Fluctuation-induced transitions in an isotropic spatially frustrated lattice model

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(Received 23 February 1990)

In this paper we discuss the effects of fluctuations on the phase diagram of an isotropic Ising model with ferromagnetic first-neighbor and antiferromagnetic second- and third-neighbor interactions. We find that, in certain cases, there are substantial deviations from mean-field behavior. Thus the phase transitions between the one-dimensional modulated phases and paramagnetic phase are found to undergo fluctuation-induced first-order phase transitions. We also discuss that point of the phase diagram which, within mean-field theory, was predicted to be an isotropic Lifshitz point. It is argued that the existence of such a point is unlikely, and some alternative scenarios are proposed.

I. INTRODUCTION

Considerable attention has been directed towards the study of frustrated lattice Hamiltonians as models of alloys, magnetic ordering, and lyotropic systems. The Hamiltonian we shall study was first introduced by Widom as a model of microemulsion,[1] but it has since been suggested that it may be appropriate for some magnetic systems.[2] The Hamiltonian has isotropic first-, second-, and third-neighbor interactions on a simple-cubic lattice.

In those parts of the parameter space that are most relevant to the study of microemulsions, the first-neighbor interactions are ferromagnetic while the second- and third-neighbor interactions are antiferromagnetic. This competition between the interactions causes spatial frustration. Thus, for certain parts of parameter space, sometimes called multistate surfaces, the Hamiltonian has many degenerate states.

One immediate consequence of this is that, at finite temperature, many ordered phases are predicted to grow from the zero-temperature multiphase lines.[3] At higher temperatures one finds the ferromagnetic, disordered, and one-dimensional ordered phases in close proximity in the phase diagram. The latter are also sometimes called periodic or lamellar phases because they consist of ferromagnetically aligned layers of positive and negative spins. It is also sometimes useful to think of these phases as consisting of a stack of interfaces, each such interface being defined by the set of all pairs of (+, -) bonds between a positive and negative layer of spins. Within mean-field theory[1] the ferromagnetic phase and the disordered phase are found to be separated by a second-order transition. The boundary between the disordered phase and the periodic phases is predicted to be second order with the wavelength of the periodic phase varying along the critical curve, eventually becoming infinite as the ferromagnetic-paramagnetic phase boundary is approached.

These ferromagnetic, lamellar, and paramagnetic phases are therefore predicted to meet at an isotropic Lifshitz point, L.[1] While much of the topology of the mean-field phase diagram is correct,[4] we will show that, for certain ranges of parameter values, the phase boundary separating lamellar states from the paramagnet becomes fluctuation-induced first order, while the ferromagnetic-disorder phase boundary remains Ising-like. This curve of induced first-order transitions must join the curve of Ising-like transitions, but simple arguments indicate that the lower critical dimensionality of an isotropic Lifshitz point should be $d_L=4$. The nature of this region of the phase diagram is, therefore, an open question, though we shall propose a few different possibilities.

We begin by applying the Hubbard transformation to the Ising model Hamiltonian,

$$H=\tfrac{1}{2}\sum_n \sigma_n L_n \sigma_n, \quad L_n=a_1\Delta_n^4+a_2\Delta_n^2+a_3 , \tag{1.1}$$

where Δ_n^2 is the usual second-order lattice-difference operator and the a_i are given in terms of the microscopic couplings. The result of this transformation is

$$Z=\sum_{\{\sigma_n\}} e^{-\beta H}=\int \mathcal{D}\phi_n e^{-\beta\mathcal{A}(\phi_n)},$$
$$\mathcal{D}\phi_n=\frac{1}{(\det L_n)^{1/2}}\prod_n d\phi_n , \tag{1.2}$$
$$\mathcal{A}=-\tfrac{1}{2}\sum_n \phi_n L_n^{-1}\phi_n-\frac{1}{\beta}\sum_n \ln(2\cosh\beta\phi_n) .$$

