumps,[10] etc.

Experiments of Aspect *et al.*,[7] based on a mixed analysis[5] (i.e., time correlations between fluorescent photons previously selected through frequency filters), have shown that the photons emitted in the two sidebands of the Mollow's triplet[11] are strongly correlated, bunched, and emitted in a given time order. These results have motivated us to investigate the quantum characteristics of the sidebands of the fluorescent spectra of a three-level atom. It is well known that for a three-level atom irradiated by two strong laser fields the fluorescence spectrum consists of a central peak at each transition frequency and three pairs of sidebands for finite detunings.[12-15] At exact resonance, that is, when the two laser frequencies are exactly equal to the respective transition frequencies of the atomic system, there are only two pairs of sidebands near each transition frequency.[12,13] Under the condition that the detunings and/or the driving fields are large enough it may be interesting to study the properties of the sidebands and the correlations between them. We present here an analytical study of the sideband correlations in a strongly driven three-level atom in the cascade configuration. We also study the photon statistics of sideband emission.

We present in Sec. II the basic theory leading to the definition of the sideband correlation functions. The derivation and discussion of the analytic expressions for the correlation functions and the sideband response function are presented in Secs. III and IV, respectively, followed by the concluding remarks in Sec. V.

II. FORMULATION OF THE PROBLEM

A. Basic equations

We consider a three-level atom with unequally spaced levels ($E_1 > E_2 > E_3$) in the cascade configuration, interacting with two monochromatic applied fields of frequencies Ω_1 and Ω_2 which are near resonant with the atomic transition frequencies $\omega_1 = (E_1 - E_2)/\hbar$ and $\omega_2 = (E_2 - E_3)/\hbar$, respectively (see Fig. 1). The dynamics of such a system is described by the master equation[16] for the reduced atomic density operator $\rho(t)$:

$$\frac{d\rho(t)}{dt} = -i[H_0,\rho(t)] - \gamma_1[A_{11}\rho(t) + \rho(t)A_{11} - 2A_{21}\rho(t)A_{12}] - \gamma_2[A_{22}\rho(t) + \rho(t)A_{22} - 2A_{32}\rho(t)A_{23}] , \tag{2.1}$$

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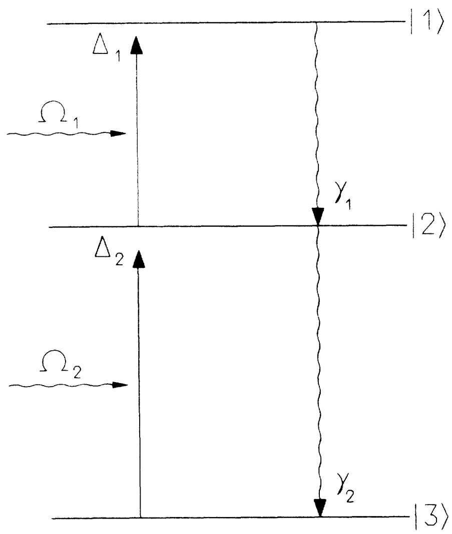

FIG. 1. Schematic diagram of a three-level atom in a cascade configuration interacting with two monochromatic fields.

where

$$H_0=\alpha_1(A_{12}+A_{21})+\alpha_2(A_{23}+A_{32})+\Delta A_{11}+\Delta_2 A_{22}\ , \tag{2.2a}$$

$$\Delta=\Delta_1+\Delta_2,\quad \alpha_j=d_jE_j/2 \quad (j=1,2)\ . \tag{2.2b}$$

Here $E_1,d_1,2\alpha_1,2\gamma_1,\Delta_1$ ($E_2,d_2,2\alpha_2,2\gamma_2,\Delta_2$) are, respectively, the amplitude of the field, induced dipole moment, Rabi frequency, Einstein A coefficient, and frequency detuning ($\Delta_i=\Omega_i-\omega_i$, $i=1,2$) corresponding to the upper (lower) transition. The operators $A_{mn}=|m\rangle\langle n|$, with $|m\rangle$ representing the mth state of the atom ($m,n=1,2,3$), satisfy the commutation relations

$$[A_{mn},A_{pq}]=A_{mq}\delta_{np}-A_{pn}\delta_{qm}\ , \tag{2.3a}$$

and the closure property

$$A_{11}+A_{22}+A_{33}=1\ . \tag{2.3b}$$

Analytical expressions for one-time expectation values of the atomic operators can be obtained under the condition that one or both the external fields are intense. We essentially follow the method of Ref. 17. A secular approximation is invoked to derive a Markovian equation for $\rho(t)$ in terms of the dressed operators $B_{ij}=|\psi_i\rangle\langle\psi_j|$ where $|\psi_i\rangle$ are the eigenstates of H_0 corresponding to an eigenvalue λ_i.[17] The dressed states $|\psi_i\rangle$ are defined in terms of the bare atom states $|i\rangle$ as

