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Compression and frequency up-conversion of an ultrashort ionizing pulse in a plasma

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The propagation of an ultrashort ionizing electromagnetic pulse in a uniform plasma is investigated. It is shown that together with frequency up-conversion there is a strong compression of the pulse. This phenomenon occurs due to an alternative mechanism involving self-steepening and field collapse with wave energy trapping into a singularity.

The recent progress in relativistic electronics[1] and femtosecond laser techniques[2] has stimulated investigations on the interaction of ultrashort electromagnetic pulses with matter. The form "ultrashort," of course, has to be defined separately for each frequency range. However, in a physical sense, this term characterizes a number of processes with evolution time scales that are smaller than the relaxation time for the medium. The nonlinear response turns out to be nonstationary. After the passage of the pulse, the medium will remain in the excited state for a long time. In that sense, with respect to the ultrashort time interval, the medium has a long memory.

One of the main nonlinear processes that dominates at high intensities of the radiation concerns the ionization of a medium by a short electromagnetic pulse. Previously, most interest in the theoretical and experimental research was focused on studies of the frequency spectrum of the radiation that ionizes the medium, at first the phenomenon of supercontinuum generation[3] and recently the strong frequency up-conversion of a laser-pulse propagating through a gas.[4] Contrary to these studies, the present paper will discuss the possibility of a significant change in the spatial structure of a pulse traveling through, and ionizing, a medium. In particular, it will be shown that during this additional ionization of a previously created plasma, a new nonlinear process will take place. It simultaneously combines the phenomena of self-steepening and wave collapse, resulting in a continuous narrowing of the pulse and wave-energy concentration in extremely compressed field bunches.

In order to understand the process of self-steepening together with electromagnetic energy capture, we introduce a perturbation in the group velocity of a wave in a plasma, i.e.,

$$\delta v_g = \delta(c(1-\omega_p^2/\omega^2)^{1/2}) = -\frac{c\omega_{p0}^2}{\omega_0^2(1-\omega_{p0}^2/\omega_0^2)^{1/2}}\left[\frac{\delta N}{2N_0}-\frac{\delta\omega}{\omega_0}\right], \quad (1)$$

where ω is the wave frequency, ω_p is the plasma frequency, c is the velocity of light, N is the plasma density, and index zero denotes unperturbed values. Both the electron density and the radiation frequency increase due to the ionization.[5] The frequency increment accumulates over the propagation path, whereas the maximum density perturbations are defined by the local action of the electric field when the pulse passes through a given point. Therefore, the density perturbations will be comparatively small for sufficiently short wave packets. In this situation, even in the absence of initial frequency modulations, the second term in (1) dominates, which means that the group velocity increases with frequency. For wave-packet collapse with nonzero energy capture, a mechanism for the nonlinearity which ensures a linear frequency self-modulation within the packet is needed. Because the frequency increase is proportional to the density growth rate, the ionization nonlinearity can play this role. It is, however, necessary to choose an optimal initial shape of the pulse for each specific type of ionization.

In order to investigate the nonlinear dynamics of the interaction we now introduce a model which will lead to a general description for both the electron impact and the field ionization processes. For this purpose we propose that the variation in plasma density during the time the pulse passes through a given point is small, i.e., $\delta N = N - N_0 \ll N_0$. Considering rapidly developing processes where relaxation and transport phenomena can be neglected, we can then use the relation

$$\frac{\partial N}{\partial t} = \nu_i N_m . \quad (2)$$

For electron impact ionization we should in (2) replace N_m by N_0, whereas for field ionization N_m stands for the neutral atom density. The ionization frequency is written in the form $\nu_i = \nu_0 f(|E|^2/E_0^2)$, where ν_0 is a constant, f is a dimensionless function which is supposed to be growing, i.e., $\partial f/\partial |E| > 0$, and E_0 is the characteristic field for which the ionization frequency changes significantly. In addition, we require that the ionization frequency is zero in the absence of the wave, i.e., $f(0)=0$. This model describes, for example, multiphoton (or tunneling) ionization. Besides, in the case of electron impact ionization, the model is also valid for some particular electron distribution functions.

