Effects of injected atomic coherence on one-photon single-mode interactions

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(Received 6 July 1989; revised manuscript received 29 March 1990)

We investigate the semiclassical electric-dipole-allowed interaction of a unidirectional plane-wave electric field with a homogeneously broadened two-level medium prepared in a coherent superposition of energy eigenstates. Atomic level pumping and the effects of phase matching the new polarization contributions are discussed. The slowly varying polarization is derived and applied to the problem of the propagation of a weak field. The polarization of the medium acquires a direct (field-independent) contribution due to the injected coherence. We find that the slowly varying envelope propagation equation retains the standard single-mode absorption coefficient but develops additional driving terms. Phase matching in both time and space is crucial for the injected coherence to couple to a field. The phase-matched case of weak-field propagation proves to be a linear superposition of two processes: the usual Beer's law exponential decay (or growth) and a constant field supported by the medium. The injected coherence enables a weak electric field to persist in an absorbing medium.

I. INTRODUCTION

A typical method for studying the interaction of radiation and atomic matter is to direct a beam of atoms into the path of an electric field. The atoms may be prepared in an energy eigenstate and injected into the region of the field. If the atoms are in their lower state, they absorb. If they are in their upper state, they undergo stimulated emission. Our goal here is to direct a beam of atoms, prepared in a coherent superposition of energy eigenstates, into the region of the field and study the effects of what we call an *injected atomic coherence.* For situations in which atoms are introduced into the interaction region at a constant rate, the initial atomic state is of considerable importance. We consider situations for which there are only two atomic levels of interest and the atoms have nonvanishing probability amplitudes in either or both of these states. The superposition state is sensitive to the relative phase of the probability amplitudes for the two levels. Since this phase can be easily disturbed, the atomic coherence for electronic levels typically has a short lifetime. The level probabilities are not sensitive to the atomic coherence and are therefore disturbed less easily.

Traditional methods of atomic state preparation for cw studies produce atoms usually in energy eigenstates or possibly in coherent superposition states. Techniques capable of preparing atoms in the same state of atomic coherence are usually not able to preserve the coherence for a large number of atoms. Weak electromagnetic interactions between atoms do not change the level probabilities, but typically alter the individual phases of the coherence in some random fashion. The net result for the assembly of atoms is that the atomic coherence averages to zero. The system behaves as if each atom were prepared in one of the energy eigenstates. We investigate the properties of systems whose net atomic coherence for the entire ensemble is nonvanishing. Each atom is prepared in a similar superposition state and we assume that the preparation process enables us to preserve the coherence.

The effect of atomic coherence on a quantized electric field is being actively studied by Scully and co-workers with regard to the quantum beat laser and the correlated emission laser.[1-9] The phase sensitivity of processes to the state of atomic coherence is also being investigated by Zaheer and Zubairy[10-12] and in squeezing.[13,14] These fully quantum-mechanical analyses provide us with the motivation to carry out the semiclassical treatment. In particular, we have studied the effects of injected atomic coherence on the one- and two-photon electric-dipole coupling of homogeneously broadened two-level atoms and a one-, two-, or three-mode electric field. We present our results in a series of papers. Since the present work is the first in this series, we briefly review the concepts of atomic-level pumping and phase matching. This discussion will suffice for all other papers. We limit the scope of the presentation to the simplest atom-field coupling, namely, the one-photon single-mode interaction.

In previous investigations concerning two-level atoms interacting with one or more modes of an electric field, expressions describing induced polarizations, populations pulsations, nonlinear absorption coefficients, and mode coupling are presented (Ref. 15 and references therein). The possibility of having two or more field modes allows a spatial grating in the population difference to form with which one or more fields may interact. The introduction of a properly phased distribution of atomic dipoles leads to an atom-field interaction similar to existing multimode treatments in that both propagating disturbances may interact coherently to form population pulsations in the two-level medium. These interferences produce sidemodes in the total polarization about the solely field-induced polarization component. A significant difference for the treatment of an injected atomic coherence results

from the fact that the coherence is a property of the medium itself and does not result from the interaction of any fields under study. We note further that this coherence cannot behave like a field and begin to saturate the atoms. The injected atomic coherence may not scatter off of any population pulsations and, therefore, the polarization is not expected to show sidebands about the atomic transition frequency.

We discuss atomic-level pumping and atomic coherence in Sec. II. The problem of phase matching is described in Sec. III. Phase matching in both time and space is crucial in order for the injected atomic coherence to have any net effect on the atom-field coupling. Section IV treats the one-photon interaction of a single-mode unidirectional electric field and an injected atomic coherence. Since the coherence between the two energy eigenstates has an associated electric-dipole moment, the process of injecting atoms with a nonvanishing two-level coherence results in a direct input polarization. The total polarization of the medium then has a contribution oscillating at the atomic transition frequency, independent of the electric field and any possible nonlinear interactions that may arise.

We obtain an expression for the polarization of the medium that was initially prepared in a coherent superposition of states. The total polarization consists of the usual field-induced polarization at the field frequency and a pair of frequency symmetric sidebands having a displacement from the field frequency given by the magnitude of the atom-field detuning. In the frequency-degenerate case (the field frequency coincides with the atomic resonance frequency), the contributions to the polarization from the injected coherence and the scattering processes are all at the same frequency as the usual polarization arising from the injected population difference. Section IV presents the necessary background for deriving the slowly varying polarization used in an earlier work[16] to study the frequency locking of a phase-matched ring-laser cavity mode. The running-wave field experiences gain since the injected coherence is itself a source of polarization. A more detailed treatment of this system is to be presented in the following paper.[17]

The electric field is specialized to a single running wave and the slowly varying field envelope propagation equation is derived in Sec. V. The equation shows additional driving terms due the injected atomic coherence. For the perfectly phase-matched case the field is driven by all three contributions produced by the injected coherence, but the coefficient of the term linear in the electric field does not depend on the coherence. In the nondegenerate case the injected coherence is phase mismatched with the field-induced polarization. For poorly phase-matched configurations the injected coherence has no effect on the polarization oscillating at the field frequency. The absorption coefficient is then the standard single-mode result.[15] The detuned case still possesses the polarization oscillating at the atomic transition frequency. The remaining contributions arise from the scattering of the electric field off of the population pulsations produced by the coherent interaction of the field and the injected atomic coherence. These nonlinear processes produce symmetrically up- and down-shifted polarization sidebands. Higher-order sidebands are not produced, in contrast to purely field-induced polarizations, since the injected atomic coherence is a property of the medium and may therefore only appear once in a given interaction. The weak-field solution of the propagation equation is given in Sec. VI and the perfectly phase-matched case is discussed in Sec. VII. Our findings are summarized in Sec. VIII.