The mean-field theory is given by those fields $\{\phi_n\}$ that minimize the Hubbard action $\mathcal{A}$. Providing one is above the lower critical dimension of the theory one can extract an effective Landau-Ginzburg-Wilson (LGW) action from (1.2) by determining those Fourier modes $\{\phi_q\}$ which at quadratic order in ϕ_n, minimize the action. One then constructs new effective fields from the critical and nearby modes. It is possible to show[1] that, in the case of the one-dimensional periodic order to disorder transition, the critical modes are $\mathbf{Q}_c=(\pm Q_c,0,0),(0,\pm Q_c,0),(0,0,\pm Q_c)$ where, within mean-field theory, $\mathbf{Q}_c$ is known in terms of the coupling parameters appearing in the microscopic Hamiltonian (1.1). We emphasize that at this stage Q_c is a parameter in the action and is therefore not determined self-consistently from the ensuing renormalization-group calculation. This is a limitation of the calculation we shall describe.

We begin by considering the critical and the "nearby" modes and define

$$\psi_1(\mathbf{r})=\sum_{\mathbf{q}}{}' \phi_{Q_c+q_1,q_2,q_3}e^{i\mathbf{q}\cdot\mathbf{r}} , \tag{1.3}$$

$$\psi_2(\mathbf{r})=\sum_{\mathbf{q}}{}' \phi_{q_1,Q_c+q_2,q_3}e^{i\mathbf{q}\cdot\mathbf{r}} , \tag{1.4}$$

$$\psi_3(\mathbf{r})=\sum_{\mathbf{q}}{}' \phi_{q_1,q_2,Q_c+q_3}e^{i\mathbf{q}\cdot\mathbf{r}} , \tag{1.5}$$

where the prime means the sum runs over momenta within a small sphere in the first Brillouin zone centered around the critical mode. From the reality condition $\phi_q^*=\phi_{-q}$ we have

$$\psi_{\mathbf{Q}_c}(\mathbf{r})=\sum_{q}{}' \phi_{-\mathbf{Q}_c+\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}} . \tag{1.6}$$

The LGW Hamiltonian is then constructed by expanding the second term of Eq. (1.2) in powers of ϕ_n and keeping only the terms up to quartic order in fields. Transforming to Fourier space we see that the coefficient of quadratic term is given by $-L_q^{-1}-\beta$. We then define

$$K_q\equiv L_q+\beta^{-1} . \tag{1.7}$$

Within mean-field theory the phase transition from the paramagnetic phase to a phase with wave vector $\mathbf{Q}_c$ occurs at $K_{\mathbf{Q}_c}=0$. Thus, in the vicinity of this transition we can expand in terms of small K_q to obtain

$$\beta\mathcal{A}=\beta\int_q|\phi_q|^2K_q + \tfrac{1}{12}\int_q\phi_{q_1}\phi_{q_2}\phi_{q_3}\phi_{q_4}\delta(q_1+q_2+q_3+q_4) , \tag{1.8}$$

where the integration is over the first Brillouin zone.

Now, in the vicinity of the phase transition, K_q is positive with its minimum at $\mathbf{Q}_c$. We now seek to include in the action only those Fourier components with wavelengths comparable to the critical modes, since these are expected to dominate near the order-disorder transition.

We thus expand K_q about each of these modes retaining terms only up to $O(q^4)$. We find

$$K_q=a+bq^2+cq^4+d\sum_{i=1}^{3}q_i^4 , \tag{1.9}$$

where a,b,c,d are given in terms of the microscopic coupling constants and the critical modes $\mathbf{Q}_c$,

$$a=24a_1(\cos Q_c-1)^2-12a_2(1-\cos Q_c)+a_3+\beta^{-1} , \tag{1.10a}$$

$$b=-4a_1[2\cos(2Q_c)+2\cos Q_c-4]-2a_2(2+\cos Q_c) , \tag{1.10b}$$

$$c=2a_1(1+2\cos Q_c) , \tag{1.10c}$$

$$d=\frac{4a_1}{3}(2\cos 2Q_c-\tfrac{5}{2}\cos Q_c+\tfrac{1}{2})+\frac{a_2}{6}(2+\cos Q_c) . \tag{1.10d}$$

