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Effect of collisions and impurities on the stability of oblique modulation of ion acoustic waves

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A nonlinear Schrödinger equation that governs the nonlinear interaction of a quasistatic plasma slow response with ion acoustic waves for a two-ion collisional plasma is derived. The effects of collisions and concentration and mass of impurity ions on the instability of obliquely modulated ion acoustic waves are discussed. The following results are found: (i) In the case of a single-ion collisional plasma, for small values of k, the value of θ above which the wave is unstable increases in comparison to the collisionless case; (ii) the presence of heavy ion impurities has no effect on the stable region in k-θ plane either for collisionless or collisional plasmas; (iii) the presence of light ion impurities changes the modulationally unstable domain drastically. The variation of this effect with the concentration and mass of impurities and collisions for the collisional plasma is discussed in detail.

I. INTRODUCTION

Modulational instability of ion acoustic waves in a dispersive and weakly nonlinear plasma has been studied by several authors theoretically[1–6] as well as experimentally.[7–10] Considering the harmonic-generated nonlinearities, the above-noted authors[1–6] derived a nonlinear Schrödinger equation for different types of plasmas, i.e., single-ion plasma, two-ion plasma, two-electron-temperature plasma, etc. The derived nonlinear Schrödinger equation governs the dynamics of nonlinear ion acoustic wave packets for different types of plasmas for parallel and oblique modulation.

The nonlinear slow quasistatic plasma response to ion acoustic waves leading to modulation of ion acoustic waves has been studied by several authors.[11–13] In a recent paper[14] we have extended this study to the case of oblique modulation. In all these studies of modulational instability, the plasma has been assumed to be collisionless. However, in all realistic situations, plasmas have some collisional dissipation, which is generally small. In one of our recent papers[15] we investigated the effect of collisions on the modulational instability caused by harmonic-generated nonlinearities.

In the present paper we investigate the effects of collisions and concentration and mass of impurity ions on the instability of obliquely modulated ion acoustic waves due to nonlinear interaction with slow quasistatic plasma response. We investigate three cases: (i) the single-ion collisional plasma, (ii) the two-ion collisionless plasma, and (iii) the two-ion collisional plasma.

Collisions affect the modulation in two ways: they give rise to (i) a slow damping of the carrier wave and (ii) a modification of the equation governing the modulational instability of the wave. We use a coordinate transformation that separates these two effects. The second effect is then investigated in detail.

The presence of impurities changes the coefficient of the nonlinear term in the nonlinear Schrödinger equation. It is found that in the presence of light ion impurities, the sign of the coefficient of nonlinear term (i.e., Q) depends on the concentration (α) of impurities. Hence the unstable domain strongly depends on the impurity concentration. In the presence of heavy ion impurities, the sign of Q does not depend on α. Hence the stable and unstable domains are independent of the impurity concentration α for this case.

Our analysis is very general in the sense that the earlier studies by Shukla,[11] and Mishra, Chhabra, and Sharma[14] are obtained as a special case of present analysis.

Basic equations are given in Sec. II. In Sec. III, we derive the nonlinear Schrödinger equation. Section IV contains some discussion and Sec. V contains conclusions.

II. BASIC EQUATIONS

We consider an obliquely modulated ion-acoustic wave traveling in the (x-y) plane in a warm two-ion plasma, (i.e., main plasma ions and impurity ions) and a hot isothermal electron, collisional plasma. We assume that the modulated amplitude of the ion acoustic wave varies in the x direction. The nonlinear interaction of finite-amplitude ion acoustic waves with the background collisional plasma is governed by the following set of normalized equations:

$$\frac{\partial n_{i1}}{\partial t}+\nabla\cdot(n_{i1}\mathbf{V}_{i1})=0\ , \tag{1}$$

$$\frac{\partial V_{i1}}{\partial t}+(\mathbf{V}_{i1}\cdot\nabla)\mathbf{V}_{i1}=-\frac{1}{\beta}\nabla\phi-\frac{1}{\beta}\frac{T_i}{T_e}\frac{1}{n_{i1}}\nabla n_{i1}-\sigma\mathbf{V}_{i1}\ , \tag{2}$$

$$\frac{\partial n_{i2}}{\partial t}+\nabla\cdot(n_{i2}\mathbf{V}_{i2})=0\ , \tag{3}$$

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$$\frac{\partial \mathbf{V}_{i2}}{\partial t}+(\mathbf{V}_{i2}\cdot\nabla)\mathbf{V}_{i2}=-\frac{\mu}{\beta}\nabla\phi-\frac{\mu}{\beta}\frac{T_i}{T_e}\frac{1}{n_{i2}}\nabla n_{i2}-\sigma\mathbf{V}_{i2}, \tag{4}$$

$$\nabla\phi=\frac{1}{n_e}\nabla n_e\ , \tag{5}$$

$$\nabla^2\phi=n_e-(1-\alpha)n_{i1}-\alpha n_{i2}\ , \tag{6}$$

where $\mathbf{V}=(V_{ix},V_{iy},0)$, $\nabla=(\partial/\partial x,\partial/\partial y,0)$, $\alpha=n_{i2}^{(0)}/n^{(0)}$, $\mu=M_1/M_2$, and $\beta=(1-\alpha+\mu\alpha)$.