The injected coherence allows a detuned single-mode electric field to develop sidebands that may grow to produce a true multimode field. The frequency spacing between the electric-field modes is equal to the magnitude of the detuning of the atomic resonance frequency from the optical field frequency. It is then natural to consider a more complicated electric field, namely, one consisting of three modes. We might imagine that the sidemodes result from the polarization sidebands produced by a detuned injected coherence. We also know that a two-mode field naturally gives rise to additional modes with an intermode frequency spacing equal to the original frequency spacing.[18,15] This multimode treatment has been carried out and will be presented in subsequent work.

II. ATOMIC-LEVEL PUMPING

Typical atomic-level-pumping processes, such as broadband optical pumping and collisional excitation, rely on relaxation at some point in the process. In some cases a pumping process may allow a coherence to exist between the levels of interest in a single atom, but since relaxation is involved, the phases of this coherence for a collection of atoms tend to be random. The net effect is that the atomic coherence is averaged out. We investigate systems in which the atomic-level pumping preserves the atomic coherence, at least to some degree.

The atomic levels of interest are assumed to be excited states that are pumped at specific rates as well as undergoing decay out of these levels, with negligible decay from the upper to lower level (as for the He-Ne laser). This pump-decay mechanism causes the system to be *open*, i.e., the number of atoms in the two levels of interest is not conserved. This scheme is well suited to the idea of injection from some external source. The atoms are injected into the field region, interact, and leave the system by decaying out of the levels of interest. The atoms may also physically leave the interaction region. This behaves as another decay channel and also avoids the problem of an intolerably high density of atoms.

Traditional treatments of atomic-level pumping are concerned with creating and sustaining a steady-state population difference. The excited-state atomic decay scheme requires the ratio of the level-pumping rate to decay rate to be different for the two levels in order for the coupling between the field and the medium to show any net effect. The typical source for level pumping in gases is considered to be collisions. Since collisions last on the order of a few picoseconds, they are treated as instantaneous over the time scales of interest (the impact or Markov approximation), i.e., the atomic coherence decay time, and the oscillation periods of the sum and difference

frequencies due to the electromagnetic coupling. The effect of collisions is introduced into the microscopic equations of motion of the medium as phenomenological pump-decay terms which inject or remove atoms from a given state. We use a density-matrix formalism to describe the dynamics. The density matrix for a given atom consists of a matrix describing the coherent interaction with the field, and another describing the incoherent pump-decay processes. A collision strong enough to change the atomic state also tends to yield a random phase for any coherence that may arise. As a result, the pump-decay density matrix is considered to be diagonal, i.e., level probabilities receive a pumping contribution but the off-diagonal coherence elements do not.

Decay processes introduced into the density-matrix equations of motion can be grouped into three categories: (i) state changing, (ii) phase interrupting, and (iii) transit time. State-changing processes such as collisions and spontaneous emission cause the atomic coherence to decay at the average of the level probability decay rates. Phase-interrupting collisions are those that do not change the level probabilities, but do disturb the phase of the oscillating dipole moment in some random fashion. For a single atom this is not of importance but for an ensemble, the net atomic coherence is reduced. The effective decay rate of the atomic coherence is then increased over the rate determined by state-changing processes. The motion of an atom across the finite beam width of the field introduces an additional decay mechanism. We expect the decay rates of all matrix elements to be similarly affected. Transit-time effects can be of importance for the case in which the atoms are injected into the system. The motion of an atom causes it to sample the transverse profile of the field and move out of the interaction region. This allows us to maintain a reasonably uniform density for a constant injection rate. If the electric field has significant transverse variations, a plane-wave description of the field is no longer valid. This must be kept in mind when analyzing the results of atomic interactions with laser beams having typical Gaussian beam profiles.

It is common in the case of atomic electronic transitions that the two-level density-matrix transverse relaxation rate is much larger than the longitudinal decay rate. In an atom there is typically no nearby set of levels that would allow collisional excitation. Hard collisions may occur without any change of state, causing only a disturbance of the phase of the atomic coherence. The atomic coherence for a collection of atoms is not able to survive for more than a few collisional time scales. For this reason collective atomic coherence was ignored in the past. It is difficult to preserve the bulk coherence long enough for studies of cw optical interactions. For the sake of contrast, we mention that in the infrared one typically studies the rotational and vibrational degrees of freedom. The transitions of interest typically have a nearby set of levels that allow a molecule to become collisionally excited. In this case a phase-interrupting collision is also a state-changing collision, so that pure phase-interrupting effects are unlikely. For a more detailed discussion see Ref. 19.

In order to model the atomic-level-pumping process simply, we assume that a number N of atoms per unit volume is injected into the system at a rate r, so that the level-pumping rates per unit volume are $\lambda_a = Nr|c_a|^2$ and $\lambda_b = Nr|c_b|^2$, where c_a and c_b are the probability amplitudes of the atomic levels shown in Fig. 1. These pumping contributions are the ones we typically encounter for excited-state systems, and they form the *phase-insensitive* part of the atomic injection. We continue by allowing the pumping to possess a nonvanishing atomic coherence $\lambda_{ab} = Nr\epsilon c_a c_b^*$, where we neglect the spatial dependence for now and ϵ denotes the statistical purity of the injected mixture. For the case of completely uncorrelated atoms ($\epsilon = 0$, mixed state) the atomic coherence is averaged out and we are left with the phase-insensitive part of the pumping. For the perfectly correlated case ($\epsilon = 1$, pure state) the maximum atomic coherence is achieved for the particular (phase-sensitive) pumping scheme used. These terms contribute directly to the population-matrix equations of motion. The terms arising from the phase-insensitive part of the pumping have the factor N_-, denoting the unsaturated population difference. Terms resulting from the phase-sensitive part appear with the factor λ_{ab}.

The technique for creating a coherent atomic state has been known for a long time in coherent transient studies.[20,19] One often thinks in terms of the Bloch vector, which is a construction for visualizing the response of a two-level medium (with equal decay rates) to an electromagnetic pulse of energy. The vector model results from a torque equation derived from the coupling of the population difference and the atomic coherence. By allowing the field to interact with the medium for a prescribed length of time, any desired superposition of the two levels can be obtained.

