Instability of a strongly inhomogeneous plasma

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The stability of a plasma boundary in the presence of a powerful p-polarized electromagnetic wave is studied. It is shown that surface waves that are coupled to plasmons localized in the plasma-vacuum transition layer can be excited. The growth rate that is calculated turns out to be larger than the corresponding value obtained without taking nonlocal effects of the transition layer into account by a factor proportional to the wavelength of the pump wave divided by the width of the boundary region. The threshold is accordingly significantly lower. We believe that the present theory will be useful for the explanation of several nonlocal optical effects arising near the surface of solid-state plasmas.

I. INTRODUCTION

The stability properties of a plasma boundary in the presence of an intense p-polarized electromagnetic wave have been of interest for many years in connection with the problem of confinement of a dense high-temperature plasma by radiation.[1] Later, in connection with laser-fusion studies,[2] the same problems appeared when a powerful heating flux of radiation was applied. Near the critical point a jump in density then arose.[3] The interaction of radiation with strongly inhomogeneous plasmas is also of interest for probing ionospheric inhomogeneities.[4] Furthermore, development of the theory of nonlinear interaction between radiation and surfaces of solid-state plasmas is important for the diagnostics of solid-state surfaces.[5,6]

Most of the previous theoretical works consider the case when the electromagnetic pump was an s-polarized wave.[7] For the p-polarized wave the theory turns out to be more complicated and less developed.[8] We will show that it is necessary to take into account the finite width of the plasma-vacuum transition layer in which volume plasmons can be localized. In this article we will also demonstrate that the nonlinear parametric interaction of a p-polarized wave with the plasma boundary in the transition layer gives rise to the excitation of volume plasmons which are coupled to the surface waves. A nonlinear dispersion relation will be derived from which we calculate the growth rate and the threshold of the instability. The growth rate thus found is larger than the corresponding value obtained without taking nonlocal effects of the transition layer into account by a factor proportional to the wavelength of the pump wave divided by the width of the boundary region. The threshold is correspondingly lower.

II. DERIVATION OF THE DISPERSION RELATION

We consider a plasma with a nonuniform density profile $n_0(z)$ that can be divided into three regions. For $z<0$ there is vacuum and for $z>d$ the unperturbed density is constant. In the transition layer $0<z<d$, $n_0(z)$ may vary arbitrarily. However, we assume d to be small compared to the wavelengths involved in the problem. Furthermore, we neglect ion motion and consider a cold electron plasma. The latter approximation is possible if the Debye length of the electrons is much smaller than d. This condition is fulfilled for many plasmas.[9]

A plane p-polarized wave is supposed to be obliquely incident from $z=-\infty$. We define the x axis to be in the polarization plane of the pump wave. By using the z component of Ampere's law and the momentum equation together with Gauss law we easily find that

$$(\nabla\times\mathbf{B})_z=\frac{m}{qc^2}\left[\left[\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}\right]^2 v_z+\omega_p^2(z)v_z\right] \quad (1)$$

holds in the transition layer. Here m and q are the mass and charge of the electron, ω_p $[=(n_0q^2/\epsilon_0 m)^{1/2}]$ is the plasma frequency, c is the velocity of light, $\mathbf{B}$ the magnetic field, and v_z the z component of the velocity of the electrons. In deducing (1) we have, according to our assumption above, noted that $\partial/\partial z$ acting on v_z and E_z ($\mathbf{E}$ is the electric field) is much larger than $\partial/\partial x$ in the transition layer. Similarly we have

$$(\nabla\times\mathbf{B})_\perp=\frac{m}{qc^2}\left[\frac{\partial^2}{\partial t^2}+\omega_p^2(z)+\frac{\partial}{\partial z}\left[\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial z}\right]\right]\mathbf{v}_\perp\ , \quad (2)$$

where the index $\perp$ denotes the component perpendicular to the z axis. It turns out to be convenient to introduce Lagrangian coordinates z_0 and τ. Accordingly we define

