

**Comment on “Phase-sensitive population decay:  
The two-atom Dicke model in a broadband squeezed vacuum”**

Z. Ficek\* and P. D. Drummond

*Department of Physics, The University of Queensland, St. Lucia, Queensland 4067, Australia*

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A recent paper [G. M. Palma and P. L. Knight, *Phys. Rev. A* **39**, 1962 (1989)] has stated that the two-atom Dicke model exhibits phase-dependent population decay. In this Comment we show that population decay is invariant with respect to the input phase angle of squeezing in the two-atom Dicke model.

In their paper<sup>1</sup> Palma and Knight have shown that the two-atom Dicke model exhibits unusual population decay behavior when irradiated with a squeezed vacuum field. The behavior, unlike that in the one-atom case, is that the total inversion decays at a rate which depends on the extent of squeezing. An interesting feature of this decay rate is that it should be easier to observe than the corresponding spectral modifications.<sup>2,3</sup> In Palma and Knight’s work, a claim is made that “the atoms interacting with a squeezed radiation field reservoir can exhibit phase-dependent population decay.” This statement, however, could be misinterpreted as implying that the decay rate itself depends on the squeezing phase. In our Comment we clarify this point, directly showing the phase invariance of the decay rate.

In fact, it is clear that the phase of the squeezed vacuum is not significant in this problem. To demonstrate this, it is enough to note that the Hamiltonian for the two-atom Dicke model<sup>1,4</sup> is time independent. A change in the vacuum reservoir squeezing phase simply corresponds to a small time translation. Since the inversion operator is form invariant under a time translation, any phase dependence could only arise from the state preparation. However, the decay rate calculated in this problem is independent of the time at which the inversion is prepared, and therefore must be phase invariant within these approximations. We note, however, that the lack of dependence on initial state preparation is related to the use of the rotating-wave and Markov approximations. A small phase dependence is in fact possible, but could only occur within a full counter-rotating term treatment.

In order to show this more quantitatively, we start from the Hamiltonian, which for the two-atom Dicke model is described by<sup>1,4</sup>

$$H = \hbar\omega_0 \sum_{i=1}^2 S_i^z + \hbar \sum_{k,s} \omega_{ks} a_{ks}^\dagger a_{ks} + \sum_{i=1}^2 \sum_{k,s} [g_{iks} a_{ks} (S_i^+ + S_i^-) + \text{H.c.}] , \quad (1)$$

where  $\omega_0$  is the atomic resonance frequency, and

$S_i^+$ ,  $S_i^-$ , and  $S_i^z$  are pseudospin operators for the atoms satisfying the following commutation relations:

$$[S_i^+, S_j^-] = 2S_i^z \delta_{ij}, \quad [S_i^z, S_j^\pm] = \pm S_i^\pm \delta_{ij} . \quad (2)$$

The coupling coefficient  $g_{iks}$  is given by

$$g_{iks} = -i \left( \frac{2\pi\hbar\omega_k}{V} \right)^{1/2} [\boldsymbol{\mu}_i \cdot \mathbf{e}_{ks}] , \quad (3)$$

where  $V$  is the normalization volume,  $\mathbf{e}_{ks}$  the unit polarization vector, and  $\boldsymbol{\mu}_i$  is the transition electric dipole moment vector. In Eq. (1) the field operators  $a_{ks}$  and  $a_{ks}^\dagger$  describe the quantized electromagnetic field, which we assume is in a broadband squeezed vacuum state with the carrier frequency at the resonant frequency  $\omega_0$  of the atomic transition. The bandwidth of the squeezing is assumed to be sufficiently broad that the squeezed vacuum appears as  $\delta$ -correlated squeezed white noise to the atoms. The correlation functions for the field operators  $a_{ks}$  and  $a_{ks}^\dagger$  can then be written as<sup>5</sup>

$$\begin{aligned} \langle a_{ks}^\dagger(\omega) a_{ks}(\omega') \rangle &= N(\omega) \delta(\omega - \omega') , \\ \langle a_{ks}(\omega) a_{ks}(\omega') \rangle &= M^*(\omega) \delta(\omega' - 2\omega_0 + \omega) . \end{aligned} \quad (4)$$