Assuming that the critical modes are well separated in the first Brillouin zone, so that the modes within one sphere do not overlap with those of another, the quartic term of (1.8) can also be simplified. Thus, in terms of fields defined in (1.3)–(1.5), the last term of Eq. (1.8) becomes

$$\int_q\phi_{q_1}\phi_{q_2}\phi_{q_3}\phi_{q_4}\delta(q_1+q_2+q_3+q_4) = \int_{\mathbf{r}}(|\psi_1|^4+|\psi_2|^4+|\psi_3|^4+4|\psi_1|^2|\psi_2|^2 +4|\psi_1|^2|\psi_3|^2+4|\psi_2|^2|\psi_3|^2) , \tag{1.11}$$

where on the right-hand side the integration is over real space down to some microscopic cutoff length.

In terms of the real fields $\{\varphi_j,\overline{\varphi}_j\}$ defined by $\psi_j(r)=\varphi_j(r)+i\overline{\varphi}_j(r)$, we can write the final effective Hamiltonian density as

$$H_{\mathrm{LGW}}=-\tfrac{1}{2}\sum_{i=1}^{3}[(\nabla\varphi_i)^2+(\nabla\overline{\varphi}_i)^2] -\tfrac{1}{2}r\sum_{i=1}^{3}[(\varphi_i)^2+(\overline{\varphi}_i)^2] -2u\left[\sum_{i=1}^{3}(\varphi_i)^2+(\overline{\varphi}_i)^2\right] +u\sum_{i=1}^{3}[(\varphi_1)^2+(\overline{\varphi}_i)^2]^2 . \tag{1.12}$$

Note that, for the moment, we have dropped the $O(q^4)$ powers appearing in expansion (1.9) and normalized the coefficient of the gradient to $\frac{1}{2}$. Equation (1.12) is certainly valid for small enough wavelengths of periodic phase, since we are above the lower critical dimension and close enough to the upper critical dimension for higher powers of the fields to be irrelevant.

However, one should also note that the above construction appears to fail when, as a function of the model parameters, all of the critical modes converge to the ferromagnetic mode $\mathbf{Q}_c=(0,0,0)$. Within the mean-field approximation this occurs upon approach to the Lifshitz point. In the present case it is not clear how to construct the Landau-Ginzburg-Wilson action since one has no well-resolved critical modes. This point in the phase diagram is further complicated by the fact that naive dimensional analysis gives upper and lower critical dimensions to be $d_u=8, d_L=4$; these results follow upon consideration of the infrared divergence of the four- and two-point functions, respectively. If the latter result is correct, then the method just described for constructing the effective action is invalid in the vicinity of L since, in the absence of long-ranged order, the dominant modes cannot be simple Fourier modes. We shall return to this matter in more detail in Sec. III.

For the moment, we merely point out that the assertion $d_L=4$ is predicated on the continuum Hamiltonian that is supposed to represent the order-disorder transition in this region of the phase diagram. However, one does expect the continuum form of the effective Hamiltonian deduced by the methods just described to be valid if $d>4$ and terms of quartic order in the wave vector are kept in Eq. (1.9). This permits the critical modes to have $Q_c \to 0$ since this would correspond to coefficient b of (1.9) approaching zero. Thus it is possible, in principle, to analyze this action in an $8-\epsilon$ expansion, indicating that L may be driven first order.[5]

This is certainly a possible scenario, though it is a little difficult to interpret the results of this analysis since, if $d_L=4$, one would need to carry the expansion below the lower critical dimension in order to access the physically relevant case $d=3$. Even if the lower critical dimension of the microscopic Hamiltonian is, in the region presently being discussed, equal to 3, the continuum theory being studied in $8-\epsilon$ expansion has $d_L=4$, so one would need to argue that the leading terms in the $8-\epsilon$ expansion will be the same for both theories. It is not clear that this is the case.