$$z=z_0+\int_0^t v_z(z(t'),t')dt' \quad (3)$$

and

$$t=\tau\ . \quad (4)$$

In this coordinate system, Eqs. (1) and (2) are written

$$\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=\frac{m}{qc^2}\left[\frac{\partial^2 v_z}{\partial \tau^2}+\omega_p^2\left[z_0+\int_0^\tau v_z dt'\right]v_z\right] \tag{5}$$

and

$$\hat{\mathbf{Z}}\times\frac{\partial \mathbf{B}_\perp}{\partial z_0}+\nabla_\perp\times\mathbf{B}_z =\frac{m}{qc^2}\left[\frac{\partial^2}{\partial\tau^2}+\omega_p^2\left[z_0+\int_0^\tau v_z d\tau'\right]+\frac{\partial^2 v_z}{\partial z_0\partial\tau}\right]\mathbf{v}_\perp , \tag{6}$$

where $\hat{\mathbf{Z}}$ is the unit vector in the z direction. We will use the lowest-order expansion of these equations, overlooking the case where the strongly nonlinear electron response, due to the steep density profile of a CO_2-laser-produced plasma, causes the generation of high harmonics of radiation.[10] Accordingly we assume that, in addition to the pump wave with frequency ω_0 and parallel wave number k_{0x}, there exist wave perturbations with indices 1 and 2 satisfying the matching conditions

$$\omega_0=\omega_1+\omega_2 , \tag{7}$$

$$k_{0x}=k_{1x}+k_{2x} , \tag{8}$$

and

$$k_{1y}+k_{2y}=0 . \tag{9}$$

Representing all quantities as

$$v_z=\sum_{j=0}^{2} v_{jz}(z_0)e^{i(k_{jx}x+k_{jy}y-\omega_j\tau)}+\text{c.c.} ,$$

etc., where c.c. denotes complex conjugate, we write Eqs. (5) and (6) as

$$ik_{1x}B_{1y}-ik_{1y}B_{1x}=-\frac{m}{qc^2}\left[\omega_1^2\epsilon_1(z_0)v_{1z}+\frac{i\omega_1}{\omega_2^*\omega_0}\frac{\partial\omega_p^2}{\partial z_0}(z_0)v_{2z}^*v_{0z}\right] \tag{10}$$

and

$$\hat{\mathbf{Z}}\times\frac{\partial \mathbf{B}_{1\perp}}{\partial z_0}+i\mathbf{k}_{1\perp}\times\mathbf{B}_{1z}=-\frac{m}{qc^2}\left[\omega_1^2\epsilon_1(z_0)\mathbf{v}_{1\perp}-i\frac{\partial\omega_p^2}{\partial z_0}\left[\frac{v_{0z}\mathbf{v}_{2\perp}^*}{\omega_0}-\frac{v_{2z}^*\mathbf{v}_{0\perp}}{\omega_2^*}\right]+i\omega_0\frac{\partial v_{0z}}{\partial z_0}\mathbf{v}_{2\perp}^*-i\omega_2^*\frac{\partial v_{2z}^*}{\partial z_0}\mathbf{v}_{0\perp}\right] , \tag{11}$$

where ϵ_1 $(=1-\omega_p^2(z_0)/\omega_1^2)$ is the dielectric function of the cold plasma and $\mathbf{k}_{1\perp}=k_{1x}\hat{\mathbf{x}}+k_{iy}\hat{\mathbf{y}}$. When deriving (10) and (11) we have neglected terms proportional to the square of the pump field. In the equations above and elsewhere we can use $v_{0z}(z_0)=v_{0z}(z)$ since higher harmonics of the pump wave are unimportant. The same approximation is *not* possible to make for the perturbations, however, and therefore our change of coordinate system is meaningful. The advantage with the Lagrangian coordinates is that Eq. (10) is very simple. In contrast, in Eulerian coordinates, additional terms proportional to $(\partial v_{2z}/\partial z_0)v_{0z}$ appear in the corresponding equation. This difference is of importance since near the zero of ϵ_1 the nonlinear terms are of the same order as the linear. A perturbational method is thus not applicable. Furthermore, the additional terms appearing in Eulerian coordinates makes the exact solution very complicated and presumably impossible to handle analytically in the further calculations.