$$|\psi_i\rangle=a_i|1\rangle+b_i|2\rangle+c_i|3\rangle\ . \tag{2.4}$$

The a_i, b_i, and c_i ($i=1,2,3$) have the explicit expressions[17]

$$a_i=\alpha_1\alpha_2N_i\ , \tag{2.5a}$$

$$b_i=\alpha_2(\lambda_i-\Delta)N_i\ , \tag{2.5b}$$

$$c_i=[(\Delta_2-\lambda_i)(\Delta-\lambda_i)-\alpha_1^2]N_i\ , \tag{2.5c}$$

where the constants N_i are defined as

$$N_i=(a_i^2+b_i^2+c_i^2)^{-1/2} \tag{2.6}$$

and λ_i ($\lambda_2>\lambda_1>\lambda_3$) are the roots of the cubic equation:

$$\lambda^3-\lambda^2(\Delta+\Delta_2)-\lambda(\Omega^2-\Delta\Delta_2)+\alpha_2^2\Delta=0\ , \tag{2.7a}$$

with

$$\Omega=(\alpha_1^2+\alpha_2^2)^{1/2}\ . \tag{2.7b}$$

Using the usual definition for the average of an operator $\hat{O}$, $\langle\hat{O}\rangle=\mathrm{Tr}(\hat{O}\rho)$, we can obtain the equations of motion for $\langle B_{ij}(t)\rangle$ and hence their solutions.[17] For the present studies we require only the diagonal components $\langle B_{ii}(t)\rangle$ which are given in the following compact form:

$$\langle B_{ii}(t)\rangle=\frac{g_i}{\nu_1\nu_2}+\sum_{\nu=\nu_1,\nu_2}\frac{1}{\nu^2-\nu_1\nu_2}\left[(\nu^2-2h_i\nu+g_i)\langle B_{ii}(0)\rangle+\sum_{\substack{j\\(j\neq i)}}(-2f_{ij}\nu+g_i)\langle B_{jj}(0)\rangle\right]\exp(-\nu t)\ , \tag{2.8}$$

where

$$f_{ij}=\gamma_1b_i^2a_j^2+\gamma_2c_i^2b_j^2 \quad (i\neq j)\ , \tag{2.9a}$$

$$f_{ii}=\gamma_1a_i^2(1-b_i^2)+\gamma_2b_i^2(1-c_i^2)\ , \tag{2.9b}$$

$$h_1=f_{13}+f_{22}+f_{23}\ , \tag{2.10a}$$

$$h_2=f_{11}+f_{13}+f_{23}\ , \tag{2.10b}$$

$$h_3=f_{11}+f_{22}\ , \tag{2.10c}$$

$$g_1=4(f_{13}f_{22}+f_{23}f_{12})\ , \tag{2.11a}$$

$$g_2=4(f_{11}f_{23}+f_{13}f_{21})\ , \tag{2.11b}$$

$$g_3=4(f_{11}f_{22}-f_{12}f_{21})\ , \tag{2.11c}$$

$$\langle B_{ii}\rangle=g_i/\nu_1\nu_2\ , \tag{2.12}$$

and ν_1,ν_2 are the two roots of the quadratic equation

$$\nu^2-(h_1+h_2+h_3)\nu+(g_1+g_2+g_3)=0\ . \tag{2.13}$$

B. Photon detection operators

To study the sideband correlations we are required to introduce the photon detection operators. For the cascade configuration under consideration the upper spectrum is characterized by the dipole operator A_{12} and the lower spectrum by the dipole operator A_{23}. We may express these original atomic operators A_{ij} in terms of the dressed operators B_{ij} by means of the relations

$$A_{12}=\sum_{i,j=1}^{3} a_i b_j B_{ij}=A_{21}^{\dagger}\ , \tag{2.14a}$$

$$A_{23}=\sum_{i,j=1}^{3} b_i c_j B_{ij}=A_{32}^{\dagger}\ . \tag{2.14b}$$

$B_{ij}(t)$ evolves with time under the action of the Hamiltonian H_0 as

$$B_{ij}(t)=B_{ij}(0)\exp[i(\lambda_i-\lambda_j)t]\quad (i,j=1,2,3)\ , \tag{2.15}$$

and gives rise to the spectral components at frequencies $\omega_\mu=\lambda_i-\lambda_j$ ($\mu=1,2,3$), with respect to the central peak. Here, μ characterizes the sideband frequencies in the following manner:

$$\omega_1=|\lambda_3-\lambda_1|,\quad \omega_2=|\lambda_2-\lambda_1|,\quad \omega_3=|\lambda_3-\lambda_2|\ , \tag{2.16}$$

with respect to the center of the spectrum, ω_3 corresponds to the farthest sidebands ($\lambda_2>\lambda_1>\lambda_3$), whereas ω_1 and ω_2 correspond to the nearest and the next-nearest sidebands if $\omega_1<\omega_2$ and vice versa if $\omega_2<\omega_1$.