One of the other possible scenarios is that the disordered phase may intrude between the periodic and ferromagnetic phases, rather like the situation in the two-dimensional axial next-nearest-neighbor Ising (ANNNI) model. Since one can show[4] that long-ranged order exists in the low-temperature limit, this does not seem to be correct. Alternatively, it is also possible that in the presence of fluctuations, the wavelength of the periodic phase does not become infinitely long as one proceeds along the curve of fluctuation-induced first-order phase transitions. With this ansatz it is possible to study the nature of the transition in the region of the lamellar, ferromagnetic, and disordered phases.

Now if Q_c remains finite, none of the problems associated with an isotropic Lifshitz point arise and it is possible to apply the same methods described in Eqs. (1.1)–(1.12) to construct an extended effective Hamiltonian. Thus, in the region where the mean-field theory predicts the periodic, ferromagnetic, and disordered phases to meet at a Lifshitz point, one can analyze the Hubbard action to find seven critical modes, six representing periodic ordering and one corresponding to a simple ferromagnetic ordering. We emphasize that this is a strong ansatz which cannot be checked within the present calculation. Nevertheless, it is worth examining its predictions.

Thus the new effective action is determined to be

$$H_{\mathrm{LGW}}=-\tfrac{1}{2}\sum_{i=1}^{3}[(\nabla\varphi_i)^2+(\nabla\bar{\varphi}_i)^2]-\tfrac{1}{2}(\nabla\varphi_0)^2 -\tfrac{1}{2}r_1\sum_{i=1}^{3}[(\varphi_i)^2+(\bar{\varphi}_i)^2]-\tfrac{1}{2}r_2(\varphi_0)^2 -2\left[\sum_{i=1}^{3}v_1[(\varphi_i)^2+(\bar{\varphi}_i)^2]+v_0(\varphi_0)^2\right]^2 +\left[\sum_{i=1}^{3}v_1^2[(\varphi_i)^2+(\bar{\varphi}_i)^2]^2+v_0^2(\varphi_0)^4\right] . \quad (1.13)$$

In Sec. II we consider the $4-\epsilon$ analysis of these two Hamiltonians. However, before we proceed it is worth making the following observations.

In principle, terms of different symmetry will flow differently under application of the renormalization group, so the two quartic terms in Hamiltonian (1.12) and the four quartic terms in Hamiltonian (1.13) should have independent coupling parameters. Under certain circumstances it may be appropriate to use the bare couplings given in (1.12) and (1.13) as initial conditions for the flow. However, arguments that this is the case are heuristic.

For example, it will transpire [see Eqs. (2.2) and (2.3)] that the ratio $u_2/u_1=2$ of the quartic couplings of (1.13) is preserved under renormalization, so $u_2=2u_1$ is a separatrix in the flow space. It is not obvious that our method of deriving the coarse-grained LGW Hamiltonian can account for those renormalizations that will occur on going from the microscopic [Eq. (1.2)] to coarse-grained form (1.13). However, since the bare couplings lie on one side of a separatrix line, it is assumed that this renormalization from the microscopic form does not permit the system to cross the separatrix. Under these circumstances it is presumed that the ratios of bare couplings can be used as initial conditions for the renormalization-group flow.

II. ORDER-DISORDER BOUNDARIES WITHIN THE EPSILON EXPANSION

We begin by considering that Hamiltonian which describes the curve of phase transitions between the paramagnetic and the modulated phases. For the moment we assume that we are far from that part of the phase diagram which, within mean-field theory, was a Lifshitz point.