Proceeding by making use of the law of generalized vortex conservation

$$i\mathbf{k}_{1\perp}\times\mathbf{v}_{1z}+\hat{\mathbf{Z}}\times\frac{\partial\mathbf{v}_{1\perp}}{\partial z_0}+\frac{q}{m}\mathbf{B}_{1\perp}=0 , \tag{12}$$

we find that $B_{1\perp}$ $[=(B_{1x}^2+B_{1y}^2)^{1/2}]$ satisfies

$$\left[\frac{\omega_1^2}{c^2}-\frac{k_{1\perp}^2}{\epsilon_1}\right]B_{1\perp}+\frac{\partial}{\partial z_0}\left[\frac{1}{\epsilon_1}\frac{\partial B_{1\perp}}{\partial z_0}\right]=\frac{m}{qc^2}\frac{\partial}{\partial z_0}\left\{\frac{\mathbf{k}_{1\perp}}{k_{1\perp}\epsilon_1}\cdot\left[i\frac{\partial\omega_p^2}{\partial z_0}\left[\frac{v_{2z}^*\mathbf{v}_{0\perp}}{\omega_2^*}-\frac{\mathbf{v}_{2\perp}^*v_{0z}}{\omega_0}\right]+i\omega_0\mathbf{v}_{2\perp}^*\frac{\partial v_{0z}}{\partial z_0}-i\omega_2^*\frac{\partial v_{2z}^*}{\partial z_0}\mathbf{v}_{0\perp}\right]\right\}+\frac{m\omega_1 k_{1\perp}}{qc^2\omega_2^*\omega_0\epsilon_1}\frac{\partial\omega_p^2}{\partial z_0}v_{2z}^*v_{0z} . \tag{13}$$

Integrating Eq. (13) twice across the transit layer, dropping terms proportional to $k_\perp d$, except the term which corresponds to the linear damping, we obtain

$$D_1B_{1\perp}(d)+\frac{\omega_0[k_{2\perp}^2-(\omega_2^{*2}/c^2)]^{1/2}\kappa v_{0z}(d)[\mathbf{k}_{1\perp}\times\mathbf{B}_{2\perp}^*(d)]_z}{k_{1\perp}\omega_2^{*2}}=-\frac{mk_{1\perp}\omega_1}{qc^2\omega_2^*\omega_0}\int_0^d\frac{\partial\omega_p^2}{\partial z_0}\frac{v_{2z}^*v_{0z}}{\epsilon_1}dz_0 , \tag{14}$$

where

$$\kappa=\left[k_{0x}^2-\frac{\omega_0^2\epsilon(\omega_0,d)}{c^2}\right]^{1/2}+\epsilon(\omega_0,d)\left[k_{0x}^2-\frac{\omega_0^2}{c^2}\right]^{1/2} , \tag{15}$$

and

$$D_1=\frac{(k_{1\perp}^2-\omega_1^2\epsilon_1(d)/c^2)^{1/2}}{\epsilon_1(d)}+\left[k_{1\perp}^2-\frac{\omega_1^2}{c^2}\right]^{1/2}-k_{1\perp}^2\int_0^d\frac{dz_0}{\epsilon_1} \tag{16}$$

is the dispersion function for surface waves. Using Eq. (14) together with the formula obtained by permuting the indices in that equation we find

