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Saha equation for a two-temperature plasma

Vivek Bakshi
Center for Energy Studies, University of Texas at Austin, Austin, Texas 78712
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A modified form of the Saha equation for a two-temperature plasma is presented, which accounts for the underpopulation of the excited states due to deviation from local thermodynamic equilibrium.

van de Sanden *et al.*[1] recently presented a thermodynamic generalization of the Saha equation for a two-temperature plasma under the assumption that the internal energy states of the heavy particles are governed by Boltzmann's law with $T=T_e$ [in the final result, Eq. (32), denominator should be g_0^0], and pointed out the invalidity of the multitemperature Saha equation (MSE).

If T_e determines the internal distribution of the heavy particles (neutral atoms and ions), summation of Boltzmann law over all excited states for a given species i yields

$$n_i^p=\left[\frac{g_i^p}{Z_i}\right]N_i\exp\left[\frac{-E_i^p}{k_bT_e}\right]\quad\{i=0,+\}\qquad(1)$$

where N_i is the total number of particles of species i and Z_i is the partition function of the internal states of species i. The rest of the notations are the same as in Ref. 1. E_i^p is the energy of the pth state of the ith species, measured from the ground state of the ith species. Substituting Eq. (1) in Eq. (32) of Ref. 1, the following equation for a plasma system composed of electrons, singly ionized ions, and neutral species is obtained:

$$\frac{N_eN_+}{N_0}=g_e\frac{Z_+}{Z_0}\left[\frac{2\pi m_ek_bT_e}{h^2}\right]^{3/2}\exp\left[\frac{-E_{\text{ion}}}{k_bT_e}\right],\qquad(2)$$

which is the Saha equation with $T=T_e$. It has recently been shown[2] that the predictions of Eq. (2) (method 1 in Ref. 2) and the predictions of MSE (method 3 in Ref. 2) for the calculation of heavy-particle temperature T_h, for a given T_e and N_e, lead to nonphysical results for plasma in partial local thermodynamic equilibrium (PLTE). The MSE is wrong in principle, whereas Eq. (32) of Ref. 1 can be used in such a way that correct results follow. This procedure is described as method 2 in Ref. 2.

In the conditions of a two-temperature plasma, deviations from LTE are indicated.[3–5] Then the Boltzmann distribution is valid among only those excited states whose principle quantum number is higher than a certain value, and only for small deviations from LTE.[6,7] Also, T_e alone does not represent the internal distribution of all the energy states of heavy particles and the population of pth level of ith species is given as

$$n_i^p=b_i^p\left[\frac{g_i^p}{Z_i}\right]N_i\exp\left[\frac{-E_i^p}{k_bT_e}\right]\quad\{i=0,+\}\qquad(3)$$

where b_i^p is called the underpopulation factor. Bakshi and Kearney[2] have recently measured b_i^p for the excited states of neutral argon in a plasma jet at atmospheric pressure. Existence of such underpopulation has also been shown by some other experimental[8,9] and theoretical works.[10–12] Drawin has calculated b_i^p for hydrogen and helium, reproduced in Ref. 4, and Biberman[3] for cesium. I suggest that the underpopulation of excited states is important and that such an effect should be included in the Saha equation for a two-temperature plasma as follows, by substituting Eq. (3) in (2):

$$\frac{N_en_+^p}{n_0^0}=g_e\frac{g_+^p}{g_0^0}\frac{b_+^p}{b_0^0}\left[\frac{2\pi m_ek_bT_e}{h^2}\right]^{3/2}\times\exp\left[\frac{-(E_+^p+E_{\text{ion}}-E_0^0)}{k_bT_e}\right].\qquad(4)$$

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