In Eq. (4) the parameters  $M(\omega_0) = M = |M| \exp(i\phi_v)$  and  $N(\omega_0) = N$  characterize the squeezing such that  $|M|^2 \leq N(N+1)$ , where the equality holds for a minimum-uncertainty squeezed states, and  $\phi_v$  is the phase of the squeezed vacuum.

Using Eqs. (1) and (4) with the Born, Markov, and rotating-wave approximations one finds a master equation for the reduced density operator  $\rho$  of the atomic system of the form<sup>6,7</sup>

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & +M\gamma \sum_{i,j} ([\rho S_j^+, S_i^+] + [S_i^+, S_j^+ \rho]) + M^* \gamma \sum_{i,j} ([\rho S_j^-, S_i^-] + [S_i^-, S_j^- \rho]) \\ & -N\gamma \sum_{i,j} (\rho S_j^- S_i^+ + S_j^- S_i^+ \rho - 2S_i^+ \rho S_j^-) - (N+1)\gamma \sum_{i,j} (\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+) - i \sum_{\substack{i,j \\ i \neq j}} \Omega_{ij} [S_i^+ S_j^-, \rho], \end{aligned} \quad (5)$$

where  $2\gamma$  is the single-atom decay rate for spontaneous emission. Here, also,  $\Omega_{ij}$  represents the dipole-dipole interaction between the atoms,<sup>8</sup> which is neglected in the Dicke approximation.

To compare our results with Palma and Knight's work we shall work with the equations of motion for the expectation value of the atomic operators. We should also stress that what we denote as the atomic operators  $S_i^+$ ,  $S_i^-$ , and  $S_i^z$  corresponds to  $\sigma_+^{(i)}$ ,  $\sigma_-^{(i)}$ , and  $\frac{1}{2}\sigma_3^{(i)}$  of Palma and Knight. We find that the master equation (5) leads to a closed set of four equations of motion of the vacuum expectation values of the atomic operators describing the total atomic population decay. This set of our equations can be written in matrix form as the inhomogeneous equation

$$\frac{d}{d\tau} \mathbf{X} = \underline{A} \mathbf{X} + \boldsymbol{\alpha}, \quad (6)$$

where  $\underline{A}$  is the real  $4 \times 4$  matrix:

$$\underline{A} = \begin{pmatrix} -n & -1 & 0 & 0 \\ -(2n-1) & -n & -2|M| & 4n \\ -4|M| & -2|M| & -n & 8|M| \\ \frac{1}{2}(n-1) & \frac{1}{2}(n-1) & |M| & -2n \end{pmatrix}. \quad (7)$$

The column vector  $\mathbf{X}$  has the following real components:

$$\begin{aligned} \langle S^z(\tau) \rangle = & -4 \frac{(n^2 - 4|M|^2)}{n(3n^2 + 1 - 12|M|^2)} + \frac{4|M|^2(3n+4)}{n(12|M|^2 - 1)} e^{-n\tau} - \frac{[(n+1)(n+1+u) + 4|M|^2]}{2u(n+u)(2n+u)} e^{-(2n+u)\tau} \\ & + \frac{[(n+1)(n+1-u) + 4|M|^2]}{2u(n-u)(2n-u)} e^{-(2n-u)\tau}, \end{aligned} \quad (12)$$

where  $u = (n^2 + 12|M|^2 - 1)^{1/2}$ .