A system of current experimental and theoretical interest is the micromaser. Recent experimental techniques[21] make it possible to study the interaction of a single atom with an electromagnetic field in a high-Q maser cavity. An atomic beam can be prepared in a coherent superposition of states by injecting Rydberg atoms in the upper state into a cavity so that a coherent microwave field builds up. The phase of the field is impressed on the

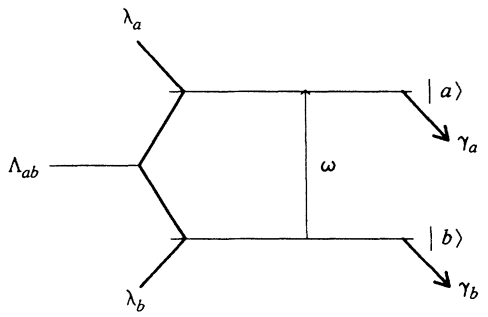

FIG. 1. Schematic energy-level diagram for a two-level atom with an angular resonance frequency ω between the upper level $|a\rangle$ and lower level $|b\rangle$. The level probabilities are pumped at rates per unit volume λ_a and λ_b and decay at rates γ_a and γ_b, respectively. The two-level coherence is pumped at a rate per unit volume Λ_{ab}.

atoms, and they leave the cavity in a phase-locked coherent superposition of states. It is crucial that the coherence be phase locked; otherwise, the effects of the atomic coherence are averaged out. The phase difference in the probability amplitudes of the atomic states corresponds to the phase of the field and is the same for all atoms.[22] The micromaser can be used for experimental preparation of an atomic beam in a predetermined coherent superposition of states.

III. PHASE MIXING

The interaction of an electric field and an injected atomic coherence leads to possible phase mismatching in both time and space. Temporal phase mismatch results from the coupling of a field and injected coherence of different frequencies. Spatial phase mismatch arises when the polarization due to the injected coherence has a different spatial profile from that of the electric field to which it attempts to couple. The equation of motion describing the atom-field coupling develops a phase-mismatch factor, which leads to a periodic modulation of the interaction. For systems of small dimensions, e.g., the micromaser, the medium behaves as "thin" so that the spatial phase-mismatch factor changes negligibly over the region of interest. The Maxwell equations require a specific relationship to be satisfied for the spatial and temporal variations of an electromagnetic field. This dispersion relation constrains the magnitude of the wave vector and the oscillation frequency. There is no such relation for the injected coherence. The time dependence is fixed by the Bohr frequency condition of the transition, but the spatial distribution of atoms may be chosen arbitrarily. If we want the injected coherence to couple to a propagating electric field, we must phase match it to the field. This constraint is also valid if we expect the atomic coherence to give rise to a field on its own. This follows directly from the fact that every field must satisfy the dispersion relation if it is to survive in the medium.

In order to phase match running- or standing-wave electric fields, a similar rapidly varying spatial profile must be constructed for the injected coherence. If the atomic injection is along the field propagation direction, the atoms trace out a running-wave profile with a wave number given by the ratio of the coherence oscillation frequency to the atomic speed. Temporal phase matching requires the atomic coherence and the field to oscillate at the same frequency. This clearly poses two problems. The first is that the atoms must travel at the speed of light in order to trace out the matching spatial profile. The second is that the resonance is destroyed by the Doppler effect. In order to achieve an acceptable degree of phase matching, we must then use the transverse degrees of freedom to carry out the atomic injection. In this way we imagine setting up a series of coherent atomic beam emitters along the field propagation direction, and direct the beams transverse to the field.

The easiest case to consider is approximately phase matching a standing wave by using identical atomic beam emitters, and spacing the beams so that they inject coherently prepared atoms at the peaks of the standing-wave pattern. We may think of this as an attempt to approximate the sinusoidal field profile with a periodic rectangular profile. The two-level medium is then not uniformly distributed in space and the profile cannot be described by a single wave number. A better approximation may be obtained if we space the atomic beam emitters as closely as possible and amplitude modulate the strength of the emitted coherence. This may be carried out by varying the purity of the statistical mixture ϵ of the atomic beams. This amounts to assigning $\epsilon=\epsilon(z)$, the wave number of the electric field. For a running wave we arrange the system as for the standing-wave case, but we modulate the atomic beams, so that each successive atomic beam apparatus uses a properly incremented phase. Another method is to incline the axis of the atomic beam emitters relative to the propagation direction. A linear phase dependence arises as a result of the varying distance the beams must cover in order to reach the interaction region. This is, of course, limited by the maximum distance over which an atomic coherence can be maintained. When the distance becomes too large, the velocity dispersion destroys the atomic coherence.

IV. SINGLE-MODE POLARIZATION

This section develops the atomic polarization for the one-photon semiclassical electric-dipole interaction of a single-mode electric field and a dilute atomic medium. The medium is assumed to be an ensemble of "two-level" atoms having identical energy-level structures in their isolated states. We call $|a\rangle$ the upper energy eigenstate and $|b\rangle$ the lower energy eigenstate of the unperturbed system. The levels are assumed to acquire population gain by the pumping rates per unit volume λ_a and λ_b, coherence gain by the rate per unit volume Λ_{ab}, and suffer population loss at rates γ_a and γ_b, respectively. These processes are taken to be independent of position and time. Figure 1 is a schematic representation of such a two-level atom. The dipole coherence retains its effective decay rate γ. The medium is assumed to be homogeneously broadened so all atoms have an angular frequency ω for the atomic line center. The two levels of interest are assumed to have a nonvanishing electric-dipole matrix element, so that the coherent effects produced by the atom-field interaction are assumed to be limited to the one-photon coupling of the states $|a\rangle$ and $|b\rangle$ arising from the unperturbed quantum-mechanical atomic Hamiltonian operator $\mathcal{H}_0$. The time-dependent electric-dipole interaction energy operator is given by

$$\mathcal{V}(\mathbf{r},t)=-e\tilde{\mathbf{r}}\cdot\mathbf{E}(\mathbf{r},t)\ ,$$

where the operator $e\tilde{r}$ is the atomic electric-dipole moment and the electric field $\mathbf{E}(\mathbf{r},t)$ is considered to be a classical scalar field for the semiclassical treatment presented in this work.