Without any loss of generality, we may restrict ourselves to the discussion of the upper spectrum, since the lower spectrum is obtained by a mere replacement of a_i by b_i and b_i by c_i. Using Eq. (2.14a) we can write A_{12} explicitly as

$$A_{12}=\sum_{i=1}^{3} a_i b_i B_{ii}+a_1b_2B_{12}+a_1b_3B_{13}+a_2b_1B_{21}+a_2b_3B_{23}+a_3b_2B_{32}+a_3b_1B_{31}\ .$$

Now we can identify our photon detection operators as

$$D_+^{1L}=a_3b_1B_{31}=(D_-^{1L})^{\dagger},\quad D_+^{1R}=a_1b_3B_{13}=(D_-^{1R})^{\dagger}\ , \tag{2.17a}$$

$$D_+^{2L}=a_1b_2B_{12}=(D_-^{2L})^{\dagger},\quad D_+^{2R}=a_2b_1B_{21}=(D_-^{2R})^{\dagger}\ , \tag{2.17b}$$

$$D_+^{3L}=a_3b_2B_{32}=(D_-^{3L})^{\dagger},\quad D_+^{3R}=a_2b_3B_{23}=(D_-^{3R})^{\dagger}\ , \tag{2.17c}$$

corresponding to the detection from left (L) and right (R) sidebands. By left (right) we mean that the particular sideband has a frequency less (more) than that of the central peak.[18]

III. SIDEBAND CORRELATION FUNCTIONS

The photon detection operators can be used to obtain the quantum properties of sidebands in terms of the second-order correlation functions:

$$G_{ij}^{(\mu)}(\tau)=\lim_{t\to\infty}\langle D_+^{\mu i}(t)D_+^{\mu j}(t+\tau)D_-^{\mu j}(t+\tau)D_-^{\mu i}(t)\rangle\quad (i,j=L,R\,;\mu=1,2,3)\ . \tag{3.1}$$

Physically the correlations $G_{LR}^{(\mu)}(\tau)$ $[G_{RL}^{(\mu)}(\tau)]$ represent the probability of detecting at time $t+\tau$ a photon of frequency ω_μ on the right [left] given that a photon of the same frequency on the left [right] has been detected at time t.

Using the solutions for $\langle B_{ii}(t)\rangle$ [Eq. (2.8)] we obtain analytical expressions for the cross- and self-correlations $G_{ij}^{(\mu)}(\tau)$ and $G_{ii}^{(\mu)}(\tau)$, respectively, for the general case of nonzero detunings. The expression for cross-correlations $G_{LR}^{(\mu)}(\tau)$ is given by

$$G_{LR}^{(\mu)}(\tau)=(a_kb_ka_lb_l)^2\left[\frac{g_k}{\nu_1\nu_2}+\sum_{\nu=\nu_1,\nu_2}\left[\frac{\nu^2-2h_k\nu+g_k}{\nu^2-\nu_1\nu_2}\right]\exp(-\nu\tau)\right]\langle B_{ll}\rangle\ . \tag{3.2}$$

The indices k and l take values depending on the pair of sidebands studied. Thus (k,l) take the values (1,3), (2,1), and (2,3), respectively, for $\mu=1$, 2, and 3. The expressions for $G_{RL}^{(\mu)}(\tau)$ are obtained by replacing $k\leftrightarrow l$ in Eq. (3.2).

The analytical expressions for the self-correlations $G_{LL}^{(\mu)}(\tau)$ are

$$G_{LL}^{(\mu)}(\tau)=(a_kb_l)^4\left[\frac{g_k}{\nu_1\nu_2}+\sum_{\nu=\nu_1,\nu_2}\left[\frac{-2f_{kl}\nu+g_k}{\nu^2-\nu_1\nu_2}\right]\exp(-\nu\tau)\right]\langle B_{kk}\rangle\ . \tag{3.3}$$

The indices (k,l) take the values (3,1), (1,2), and (3,2) for $\mu=1$, 2, and 3, respectively. The expressions for $G_{RR}^{(\mu)}(\tau)$ are obtained by replacing $k\leftrightarrow l$ in Eq. (3.3).