Because the dielectric constant is almost unchanged within the pulse, all variations of the wave-packet parameters are important only at distances exceeding the initial pulse length. Significant modifications of the shape of the envelope are possible even for slight changes of the frequency. This fact permits us to adopt a simple parabolic equation for the electric-field envelope. Thus

$$-2ik_0\frac{\partial E}{\partial z}-\frac{(1-\epsilon)}{c^2\epsilon}\frac{\partial^2 E}{\partial\tau^2}+\frac{\omega_{p0}^2}{c^2}\frac{(N-N_0)}{N_0}E=0\,, \quad (3)$$

where $\epsilon\equiv 1-\omega_{p0}^2/\omega_0^2$ is the dielectric permittivity of the unperturbed plasma, and where the local time $\tau=t-z/v_{g0}$ is supposed to start from the time when the pulse arrives at a given point z in the plasma. Equation (3) neglects wave-energy dissipation connected with the ionization and the electron collisions. This is correct if the imaginary part of the dielectric permittivity is less than the variations in ϵ that are caused by the frequency up-conversion, i.e., ν/ω_0 has to be much smaller than $\delta\omega/\omega_0$, where ν is the effective dissipative frequency.

We now normalize τ by $(N_0/\epsilon N_m v_0\omega_0^2)^{1/3}$, z by $2c\omega_0^{5/3}\epsilon^{5/6}N_0^{2/3}/N_m^{2/3}\omega_p^2 v_0^{2/3}$, and introduce

$$n\equiv[(N-N_0)/N_m](N_0v_0^2/\epsilon N_m\omega_0^2)^{-1/3}$$

and $A\equiv E/E_0$. Equations (2) and (3) can then be rewritten in the dimensionless form

$$\frac{\partial n}{\partial\tau}=f(|A|^2) \quad (4)$$

and

$$-i\frac{\partial A}{\partial z}=\frac{\partial^2 A}{\partial\tau^2}-nA\,. \quad (5)$$

Let us now define the position in time of the intensity distribution center, as well as the shift of the central frequency, by means of the expressions

$$\bar\tau(z)=w_0^{-1}\int_{-\infty}^{\infty}dt\,t\,|A(z,t)|^2$$

and

$$\Delta\bar\omega=(2\pi w_0)^{-1}\int_{-\infty}^{\infty}d\Omega\,\Omega\,|A_\Omega|^2\,,$$

where $A_\Omega=\int_{-\infty}^{\infty}dt\,A\exp(i\Omega t)$ is the Fourier spectrum of the electric-field envelope. The pulse energy $w_0=\int_{-\infty}^{\infty}dt\,|A|^2$ is conserved in the parabolic approximation. These integrals are related as

$$\frac{d}{dz}\Delta\bar\omega=-\frac{1}{2}\frac{d^2\bar\tau}{dz^2}=\frac{1}{w_0}\int_{-\infty}^{\infty}dt\,|A(z,t)|^2 f(|A(z,t)|^2)\,. \quad (6)$$

Thus, the rate of central frequency shift along the z coordinate is proportional to the intensity center "acceleration." The signs of these values are fixed for the ionization nonlinearity. With increasing frequency the electromagnetic pulse travels with increasing velocity.

The possibilities for pulse compression during the acceleration process can be studied within the nonlinear geometrical optics approximation. By writing the complex electric-field amplitude in the form $A(z,t)=\psi\exp(-i\phi)$, we obtain a system of two equations for the two real functions $\rho=\psi^2/2$ and $V=-2\partial\phi/\partial\tau$. Thus

$$\frac{\partial\rho}{\partial z}+\frac{\partial}{\partial\tau}(\rho V)=0 \quad (7)$$

and

$$\frac{\partial V}{\partial z}+V\frac{\partial V}{\partial\tau}\approx-2f(2\rho)\,. \quad (8)$$

Equations (7) and (8) are valid if the wave amplitude is not too small. Specifically, we require that $f(2\rho)\gg\partial(\rho^{-1/2}\partial^2\rho^{1/2}/\partial\tau^2)/\partial\tau$. This inequality is not satisfied at the periphery of the field distribution where the solutions of (7) and (8) must be matched to those of the full parabolic equations.

If we imagine that the space and time coordinates in Eqs. (7) and (8) are interchanged, we describe a one-dimensional ideal-fluid model. The right-hand side in (8) can then be interpreted either as an external force of the form $-2\rho f$ or as an internal "gas" pressure, $P=2\int d\tau\rho f$, that is nonlocally related to the density. Due to this nonlocality, Eqs. (7) and (8) do not have Riemann wave solutions. However, similar to what happens in ideal-gas dynamics, the nonlinear evolution is characterized by the formation of singularities. When the frequency within a packet, corresponding to the gas velocity V, is a linear function of τ, i.e, $V\sim(\tau-\tau_0)$, the field distribution condenses at the point $\tau=\tau_0$ whereas z grows so that a finite part of the wave energy is captured within the singularity, i.e., the gas density approaches infinity locally.