To each atom of the ensemble a density operator $\hat{\rho}^{(i)}$ may be assigned, which together with all other such operators form a population operator[23] $\rho(\mathbf{r},t)$. The Hermitian population operator contains all the information we may obtain about the system and obeys the master

equation (obtained from the Schrödinger or equivalent Heisenberg equation of motion)

$$i\hbar\frac{d\rho}{dt}=[\mathcal{H},\rho]+\Gamma(\rho)\ ,$$

where the total semiclassical Hamiltonian $\mathcal{H}=\mathcal{H}_0+\mathcal{V}(t)$, and Γ consists of any possible phenomenological contributions to the evolution, e.g., decay processes and external pumping mechanisms. For arbitrary state vectors $|i\rangle$ and $|j\rangle$ the matrix element of the atomic electric-dipole operator between these states is

$$\wp_{ij}\equiv\langle i|e\vec{r}|j\rangle=\wp_{ji}^{*}\ ,$$

and ρ_{ij} is the corresponding element for the population matrix. The general equation of motion for the population-matrix elements is given by

$$\dot{\rho}_{ij}=\Lambda_{ij}-(\gamma_{ij}+i\omega_{ij})\rho_{ij}-\frac{i}{\hbar}[\mathcal{V},\rho]_{ij}\ . \tag{1}$$

The polarization of the medium is the expectation value of the total dipole moment per unit volume that can be calculated using the relation

$$P(\mathbf{r},t)=\langle e\vec{r}\rangle=\mathrm{Tr}\{e\vec{r}\rho(\mathbf{r},t)\}=\sum_{i,j}\wp_{ij}\rho_{ji}\ , \tag{2}$$

where the sums are over complete sets of states. We need only consider those states $\{|i\rangle\}$ that play a role in the electromagnetic interaction. In the one-photon two-level model the energy-eigenstate representation of interest is $\{|i\rangle\}=|a\rangle,|b\rangle$. We define $\wp\equiv\wp_{ab}$ and obtain the polarization expression

$$P(\mathbf{r},t)=\wp^{*}\rho_{ab}(\mathbf{r},t)+\mathrm{c.c.} \tag{3}$$

The off-diagonal population-matrix element is directly related to the polarization of the two-level medium and depends on the level-population difference. This coupling of the population difference and the polarization causes the population-matrix element equations of motion to yield a nonlinear relation between the electric field and the polarization of the medium.

The simplest case of an electric field for our studies is one which oscillates harmonically in time at a single angular frequency ν and has the form

$$E(\mathbf{r},t)=\tfrac{1}{2}\tilde{\mathcal{E}}(\mathbf{r},t)e^{-i\nu t}+\mathrm{c.c.}\ , \tag{4}$$

where $\tilde{\mathcal{E}}(\mathbf{r},t)$ is the complex mode amplitude and ν is an optical angular frequency. The amplitude $\tilde{\mathcal{E}}$ is considered to be slowly varying in time so that negligible variations occur for times on the order of the optical temporal period $2\pi/\nu$. We assume the injected atomic coherence to have the form

$$\Lambda_{ab}=\tilde{\lambda}_{ab}e^{-i\omega t}\ , \tag{5}$$

which is a dipole coherence oscillating at the resonance frequency of the atomic transition with a complex amplitude $\tilde{\lambda}_{ab}$ that is slowly varying in time but may have a spatial dependence that is rapidly varying—in particular, that defined in Eq. (50) below. The spectrum of the electric field and the injected atomic coherence is illustrated in Fig. 2. The matrix elements of the interaction energy in this representation using the rotating-wave approximation (RWA) are

$$\mathcal{V}_{ab}=\mathcal{V}_{ba}^{*}=-\tfrac{1}{2}\wp\tilde{\mathcal{E}}e^{-i\nu t}\ .$$

The response of the medium is described in terms of a population matrix $\rho(\mathbf{r},t)$ using the equations of motion

$$\dot{\rho}_{aa}=\lambda_a-\gamma_a\rho_{aa}-(i\hbar^{-1}\mathcal{V}_{ab}\rho_{ba}+\mathrm{c.c.})\ , \tag{6}$$

$$\dot{\rho}_{bb}=\lambda_b-\gamma_b\rho_{bb}+(i\hbar^{-1}\mathcal{V}_{ab}\rho_{ba}+\mathrm{c.c.})\ , \tag{7}$$

$$\dot{\rho}_{ab}=\Lambda_{ab}-(\gamma+i\omega)\rho_{ab}+i\hbar^{-1}\mathcal{V}_{ab}(\rho_{aa}-\rho_{bb})\ . \tag{8}$$

The level decay rates γ_a and γ_b are introduced phenomenologically and γ is the dipole decay rate. We define the two-level population sum and difference as

$$S(\mathbf{r},t)=\rho_{aa}(\mathbf{r},t)+\rho_{bb}(\mathbf{r},t)\ , \tag{9}$$

$$D(\mathbf{r},t)=\rho_{aa}(\mathbf{r},t)-\rho_{bb}(\mathbf{r},t)\ , \tag{10}$$

respectively. It is convenient to express the above equations of motion in the temporally slowly varying (interaction picture) forms

$$\dot{\rho}_{aa}=\lambda_a-\gamma_a\rho_{aa}-(i\mathcal{V}\tilde{\rho}_{ba}+\mathrm{c.c.})\ , \tag{11}$$

$$\dot{\rho}_{bb}=\lambda_b-\gamma_b\rho_{bb}+(i\mathcal{V}\tilde{\rho}_{ba}+\mathrm{c.c.})\ , \tag{12}$$

$$\dot{\tilde{\rho}}_{ab}=B_t\tilde{\lambda}_{ab}-(\gamma+i\delta)\tilde{\rho}_{ab}+i\mathcal{V}(\rho_{aa}-\rho_{bb})\ , \tag{13}$$

where the atom-field coupling factor

$$\mathcal{V}=-\wp\tilde{\mathcal{E}}/2\hbar\ , \tag{14}$$

the slowly varying coherence matrix element

$$\tilde{\rho}_{ab}=\rho_{ab}e^{i\nu t}\ , \tag{15}$$

the temporal injected-coherence-field phase-mismatch factor

$$B_t=e^{-i\delta t}\ , \tag{16}$$

and the detuning of the atomic resonance frequency ω from the field frequency

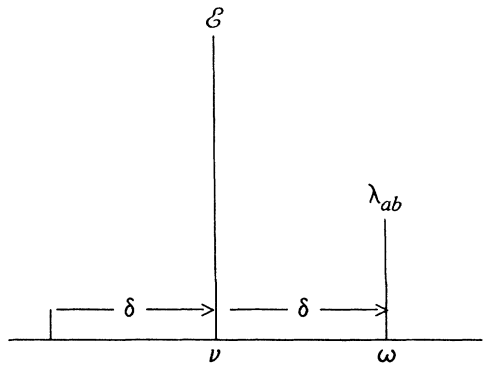

FIG. 2. Spectrum for a single-mode electric field $\mathcal{E}$ with an injected atomic coherence λ_{ab} for a one-photon transition. The detuning of the atomic transition angular frequency from the optical angular frequency ν is $\delta=\omega-\nu$. For nonlinear interactions a sideband at frequency $\nu-\delta$ is anticipated.