In the above equations, $n_{i1},\mathbf{V}_{i1}$ and $n_{i2},\mathbf{V}_{i2}$ are the density and fluid velocity of the two-ion species, n_e is the electron density, ϕ is the electrostatic potential, σ is the ion-electron collision frequency, μ is the mass ratio of the main ions to the impurity ions, and α is the fractional concentration of impurity ions. As a simplification, we have taken the same collision frequency for both the ions. In Eq. (5), we have neglected the electron inertia. The quantities $\mathbf{V}$, ϕ, t, (x,y), σ, and (n_{i1},n_{i2},n_e) are normalized with respect to the ion acoustic wave speed in the mixture $C_s=(T_e\beta/M_1)^{1/2}$, thermal potential (T_e/e), inverse of the ion plasma frequency ω_{pi}^{-1}, Debye length λ_D, ω_{pi}, and the unperturbed plasma density n_0, respectively.

III. DERIVATION OF THE NONLINEAR SCHRÖDINGER EQUATION

We are interested in investigating the slow quasistatic plasma response to the ion acoustic waves. Therefore, we write the field quantities in normalized form as follows:

$$n_j=1+n_j^h+n_j^l\ , \tag{7}$$

$$\mathbf{V}_j=\mathbf{V}_j^h+\mathbf{V}_j^l\ , \tag{8}$$

$$\phi=\phi^h+\phi^l\ , \tag{9}$$

where $n_j^{h(l)}\ll 1$. The superscripts h and l represent the corresponding quantities associated with the ion wave (high frequency) and with the quasistatic plasma slow motion (low frequency), respectively.

Using Eqs. (7)–(9) in Eq. (5), the electron density perturbation associated with the ion acoustic waves in the presence of the plasma slow motion is given by

$$n_e^h=(1+n_e^l)\phi^h\ . \tag{10}$$

Now we combine Eqs. (1) and (2), and (3) and (4). Then introducing Eqs. (6)–(9), we obtain the following nonlinear equation for the ion acoustic waves in the presence of the plasma slow response:

$$\left[\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}+\sigma\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial}{\partial t}\right]\phi^h+\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]+\sigma\frac{\partial}{\partial t}\right]n_e^l\phi^h=0\ . \tag{11}$$

In deriving Eq. (11), ions are assumed to be much colder than electrons, i.e., $T_i/T_e\ll 1$. Also, we used the quasineutral and quasistatic behaviors of plasma towards slow response, i.e.,

$$[(1-\alpha)n_{i1}^l+\alpha n_{i2}^l]=n_e^l \text{ and } \mathbf{V}_i^l=\mathbf{V}_e^l\approx\mathbf{0}\ .$$

In the absence of nonlinear interaction, the linearization of (11) yields the following dispersion relation:

$$\omega(\omega+i\sigma)=\frac{k^2}{1+k^2}\ , \tag{12}$$

where $k^2=k_x^2+k_y^2$, with k_x and k_y being the x and y components of the wave vector $\mathbf{k}$ of the ion acoustic wave. The modulation group velocity (i.e., the velocity with which the modulation propagates) of the wave is given by

$$V_g=\frac{\partial\omega_r}{\partial k_x}=\frac{k\cos\theta}{\omega_r(1+k^2)^2}=\frac{\partial\omega_r}{\partial k}\cos\theta\ , \tag{13}$$

which is the component of the group velocity $(\partial\omega_r/\partial k)$ along the direction of modulation. Here θ is the angle between the wave vector of the ion acoustic wave and the x axis, the direction in which the modulation of the wave amplitude propagates and ω_r is the real part of ω, which, according to the dispersion relation (12) is given by

$$\omega_r=\left[\frac{k^2}{(1+k^2)^2}-\frac{\sigma^2}{4}\right]^{1/2}\ . \tag{14}$$