Figure 2 displays the general behavior of the cross- and self-correlations for the case of unequal detunings ($\Delta_1\neq\Delta_2$). The inset of Fig. 2(a) displays the seven-peak fluorescence spectrum for the upper transition. A general conclusion is that the photons from a pair of sidebands are strongly correlated. Further, the cross-correlations show a bunching property, $G_{ij}^{(\mu)}(0)>0$ [see Fig. 2(a)], whereas the self-correlations show an antibunching property, $G_{ii}^{(\mu)}(0)=0$ [see Fig. 2(b)]. The antibunching property is physically obvious as the emission of a photon of a particular sideband frequency precludes the possibility of a simultaneous observation of another photon of the same frequency. In fact, detection of the first photon of a certain frequency implies a quantum jump of the atom from the lower excited state to the upper excited state; the atom can emit the same frequency photon only after a finite interval of time during which it gets deexcited and

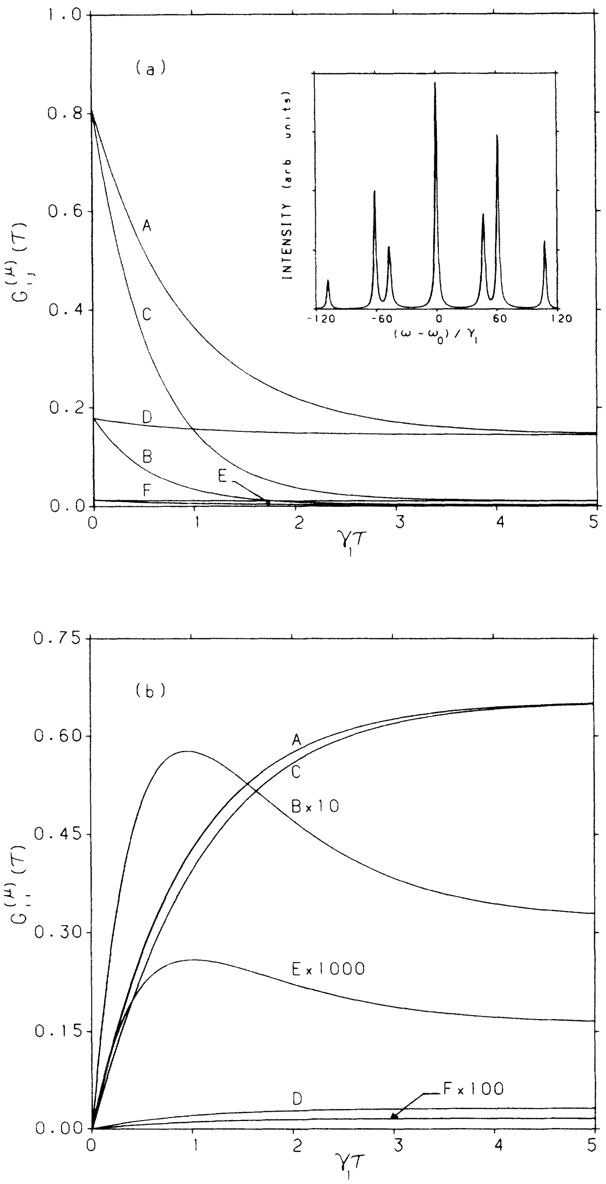

FIG. 2. (a) Cross-correlations $G_{LR}^{(\mu)}(\tau)$ and $G_{RL}^{(\mu)}(\tau)$ as a function of $\gamma_1\tau$ for $\alpha_1=20\gamma_1$, $\alpha_2=30\gamma_1$, $\gamma_2=\gamma_1$, $\Delta_1=50\gamma_1$, and $\Delta_2=30\gamma_1$. Curves $A-C$ are $G_{LR}^{(1)}$, $G_{LR}^{(2)}$, and $G_{LR}^{(3)}$, respectively, and curves $D-F$ are the corresponding RL correlations. The inset shows the fluorescent spectrum from the upper transition. ω_0 is the upper transition frequency. (b) Self-correlations $G_{LL}^{(\mu)}(\tau)$ and $G_{RR}^{(\mu)}(\tau)$ as a function of $\gamma_1\tau$. Other data as in (a). Curves $A-C$ are $G_{LL}^{(1)}$, $G_{LL}^{(2)}$, and $G_{LL}^{(3)}$, respectively, and curves $D-F$ are the corresponding RR correlations.

excited again. In the usual antibunching experiments with a two-level atom, carried out in the time domain, detection of a photon is associated with the atom making a jump from a upper to the lower state rather than the lower to the upper state.[7]