$$\delta=\omega-\nu \ . \tag{17}$$

The corresponding equations of motion for the population difference and sum are

$$\dot{D}=-\gamma_+ D-\gamma_- S+2\lambda_- -2(i\mathcal{V}\tilde{\rho}_{ba}+\text{c.c.}) \ , \tag{18}$$

$$\dot{S}=-\gamma_+ S-\gamma_- D+2\lambda_+ \ , \tag{19}$$

where the incoherent exchange constants are defined as

$$\gamma_\pm=\tfrac{1}{2}(\gamma_a\pm\gamma_b) \ , \tag{20}$$

$$\lambda_\pm=\tfrac{1}{2}(\lambda_a\pm\lambda_b) \ . \tag{21}$$

In order to obtain an expression for the polarization, we must solve Eq. (8) [or Eq. (13)] for the off-diagonal element ρ_{ab}. The contribution of the injected atomic coherence to the coherent terms of Eq. (18) gives rise to temporal oscillations at angular frequencies $\pm\delta$ in the population difference. The detuning off line center determines the rate of these population pulsations. The solutions for the population-matrix elements of Eqs. (6)–(8) can be obtained by describing the matrix elements in terms of temporal frequency factors arising from the coherent interaction of the electric field and the atomic oscillation. We Fourier analyze both the polarization (dipole coherence) ρ_{ab} and the level populations ρ_{aa} and ρ_{bb} as

$$\rho_{ab}(\mathbf{r},t)=e^{-i\nu t}\sum_{m=-\infty}^{\infty} p_m e^{im\delta t} \ , \tag{22}$$

$$\rho_{\alpha\alpha}(\mathbf{r},t)=\sum_{k=-\infty}^{\infty} n_{\alpha k}e^{ik\delta t}, \quad \alpha=a,b \ . \tag{23}$$

We extend these expansions to the population difference and write

$$D(\mathbf{r},t)=\sum_{k=-\infty}^{\infty} d_k e^{ik\delta t} \ . \tag{24}$$

Substituting the expansions of Eqs. (22) and (23) into Eq. (8), we find

$$p_m=i\mathcal{D}_m\mathcal{V}d_m+\mathcal{D}_{-1}\tilde{\lambda}_{ab}\delta_{m,-1} \ , \tag{25}$$

where the complex-Lorentzian denominator

$$\mathcal{D}(\delta)=1/(\gamma+i\delta) \ , \tag{26}$$

with the convenient notation

$$\mathcal{D}_k=\mathcal{D}[(k+1)\delta] \tag{27}$$

and $\mathcal{V}$ as in Eq. (14) above. We continue the decomposition by using the expansions in Eqs. (6) and (7) to obtain the general relations

$$d_k=-i2T_1\mathcal{F}_k^\dagger(\mathcal{V}p^*_{-k}-\mathcal{V}^*p_k)+N_-\delta_{k0} \ , \tag{28}$$

where the unsaturated population difference

$$N_-=\lambda_a/\gamma_a-\lambda_b/\gamma_b \ , \tag{29}$$

the time scales for the incoherent processes are given by the average level lifetime

$$T_1=\tfrac{1}{2}(\gamma_a^{-1}+\gamma_b^{-1}) \ , \tag{30}$$

the dipole decay time $T_2=1/\gamma$, and the dimensionless complex-population-pulsation factor

$$\mathcal{F}_k^+(\delta)=\frac{1}{2T_1}\left[\frac{1}{\gamma_a+ik\delta}+\frac{1}{\gamma_b+ik\delta}\right] \ . \tag{31}$$

The Fourier dipole coherence components p_m can be eliminated from the population difference expression by substituting Eq. (25) into Eq. (28). We find

$$d_k=\mathcal{S}_k[2T_1\mathcal{F}_k^+(q\delta_{k,1}+q^*\delta_{k,-1})+N_-\delta_{k,0}] \ , \tag{32}$$

where the injected coherence coupling factor

$$q=-i\mathcal{V}\mathcal{D}^*_{-1}\tilde{\lambda}^*_{ab} \ , \tag{33}$$

the saturation factor

$$\mathcal{S}_k=\left[1+\tilde{I}\mathcal{F}_k^+\frac{\gamma}{2}(\mathcal{D}_k+\mathcal{D}^*_{-k})\right]^{-1} \ , \tag{34}$$

and the dimensionless Lorentzian

$$\mathcal{L}(\delta)=\gamma^2/(\gamma^2+\delta^2) \ . \tag{35}$$

Since the physical intensity is defined as the average irradiance of the field, we write it as $c\epsilon|\mathcal{E}|^2/2$ (in mks units). A convenient parameter for our analysis is the dimensionless intensity $I=|\mathcal{E}|^2/|\mathcal{E}_s|^2$, which corresponds to the intensity that produces a steady-state population difference which is one-half the unsaturated value. For the one-photon coupling the corresponding field amplitude is given by the expression for its inverse

$$\mathcal{E}_s^{-1}=\frac{\wp}{\hbar}(T_1T_2)^{1/2} \ , \tag{36}$$

and we write $I_s=|\mathcal{E}_s|^2$. The dimensionless intensity is then expressed as

$$\tilde{I}=|\wp\tilde{\mathcal{E}}/\hbar|^2T_1T_2 \ . \tag{37}$$

We note that $d_{-k}=d_k^*$ as expected since the population difference is real valued. The self-consistent form for the p_m components is derived by using Eq. (32) to eliminate the population-difference components in Eq. (25). This results in

$$p_m=i\mathcal{D}_m\mathcal{V}\mathcal{S}_m[2T_1\mathcal{F}_m^+(q\delta_{m,1}+q^*\delta_{m,-1})+N_-\delta_{m,0}] +\mathcal{D}_{-1}\tilde{\lambda}_{ab}\delta_{m,-1} \ . \tag{38}$$

It is worthwhile to consider what the general relations of Eqs. (32) and (38) tell us about developing polarizations in our one-photon two-level (excited-state) model. If there is no pumping of the levels the system is "shut down" so that no polarization persists. Once the pumping is turned on, there is a possibility of polarization. For pumping allowing no coherence ($N_-\neq0, |\lambda_{ab}|=0$), electric-field-induced polarizations develop. This is the standard result of an electric field interacting with a steady-state (saturated) population difference: the $m=0$ case of Eq. (38). If we now allow the coherence ($N_-\neq0\neq|\lambda_{ab}|$), a field-independent polarization exists. Whenever a field is present, it couples to this polarization and drives the population difference, thus modifying the

steady-state saturation of the medium. For systems tuned off resonance, a beating occurs as indicated by the $k\neq 0$ terms in Eq. (32). These effects are the population pulsations which may, in turn, interact with the field to produce coherent nonlinear polarizations—terms in Eq. (38) containing the population pulsation factor $\mathcal{F}_m^+$. One could imagine eliminating the effect of the unsaturated population difference by pumping so that $N_- =0$, leaving only contributions requiring the injected coherence.