It is obvious from eq. (12) that the extent of squeezing, which is characterized by the parameter  $M$ , changes the population decay constants. However, this time evolution of the total atomic population is completely independent of the squeezing phase  $\phi_v$ . Our result (12) generalizes the result of Palma and Knight to the more realistic case of a partially squeezed vacuum with  $|M|^2 < N(N+1)$ . It agrees with the earlier result in the minimum uncertainty limit of  $|M|^2 = N(N+1)$  and for a real  $M$ , i.e., for  $|M| = M$ . It also agrees with these results for a thermal reservoir, when  $|M| = 0$ . In practice, some intermediate value of squeezing, with  $0 < |M|$

$$\begin{aligned} X_1 &= \langle S_1^+ S_1^- + S_2^+ S_2^- \rangle, \\ X_2 &= \langle S_1^+ S_2^- + S_2^+ S_1^- \rangle, \\ X_3 &= (\langle S_1^+ S_2^+ + S_1^- S_2^- \rangle \cos \phi_v \\ &\quad - i \langle S_1^+ S_2^+ - S_1^- S_2^- \rangle \sin \phi_v), \\ X_4 &= \langle S_1^+ S_2^+ S_1^- S_2^- \rangle, \end{aligned} \quad (8)$$

while the vector  $\boldsymbol{\alpha}$  has the components

$$\alpha_1 = \alpha_2 = (n-1), \quad \alpha_3 = 2|M|, \quad \alpha_4 = 0. \quad (9)$$

For simplicity, in Eqs. (6)–(9) we have introduced the notation

$$\tau = 2\gamma t, \quad n = (1 + 2N). \quad (10)$$

The time evolution of the total population of the two-atom system is described by the  $X_1$  component of the vector  $\mathbf{X}$  in the form

$$\langle S^z(\tau) \rangle = (\langle S_1^+ S_1^- + S_2^+ S_2^- \rangle - 1). \quad (11)$$

The system of equations (6) can be easily solved using the Laplace transform technique. Assuming that initially both atoms were in their excited state, from (6) and (11) we obtain the following solution for the time evolution of the total atomic population as

$<[N(N+1)]^{1/2}$  is likely.

From the form of Eqs. (6)–(8) it is easy to see that the total atomic population, described by the  $X_1$  component of the vector  $\mathbf{X}$ , is coupled with linear combinations of the  $S_1^+ S_2^+$  and  $S_1^- S_2^-$  operators. These correlation products are individually phase dependent so in this sense the population decay could be interpreted as having phase-sensitive properties. Despite this, the linear combination involved  $X_3$  has no phase-dependent behavior itself. Thus these correlations do not transfer their phase dependence to the population decay rate, which is phase invariant.<sup>9</sup>

\*On leave from Institute of Physics, A. Mickiewicz University, Poznań, Poland.

<sup>1</sup>G. M. Palma and P. L. Knight, *Phys. Rev. A* **39**, 1962 (1989).

<sup>2</sup>H. J. Carmichael, A. S. Lane, and D. F. Walls, *Phys. Rev. Lett.* **58**, 2539 (1987); *J. Mod. Opt.* **34**, 821 (1987).

<sup>3</sup>T. Quang, M. Kozierowski, and L. H. Lan, *Phys. Rev. A* **39**, 644 (1989).

<sup>4</sup>R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).

<sup>5</sup>C. W. Gardiner and M. J. Collett, *Phys. Rev. A* **31**, 3761 (1985).

<sup>6</sup>A. S. Shumovsky and T. Quang, *J. Phys. B* **22**, 131 (1989).

<sup>7</sup>M. R. B. Wahiddin, S. S. Hassan, and R. K. Bullough, in *Proceedings of the Sixth Rochester Conference on Coherence and Quantum Optics, 1989*, edited by J. H. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1990).

<sup>8</sup>M. J. Stephen, *J. Chem. Phys.* **40**, 669 (1964).

<sup>9</sup>In Ref. 1, Appendix A, the authors state that there is no loss of generality in using  $\phi_v=0$ , thus implicitly agreeing with our phase-invariant result.