Population trapping in the Jaynes-Cummings model via phase coupling

J. I. Cirac and L. L. Sánchez-Soto
Departamento de Óptica, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain
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An expression for the population inversion in the Jaynes-Cummings model and the most general initial state of the atom-field system is presented. States that maintain the same level of population inversion when the atom and the field are initially decoupled are found. When the atom is prepared in a coherent superposition of its states and the field is in an eigenstate of the Susskind-Glogower phase operator, for a certain choice of the relative phase between the atomic dipole and the field, coherent trapping occurs. The evolution of the squeezing of the field for these states is given, and the dependence on the phase and on the mean photon number of the field is studied.

I. INTRODUCTION

The Jaynes-Cummings model[1] (JCM) of a single two-level atom interacting with a single mode of the quantized radiation field in a lossless cavity is one of the most thoroughly examined models in quantum optics. In the framework of this model many nonclassical effects, such as vacuum-field Rabi oscillations,[2] antibunching,[3] and squeezing[4] of the radiation field, or collapse-revival phenomena,[5] have been predicted. This last phenomenon provides unquestionable evidence for the discreteness of the radiation field, and has been studied in detail when the field is initially in one of the following states: coherent,[6] chaotic,[7] squeezed,[8] binomial[9] thermocoherent,[10] logarithmic,[11] multiphoton Holstein-Primakoff SU(2) group,[12] and also for the superposition of coherent and chaotic fields.[13] The effects of cavity damping have also been discussed.[14]

In spite of their extreme difficulty, recent experiments with Rydberg atoms in high-Q microwave cavities have allowed experimental observations of the dynamical properties of the model.[15]

The phenomenon of collapses and revivals of the atomic inversion depends on the statistics of the photon-number distribution, but not on the phase of the field if the atom is initially in one of its two states. However, if the atom is prepared in a coherent superposition of the excited and ground levels, the excitation probability depends on the relative phase between the atomic coherence and the exciting field.[16] This phenomenon of phase sensitivity in the atom-field interaction has attracted a lot of attention since it provides a useful means of testing the predictions of the quantum theory of radiation against those of semiclassical and neoclassical theories, as well as having applications to noise quenching by correlated spontaneous emission,[17] quantum beats,[18] and noise-free amplification.[19]

The spontaneous decay[20] and the fluorescence spectrum[21] of an atom in a broadband squeezed vacuum are two additional examples where the phase dependence plays a predominant role, and novel effects appear when the phase is changed.

Agarwal and Puri[22] have demonstrated that the effects due to the quantum nature of the field in the cavity can be isolated by setting the initial phase of the atomic dipole moment. Recently, Zaheer and Zubairy[23] have shown that in the interaction of a two-level atom, initially prepared in a coherent superposition of its two states, with a coherent field, it is possible to obtain coherent trapping for a particular choice of the phase and in the semiclassical limit. This can be interpreted as a result of a destructive interference between the atomic dipole and the cavity eigenmode. Slosser, Meystre, and Braunstein[24] have studied the evolution of a single mode of the field driven by a current of two-level atoms, each interacting with the mode for a time τ, a problem widely studied in the context of micromaser theory,[25] showing that under certain trapping condition it evolves towards a new class of pure states which remain unchanged after each interaction time τ.

In the present work we focus our attention on the phenomenon of trapping. We show that, for a set of initial conditions of the atom-field system, the atomic inversion, far from exhibiting revivals, remains constant. Some of those initial states were well known, but we find pure and decoupled states for which exact coherent trapping occurs, i.e., the population of each atomic level and of the field does not evolve. This is in sharp contrast with the belief that a pure two-level system cannot exhibit coherent trapping.[5] Note that, although other observables like dipole moment or squeezing are more phase sensitive, it is the atomic inversion that is best suited for experimental investigations.