Now we calculate the electron density perturbation n_e^l associated with the quasistatic plasma slow motion. Taking the x component of the momentum balance equations for ions and electrons,

$$\frac{\partial V_{i1x}}{\partial t}+V_{i1x}\frac{\partial V_{i1x}}{\partial x}+V_{i1y}\frac{\partial V_{i1x}}{\partial y}=-\frac{1}{\beta}\frac{\partial\phi}{\partial x}-\frac{1}{\beta}\frac{T_i}{T_e}\frac{1}{n_{i1}}\frac{\partial n_{i1}}{\partial x}-\sigma V_{i1x}\ , \tag{15}$$

$$\frac{\partial V_{i2x}}{\partial t}+V_{i2x}\frac{\partial V_{i2x}}{\partial x}+V_{i2y}\frac{\partial V_{i2x}}{\partial y}=-\frac{\mu}{\beta}\frac{\partial\phi}{\partial x}-\frac{\mu}{\beta}\frac{T_i}{T_e}\frac{1}{n_{i2}}\frac{\partial n_{i2}}{\partial x}-\sigma V_{i2x}\ , \tag{16}$$

$$\frac{\partial\phi}{\partial x}=\frac{1}{n_e}\frac{\partial n_e}{\partial x}\ . \tag{17}$$

Using Eqs. (7)–(9) in Eqs. (15)–(17) and averaging over the ion acoustic wave periods we get

$$\tfrac{1}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle=-\frac{1}{\beta}\frac{\partial\phi^l}{\partial x}-\frac{1}{\beta}\frac{T_i}{T_e}\frac{\partial n_{i1}^l}{\partial x}\ , \tag{18}$$

$$\tfrac{1}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\frac{\mu}{\beta}\frac{\partial\phi^l}{\partial x}-\frac{\mu}{\beta}\frac{T_i}{T_e}\frac{\partial n_{i2}^l}{\partial x}\ , \tag{19}$$

$$\frac{\partial\phi^l}{\partial x}=\frac{\partial n_e^l}{\partial x}\ , \tag{20}$$

where we have assumed that the phase velocity of the modulation is much smaller than the electron and ion thermal velocities. In the above equations angular brackets denote averaging over the ion acoustic wave period and $|(V_{i1x}^h)^2|$ denotes the amplitude of $(V_{i1x}^h)^2$. The coefficient $\exp(-\sigma t)$ represents the damping due to the presence of collisions. The left-hand side of Eqs. (18) and (19) represent the ion ponderomotive force. Equations (18) and (19) can be written as

$$\frac{\beta}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle=-\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_{i1}^l}{\partial x}\ , \tag{21}$$

$$\frac{\beta}{4\mu}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_{i2}^l}{\partial x}\ . \tag{22}$$

Now multiplying Eq. (21) by $(1-\alpha)$ and Eq. (22) by α, and then adding, we get

$$\frac{(1-\alpha)\beta}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle+\frac{\alpha\beta}{4\mu}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\frac{\partial\phi^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_e^l}{\partial x}\ , \tag{23}$$

where we have used $[(1+\alpha)n_{i1}^l+\alpha n_{i2}^l]=n_e^l$. Using Eq. (20) in Eq. (23), we get

$$\frac{(1-\alpha)\beta}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle+\frac{\alpha\beta}{4\mu}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\frac{\partial n_e^l}{\partial x}-\frac{T_i}{T_e}\frac{\partial n_e^l}{\partial x}$$

or

$$\frac{(1-\alpha)\beta}{4}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i1x}^h)^2|\rangle+\frac{\alpha\beta}{4\mu}\exp(-\sigma t)\frac{\partial}{\partial x}\langle|(V_{i2x}^h)^2|\rangle=-\left[1+\frac{T_i}{T_e}\right]\frac{\partial n_e^l}{\partial x}\ . \tag{24}$$

Now from the x component of the ion momentum equations, i.e., Eqs. (15) and (16), we get

$$V_{i1x}^h=\frac{1}{\beta}\frac{1}{(\omega+i\sigma)}k_x\phi^h \tag{25}$$

and

$$V_{i2x}^h=\frac{\mu}{\beta}\frac{1}{(\omega+i\sigma)}k_x\phi^h\ . \tag{26}$$

Using Eqs. (25) and (26) in Eq. (24), we get

$$n_e^l=-\frac{1+k^2}{4(1+\gamma)}|\phi^h|^2\cos^2\theta\exp(-\sigma t)\ , \tag{27}$$

where $\gamma=T_i/T_e$ is the ratio of the ion to electron temperatures. Substituting Eq. (27) in Eq. (11) we get

$$\left[\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}+\sigma\left[1-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\frac{\partial}{\partial t}\right]\phi^h-\left[\frac{\partial^2}{\partial t^2}-\frac{1}{\beta}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]+\sigma\frac{\partial}{\partial t}\right]\times\frac{1+k^2}{4(1+\gamma)}\cos^2\theta\exp(-\sigma t)|\phi^h|^2\phi^h=0\ . \tag{28}$$