$$\left[D_1D_2^*-\frac{[k_{1\perp}^2-(\omega_1^2/c^2)]^{1/2}[k_{2\perp}^2-(\omega_2^{*2}/c^2)]^{1/2}(\mathbf{k}_{1\perp}\cdot\mathbf{k}_{2\perp})^2\omega_0^2\kappa^2|v_{0z}|^2}{k_{1\perp}^2k_{2\perp}^2\omega_1^2\omega_2^{*2}}\right]B_{1\perp}$$
$$=-\frac{mk_{1\perp}\omega_1D_2^*}{qc^2\omega_2^*\omega_0}\int_0^d\frac{\partial\omega_p^2}{\partial z_0}\frac{v_{2z}^*v_{0z}}{\epsilon_1}dz_0+\frac{m\mathbf{k}_{1\perp}\cdot\mathbf{k}_{2\perp}[k_{1\perp}^2-(\omega_1^2/c^2)]^{1/2}\kappa v_{0z}}{qc^2k_{2\perp}\omega_1\omega_2^*}\int_0^d\frac{\partial\omega_p^2}{\partial z_0}\frac{v_{1z}v_{0z}^*}{\epsilon_2^*}dz_0\ . \tag{17}$$

Furthermore, from (10) and the corresponding equation for index 2 we have

$$\left[\epsilon_1\epsilon_2^*-\left[\frac{\partial\omega_p^2}{\partial z_0}\right]^2|v_{0z}|^2\Big/\omega_0^2\omega_1^2\omega_2^{*2}\right]v_{2z}^*$$
$$=\frac{qc^2}{m\omega_2^*}\left[\frac{ik_{2\perp}\epsilon_1B_{2\perp}^*(d)}{\omega_2^*}+\frac{k_{1\perp}(\partial\omega_p^2/\partial z_0)v_{0z}^*B_{1\perp}}{\omega_1^3\omega_0}\right], \tag{18}$$

where we have approximated $B_{1\perp}$ and $B_{2\perp}$ by constants in the boundary region. The left-hand side of Eq. (18) describes the nonlinear decay of the pump wave into plasmons localized in the transition layer, whereas the right-hand side is due to the coupling to the global surface waves. Similarly the left-hand side of Eq. (17) describes the decay into surface waves and the right-hand side represents the coupling to the plasmons. If we disregard the right-hand side terms in (17) the resulting expression agrees qualitatively with previous works for plasmas with sharp boundaries.[11] We have here neglected terms of the same order as the nonlinear terms of the left-hand side of Eq. (17) when deriving (13). The agreement with the earlier papers is thus not exact. Our reason for keeping such terms in (17) was just to make the physical picture of the present process clearer. Instead we have to use the opposite approximation, i.e., we omit all the nonlinear terms on the left-hand side of Eq. (17), since we have the maximum growth rate for perturbations having equal frequencies $\omega_r=\text{Re}\omega_1=\text{Re}\omega_2$. In this case the nonlinear terms in (17) that represent the coupling to the plasmons are larger than those of previous works by a factor of the order of $1/kd$, as will be demonstrated below. Thus we note that the results obtained in the sharp boundary limit cannot be recovered when d approaches zero. The reason is that it is very difficult to analyze such limits in a proper way since the excursion length of the electrons in the boundary layer then cannot be treated as a small parameter.

Considering the situation when $\text{Re}\omega_1=\text{Re}\omega_2$, we find from (17), (18), and the corresponding equations with permuted indices the dispersion relation

$$D_{1n}D_{2n}^*=\frac{k_{1\perp}^2k_{2\perp}^2}{4\omega_r^6}\left|\int_0^d\frac{(\partial\omega_p^2/\partial z_0)v_{0z}dz_0}{\epsilon_1\epsilon_2^*-(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2/4\omega_r^6}\right|^2, \tag{19}$$

where

$$D_{1,2n}=D_{1,2}+\frac{k_{1,2\perp}^2}{4\omega_r^6}\int_0^d\frac{(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2dz_0}{\epsilon_{1,2}[\epsilon_1\epsilon_2^*-(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2/4\omega_r^6]}\ . \tag{20}$$