The LGW Hamiltonian for this region of the phase diagram may be written

$$H_{\mathrm{LGW}}=-\tfrac{1}{2}\sum_{i=1}^{3}[(\nabla\varphi_i)^2+(\nabla\bar{\varphi}_i)^2]-\tfrac{1}{2}r\sum_{i=1}^{3}[(\varphi_i)^2+(\bar{\varphi}_i)^2] -u_1\sum_{i=1}^{3}[(\varphi_i)^4+(\bar{\varphi}_i)^4]-2u_1\sum_{i=1}^{3}\varphi_i^2\bar{\varphi}_i^2 -\frac{u_2}{2}\sum_{i\neq j}^{3}(\varphi_i^2\varphi_j^2+\bar{\varphi}_i^2\bar{\varphi}_j^2+2\bar{\varphi}_i^2\varphi_j^2) . \quad (2.1)$$

The ϵ expansion of this Hamiltonian has been previously carried out by Mukamel and Krinsky[6] (MK). In the limit of large rescaling factor b, they obtain the following flow equations:

$$u_1' = b^{\epsilon}[u_1 - (40u_1^2 + 4u_2^2)K_4 \ln b] , \quad (2.2)$$

$$u_2' = b^{\epsilon}[u_2 - (32u_1u_2 + 12u_2^2)K_4 \ln b] . \quad (2.3)$$

Following Mukamel and Krinsky we define $x_1 = (K_4/\epsilon)u_1, x_2 = (K_4/\epsilon)u_2$. One can show that these flow equations possess the four fixed points given in Table I (see also Fig. 1).

Now the line $x_2 = 2x_1$ is a separatrix, so that the flow preserves the ratio of coupling constants $x_2/x_1 = 2$. Thus, if one chooses the initial conditions to the left of the separatrix, there will not be any accessible stable fixed points, and we conclude that the transition is fluctuation induced first order. On the other hand, if the initial conditions place us to the left of the separatrix, we will obtain a second-order transition with the exponents in the universality class of the stable fixed point (MK). The fixed point along the separatrix would then correspond to a tricritical point.

As we argued at the end of Sec. I, one should be able to use the bare ratio of coupling constants as initial conditions for the renormalization-group flow. In this case we find $x_2^b = 4x_1^b$ and this places us well into the region where a fluctuation-induced first-order phase transition is to be expected. This conclusion is consistent with earlier Monte Carlo simulations.[4] It is to be contrasted with the situation for the ANNNI model where the periodic order to paramagnet transition is in the X-Y universality class. It is possible that magnetic materials will exhibit these different behaviors depending on the degree of anisotropy in the Hamiltonian.

The analysis carried out above is valid only for regions of the phase diagram well away from the mean-field Lifshitz point. If, in the presence of fluctuations, the wavelength of the lamellar phase does not diverge, then it is possible to analyze the Hubbard action in the vicinity of the mean-field Lifshitz point in the manner just described for lamellar ordering. It is found that there are now seven dominant critical modes (0,0,0), $(\pm Q_c,0,0),(0,\pm Q_c,0),(0,0,\pm Q_c)$. The effective Hamiltonian density is then given by Eq. (1.13).

TABLE I. Fixed points for the six-component model.

X_1	X_2
0	0
$\frac{1}{56}$	$\frac{1}{28}$
$\frac{1}{44}$	$\frac{1}{44}$
$\frac{1}{40}$	0