Self-similar collapsing solutions with increasing linear frequency self-modulation can be found if the ionization frequency has a power-law dependence on the amplitude, i.e., $f=\rho^p$, where $p>0$. The solutions can then be written as $\rho=(\tau-\tau_0)^{1/p}u(z)$ and $V=-(\tau-\tau_0)v(z)$, where the functions $u(z)$ and $v(z)$ satisfy the equations

$$\frac{du}{dz}=[(p+1)/p]uv \quad (9)$$

and

$$\frac{dv}{dz}=v^2+2u^p\,. \quad (10)$$

Below we shall just analyze the case $p=1$. The intensity distribution then has an area conserved triangular form. The solution with the boundary conditions $u(z_0)=u_0$ and $v(z_0)=0$ is

$$(2u_0)^{1/2}(z-z_0)=\int_0^{[(1/2)\ln(u/u_0)]^{1/2}}ds\exp(-s^2)\,. \quad (11)$$

Near the coordinate $z=z^*=z_0+(\pi/8u_0)^{1/2}$ where collapse occurs, we have

$$u(z)\approx\{8(z^*-z)^2\ln[1/(z^*-z)]\}^{-1}$$

and $v(z)\approx(z^*-z)^{-1}$. The duration T of the pulse and the central frequency shift $\Delta\bar\omega$ are then determined by the asymptotic expressions

$$T(z\to z^*)=(z^*-z)\{8w_0\ln[1/(z^*-z)]\}^{1/2} \quad (12)$$

and

$$\Delta\bar\omega(z\to z^*)=(2^{3/2}/3)\{w_0\ln[1/(z^*-z)]\}^{1/2}\,. \quad (13)$$

Now we must reconsider the conditions for the geometrical optics approximation, and by means of the self-similar solution, the scale for the critical-field localization. It then turns out that the dispersive term in (8) cannot be neglected at the front of the pulse where $(\tau_0-\tau)^4 \lesssim 1/2u$. During the field collapse the approximation improves, but as the pulse duration T simultaneously decreases as $(w_0/u)^{1/2}$, a limitation on the minimum pulse duration is $T_{\min} \approx 1/(2w_0)^{1/2}$. The maximum frequency shift for the critical compression remains finite, $\Delta\bar{\omega}_{\max} \sim [w_0 \times \ln(16w_0)]^{1/2}$, so that our parabolic approximation is correct. Similar estimates of the critical values can be made for any power dependence $(p \neq 1)$ of the ionization frequency.

In addition to the collapsing structures discussed above, there are also self-similar solutions to Eqs. (7) and (8). They describe extending triangular wave intensity packets with linear frequency self-modulation. In this case the expressions for $\bar{\tau}$ and the frequency shift can be written in the asymptotic $(z \to \infty)$ form

$$\bar{\tau}(z) \approx (4\sqrt{2}/3) w_0^{1/2} z \ln z \tag{14}$$

and

$$\Delta\bar{\omega}(z) \approx \tfrac{2}{3} (2w_0 \ln z)^{1/2} . \tag{15}$$

We have also complemented our analytical work by means of a numerical analysis of the evolution of a Gaussian wave packet, using Eqs. (4) and (5) with $f(|A|^2) = |A|^2$. The behavior of the front part of the pulse then confirmed the above analysis, and thus described a collapse of the intensity distribution with capture of about 30% of the wave energy within a very short time scale. The dispersive term stopped the self-compression process at $T_{\min} \approx T_0/15$, and was also responsible for a slight modulational instability. The dispersive effects of the plasma on the lengthening of the pulse were of little importance. The pulse evolution beyond the coordinate z^* where collapse occurs, resembled qualitatively the dynamics of an expanding triangular packet formed by the remaining part of the Gaussian pulse.

Finally, let us propose some experimental situations of relevance to our presented analysis. Considering free-electron laser radiation with wavelength $\lambda = 2\pi/k_0 = 1$ cm, pulse duration $T_0 = 3$ ns, and flux intensity 10^6 W/cm^2 in a plasma with $\nu_0 = 10^7$ s^{-1} and $\epsilon = 0.25$, we obtain a compression of the pulse up to 200 ps at a distance $z^* = 5$ m. Similarly, for a Ne laser with $\lambda = 1$ μm, $T_0 = 0.5$ ps, and flux intensity 10^{14} W/cm^2 in a gas with $\nu_0 = 10^{10}$ s^{-1}, $\epsilon = 0.3$, pressure 100 atm, and $N_m \approx N_0$, we find the values $T_{\min} = 30$ fs and $z^* = 0.2$ cm. Collisional absorption may begin to play a role in limiting the pulse power at higher densities than those above.

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