Stopping power of an electron gas for a slow antiproton

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The screening of a slow antiproton in an electron gas is discussed as an analogy of dielectric response formulation. A simple parametric form of the dielectric function is proposed that leads to analytic expressions for the effective potential and induced hole density. The parameter is fixed via the cusp condition for the total electron density at the position of the probe charge. The stopping power is calculated by making use of the transport cross section. The results are in fair agreement with those given by a recent self-consistent calculation.

A question that recently has raised much interest in the field of charged-particle interaction with matter is that of antiproton stopping power.[1-3] The availability of low-energy antiparticles[4] gives a real challenge for theoretical considerations. In the slowing-down process the projectile interacts with the constituents of the target. The intrinsic difficulty of the problem resides in the many-body character of the interaction.

The theoretical calculation presupposes a model for the stopping material. In the electron-gas model the electrons respond collectively to the perturbing potential. Because of the antiproton-electron scattering there is a time delay in the individual electron motion. When these motions are averaged, there is a smaller electron density near the antiparticle than elsewhere, and this is the screening hole. When the velocity of the projectile v is small compared with the Fermi velocity v_F of the electrons, the relevant excitations responsible for the energy dissipation are electron-hole pairs. The amount of phase space available for the creation of these pairs is determined by kinematical constraints. It is therefore independent of the details of the interaction between the probe charge and the system.[5] These statements allow us to use the friction force formulation of the stopping power.[6,7]

For low velocity of a massive projectile the energy loss per unit path length can be written as[6,7]

$$\frac{dE}{dR}=n_0 v v_F \sigma_{\rm tr}(v_F) , \tag{1}$$

where $n_0=v_F^3/3\pi^2$ is the density of the background electron gas and $\sigma_{\rm tr}$ is the momentum-transfer cross section. (We use atomic units throughout this work.)

Equation (1) is based on the adiabatic picture, i.e., the momentum transfer to the projectile per second is taken to act as a time-independent force. This picture is expected to be valid if the momentum transfer per collision is very small compared to the momentum of the probe charge. In terms of the phase shifts δ_l generated by a statically screened spherically symmetric potential one can write for the cross section

$$\sigma_{\rm tr}(v_F)=\frac{4\pi}{v_F^2}\sum_{l=0}^{\infty}(l+1)\sin^2[\delta_l(v_F)-\delta_{l+1}(v_F)] . \tag{2}$$

The problem of the low-velocity stopping is then reduced to the determination of the effective scattering potential. The electron gas is repelled by the antiproton and a depletion hole is created. The electron density around a repulsive impurity may be reduced at any point only by an amount n_0 and the range of the induced hole is at least r_s defined by $r_s=(3/4\pi n_0)^{1/3}$.

In a recent paper[8] the screening and stopping problem of an antiproton have been discussed within the framework of density-functional formalism. In this scheme the theory of potential scattering can be applied directly. The self-consistency condition of this formalism is the Friedel sum rule,[9] which relates the scattering phase shifts to the total impurity charge Z by the formula

$$Z=\frac{2}{\pi}\sum_{l=0}^{\infty}(2l+1)\delta_l(v_F) \tag{3}$$

and for antiprotons $Z=-1$.

Here we propose a simple model calculation for the induced hole density $\Delta n(r)$ and effective scattering potential $V(r)$. We use the following form for a parametric dielectric function:[10]

$$\epsilon(q)=1+\frac{q_{\rm TF}^2}{q^2}\frac{1}{1+\lambda 3(q/2v_F)^2} , \tag{4}$$

where $q_{\rm TF}^2=4v_F/\pi$ is the Thomas-Fermi expression and λ will be fixed below. By choosing this particular form we can reproduce, apart from the long-range Friedel oscillation, the high-density results in a very natural manner. The bare Coulomb potential $-Z/r$ of the antiproton is screened[11] according to

$$V(q)=-Z\frac{4\pi}{q^2\epsilon(q)} . \tag{5}$$

TABLE I. Calculated values of the cusp parameter λ, the induced hole density at the origin $\Delta n(0)$, and the stopping power $(dE/dR)v^{-1}$ for $Z=-1$. The values predicted by the self-consistent calculation (Ref. 8) are listed for comparison. See the text for the details.

		This work		Ref. 8	
r_s	λ	$-\Delta n(0)$	$(dE/dR)v^{-1}$	$-\Delta n(0)$	$(dE/dR)v^{-1}$
0.5	1.790	0.8431	0.372	1.1581	0.355
1	2.610	0.1473	0.242	0.1947	0.220
1.5	3.452	0.0502	0.173	0.0641	0.154
2	4.304	0.0229	0.131	0.0283	0.114
3	6.025	0.0074	0.084	0.0087	0.070
4	7.758	0.0033	0.059	0.0037	0.048

The induced hole density is then calculated from the Poisson equation to be

$$\Delta n(r)=Z\int d^3q\frac{1}{(2\pi)^3}e^{iq\cdot r}\frac{\epsilon(q)-1}{\epsilon(q)} . \tag{6}$$