The paper is organized as follows. In Sec. II we present the expression for the evolution of population inversion for the most general initial state of the global atom-field system. In Sec. III we study trapping phenomenon, and we obtain pure and decoupled states of the atom and the field which exhibit coherent trapping. These states can be identified as the eigenstates of the well-known Susskind-Glogower phase operator.[26] In Sec. IV we show that, for a certain choice of their parameters, these states present squeezing. We also investigate how the degree of squeezing of the field in the cavity evolves with time, and we find nearly the same results obtained when the initial state of the field is a squeezed vacuum:[27] Only for low values of the mean photon number the field is squeezed at any times beyond the first few instants. This is in contrast with the results of Ref. 4, where the initial state was taken to be a coherent state and where some small amount of squeezing appeared and disappeared when the population inversion was in the revival or in the collapse region, respectively. Our conclusions are summarized in Sec. V.

II. POPULATION INVERSION FOR AN ARBITRARY INITIAL STATE

The Hamiltonian of the exactly soluble JCM in the rotating-wave approximation (RWA) reads

$$H=\tfrac{1}{2}\hbar\omega_0\sigma_3+\hbar\omega a^\dagger a+\hbar\lambda(a^\dagger\sigma_-+\sigma_+a)\ , \tag{1}$$

where the σ's are the usual pseudospin operators acting in the space of atomic states, a and $a^\dagger$ are the annihilation and creation operators for the field mode, and λ is a coupling constant containing the transition dipole moment. The natural transition frequency ω_0 of the atom need not to coincide with the mode frequency ω, although the RWA is reliable only if $|\omega_0-\omega|\ll\omega_0,\omega$.

We first consider the most general initial state for the system,

$$\rho(0)=\sum_{n,m=0}^{\infty}\sum_{\alpha,\beta=g}^{e}P(\alpha,\beta,n,m)|n,\alpha\rangle\langle m,\beta|\ . \tag{2}$$

Here $|n,\alpha\rangle$ denotes a state with n photons and the atom in the ground ($\alpha=g$) or in the excited state ($\alpha=e$), and $P(\alpha,\beta,n,m)$ are c numbers constrained by the conditions

$$\mathrm{Tr}\rho(0)=1,\quad [\rho(0)]^\dagger=\rho(0)\geq 0\ . \tag{3}$$

For the initial state given in (2), the population inversion is given by

$$\langle\sigma_3(t)\rangle=\sum_{n=0}^{\infty}\left[\left[\frac{\lambda_n^2}{\mu_n^2}\cos(2\mu_n t)+\frac{1}{4}\frac{\Delta^2}{\mu_n^2}\right][P(e,e,n,n)-P(g,g,n+1,n+1)]+\frac{2\lambda_n}{\mu_n}\sin(2\mu_n t)\mathrm{Im}[P(g,e,n+1,n)]\right.$$
$$\left.+\frac{\lambda_n\Delta}{\mu_n^2}[\cos(2\mu_n t)-1]\mathrm{Re}[P(g,e,n+1,n)]\right]-P(g,g,0,0)\ , \tag{4}$$

where the detuning parameter is $\Delta=\omega_0-\omega$ and

$$\lambda_n=\lambda\sqrt{n+1}\ , \tag{5a}$$

$$\mu_n=\left[\lambda_n^2+\tfrac{1}{4}\Delta^2\right]^{1/2}\ . \tag{5b}$$

In the following we shall assume, as usual, that at time $t=0$ the density operator factors into its atomic and field parts

$$\rho(0)=\rho_A(0)\otimes\rho_F(0)\ , \tag{6}$$

with

$$\rho_A(0)=p_g|g\rangle\langle g|+(1-p_g)|e\rangle\langle e|+q(e^{-i\psi}|g\rangle\langle e|+e^{i\psi}|e\rangle\langle g|)\ , \tag{7a}$$

$$\rho_F(0)=\sum_{n=0}^{\infty}P(n)|n\rangle\langle n|+\sum_{n<m}P(n,m)(e^{i\phi(m,n)}|m\rangle\langle n|+e^{-i\phi(m,n)}|n\rangle\langle m|)\ , \tag{7b}$$

where $\frac{1}{4}\geq p_g(1-p_g)\geq q^2$ (the second equality is satisfied only for a pure state). In this case the atomic inversion can be expressed as

$$\langle\sigma_3(t)\rangle=h_0+h_1(t)+h_2(t)\ , \tag{8}$$

where

$$h_0=-p_gP(0)+\sum_{n=0}^{\infty}\left[\frac{\frac{1}{4}\Delta^2}{\mu_n^2}[(1-p_g)P(n)-p_gP(n+1)]-\frac{\lambda_n\Delta}{\mu_n^2}qP(n,n+1)\cos[\phi(n+1,n)-\psi]\right]\ , \tag{9a}$$