Formally we have introduced an ambiguity by considering the case $\text{Re}\omega_1=\text{Re}\omega_2$ since the poles of the integrals in Eqs. (18) and (19) now will appear on the real z_0 axis for certain parameters values. If we instead let $\text{Re}\omega_1=\text{Re}\omega_2+\Delta\omega$ and we take the limit $\Delta\omega\to0$, this difficulty could be handled, although the sign of the nonlinear part of the dispersion function in (20) may depend on whether $\Delta\omega\to0$ from the plus or minus side. The solutions to the dispersion relation are of course not affected by this choice.

To solve the integrals in (19) and (20) we first observe that the main contribution comes from z_0 values close to the resonant surface z_r defined by $\epsilon(\omega_r,z_r)=0$. This allow us to extend the limits of the integrals to infinity, use $\epsilon_{1,2}=(\partial\omega_p^2/\partial z_0)(z_0-z_r)/\omega_r^2+2i\gamma/\omega_r$, where $\gamma=\text{Im}\omega_{1,2}$, and approximate the other functions of z_0 by constants. The dispersion relation (19) then reduces to

$$(D_1-k_{1\perp}^2A)(D_2^*+k_{2\perp}^2A)-|F^2|=0\ , \tag{21}$$

where

$$A=\frac{\pi(\partial\omega_p^2/\partial z_0)|v_{0z}|^2}{16\omega_r^2((\gamma/\omega_r)-i\{[(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2/16\omega_r^6]-(\gamma^2/\omega_r^2)\}^{1/2})}\frac{1}{\{[(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2/16\omega_r^6]-(\gamma^2/\omega_r^2)\}^{1/2}} \tag{22}$$

and

$$F^2=\frac{\pi k_{1\perp}^2k_{2\perp}^2|v_{0z}|^2}{16[\gamma^2-(\partial\omega_p^2/\partial z_0)^2|v_{0z}|^2/16\omega_r^6]}\ . \tag{23}$$

In the formulas above all z_0-dependent functions are to be evaluated at the resonance surface $z_0 = z_r$. Furthermore, $v_{0z}(z_r)$ is given by

$$v_{0z}(z_r) = \frac{2iqck_{0x}E_0}{m\omega_0^2\epsilon(\omega_0, z_r)} \left[1 + \frac{\{[\omega_0^2\epsilon(\omega_0, z>d)/c^2] - k_{0x}^2\}^{1/2}}{\epsilon(\omega_0, z>d)\{[\omega_0^2\epsilon(\omega_0, z<0)/c^2] - k_{0x}^2\}^{1/2}} \right]^{-1}, \tag{24}$$

where E_0 is the electric-field strength of the incident pump wave.

To find the solutions of the dispersion relation we first observe that for $\gamma^2 \approx (\partial\omega_p^2/\partial z_0)^2 |v_{0z}|^2/16\omega_r^4$, (21) reduces to

$$D_1 D_2^* \{[(\partial\omega_p^2/\partial z_0)^2 |v_{0z}|^2/16\omega_r^6] - \gamma^2\}^{1/2} + \frac{\pi(k_{2\perp}^2 D_1 - k_{1\perp}^2 D_2^*)|v_{0z}|}{8} = 0 . \tag{25}$$

If both D_1 and D_2^* are nonresonant, the last term is a factor of order $k_\perp d$ smaller than the first term and the approximate expression for the growth rate used to derive (25) is thus valid. The mechanism of this instability is the same as for the two-plasmon decay.[12] However, previous authors considering this process in inhomogeneous plasmas[13] have assumed the opposite ordering of wavelengths and inhomogeneity scale lengths. Thus our result described by Eq. (25) is new.