On the other hand, the nonzero value of the cross-correlations initially (at $\tau=0$) implies that the emission of a photon of a particular frequency from the left (right) sideband does not necessarily preclude the probability of observing a photon of the corresponding frequency from the right (left) sideband simultaneously. A simple physical interpretation of this result may be given by invoking the picture of the energy levels of a combined system of the three-level and the laser photons interacting together.[13] As a consequence of the quantization of the laser field, this "dressed atom" has an infinite number of levels. After absorbing a photon, the dressed atom can cascade downwards from the excited state either to its original state (elastic Rayleigh scattering) or to any of the perturbed states of the original state (Raman scattering) with the emission of a photon at the transition frequency Ω_i $(i=1,2)$ or at the sideband frequencies $\Omega_i \pm \omega_\mu$ $(\mu=1,2,3)$, respectively. The former contributes to the central peak of the spectrum. The correlations between the left and right sideband photons may be viewed as a manifestation of such cascade emission. In our formulation in terms of the dressed state of the atom we have inherently summed over all these microscopic processes involving photon absorptions and emissions. The temporal correlations between the photons belonging to a pair of sidebands are, however, reflected in the behavior of theoretically calculated correlation function $G_{ij}^{(\mu)}(t)$. Hence $G_{ij}^{(\mu)}(\tau)\neq 0$ for all times. The time-ordered nature of $G_{ij}^{(\mu)}(\tau)$ and $G_{ii}^{(\mu)}(\tau)$ is clearly seen from Figs. 2(a) and 2(b), respectively. In general, it is observed that $G_{LR}^{(\mu)}(\tau)\neq G_{RL}^{(\mu)}(\tau)$ and $G_{LL}^{(\mu)}(\tau)\neq G_{RR}^{(\mu)}(\tau)$. Moreover, $G_{LR}^{(\mu)}(\tau) \gg G_{RL}^{(\mu)}(\tau)$ and $G_{LL}^{(\mu)}(\tau) \gg G_{RR}^{(\mu)}(\tau)$. With respect to the cross-correlations, this implies that, for a μth pair of sidebands, if an L photon has a high probability of being followed by an R photon after time τ then, necessarily, the probability of the R photon being followed by an L photon after time τ is very low. A similar pattern is observed in case of self-correlations also, i.e., if $G_{LL}^{(\mu)}(\tau)$ is very high, then, necessarily, $G_{RR}^{(\mu)}(\tau)$ is very low. This behavior can be seen clearly from Fig. 2. However, whether LR,LL correlations are greater than RL,RR correlations or vice versa depends solely on the value and sign of the detunings. For the data considered in Fig. 2, the curves show that the values of $G_{LR}^{(1,2,3)}(\tau)$ and $G_{LL}^{(1,2,3)}(\tau)$ are much greater than those of $G_{RL}^{(1,2,3)}(\tau)$ and $G_{RR}^{(1,2,3)}(\tau)$, respectively, implying that the time ordering is such that the L sideband photon always precedes the R sideband photon. A further interesting aspect of the self-correlations is that one pair of sideband correlations exhibits a humped behavior [curves B and E of Fig. 2(b)]. This is reflected as a super-Poissonian characteristic of photon statistics in the response function $Q(T)$ and shall be discussed in Sec. IV.

We next consider the case when the two driving fields have opposite detunings $(\Delta_1=-\Delta_2)$. In this case, the nearest sideband on one side of the center and the next-nearest sideband on the other side of the center disappear, reducing the normal seven-peak spectrum to an asymmetric Stark quintuplet [see inset of Fig. 3(a)]. It can be seen from Eq. (2.7) that one of the roots of the cubic equation is $\lambda_1=0$ and, hence, the coefficient $b_1=0$. Thus the dipole operator A_{12} [Eq. (2.14a)] now contains only six terms, two of which contribute to the central peak. Therefore we now define our photon detector operators in the following manner:

$$D_{+}^{1R}=a_1b_3B_{13}=(D_{-}^{1R})^{\dagger},\quad D_{+}^{1L}=a_1b_2B_{12}=(D_{-}^{1L})^{\dagger}\ , \tag{3.4a}$$

$$D_{+}^{2R}=a_2b_3B_{23}=(D_{-}^{2R})^{\dagger},\quad D_{+}^{2L}=a_3b_2B_{32}=(D_{-}^{2L})^{\dagger}\ . \tag{3.4b}$$

We must qualify here that the operators $D_{\pm}^{1R}$ and $D_{\pm}^{1L}$, though referring to nearest sidebands on the right and left, respectively, are, however, not placed symmetrically about the central peak as in the general case of unequal detunings discussed earlier. But the operators $D_{\pm}^{2R}$ and $D_{\pm}^{2L}$ still refer to the outermost sidebands of the spectrum and these are placed symmetrically about the center.

The pattern of behavior for the cross- and the self-correlations for the optical double resonance $\Delta_1+\Delta_2=0$ is similar to that of the general case. This is displayed in Figs. 3(a) and 3(b). The notable difference here is that $G_{LR}^{(1)}$ and $G_{RL}^{(1)}$ show an antibunching property, $G_{ij}^{(1)}(0)=0$ [curves A and C of Fig. 3(a)].

In the resonance case ($\Delta_1=\Delta_2=0$), the asymmetric Stark quintuplet becomes a symmetric Stark quintuplet, and is shown in the inset of Fig. 4(a). The detection operators are still given by the relations (3.4a) and (3.4b). As can be seen from Figs. 4(a) and 4(b) there is no time lag between the appearance of the left and right sidebands under pure resonance conditions. The other general aspects of their behavior are much the same as for the optical double resonance case.