$$h_1(t)=\sum_{n=0}^{\infty}\frac{\lambda_n^2}{\mu_n^2}\cos(2\mu_n t)\left[(1-p_g)P(n)-p_gP(n+1)+\frac{\Delta}{\lambda_n}qP(n,n+1)\cos[\phi(n+1,n)-\psi]\right]\ , \tag{9b}$$

$$h_2(t)=\sum_{n=0}^{\infty}\frac{2\lambda_n}{\mu_n}\sin(2\mu_n t)qP(n,n+1)\sin[\phi(n+1,n)-\psi]\ . \tag{9c}$$

Note that $h_2(t)$ is an interference term that depends on both the relative phase between the atomic dipole and the field, and the purity of the initial state. Contrary, $h_1(t)$ has two parts: the last term is similar to $h_2(t)$ but is due to the existence of a detuning Δ between the atomic transition and the frequency of the cavity mode; the first and second term on the right-hand side of (9b) express nothing but the possibility of exchange of a photon between the atom and the field, and hence they are phase independent. When $h_1(t)$ and $h_2(t)$ vanish, the atomic inversion remains constant, the atom will be trapped by the field, and there will be not any transition between its levels, as occurs when it is isolated.

III. COHERENT AND INCOHERENT TRAPPING

In this section we discuss the steady-state behavior of the atomic populations in the excited and ground levels. We consider (6) as the initial state of the system, i.e., we do not study some coupled states of the atom and the field, such as the dressed states,[22] that exhibit trapping. We shall use this term to refer to a persistent probability for occupying a given level in spite of the existence of both the radiation field and transitions to the other level.[5] Under such conditions, level population takes on a steady value.

Equation (8) shows that the atomic inversion remains constant only when both $h_1(t)$ and $h_2(t)$ vanish. We consider trapping phenomenon resulting from two different sets of initial conditions that cancel $h_2(t)$ out. In the first, there is not any phase coupling between the atom and the field, i.e., q or $P(n,n+1)$ are zero. In the second, the atomic dipole and the field are coupled through their phases in such a way that $h_2(t)$ vanishes. In this last case we shall only consider pure states since mixed states are intermediate situations between pure and chaotic states and add nothing new to the JCM.

A. Incoherent pumping

For q or $P(n,n+1)$ vanishing, $h_1(t)=0$ only when

$$P(n)=\left[1-\frac{p_e}{p_g}\right]\left[\frac{p_e}{p_g}\right]^n , \tag{10}$$

with

$$p_e=1-p_g, \quad \tfrac{1}{2}<p_g<1 . \tag{11}$$

Thus, in this case, at least either $\rho_A(0)$ or $\rho_F(0)$ must be a mixed state. Trapping does not depend on the phases and coherences, and so there is *incoherent trapping*. Note that for squeezed vacuum $h_2(t)$ is zero and there is no interference term in the atomic inversion because of a lack of coherent coupling between the one-photon transition of the atom and the two-photon squeezed state. This is the reason why the model considered in Ref. 27 of an atom in a coherent superposition of its two states interacting with a squeezed vacuum does not exhibit phase sensitivity, and the results are very similar to those for a thermal field. For a chaotic state

$$P(n,m)=\left[1-\frac{p_e}{p_g}\right]\left[\frac{p_e}{p_g}\right]^n \delta_{n,m} , \tag{12}$$

Eq. (10) holds and the atomic inversion is constant. In addition, when q is zero, any observable of the system does not evolve. In this case, the light and the atom does not interact because $[H,\rho(0)]=0$, and hence $\rho(0)$ is a constant of motion. This is not the only state that is a constant of motion, but it is the one that can be expressed in the form (6). There is an infinite set of states of the form

$$\rho(0)\propto f(C) ,$$

that commute with the JC Hamiltonian, $C=\frac{1}{2}\sigma_3+a^\dagger a$ being the excitation number, which is a constant of motion for the JCM. Note that these results hold for arbitrary values of the detuning Δ, in the range in which the RWA is valid.