The free parameter λ is determined by making use of the cusp condition. This condition for the total electron density $n(r)=n_0+\Delta n(r)$ at the position of an impurity with charge Z reads[12,13]

$$\left.\frac{n'(r)}{n(r)}\right|_{r=0}=-2Z\mu , \tag{7}$$

where μ denotes the reduced mass of the electron-impurity (two-body) system. It is easy to show that in our model calculation ($\mu=1$)

$$\Delta n'(r=0)=-2Zn_0\frac{1}{\lambda} .$$

Consequently, λ tends to unity in the very-high-density limit ($n_0\gg|\Delta n|$). This value of λ represents the lower bound for the antiproton case. Furthermore, for this density range the Friedel sum rule is satisfied exactly[14,15] by the present model, independently of λ. Note that $\lambda=1$ leads to unphysical results for $r_s>0.8$ yielding $|\Delta n(0)|>n_0$. The numerical results we shall present will show that the correction to the linear response ($\lambda=1$) is actually very large even for $Z=-1$. Via the cusp condition we take into account the strong Coulomb interaction between the antiproton and electrons. In other words the nonlinearity in the response of the electron density to the impurity charge Z enters into our treatment through the parameter $\lambda=\lambda(Z,r_s)$. The effective potential and induced hole density is given by

$$V(r)=-\frac{Z}{r}e^{-\alpha r}\left[\cos(\beta r)+\frac{\alpha^2-\beta^2}{2\alpha\beta}\sin(\beta r)\right] , \tag{8}$$

$$\Delta n(r)=Z\frac{2n_0}{\lambda\alpha\beta}\frac{e^{-\alpha r}}{r}\sin(\beta r) . \tag{9}$$

In these expressions the unknown quantities are defined by

$$\alpha=\alpha(Z,r_s)=(\omega_p/\sqrt{\lambda}+v_F^2/3\lambda)^{1/2} , \tag{10a}$$

$$\beta=\beta(Z,r_s)=(\omega_p/\sqrt{\lambda}-v_F^2/3\lambda)^{1/2} , \tag{10b}$$

where $\omega_p=(4\pi n_0)^{1/2}$ is the classical plasma frequency. For completeness we note that from Eqs. (5) and (6) together with Eq. (4) one can obtain analytic expressions for arbitrary values of λ. Our special forms are valid for $\lambda>\pi v_F/12$.

We investigate the capability of our parametric formulation in the density range $0.5\le r_s\le 4$. Table I contains the calculated values of $\lambda,\Delta n(0)$ and $(dE/dR)v^{-1}$ as a function of the density parameter r_s at $Z=-1$. The phase shifts have been calculated numerically by a high-accuracy variational method.[16–18] For comparison we also include the values of $\Delta n(0)$ and the stopping power predicted by the self-consistent calculation.[8] Despite the simplicity of the model, the results are in good agreement with those obtained by more elaborated calculation. Consequently, it seems that the essential physics is treated properly by the cusp condition.

It is instructive to calculate the stopping power for different values of the "antiparticle" charge. In Fig. 1 we plot the reduced stopping power $Q=(dE/dR)(vZ^2)^{-1}$ as a function of Z for a particular value of the density pa-

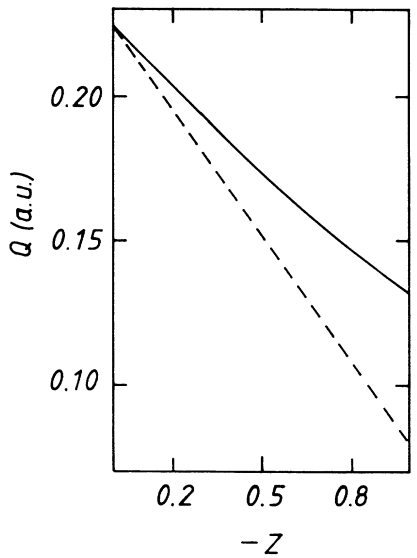

FIG. 1. Reduced stopping power $Q=(dE/dR)(vZ^2)^{-1}$ as a function of the "antiparticle" charge Z. The density parameter is $r_s=2$. The solid curve corresponds to the present scheme. The dashed curve is obtained from Eq. (11). See text for further details.

rameter ($r_s=2$). The solid line corresponds to the present scheme with numerical calculation for the phase shifts. It is worth mentioning that we can describe this curve by a simple relation

$$Q=Q^B[1+0.67Z\alpha(Z,2)] , \tag{11}$$

where $Q^B=0.224$ is the first-order Born (or linear-response) result at $Z=0$. Equation (11) might be regarded as a description of the *nonlinear* version of the Barkas effect.[19,20] The dashed curve of Fig. 1 is obtained by setting $\lambda=1$ in Eqs. (10a) and (11). The deviation between the two curves is significant, except for a very restricted range of Z. Therefore, a simple higher-order-perturbation interpretation of the nonlinearity for the linear scattering potential has a limited validity. This statement coincides with the conclusion of detailed calculations for positive "particles".[21]

We finish our consideration with an energy investigation. The interaction energy between the antiproton and its hole is given by

$$W=-\frac{\alpha^2+\beta^2}{2\alpha} \tag{12}$$

in our parametrization scheme. In the low-density limit ($r_s\to\infty$) the quantities α and β are equal and given by the relation $\alpha=\beta=(\frac{3}{2})^{1/3}/r_s$. Thus the result resembles to the total potential energy of a fixed electron in a Wigner lattice.[11]

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