IV. RESPONSE FUNCTION

The photon statistics of the sideband emission can be determined from the detector response function $Q_i^{(\mu)}(T)$ which is directly related to $G_{ii}^{(\mu)}(\tau)$ in the steady state

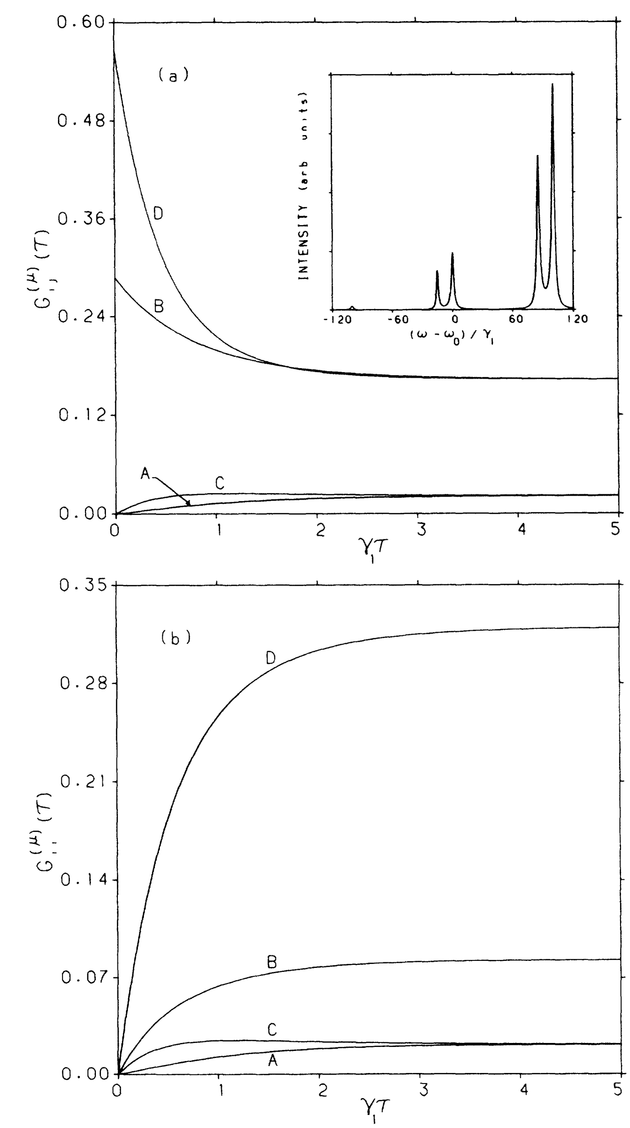

FIG. 3. (a) Same as Fig. 2(a) but for $\Delta_1=-\Delta_2=70\gamma_1$. Curves A–D are $G_{LR}^{(1)}$, $G_{LR}^{(2)}$, $G_{RL}^{(1)}$, and $G_{RL}^{(2)}$, respectively. (b) Same as Fig. 2(b) but for $\Delta_1=-\Delta_2=70\gamma_1$. Curves A–D are $G_{LL}^{(1)}$, $G_{LL}^{(2)}$, $G_{RR}^{(1)}$, and $G_{RR}^{(2)}$, respectively.