B. Coherent pumping

We now assume that the initial state is pure and the phase of the atomic dipole and the field are identical. For a pure state, we have in (6)

$$\begin{aligned} q&=[p_g(1-p_g)]^{1/2} ,\\ P(n,m)&=\sqrt{P(n)}\sqrt{P(m)} , \\ \phi(m,n)&=\phi(m)-\phi(n) . \end{aligned} \tag{13}$$

It is easily shown that $\langle\sigma_3(t)\rangle$ remains constant when the photon-number distribution is given by

$$P(n)\propto\left[\frac{p_e}{p_g}\right]^n\prod_{i=0}^{n-1}\left[\frac{\frac{1}{2}\Delta+\mu_i}{\lambda_i}\right]^2, \quad n=1,2,\ldots \tag{14a}$$

and

$$\phi(n)=n\psi . \tag{14b}$$

With this last condition, the mean electric field is

$$\langle E(t)\rangle\propto\sin(\omega t-\psi) ,$$

and so we can identify the angle ψ with the phase of the classical field.

For this coherent pumping situation we shall consider only the case $\Delta=0$ since it keeps the expressions simpler and retains all the relevant physical features. Now the condition (14a) is transformed in (10). As a consequence, we have that for the initial state given by

$$|\Psi(0)\rangle=|\Psi(0)\rangle_A\otimes|\Psi(0)\rangle_F , \tag{15}$$

where

$$|\Psi(0)\rangle_A=\left[1+|z|^2\right]^{-1/2}\left[|e\rangle+z^*|g\rangle\right] , \tag{16a}$$

$$|\Psi(0)\rangle_F=\left[1-|z|^2\right]^{1/2}\sum_{n=0}^{\infty}z^n|n\rangle , \tag{16b}$$

and

$$z=|z|e^{i\phi}, \quad |z|<1 , \tag{17}$$

coherent trapping occurs. Here the condition $|z|<1$ means that, as in the case of incoherent pumping, the upper state population must be less than the lower state population. It can be shown that all the diagonal elements of the density operator expressed in the basis $|n,\alpha\rangle$ remain unchanged. A possible explanation for such behavior can be as follows. Whereas in an N-level atom ($N>2$) coherent trapping occurs due to the coherent interference between two or more transition channels, in the case of a two-level atom the atomic dipole interferes destructively with the cavity eigenmode, inhibiting the transition between the two levels[23] and forcing $h_2(t)$ to vanish. When the atom and the field are in thermal equilibrium, Eq. (10) holds and $h_1(t)$ is zero. These two conditions are fulfilled only for the state (15). Other states which satisfy (14), and for a certain choice of the mean photon number of the field, can make $h_1(t)$ extremely small, and hence the amplitude of the oscillations of the population inversion becomes nearly zero. This is the coherent state case studied in Ref. 23. Note that the density operator of the state (15) does not commute with the Hamiltonian (1) and so it is not a constant of motion. In fact, the atomic dipole and the off-diagonal elements of the density operator of the field oscillate. In this context we should mention the tangent and cotangent states introduced by Slosser, Meystre, and Braunstein,[24] which not only leave the atomic inversion unchanged but also the field itself, even if the upper state population is larger than the lower state population, but this trapping is achieved only for some fixed interaction times.

The states of the electromagnetic field defined in (16b) are eigenstates of the Susskind-Glogower phase operator[26]$(N+1)^{-1/2}a$. When we substitute $z=|z|e^{i\phi}$ in (16b) they are very reminiscent of the coherent states. In fact, it has been recently pointed out that they may be viewed as generalized coherent states of the SU (1,1) group.[28] We also note that the properties of the diagonal elements of its density operator are the same as those for the chaotic field. For these states we have

$$\langle n\rangle=\frac{|z|^2}{1-|z|^2}\ , \tag{18a}$$

$$\langle n^2\rangle=\langle n\rangle(1+2\langle n\rangle)\ . \tag{18b}$$

For $\langle n\rangle>0$, $(\Delta n)^2>\langle n\rangle$ and so they exhibit a super-Poissonian photon-number distribution. The degree of rth order coherence is

$$g^r(0)=\frac{\langle a^\dagger a^\dagger\cdots aa\cdots\rangle}{\langle a^\dagger a\rangle^r}=r!\ . \tag{18c}$$