We assume that the nonlinear interaction of the quasistatic plasma slow response with the ion acoustic waves gives rise to an envelope of wave whose amplitude varies on time and space scales much more slowly than those of the ion acoustic oscillations. Accordingly, we let

$$2\phi^h=\epsilon^{1/2}\phi^h(\xi,\tau)\exp(-i\omega t+ik_xx+ik_yy)+\text{c.c.}\ , \tag{29}$$

where ϵ indicates the magnitude of small but finite amplitude ϕ^h, ξ and τ are defined such that[15]

$$\xi=\epsilon^{1/2}(x-V_gt)\exp\left[-\frac{\sigma t}{2}\right]\ , \tag{30a}$$

and

$$\tau=\epsilon t\exp(-\sigma t)\ . \tag{30b}$$

Substituting Eqs. (29) and (30) in Eq. (28) and using Eqs. (12) and (13), we get, to $O(\epsilon^{3/2})$, the following nonlinear Schrödinger equation:

$$i\frac{\partial\phi^h}{\partial\tau}+\left[\frac{1}{2\omega_r(1+k^2)^2}-\frac{2k^2\cos^2\theta}{\omega_r(1+k^2)^3}-\frac{k^2\cos^2\theta}{2\omega_r^3(1+k^2)^4}\right]\frac{\partial^2\phi^h}{\partial\xi^2}+\frac{\omega_rk^2}{8(1+\gamma)}\left[1+\frac{\sigma^2}{4\omega_r^2}+\frac{\alpha(1-\mu)}{\omega_r^2\beta}\right]|\phi^h|^2\phi^h=0$$

or

$$i\frac{\partial\phi^h}{\partial\tau}+P\frac{\partial^2\phi^h}{\partial\xi^2}+Q|\phi^h|^2\phi^h=0\ , \tag{31}$$

where P and Q are the dispersive and nonlinear terms, respectively. The dispersion term $2P$ is the component of the modulation group velocity dispersion $(\partial\omega_r/\partial k)$ along the direction of modulation, i.e.,

$$P=\frac{1}{2}\frac{\partial V_g}{\partial k}\cos\theta=\frac{1}{2}\frac{\partial V_g}{\partial k_x}=\frac{1}{2}\frac{\partial^2\omega_r}{\partial k_x^2}=\frac{1}{2}\left[\cos^2\theta\frac{\partial^2\omega_r}{\partial k^2}+\sin^2\theta\frac{1}{k}\frac{\partial\omega_r}{\partial k}\right]$$

or

$$P=\left[\frac{1}{2\omega_r(1+k^2)^2}-\frac{2k^2\cos^2\theta}{\omega_r(1+k^2)^3}-\frac{k^2\cos^2\theta}{2\omega_r^3(1+k^2)^4}\right] \tag{32}$$

and

$$Q=\frac{\omega_r k^2}{8(1+\gamma)}\left[1+\frac{\sigma^2}{4\omega_r^2}+\frac{\alpha(1-\mu)}{\omega_r^2\beta}\right]. \tag{33}$$

It may be pointed out that our expressions for P and Q reduce to those obtained by earlier workers in the corresponding special cases. For example, for $\alpha=0$ or 1 and $\sigma=0$, these reduce to those obtained by Mishra, Chhabra, and Sharma,[14] while for $\alpha=0$ or 1, $\sigma=0$, and $\theta=0$, they reduce to those obtained by Shukla.[11]

We notice from Eqs. (32) and (33) that the coefficient of dispersive term P depends on σ (through ω_r), i.e., collisions, whereas it does not depend on μ and α, i.e., impurities. But the coefficient of nonlinear term Q depends on σ as well as on μ and α. We also notice that for heavy impurities ($\mu<1$), Q always remains positive; thus the presence of heavy impurities (say, helium or argon in a hydrogen plasma) would not affect the instability domain, the condition for this being $P>0$, in this case. However, for light impurities ($\mu>1$), the sign of Q changes as k passes through k_c, where k_c, the critical value of k, is given by $Q=0$ or $k_c=[\alpha(\mu-1)]^{1/2}$. Therefore we expect that the presence of light impurities (say, hydrogen in a helium or argon plasma) would affect the stable and unstable domains significantly.

IV. DISCUSSION

The amplitude of the obliquely modulated ion acoustic wave, defined by the nonlinear Schrödinger equation, i.e., Eq. (31), will be modulationally stable or unstable according as $PQ<0$ or $PQ>0$.[16] To investigate the effect of collisions, concentration, and mass of impurity ions on the modulationally unstable domain in the k-θ plane, we plot curves $P=0$ and $Q=0$ in a polar (k-θ) plane for each set of values of the parameters σ, α, and μ. In the case of single-ion plasma and two-ion plasma having heavy ion impurities, we plot only one curve, i.e., $P=0$, because in these cases Q always remains positive. In the following we consider three different cases.