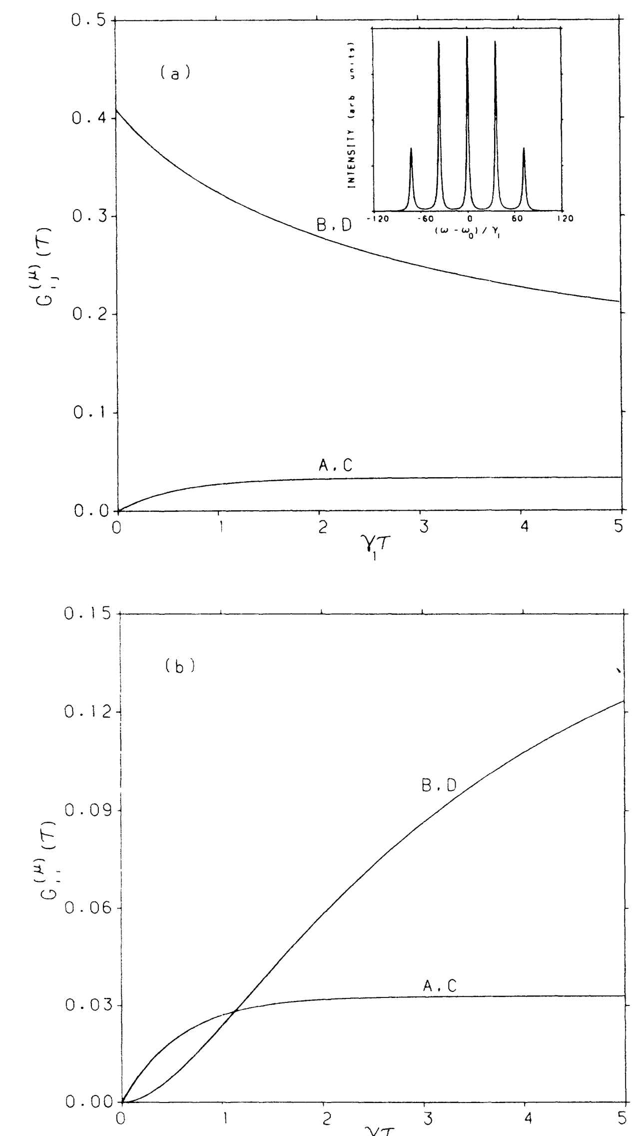

FIG. 4. (a) Same as Fig. 3(a) but for $\Delta_1=\Delta_2=0$. (b) Same as Fig. 3(b) but for $\Delta_1=\Delta_2=0$.

$$Q_i^{(\mu)}(T)=\frac{2q}{T}\int_0^T d\tau(T-\tau)[G_{ii}^{(\mu)}(\tau)-\langle D_+^{\mu i}D_-^{\mu i}\rangle^2]/\langle D_+^{\mu i}D_-^{\mu i}\rangle \quad (\mu=1,2,3 \text{ and } i=L,R)\ , \tag{4.1}$$

where q is a parameter related to detector efficiency and has the dimension of frequency. Using the expressions for $G_{ii}^{(\mu)}(\tau)$ and $D_\pm^{\mu i}$ we obtain

$$Q_i^{(\mu)}(T)=2q[K_i^{(\mu)}(\nu_1)+K_i^{(\mu)}(\nu_2)]\ , \tag{4.2}$$

with

$$K_i^{(\mu)}(\nu)=\frac{(a_k b_l)^2}{\nu}\left[\frac{-2f_{kl}\nu+g_k}{\nu^2-\nu_1\nu_2}\right]\left[1-\frac{[1-\exp(-\nu T)]}{\nu T}\right]\ , \tag{4.3}$$

where for the general case of detunings we have for $i=L$, $(k,l)=(3,1)$, (1,2), and (3,2) when $\mu=1$, 2, and 3, respectively. For $i=R$ the corresponding values of (k,l) are interchanged. For the case when $\Delta_1=-\Delta_2$ we have for $i=L$, $(k,l)=(1,2)$ and (3,2) for $\mu=1$ and 2, respectively. For $i=R$, $(k,l)=(1,3)$ and (2,3) for $\mu=1$ and 2, respectively. For the special case of resonance, i.e., $\Delta_1=\Delta_2=0$, we have the following explicit expressions for $Q_i^{(\mu)}(T)$:

$$Q_L^{(1)}(T)=Q_R^{(1)}(T)=\frac{4qf_{12}}{\nu_1^2}\left[\frac{\alpha_2^2}{2\Omega^2}\right]\left[\frac{1-\exp(-\nu_1 T)}{\nu_1 T}-1\right]\ , \tag{4.4}$$

$$Q_L^{(2)}(T)=Q_R^{(2)}(T)=2q\left[\frac{\alpha_1^2}{4\Omega^2}\right]\left[\frac{f_{12}}{\nu_1^2}\left[1-\frac{1-\exp(-\nu_1 T)}{\nu_1 T}\right]-\frac{1}{2\nu_2}\left[1-\frac{1-\exp(-\nu_2 T)}{\nu_2 T}\right]\right]\ . \tag{4.5}$$

Figure 5 shows the variation of $Q_i^{(\mu)}(T)$ with time for unequal detunings. It is observed that two pairs of sidebands exhibit purely sub-Poissonion statistics (see curves A, C, D, and F in Fig. 5). However, curve B in Fig. 5 indicates that the photons belonging to the third pair of sidebands show sub-Poissonian character for very short times. For longer times the L photon exhibits a super-Poissonian nature (curve B) while the R photon tends to show a pure Poissonian behavior (curve E). Moreover, this super-Poissonian behavior is highly dependent on the