The existence of a nonvanishing coherence from the pumping process leads to a direct input polarization at the atomic transition frequency. This is not due to a saturation phenomenon, as evidenced by the lack of a saturation factor in the last term of Eq. (38). Equations (32) and (38) restrict the values of the indices to be $k,m=0,\pm 1$. The only Fourier components that appear are p_0, $p_{\pm 1}$, d_0, and $d_{\pm 1}$. No other components are needed and no restriction on the strength of the electric field has been imposed. The polarization of the medium can be written as

$$\begin{aligned} P(\mathbf{r},t)&=\tfrac{1}{2}\tilde{\mathcal{P}}(\mathbf{r},t)e^{-i\nu t}+\text{c.c.}\\ &=\tfrac{1}{2}[\tilde{\mathcal{P}}_1(\mathbf{r},t)e^{i\delta t}+\tilde{\mathcal{P}}_2(\mathbf{r},t)\\ &\quad+\tilde{\mathcal{P}}_3(\mathbf{r},t)e^{-i\delta t}]e^{-i\nu t}+\text{c.c}\ , \end{aligned} \tag{39}$$

where the temporally slowly varying complex-polarization amplitudes are

$$\tilde{\mathcal{P}}_m=2\wp^* p_{2-m}\ . \tag{40}$$

The expressions for the polarization amplitudes are

$$\tilde{\mathcal{P}}_1=4\wp^* T_1\mathcal{F}_1^+\mathcal{D}_1\mathcal{V}^2\mathcal{D}_{-1}^*\tilde{\lambda}_{ab}^*\mathcal{S}_1\ , \tag{41}$$

$$\tilde{\mathcal{P}}_2=i2\wp^* N_-\mathcal{D}\mathcal{V}\mathcal{S}_0\ , \tag{42}$$

$$\tilde{\mathcal{P}}_3=2\wp^*(1-\tfrac{1}{2}\tilde{I}\mathcal{F}_1^+\mathcal{S}_1)^*\mathcal{D}_{-1}\tilde{\lambda}_{ab}\ , \tag{43}$$

where the special cases of the saturation factor $\mathcal{S}_k$ are the real valued

$$\mathcal{S}_0=[1+\tilde{I}\mathcal{L}(\delta)]^{-1}\ , \tag{44}$$

and the complex valued

$$\mathcal{S}_1=\left[1+\tilde{I}\mathcal{F}_1^+\frac{\gamma}{2}[\mathcal{D}(2\delta)+\mathcal{D}^*(0)]\right]^{-1}\ . \tag{45}$$

The polarization $\tilde{\mathcal{P}}_2$ is induced solely by the electric field. The sideband polarization $\tilde{\mathcal{P}}_3$ has a direct contribution from the injected atomic coherence given by the first term in Eq. (43), while the second term results from a nonlinear interaction with the field. The other sideband $\tilde{\mathcal{P}}_1$ consists only of a nonlinear contribution.

V. SINGLE-RUNNING-WAVE FIELD

In order to develop a propagation equation for the electric-field mode, we must specify the spatial dependence of the field and the injected atomic coherence. We consider the simple case of a unidirectional running-wave field by modifying Eq. (4) as

$$E(\mathbf{r},t)=\tfrac{1}{2}\mathcal{E}(\mathbf{r},t)e^{i(\mathbf{K}\cdot\mathbf{r}-\nu t)}+\text{c.c.}\ , \tag{46}$$

$$\tilde{\mathcal{E}}(\mathbf{r},t)=\mathcal{E}(\mathbf{r},t)e^{i\mathbf{K}\cdot\mathbf{r}}\ , \tag{47}$$

where $\mathbf{K}$ is the wave vector satisfying the dispersion relation $K=\nu/c$, and $\mathcal{E}$ is an electric-field mode amplitude, which is slowly varying over an optical wavelength $2\pi/K$ as well as slowly varying in time. For the running-wave case $|\tilde{\mathcal{E}}|=|\mathcal{E}|$, so the dimensionless intensity $\tilde{I}$ of Eq. (37) becomes

$$I=|\wp\mathcal{E}/\hbar|^2 T_1 T_2\ . \tag{48}$$

The injected coherence of Eq. (5) is also chosen to have a unidirectional running-wave form,

$$\Lambda_{ab}=\lambda_{ab}e^{i(\mathbf{K}_\Lambda\cdot\mathbf{r}-\omega t)}\ , \tag{49}$$

$$\tilde{\lambda}_{ab}=\lambda_{ab}e^{i\mathbf{K}_\Lambda\cdot\mathbf{r}}\ , \tag{50}$$

where λ_{ab} is a slowly varying complex amplitude, just as $\mathcal{E}$ is, and $\mathbf{K}_\Lambda$ is the wave vector. We express the injected coherence in terms of its phase mismatch relative to the field as

$$\Lambda_{ab}=B\lambda_{ab}e^{i(\mathbf{K}\cdot\mathbf{r}-\nu t)}\ , \tag{51}$$

where the injected-coherence-field phase-mismatch factor

$$B=e^{i[(\mathbf{K}_\Lambda-\mathbf{K})\cdot\mathbf{r}-\delta t]}\ . \tag{52}$$

If the space-time dependence of the injected coherence is sufficiently similar to the field, then we may treat the phase-mismatch factor as slowly varying in both time and space. We assume that the electric field induces a polarization

$$\begin{aligned} P(\mathbf{r},t)&=\tfrac{1}{2}[\mathcal{P}_1(\mathbf{r},t)B^*+\mathcal{P}_2(\mathbf{r},t)\\ &\quad+\mathcal{P}_3(\mathbf{r},t)B]e^{i(\mathbf{K}\cdot\mathbf{r}-\nu t)}+\text{c.c.}\ , \end{aligned} \tag{53}$$

where the component complex-polarization amplitudes are

$$\tilde{\mathcal{P}}_1=\mathcal{P}_1 e^{-i\mathbf{K}_\Lambda\cdot\mathbf{r}}\ , \tag{54}$$

$$\tilde{\mathcal{P}}_2=\mathcal{P}_2 e^{i\mathbf{K}\cdot\mathbf{r}}\ , \tag{55}$$

$$\tilde{\mathcal{P}}_3=\mathcal{P}_3 e^{i\mathbf{K}_\Lambda\cdot\mathbf{r}}\ , \tag{56}$$

with the polarization amplitudes $\mathcal{P}_m$ slowly varying in the same sense as $\mathcal{E}$.