A. Single-ion collisional plasma

This case is obtained by taking $\alpha=0$ or 1. Since, in this case, Q remains positive throughout the whole region, the stability and instability are decided by $P<0$ and $P>0$, respectively. For values of k and θ in the domain lying below the curve $P=0$, the wave will be stable. And for values of k and θ in the domain lying above the curve $P=0$, the wave will be unstable. Since the dispersive term P depends on the collision frequency (σ), the unstable domain is modified by the presence of collisions.

For different values of collision frequency (σ), the stable and unstable regions are shown in Fig. 1. It is found that for every value of σ, there is a minimum value of k, i.e., $k_{\min}$ for real ω, below which k is imaginary. Hence, for every σ, an ion acoustic wave does not exist for $k<k_{\min}$ and on increasing σ, the value of $k_{\min}$ increases. It is also found that for a given small value of $k>k_{\min}$ the value of θ, above which the wave becomes unstable, increases in comparison to the collisionless case, while for the large value of k, the value of θ above which the wave becomes unstable, is almost equal to the corresponding value for collisionless case.

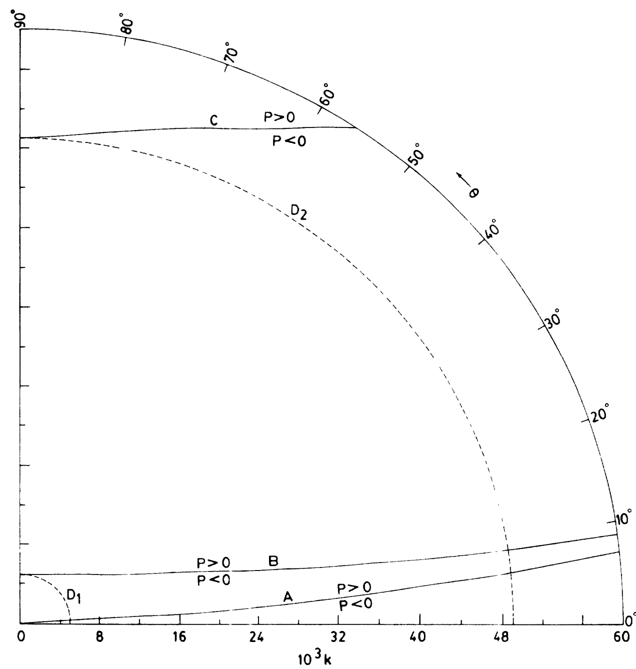

FIG. 1. Plot of $P=0$ in the k-θ plane for single-ion collisional plasma for different values of σ. The curves A, B, and C refer to $P=0$ for $\sigma=0$, 0.01, and 0.1, respectively. The dashed curves D_1 and D_2 represent $\omega_r=0$ (for $\sigma=0.01$ and 0.1), below which the wave does not exist. Q is always positive.

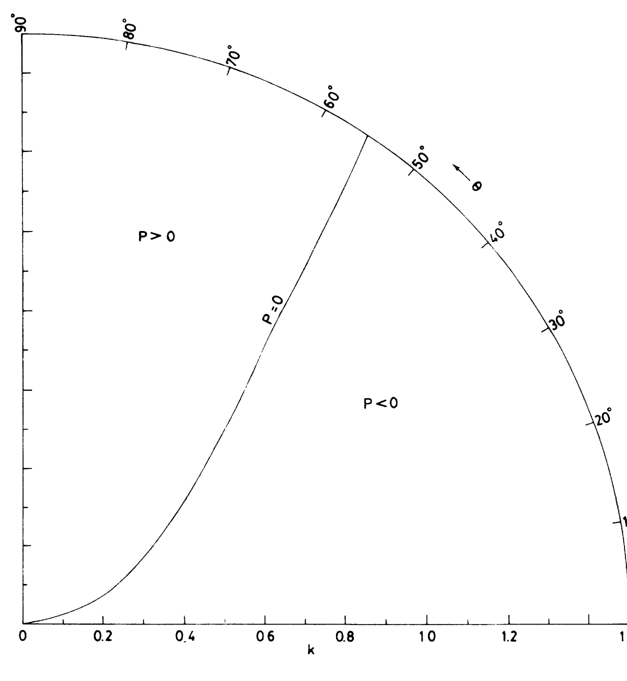

FIG. 2. Plot of $P=0$ in the k-θ plane for two-ion collisionless plasma for heavy-impurity case ($\mu<1$). Q is always positive.