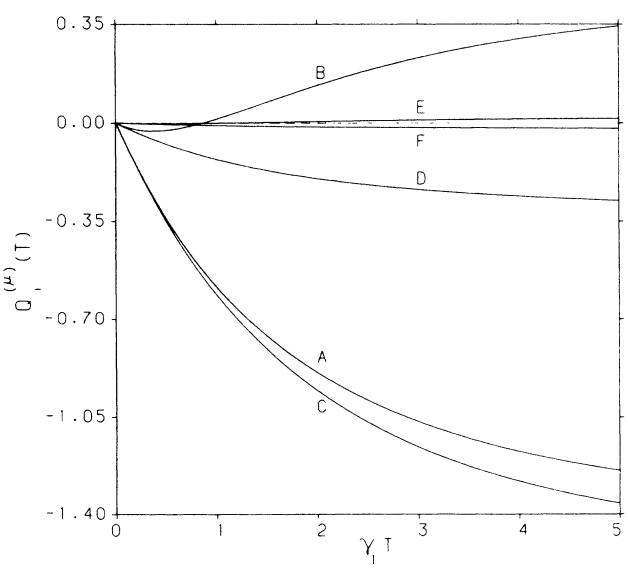

FIG. 5. Response functions $Q_L^{(\mu)}(\tau)$ and $Q_R^{(\mu)}(\tau)$ as a function of $\gamma_1 T$. Curves $A-C$ are $Q_L^{(1)}$, $Q_L^{(2)}$, and $Q_L^{(3)}$, respectively, and curves $D-F$ are the corresponding R response functions. Data as in Fig. 2(a).

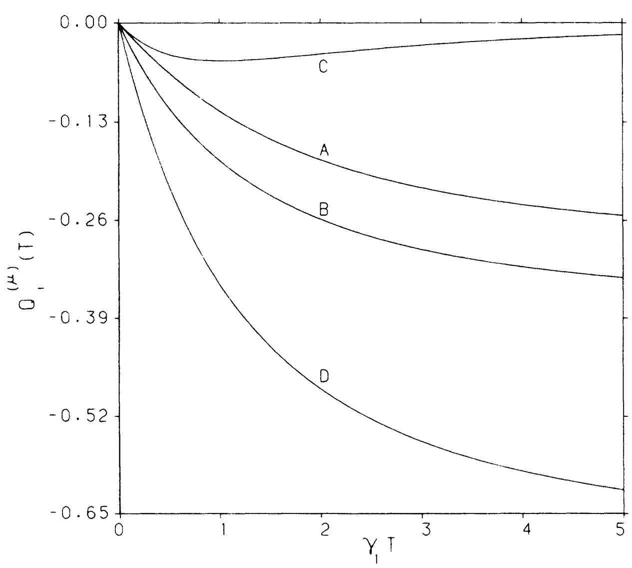

FIG. 6. Same as Fig. 5 but for $\Delta_1=-\Delta_2=70\gamma_1$. Curves $A-D$ are $Q_L^{(1)}$, $Q_L^{(2)}$, $Q_R^{(1)}$, and $Q_R^{(2)}$, respectively.

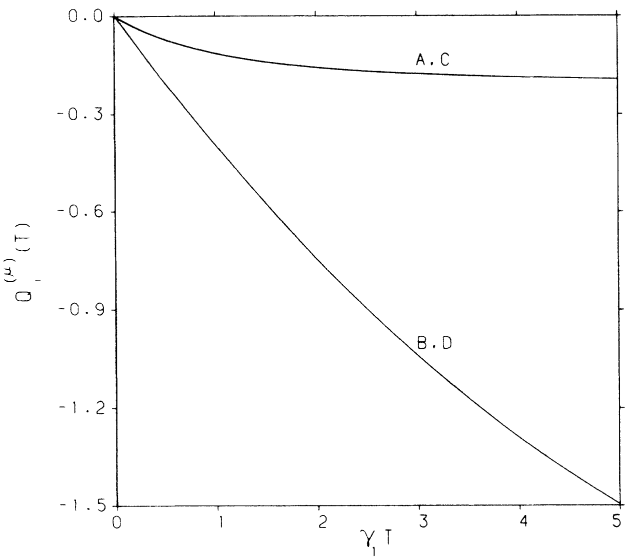

FIG. 7. Same as Fig. 6 but for $\Delta_1 = \Delta_2 = 0$.

value of the detuning. The super-Poissonian behavior is a reflection of the humped nature of the corresponding self-correlation. It is further observed that in the case when the detuning is comparable to the field, the humped nature and hence the super-Poissonian behavior disappears.

For the case of equal and opposite detunings (including exact resonance) all sidebands display a sub-Poissonian behavior as seen in Figs. 6 and 7.

V. CONCLUSIONS

We have examined in this paper the quantum nature of the sidebands of the fluorescent spectra of a cascade system. We conclude that there is a definite correlation and time ordering between photons from corresponding pairs of sidebands. Whether the sideband of frequency greater than central is followed by the corresponding sideband of frequency lesser than the central depends mainly on the value and sign of detunings. At the microscopic level this may be interpreted as due to a "dressed three-level atom" cascading through an infinite quasiperiodic array of energy levels. Further, the quantum nature of the sideband correlations is reflected in the sub- and super-Poissonian nature of the response functions $Q(T)$.

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