The steady-state slowly varying Maxwell equation for the electric-field amplitude reads[24]

$$\frac{d\mathcal{E}(z)}{dz}=i\frac{K}{2\epsilon}\mathcal{P}(z)\ , \tag{57}$$

where the z direction is taken to be along $\mathbf{K}$ and the time dependences of the polarization amplitudes are assumed to be negligible. We use the Fourier polarization amplitudes of Eqs. (41)–(43) to write the propagation equation

$$\frac{d\mathcal{E}}{dz}=-\alpha\mathcal{E}+\beta_1 B^*\lambda_{ab}^*+\beta_3 B\lambda_{ab}\ , \tag{58}$$

where

$$\alpha=\alpha_0\gamma\mathcal{D}(\delta)\mathcal{S}_0 , \tag{59}$$

$$\beta_1=i\beta_0 I\mathcal{T}_1^{+}\gamma\mathcal{D}(2\delta)\mathcal{S}_1 e^{-i2\Phi} , \tag{60}$$

$$\beta_3=i\beta_0^{*}(2-I\mathcal{T}_1^{+}\mathcal{S}_1)^{*} , \tag{61}$$

$$\alpha_0=-N_{-}K|\wp|^2/2\epsilon\hbar\gamma , \tag{62}$$

$$\beta_0=K\wp/2\epsilon\gamma , \tag{63}$$

$$e^{-i\Phi}=\wp\mathcal{E}/|\wp\mathcal{E}| . \tag{64}$$

If we ignore the injected coherence, we obtain the familiar Beer's law equation with the same single-mode complex-absorption coefficient. When the injected coherence is turned on, it contributes to the polarization directly at the atomic resonance frequency and through population pulsation processes. The latter polarization contributions are nonlinear in the electric field (aside from saturation), causing the propagation equation to become even more nonlinear. The propagation equation is a nonlinear differential equation that is not easily solved. When we tune the field off line center, the input field induces a polarization at a different frequency than that provided by the injected atomic coherence. The injected coherence produces frequency-symmetric sidebands about the field frequency. The magnitude of the sideband displacement equals the detuning.

In general, the coupling of the injected coherence to the electric field suffers a phase mismatch in both time and space. A weak phase mismatch reduces the coupling, thus making it more difficult to observe experimentally. For a temporal mismatch the integration time of the detector becomes important. If the detector has an integration time τ, then the coherence shows little change with respect to the field for $\tau\ll\delta^{-1}$. For times such that $\tau\gtrsim\delta^{-1}$, the oscillations in the terms arising from the injected coherence wash out. The observer then must tune the spectrometer to the atomic resonance frequency in order to see the effects of the injected coherence, but these are unrelated to the original electric field. A similar situation occurs for a spatial phase mismatch. If the interaction region has a length in the propagation direction that allows many oscillations of the phase-mismatch factor, then the coupling averages to zero. For interaction lengths much less than the spatial oscillation period, the effect of the injected coherence is preserved, i.e., it is not washed out.

We examine the properties of the propagation coefficients of Eq. (58). The single-wave complex-absorption coefficient of Eq. (59) is the standard complex-Lorentzian response of an atomic dipole behaving as a classical oscillator. The absorption profiles (real part) are power-broadened Lorentzians with peak values being saturated down due to a reduction in the population difference. The index profiles show the typical frequency dependence of the relative phase shift arising from driving an oscillator off resonance.

The normalized conjugate-sideband injected coherence coefficient β_1/β_0 of Eq. (60) as a function of detuning for various intensities is illustrated in Fig. 3. Please note that the parameter values corresponding to the set of curves on a single plot are listed in order of solid, dashed, dot-dashed, and dotted curves. The dependence on detuning proves to be only significant near resonance ($\delta=0$). The large scale structure of the curves near resonance shows that the real and imaginary parts are similar in form to those of α with the real and imaginary parts interchanged (with a sign change for the imaginary part). This is not surprising if we view the injected coherence as a polarization and remember that it drives the field in a manner that may be called *in-quadrature*, i.e., the real part couples to the imaginary part and vice versa. This coefficient depends entirely on the coherent nonlinear interaction of the field and the injected coherence. $\mathrm{Im}(\beta_1)$ shows shoulders which lead into wings that are reminiscent of coupling coefficients.

The normalized injected coherence coupling coefficient β_3/β_0 of Eq. (61) is plotted in Fig. 4 as a function of de-