If we had included electron-ion collisions, then γ would have been replaced by $\gamma + \nu_{e\text{-}i}/2$, where $\nu_{e,i}$ is the electron-ion collision frequency. The threshold electric field could then be determined from Eq. (25) together with (24). Considering now the case when both perturbations are resonant, i.e., $D_{1,2}(\omega_0/2, k_{0x}/2, k_{1,2y}) = 0$, we find that (21) reduces to

$$\frac{\partial D_1}{\partial\omega_1}(\gamma - \gamma_d) = -\frac{\pi k_{1\perp}^2 |v_{0z}|}{4\{\gamma^2 - [(\partial\omega_p^2/\partial z_0)^2 |v_{0z}|^2/16\omega_r^6]\}^{1/2}} \left[1 + \frac{(\partial\omega_p^2/\partial z_0) v_{0z}|}{4\omega_r^2(\gamma + \{\gamma^2 - [(\partial\omega_p^2/\partial z_0)^2 |v_{0z}|^2/16\omega_r^2]\}^{1/2})} \right], \tag{26}$$

where the linear damping rate of the surface wave γ_d is

$$\gamma_d = -\frac{k_{1\perp}^2}{\partial D_1/\partial\omega_1} \mathrm{Im} \int_0^d \frac{dz_0}{\epsilon_1} . \tag{27}$$

We cannot solve (26) analytically. However, it is clear that we now have a slightly higher growth rate than for the nonresonant case and that it is increasing with $k_{1\perp}^2$. Furthermore, for large $k_{1\perp}^2$ the perturbations are electrostatic. The maximum value of γ is obtained for $k_{1\perp} \gg (\partial\omega_p^2/\partial z_0)^2 |v_{0z}|/\omega_r^5$. Numerically we find that the growth rate here may be enhanced by a factor ≈ 1.7 as compared to the nonresonant case, i.e.,

$$\gamma_m \approx \frac{1.7}{4\omega_r^2} \frac{\partial\omega_p^2}{\partial z_0} |v_{0z}| , \tag{28}$$

where γ_m is the maximum value of the growth rate. We note that this means that, since we do not have $\gamma \gg (\partial\omega_p^2/\partial z_0)|v_{0z}|/\omega_r^2$ for any k values, the excitation of surface waves for the present conditions is always coupled to the two-plasmon decay process as can be seen from Eq. (26). We stress that it is not sufficient to take the linear damping of the surface waves into account to obtain the threshold of this instability, as electron-ion collisions also must be included. The result for the threshold is

$$|v_{0z}|_{\mathrm{thr}} \approx \frac{2\omega_r^2 \nu_{e\text{-}i}}{1.7(\partial\omega_p^2/\partial z_0)} , \tag{29}$$

as follows from (28).

III. CONCLUSIONS

In this paper we have found an instability mechanism of a diffuse plasma boundary excited by electromagnetic radiation. The thickness of the transition layer has been assumed to be much smaller than the wavelength of the pump wave. Nevertheless, in this layer, localized volume plasma waves can be parametrically excited. We have shown that this microprocess is coupled to the global surface wave instability. The growth rate that we have found is much larger than the well-known result for the surface wave instability when the boundary is sharp. We have only considered the special case of two-plasmon decay in the cold-plasma approximation when the plasmons are localized. Evidently, it would be very interesting to take thermal effects into account to introduce space dispersion of the plasmons and the possibility of ion-sound propagation. In the future we intend to investigate the connection of other microinstabilities in the transition layer (induced Brillouin scattering, induced Raman scattering, decay into ion-sound and plasma waves, aperiodical two-stream instability) with global surface wave for both polarizations of the pump wave. We believe that the small threshold value for excitation of surface waves that can be radiated due to scattering by a rough boundary due to the new mechanism pointed out here may provide the main explanation of several anomalous optical phenomena such as enlarged Raman scattering[6] connected with surface effects. However, for solid-state plasmas our theory can only be considered as a prerequisite to improved models including quantum mechanical as well as thermal effects.[5]

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