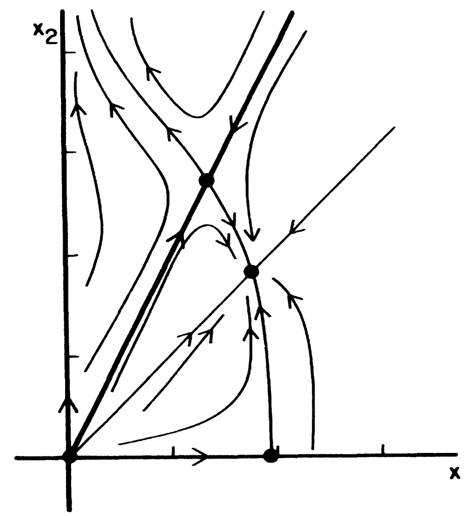

FIG. 1. Flow diagram for the six-component model. The stable fixed point (MK) lies to the left of the separatrix $x_2 = 2x_1$ (bold line). There are two fixed points along the separatrix. At the origin one has the Gaussian fixed point, the other one is a Heisenberg $O(6)$ fixed point. The initial conditions for the flow of Hamiltonian (1.12) are believed to lie to the right of the separatrix.

We rewrite the interaction part of the Hamiltonian in a more convenient form

$$H_{\text{int}} = -u_1 \sum_{i=1}^{3} [(\varphi_i)^4 + (\overline{\varphi}_i)^4] - 2u_1 \sum_{i=1}^{3} \varphi_i^2\overline{\varphi}_i^2 - \frac{u_2}{2} \sum_{i \neq j}^{3} (\varphi_i^2\varphi_j^2 + \overline{\varphi}_i^2\overline{\varphi}_j^2 + 2\overline{\varphi}_i^2\varphi_j^2) - u_3\varphi_0^2 \sum_{i=1}^{3} [(\varphi_i)^2 + (\overline{\varphi}_i)^2] - u_4\varphi_0^4 . \quad (2.4)$$

In the limit of large rescaling factor b, we obtain to first order in ϵ,

$$u_1' = b^{\epsilon}[u_1 - (40u_1^2 + 4u_2^2 + u_3^2)K_4 \ln b] , \quad (2.5)$$

$$u_2' = b^{\epsilon}[u_2 - (32u_1u_2 + 12u_2^2 + 2u_3^2)K_4 \ln b] , \quad (2.6)$$

$$u_3' = b^{\epsilon}[u_3 - (16u_1u_3 + 12u_3u_4 + 8u_2u_3 + 8u_3^2)K_4 \ln b] , \quad (2.7)$$

$$u_4' = b^{\epsilon}[u_4 - (36u_4^2 + 6u_3^2)K_4 \ln b] . \quad (2.8)$$

In terms of the parameters $x_i = (K_4/\epsilon)u_i$, the fixed points are given in Table II.

All of the fixed-point Hamiltonians, implied by Table II, except for the last one, lie in the direct-product group of the earlier six-component Hamiltonian and the Ising Hamiltonian. However, examination of the stability of these fixed points indicates that they correspond to the fixed points of the previous six-component Hamiltonian with Ising-like crossover. The final fixed point lies in the $O(7)$ universality class.

Note that most of these fixed points are on the now three-dimensional separatrix $x_2 = 2x_1$, and the rest are to

TABLE II. Fixed points for the seven-component model.

X_1	X_2	X_3	X_4
0	0	0	0
$\frac{1}{40}$	0	0	0
0	0	0	$\frac{1}{36}$
$\frac{1}{40}$	0	0	$\frac{1}{36}$
$\frac{1}{56}$	$\frac{1}{28}$	0	0
$\frac{1}{44}$	$\frac{1}{44}$	0	0
$\frac{1}{56}$	$\frac{1}{28}$	0	$\frac{1}{36}$
$\frac{1}{44}$	$\frac{1}{44}$	0	$\frac{1}{36}$
$\frac{1}{60}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{60}$

the right of the separatrix. This means that one must choose initial conditions that lie on the separatrix or to the right of it, if we are to access one of them. Of the fixed points that are located on the separatrix, $O(6)$ (with Ising crossover), and $O(7)$ are the most stable. If one chooses the initial conditions on the right-hand side of the separatrix, then one will find an even more stable fixed point corresponding to (MK) with Ising crossover.