In the semiclassic limit ($\langle n\rangle\to\infty$) these states approach the *phase states*,[26] which have been extensively used in quantum optics and have been shown to have the properties expected of a state of well-defined phase. They are most conveniently written in terms of the limit

$$|\theta\rangle=\lim_{s\to\infty}(s+1)^{-1/2}\sum_{n=0}^{s}e^{in\theta}|n\rangle\ . \tag{19}$$

This limit has to be taken, as pointed out by Barnett and Pegg,[29] when expected values are calculated. We have plotted in Fig. 1 the evolution of the atomic inversion for different values of s, the maximum number of photons of the state (19). Note that the atomic inversion oscillates with Rabi frequency $2\lambda_s$ and the amplitude approaches zero when $s\to\infty$. So these phase states can exhibit coherent trapping when the atom is prepared in a state with infinite temperature ($p_g=p_e=\frac{1}{2}$). Recently, Barnett and Pegg have constructed a well-behaved Hermitian optical phase operator through these phase states.[29]

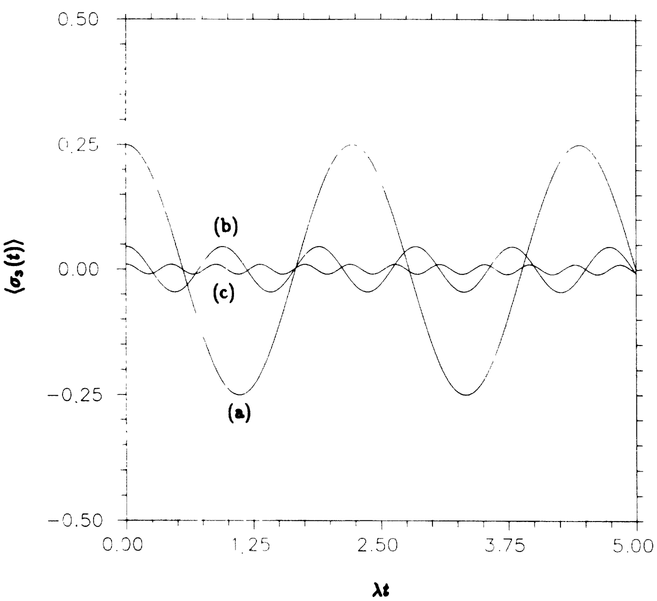

FIG. 1. Evolution of the atomic inversion for an atom prepared in a coherent superposition of its states ($\rho_g=\rho_e=\frac{1}{2}$) and the phase state defined in (19) before taking the limit. The values of the maximum photon-number Fock state for this state are (a) 1, (b) 10, and (c) 50.

C. Semiclassical limit

Finally, to conclude this section, we investigate the similarities between the quantum and the semiclassical regimes in the study of the trapping phenomenon in the JCM. Coherent trapping can be explained semiclassically in a three-level atom.[5] We might ask if in a two-level atom the semiclassical steady state for the atomic inversion coincides with that obtained in (15). The response of a two-level system driven by a nearly resonant field is described by the Bloch equations,[30] which may be written (neglecting relaxation) as

$$\frac{d\mathbf{B}}{dt}=\mathbf{\Omega}_B\times\mathbf{B}\ , \tag{20}$$

where $\Omega_B=(-\xi,0,\Delta)$ is the driving field vector and $\mathbf{B}=(B_1,B_2,B_3)$ is the Bloch vector. The quantity $\xi=Ed/\hbar$ is the Rabi frequency associated with a driving field of the form $E\sin(wt-\theta)$ and has been chosen to be real; Δ is the detuning parameter; d is the atomic dipole-moment-matrix element; and B_1, B_2, and B_3 are related to the atomic density-matrix elements by

$$B_1=\rho_{ge}e^{-i(\omega t-\theta)}+\text{c.c.}\ , \tag{21a}$$

$$B_2=i(\rho_{ge}e^{-i(\omega t-\theta)}-\text{c.c.})\ , \tag{21b}$$

$$B_3=\rho_g-\rho_e\ . \tag{21c}$$

When initially $\mathbf{B}$ and $\mathbf{\Omega}_B$ are parallel, B remains stationary. It has been pointed out that these configurations correspond to atoms in particular dressed states of the atom-field system.[31]