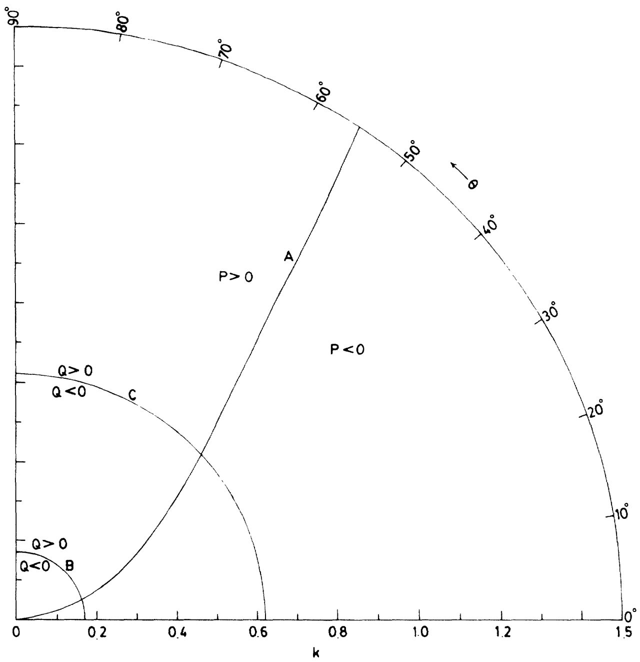

FIG. 3. Plot of $P=0$ and $Q=0$ in the k-θ plane for two-ion collisionless plasma having light impurities for $\alpha=0.01$ at different values of μ. The curve A refers to $P=0$ (for all values of μ), whereas B and C refer to $Q=0$ for $\mu=4$ and 40, respectively.

B. Two-ion collisionless plasma

This case is obtained by taking $\sigma=0$. In this case we have two subcases: (1) heavy-ion impurity and (2) light-ion impurity.

1. *Heavy-ion-impurity case*

Although the coefficient of the nonlinear term (Q) is a function of μ and α (i.e., impurities), for the heavy-ion-impurity case ($\mu<1$) its sign always remains positive throughout the whole region. Therefore, in this case also the stable and unstable regions are decided by $P<0$ and $P>0$, respectively. Whereas the dispersive coefficient P does not depend on μ and α, therefore the unstable domain is not affected by the presence of impurities (i.e., μ and α). Figure 2 shows the stable and unstable regions for this case. Hence, in the case of heavy-ion impurities (say, helium or argon in a hydrogen plasma), the stable and unstable regions remain the same as in the case of single-ion collisionless plasma, which is qualitatively in agreement with the results obtained by Chhabra and Sharma[5] in which they have considered harmonic-generated nonlinearities.

2. *Light-ion-impurity case*

In this case ($\mu>1$), the sign of Q changes as k passes through k_c, where k_c is the critical value of k and is given

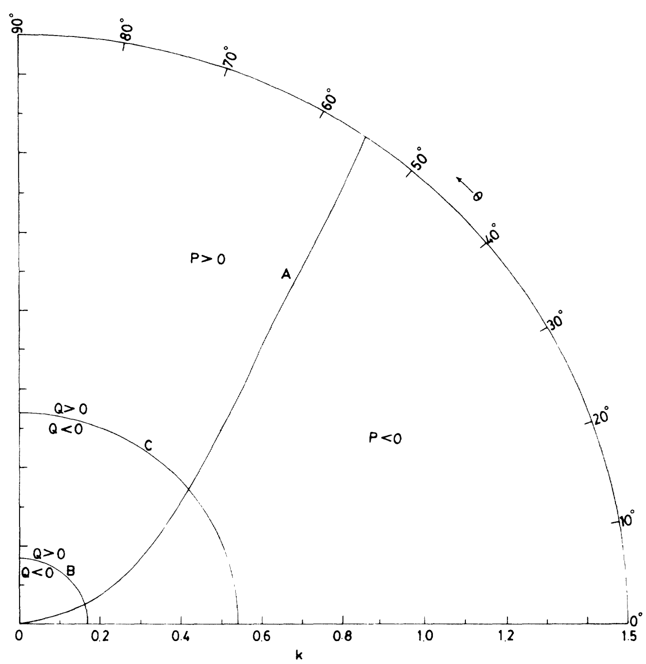

FIG. 4. Plot of $P=0$ and $Q=0$ in the k-θ plane for two-ion collisionless plasma having light impurities for $\mu=4$ at different values of α. The curve A refers to $P=0$ (for all values of α), whereas B and C refer to $Q=0$ for $\alpha=0.01$ and 0.1, respectively.