The problem with this analysis is that the nature of the ansatz leading to (2.4) does not permit one to fix these initial conditions, even heuristically. The most likely conclusion is that, as a function of the microscopic interactions, one chooses those initial conditions on the separatrix that lead to the $O(7)$ fixed point. One then finds the following topology for the phase diagram. A curve of fluctuation-induced phase transitions is terminated by the $O(7)$ second-order transition and the order-disorder transition curve then continues as an Ising-like set of second-order transitions. The $O(7)$ fixed point would, in this case, also be a critical end point of a curve of conventional first-order phase transitions between the broken-symmetry, ferromagnetic, and lamellar phases.

It is worth pointing out that extensive Monte Carlo simulations have been carried out for some points along this order-disorder boundary.[4] It seems clear that at least part of the ferromagnetic-paramagnetic boundary is of the Ising universality class, while the lamellar-paramagnetic boundary is weakly first order. The simulations in the region between these limits are inconclusive. In Sec. III we make some further comments about this region.

III. CONCLUSIONS

Assuming the forms of the LGW Hamiltonians given by Eqs. (1.12) and (1.13), we have carried out a $4-\epsilon$ expansion to determine the topology of the phase diagram near the order-disorder boundaries for one-dimensional periodic and ferromagnetic ordering. The conclusions are that, for sufficiently short wavelengths of periodic order, the lamellar-disorder boundary is driven fluctuation-induced first order since the stable (MK) fixed point is inaccessible, while the ferromagnetic-disordered boundary remains a curve of Ising-like transitions. The situation becomes very unclear for that region of the phase diagram where the period of lamellar ordering is very long. Within mean-field theory this point is described by an isotropic Lifshitz point. One model that we have constructed suggests that transitions near this point will be in the $O(7)$ universality class, but this requires that Q_c remains finite. If Q_c vanishes, then an $8-\epsilon$ expansion may be carried out. However, an elementary argument indicates that $d_L=4$,[7] so the results are difficult to interpret. Given the inconclusiveness associated with this analysis it seems worth discussing other alternatives for this region of the phase diagram. The first point that must be emphasized is that, unlike the situation for the three-dimensional ANNNI model, the present lattice model is isotropic, and the periodic phases arise as a consequence of spontaneous symmetry breaking. One is then led to question the influence of the lattice on the lamellar phases. In the case where the layers are widely separated it is possible that the layers roughen. There is some evidence that, in the vicinity of the putative isotropic Lifshitz point, loss of long-ranged lamellar order may be associated with roughening. Thus a Padé approximation of a long low-temperature expansion of an isolated interface has been used to calculate the interfacial height fluctuations.[8] In that region of the phase diagram presently under discussion, the roughening curve is, within numerical error, identical to the disordering transition of the lamellar phase as calculated by simulation. If this transition were to be associated with interfacial roughening, then one might, on the basis of our information on the Laplacian roughening model,[9] predict two translationally disordered phases, one corresponding to an orientationally broken symmetry. This possibility that the model may possess a number of disordered phases is intriguing. However, our analysis has been hampered by the fact that the Kosterlitz-Thouless fixed point requires a calculation up to the second order in the fugacity, while the functional flow for the interlayer potential is poorly understood beyond linear order in the potential.[10,11] More details of this situation will be presented at a later date.[10] In any case, if the disordering takes place by roughening, then the layers are already fluidlike and another continuum Hamiltonian, appropriate for such a situation, may be extracted from the Hubbard action. Thus one no longer has single Fourier modes (density wave excitations) being pinned to the lattice beneath the paramagnetic-lamellar phase transition. One then conjectures that the dominant excitations in the Hubbard action are of form

$$S(\mathbf{r})=\chi(\mathbf{r})\cos[Q_c z+\theta(\mathbf{r})] , \tag{3.1}$$

where θ is a slowly varying phase factor that permits the layers to have transverse fluctuations and $\chi(\mathbf{r})$ is a slowly varying density variable. If the spatial variations in the density variables are slow, then one can show that the quadratic term in the effective Hamiltonian is