For $\Delta=0$, trapping occurs when

$$\rho_{ge}(0)=qe^{-i\theta}\,, \qquad \text{(22a)}$$

$$\tfrac{1}{4}\ge q \ge 0\,,$$

and

$$\rho_g(0)=\rho_e(0)=\tfrac{1}{2}\,. \qquad \text{(22b)}$$

Here, even when the atom is prepared in a mixed state ($q<\frac{1}{4}$), the first equation expresses the same condition obtained in (14b): The atomic dipole and the field must have identical phases at $t=0$. Equation (22b) is obtained for the quantum case when in (10) the average photon number tends to infinite (semiclassical limit), i.e., for a phase state.

When $\Delta\neq 0$ it is easily shown that $\langle \sigma_3(t)\rangle$ is constant if the initial state of the atom is such that either $q=0$ and $p_g=\frac{1}{2}$ or $(1-2p_g)\xi=2q\Delta$ and $\rho_{ge}(0)=qe^{-i\theta}$. The first conditions are the same studied in the beginning of this section when $\langle n\rangle\to\infty$, and the second ones are the same for a coherent state in the semiclassical limit too.

IV. TIME EVOLUTION OF THE SQUEEZING

We have found in Sec. III that all the diagonal elements of the eigenstates of the Susskind-Glogower phase operator remain constant for a certain initial state of the atom, when the atom and the field are on resonance. In this section we discuss the magnitude and evolution of the possible squeezing, given the appropriate parameters for these states. The quadratures of the field are defined as

$$a_1=a+a^{\dagger}\,, \qquad \text{(23a)}$$

$$a_2=i(a-a^{\dagger})\,. \qquad \text{(23b)}$$

For the state given by Eq. (16b) we have for the variances in a_1 and a_2,

$$\langle(\Delta a_1)^2\rangle-1=\frac{2|z|^2}{1-|z|^2}+2|z|^2(1-|z|^2)\cos(2\phi)\sum_{n=0}^{\infty}|z|^{2n}\sqrt{n+1}\sqrt{n+2}-\left[2|z|(1-|z|^2)\cos(\phi)\sum_{n=0}^{\infty}|z|^{2n}\sqrt{n+1}\right]^2, \qquad \text{(24a)}$$

$$\langle(\Delta a_2)^2\rangle-1=\frac{2|z|^2}{1-|z|^2}-2|z|^2(1-|z|^2)\cos(2\phi)\sum_{n=0}^{\infty}|z|^{2n}\sqrt{n+1}\sqrt{n+2}-\left[2|z|(1-|z|^2)\sin\phi\sum_{n=0}^{\infty}|z|^{2n}\sqrt{n+1}\right]^2, \qquad \text{(24b)}$$

Note that $\langle(\Delta a_1)^2(\phi)\rangle=\langle(\Delta a_2)^2(\phi+\pi/2)\rangle$ and the dependence on ϕ is through $\cos(2\phi)$. So we may restrict ourselves to study $\langle(\Delta a_1)^2(\phi)\rangle$ in the first quadrant. For $\phi=\pi/2$ we get

$$\langle(\Delta a_1)^2(\pi/2)\rangle-1 =2|z|^2\left[\frac{1}{1-|z|^2}-(1-|z|^2)\times\sum_{n=0}^{\infty}|z|^{2n}\sqrt{n+1}\sqrt{n+2}\right], \qquad \text{(25a)}$$

and it may be written as

$$\langle(\Delta a_1)^2(\pi/2)\rangle-1 =-2|z|^2(1-|z|^2)\times\sum_{n=0}^{\infty}|z|^{2n}[\sqrt{n+1}\sqrt{n+2}-(n+1)]<0\,. \qquad \text{(25b)}$$