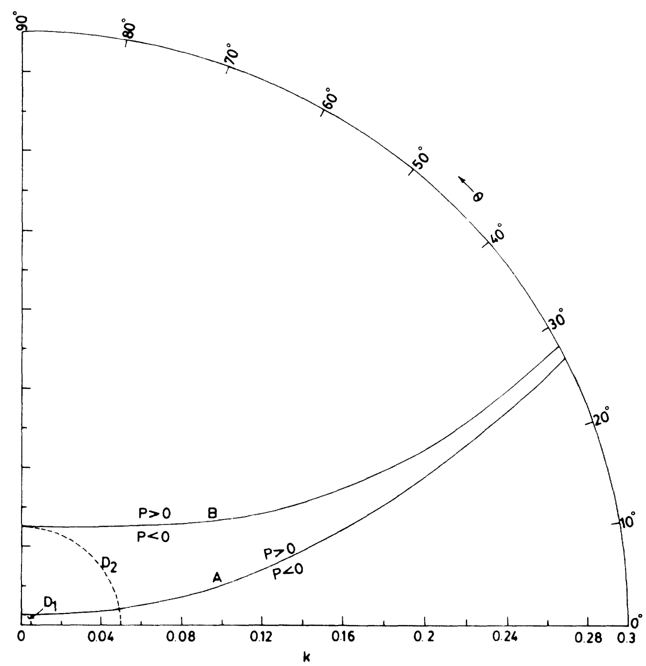

FIG. 5. Plot of $P=0$ in the k-θ plane for two-ion collisional plasma for heavy-impurity cases ($\mu<1$) at different values of σ. The curves A and B refer to $P=0$ for $\sigma=0.01$ and 0.1, respectively. The dashed curves D_1 and D_2 refer to $\omega_r=0$ (for $\sigma=0.01$ and 0.1), below which the wave does not exist. Q is always positive.

by $Q=0$ or $k_c=[\alpha(\mu-1)]^{1/2}$. Below k_c, Q is negative and above k_c, Q is positive. In this case the entire k-θ plane gets divided by the curves $P=0$ and $Q=0$ into four regions; in two of them the wave will be stable ($PQ<0$) and in the remaining two the wave will be unstable ($PQ>0$), as shown in Figs. 3 and 4. The stable and unstable regions below k_c get interchanged in comparison to the heavy-ion-impurity case, while above k_c, the stable and unstable regions remain the same as in the heavy-ion-impurity case.

C. Two-ion collisional plasma

Equation (33) shows that Q is a function of σ, μ, and α; but for the heavy-impurity case ($\mu<1$), it remains always positive. Therefore, in this case also the stable and unstable regions are decided by $P<0$ and $P>0$, respectively. Equation (32) shows that P is a function of σ, while it does not depend on μ and α. Therefore, in the case of heavy-ion-impurity collisional plasma, there is no effect of impurities on the unstable region. The effect of collisions for the heavy-impurity case is shown in Fig. 5, which is similar to the case of single-ion collisional plasma.

For light-ion-impurity collisional plasma, the sign of Q also changes as k passes through $k_c=[\alpha(\mu-1)]^{1/2}$. In this case the entire k-θ plane gets divided into five regions, in two of them the wave will be stable ($PQ<0$), while in another two regions the wave will be unstable

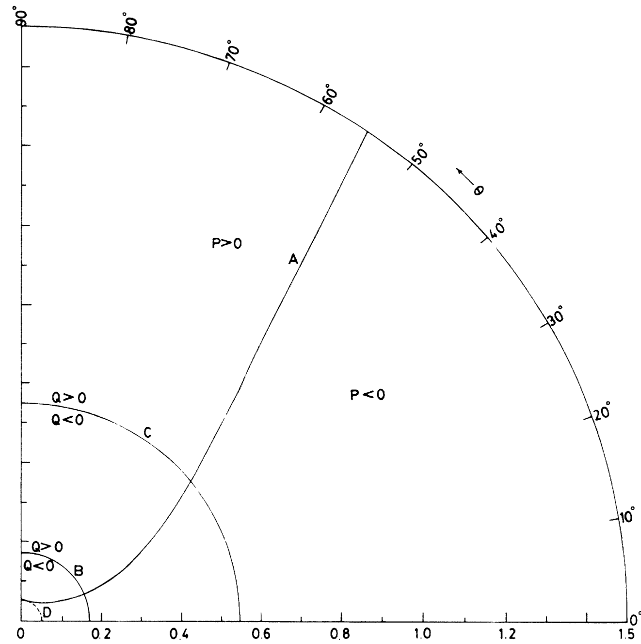

FIG. 7. Plot of $P=0$ and $Q=0$ in the k-θ plane for two-ion collisional plasma having light impurity for $\sigma=0.1$, $\mu=4$, and at different values of α. The curve A refers to $P=0$ (for all values of α), whereas B and C refer to $Q=0$ for $\alpha=0.01$ and 0.1, respectively. The dashed curve D refers to $\omega_r=0$, below which the wave does not exist.