$$\mathcal{H}=\int_r \chi^2(\mathbf{r})\{a_1[\nabla_t^2\theta(\mathbf{r})]^2+(2a_1Q_c^2-a_2)(\nabla_t\theta)^2 -(a_2-6a_1Q_c^2)(\partial_z\theta)^2+2a_1(\partial_z\nabla_t\theta)^2\} \tag{3.2}$$

where $\mathbf{r}$ and $\mathbf{t}$ are, respectively, three- and two-dimensional vectors. The physical origin of these terms is immediately evident. They are, respectively, a curvature term (an excess in energy resulting from bends of the interfaces in the lamellar phase), a surface-tension term, a simple bulk modulus compression term, and the additional effective compression term. All terms except the final one have been previously discussed in phenomenological models of interacting layers, though the surface tension is often considered irrelevant for lyotropic systems. Indeed, using the bare values from microscopic Hamiltonian for a_1 and a_2 we find that in the vicinity of the phase-transition line the term is vanishingly small. It is interesting to note that, on independent grounds, we have earlier identified this as the microemulsion region of the lattice model.[1,4] On the other hand, the final term of Eq. (3.2) is of comparable magnitude to the first, and on the basis of the underlying microscopic model cannot be dropped. In fact, it can be viewed as an additional term in the bulk compression energy resulting from the fact that stretched interfaces have larger surface area. There seems to be no justifiable reason for neglecting this term.

We can see that the detailed nature of this region of the phase diagram is not yet clear. It seems likely that a number of novel phenomena may be present.

ACKNOWLEDGMENTS

One of us (Y.L.) gratefully acknowledges support from the Regents of the University of California. K.A.D. gratefully acknowledges financial support from the Dreyfus Foundation, AT&T, and the Sloan Foundation. The authors had helpful conversation and correspondence with Professor M. E. Fisher and Dr. M. Barbosa. We are grateful to Dr. Barbosa for sending us a copy of her unpublished work. The authors wish to thank A. Berera for a correction to Eq. (2.8).

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[1]B. Widom, J. Chem. Phys. **84**, 6943 (1986); K. A. Dawson, M. D. Lipkin, and B. Widom, *ibid.* **88**, 5149 (1988); K. A. Dawson, Phys. Rev. A **36**, 3383 (1987). An extensive review article that discusses related issues is W. Selke, Phys. Rep. **170**, 213 (1988).

[2]P. J. Upton and J. M. Yeomans, Europhys. Lett. **5**, 575 (1988); P. J. Upton and J. M. Yeomans, Phys. Rev. B **40**, 439 (1989).

[3]A. Berera and K. A. Dawson, Phys. Rev. A **41**, 627 (1990).

[4]K. A. Dawson, B. Walker, and A. Berera, Physica (to be published); see also N. Jan and D. Stauffer, J. Phys. (Paris) **49**, 623 (1988).

[5]Marcia C. Barbosa (unpublished).

[6]In this paper [D. Mukamel and S. Krinsky, Phys. Rev. B **13**, 5078 (1976)] the authors study a more general form than that of Eq. (2.1), only four of their fixed points (a,b,e,f) are relevant to our study. See also, S. Krinsky and D. Mukamel, Phys. Rev. B **16**, 2313 (1977).

[7]R. M. Hornreich, M. Luban, and S. Shtrikman, Phys. Rev. Lett. **35**, 1678 (1975); R. M. Hornreich Phys. Rev. B **19**, 5914 (1979); N. S. Tonchev and D. I. Uzunov, Physica A **134**, 265 (1985).

[8]B. Khang, A. Berera, and K. A. Dawson (unpublished).

[9]D. Nelson, Phys. Rev. A **26**, 269 (1982).

[10]K. A. Dawson and Y. Levin (unpublished).

[11]B. Khang, Y. Levin, and K. A. Dawson (unpublished).