Thus there is squeezing in the field. We see that for $\phi=\pi/4$

$$\langle(\Delta a_1)^2(\pi/4)\rangle-1=\langle(\Delta a_2)^2(\pi/4)\rangle-1 =2|z|^2\langle[\Delta(\sqrt{a^{\dagger}a+1})]^2\rangle>0\,, \qquad \text{(26)}$$

and there is not any quadrature of the field squeezed. For $\phi=0$, $\langle(\Delta a_2)^2(0)\rangle$-1 is given by (25b), and provided $\langle(\Delta a_1)^2\rangle\langle(\Delta a_2)^2\rangle\ge 1$, a_1 cannot exhibit squeezing. In the Fig. 2 we have plotted $\langle(\Delta a_1)^2\rangle-1$ as a function of ϕ and $|z|^2$. Note that the squeezing in a_1 increases (decreases) when $|z|^2$ increases for $\phi\sim\pi/2$ ($\phi\sim 0$). In fact, for a phase state with $\phi=\pi/2$, the quadrature a_1 is completely squeezed.

Now we investigate how the properties of squeezing evolve when the field in the state defined in (16b) interacts with an atom in the state given by (16a). We consider only the quadrature a_1 because the squeezing in a_2 is obtained from the same expressions that for a_1, by changing

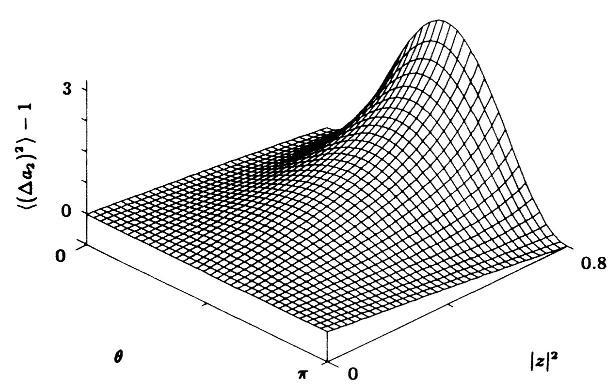

FIG. 2. Squeezing in the quadrature a_2 for a state defined in (16b) as a function of the mean photon number and the phase of the field. Note that squeezing increases (decreases) with the mean photon number and for $\theta\sim 0$ ($\theta\sim\pi/2$).

θ by $\theta+\pi/2$. In terms of the photon operators we get for the squeezing in a_1 the expression

$$\langle[\Delta a_1(t)]^2\rangle-1=2\langle a^\dagger a\rangle+2\,\mathrm{Re}\langle a^2(t)\rangle -[2\,\mathrm{Re}\langle a(t)\rangle]^2\;, \qquad (27)$$

and, after some lengthy algebra, we find

$$\langle a^\dagger a\rangle=\frac{|z|^2}{1-|z|^2}\;, \qquad (28a)$$

$$\mathrm{Re}\langle a\rangle=\frac{1-|z|^2}{1+|z|^2}|z|\sum_{n=0}^{\infty}|z|^{2n}(\sqrt{n+1}+\sqrt{n}\,)\times\cos[(\lambda_{n-1}-\lambda_n)t+\phi]\;, \qquad (28b)$$

$$\mathrm{Re}\langle a^2\rangle=\frac{1-|z|^2}{1+|z|^2}|z|^2\times\sum_{n=0}^{\infty}|z|^{2n}[\sqrt{(n+2)(n+1)}+\sqrt{(n+1)n}\,]\times\cos[(\lambda_{n-1}-\lambda_{n+1})t+2\phi]\;, \qquad (28c)$$

with λ_n is defined in (5b). In Figs. 3(a) and 3(b) we have plotted the evolution of the squeezing in the cavity for different values of $|z|$ and $\phi=0$ ($\phi=\pi/2$). We see that when $|z|$ is small, i.e., for low values of $\langle n\rangle$, the field is periodically squeezed, with the same period for all values of ϕ. For large values of $\langle n\rangle(|z|\sim 1)$, after the initial squeezing diappears, the oscillations of $\langle(\Delta a_1)^2\rangle$ have the same period and are far from the value of squeezing independently of the initial phase of the field. This coincides with the results obtained in Ref. 27, where the initial state of the field was taken to be a squeezed vacuum, and differ with those obtained for a coherent state in Ref. 4. When the mean photon number is small, the atom and the field are almost decoupled; the atomic dipole and the coherences of the field oscillates with the Rabi frequency, the amplitude of the oscillations becomes small, and, after each oscillation, the initial situation is reached. For a large mean photon number in the initial field, the atomic dipole and the coherences of the field are strongly coupled through their phases and the first modifies the second, destroying the squeezing of the field.