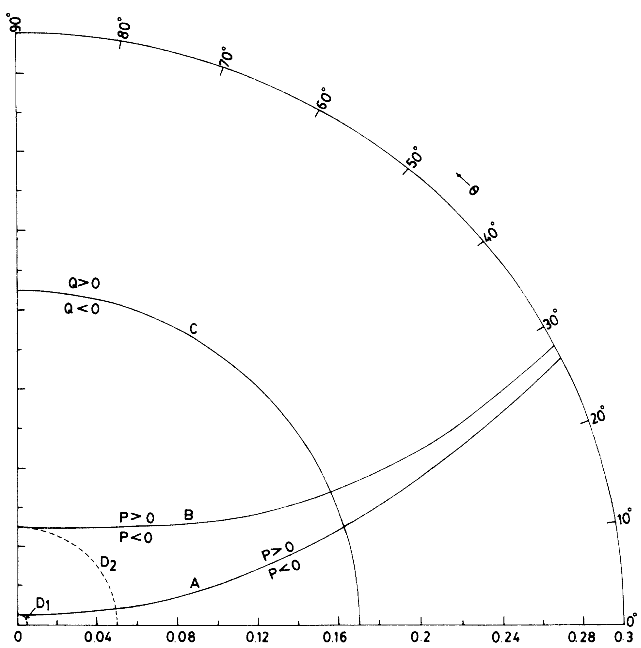

FIG. 6. Plot of $P=0$ and $Q=0$ in the k-θ plane for two-ion collisional plasma having light impurities for $\mu=4$, $\alpha=0.01$, and at different values of σ. The curves A and B refer to $P=0$ for $\sigma=0.01$ and 0.1, respectively, whereas C refers to $Q=0$ (for all values of σ). The dashed curves D_1 and D_2 refer to $\omega_r=0$ (for $\sigma=0.01$ and 0.1), below which the wave does not exist.

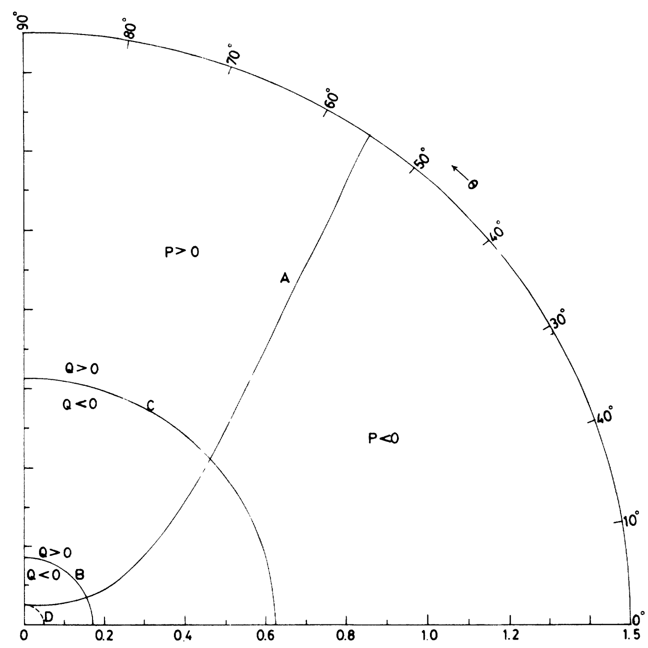

FIG. 8. Plot of $P=0$ and $Q=0$ in the k-θ plane for two-ion collisional plasma having light impurity for $\sigma=0.1$, $\alpha=0.01$, and at different values of μ. The curve A refers to $P=0$ (for all values of μ), whereas B and C refer to $Q=0$ for $\mu=4$ and 40, respectively. The dashed curve D refers to $\omega_r=0$, below which the wave does not exist.

($PQ > 0$), and in the remaining region ($k < k_{\min}$) the wave would not exist. These cases are shown in Figs. 6–8. In the region below k_c, the stable and unstable regions are interchanged in comparison to the heavy-ion-impurity collisional plasma case. On increasing the value of μ and/or α, the value of k_c increases.

V. CONCLUSIONS

Our main conclusions are as follows: (i) Due to the presence of collisions there exists a minimum value of k for every σ below which the ion acoustic wave does not exist. (ii) For a single-ion collisional plasma for a given small value of k, $k > k_{\min}$, the value of θ above which the wave is unstable increases in comparison to the collisionless case. (iii) The presence of heavy-ion impurities has no effect on the unstable region in the k-θ plane either for collisionless or for collisional plasma. (iv) The presence of light-ion impurities changes the modulationally unstable region drastically. In this case, below the critical value of k, i.e., k_c, the stable and unstable regions get interchanged in comparison to the heavy-impurity case. While above k_c, the stable and unstable regions remain the same as in the heavy-impurity case.

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