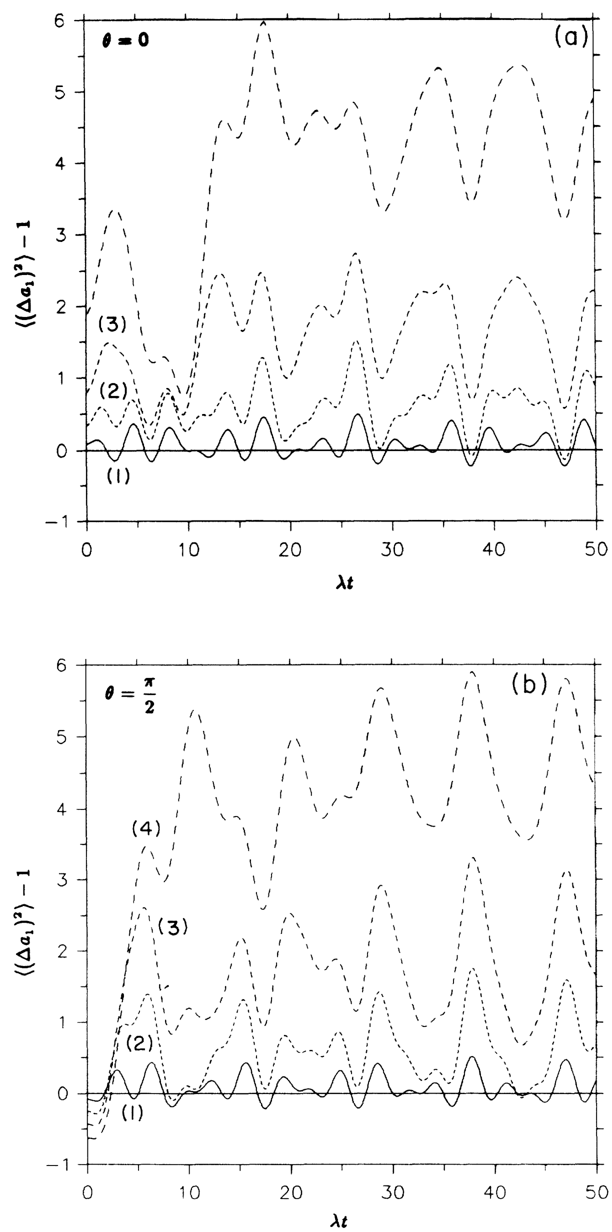

FIG. 3. Evolution of the squeezing in a_1 as a function of the interaction time for the state defined in (16b). The values of $|z|^2$ are (1) 0.1, (2) 0.3, (3) 0.5, (4) 0.7. The phase of the field is 0 for (a) and $\pi/2$ for (b).

V. CONCLUSIONS

The atomic inversion in the semiclassical JCM does not evolve when the system is prepared in a dressed state. In the case of a two-level atom, initially in a coherent superposition of the ground and excited states, interacting with a coherent state of the field, the atomic inversion almost remains constant for a certain choice of the relative phase between the atomic dipole and the cavity field, and hence coherent trapping occurs. We have shown that, when the initial state of the field is an eigenstate of the Susskind-Glogower phase operator and the phases of the field and the dipole moment are identical, not only the atomic inversion remains constant, but so does every diagonal element of the density matrix of the atom-field system expressed in the basis $|n,\alpha\rangle$, and hence exactly coherent trapping occurs. This can be interpreted as the result of the existence of thermodynamic equilibrium between the atom and the field and a destructive interference between the atomic dipole and the electric field. Other states of the atom-field system can maintain the atomic inversion constant, but they are mixed or they are not decoupled.

The degree of squeezing strongly depends on the initial phase of the atomic dipole for short times, but it does not for long times, when the initial state for the field is that given in (16b) and for the atom an appropriate coherent superposition of its states. Given the well-known behavior of the squeezed vacuum, only for low values of the average photon-number the field is squeezed at any times beyond the first few instants. This is in contrast to the squeezing obtained for the evolution of a coherent state in the JCM.

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