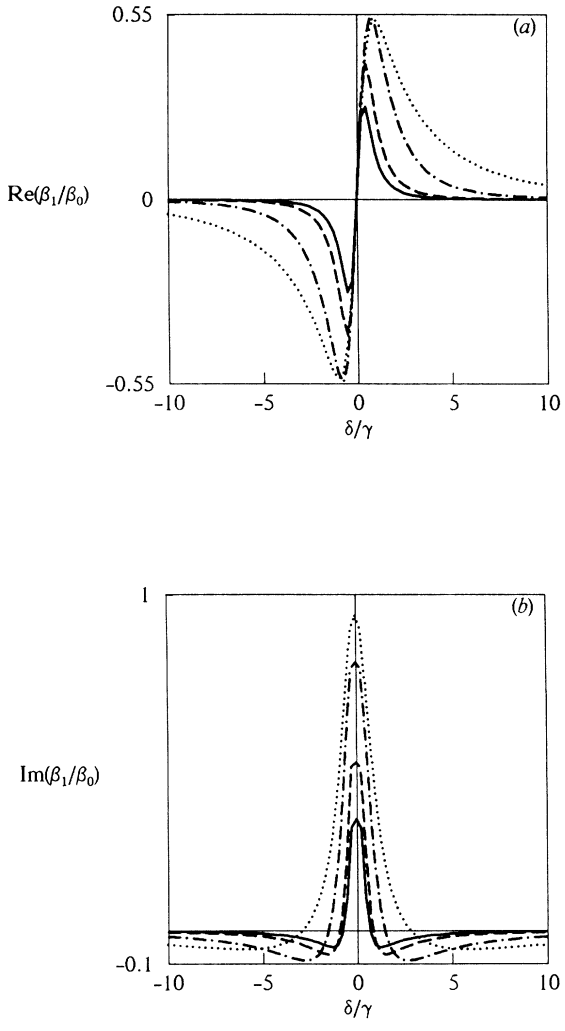

FIG. 3. The conjugate-sideband injected coherence coefficient β_1 of Eq. (60) as a function of detuning for $I=0.5$, 1, 4, and 16, with $\Phi=0$. In (a) the curve has indexlike profiles similar to $\mathrm{Im}(\alpha)$, but β_1 increases in strength with I since it depends solely on the nonlinear scattering processes. Plot (b) shows a Lorentzian-like feature similar to $\mathrm{Re}(\alpha)$ except for the wings at large $|\delta|$ and the increase with field intensity.

tuning for the same intensities as before. The gross structure seen here is again in-quadrature to α but we notice that the imaginary part has wings approaching a nonzero value, as if a resonant weak field were probing the unsaturated transition. This is a valid way of considering the plot since the injected coherence contributes a constant value independent of how strongly the transition is saturated. The coherent contributions to β_3 show up clearly near resonance as they do for β_1, yet the curves look very different. β_1 results from the interaction of $\mathcal{E}$ with the grating formed by $\mathcal{E}\lambda_{ab}^{*}$ leading to an oscillator response $\gamma\mathcal{D}(2\delta)$, which quickly damps the profile in δ. The coherent part of β_3 results from $\mathcal{E}$ scattering off of $\mathcal{E}^{*}\lambda_{ab}$ leading to an overall modulating function $\gamma\mathcal{D}(0)$, which is unity. This allows the central coherent features of the interaction contained in $\mathcal{T}_1^{+}\mathcal{S}_1$ to fully develop.

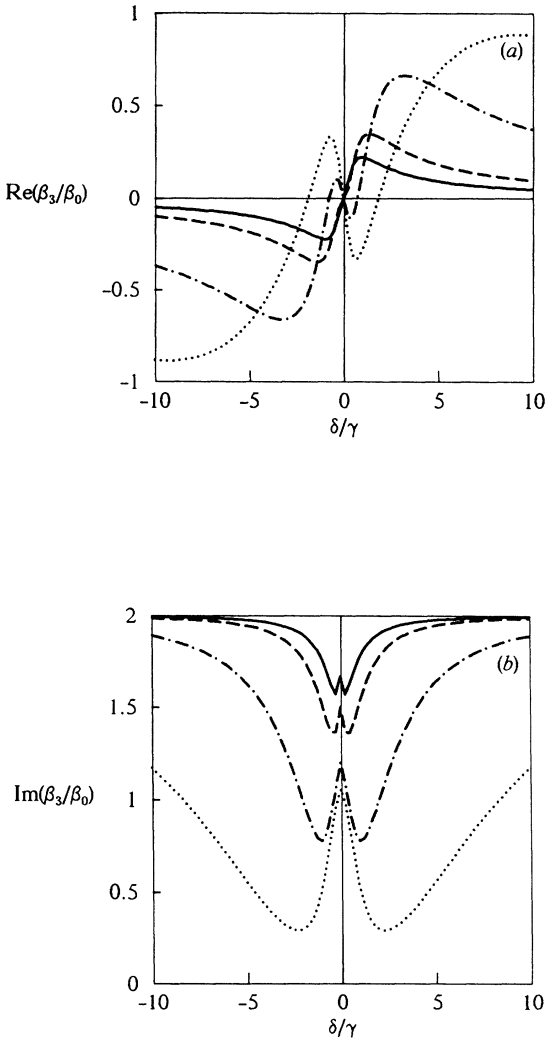

FIG. 4. Plots of the real part (a) and imaginary part (b) of the injected coherence coefficient β_3 of Eq. (61) vs the detuning, for the same values of the intensity I as listed in Fig. 3. For small detunings the effects of the population pulsations can be seen. The imaginary part has wings that approach a nonzero value since the injected coherence itself is a polarization and contributes directly to the propagation.

The "inverted dip" structures in Figs. 3(b) and 4(b) are commonly seen in the spectra of inhomogeneously broadened media or by having the atomic transition profile scanned through the saturator frequency, as effectively done here.

The appearance of the sideband polarization in Eq. (53) oscillating with a phase factor $\exp[i(\mathbf{K}_\Lambda\cdot\mathbf{r}-\omega t)]$ indicates that the growth of a sidemode $\mathcal{E}_1\exp[i(\mathbf{K}_1\cdot r-\nu_1 t)]$ is possible, where $\nu_1=\omega$ and $K_1=\nu_1/c$ as required by the Maxwell equations. The first term in Eq. (43) for the sideband amplitude $\mathcal{P}_3$ simply indicates that there is a polarization oscillating as $\exp[i(\mathbf{K}_\Lambda\cdot\mathbf{r}-\omega t)]$ since it has been injected as such. This contribution to the polarization does not involve the response of the medium to any coherent superposition of fields. The term containing the complex-population-pulsation factor $\mathcal{T}_1^{+}$ can be thought of as a coherent contribution since it results from the scattering of the field off the fringe grating formed by the coherent superposition of the field and the injected atomic coherence. The ability of the field to propagate depends on the phase-match condition $|\mathbf{K}_\Lambda-\mathbf{K}|\cong 0$.

VI. WEAK-FIELD SOLUTION

We calculate the solution for the slowly varying electric-field-mode envelope $\mathcal{E}$ for the weak-field case. The linearized propagation equation is

$$\frac{d\mathcal{E}}{dz}=-\alpha_0\gamma\mathcal{D}(\delta)\mathcal{E}+i2\beta_0 B_t\lambda_{ab}e^{i\Delta Kz}\,, \tag{65}$$

where we assume that the field and injected coherence travel nearly parallel to the z axis with the phase mismatch $\Delta K=(\mathbf{K}_\Lambda-\mathbf{K})\cdot\hat{\mathbf{z}}$. From this equation we learn that the first-order spatial dependence of an electric-field mode is determined by the standard linear complex-absorption coefficient and the injected atomic coherence amplitude modulated by its phase mismatch with the field. The weak-field solution is

$$\mathcal{E}(z)=[\mathcal{E}(0)-\mathcal{E}_f]e^{-\alpha z}+\mathcal{E}_f e^{i\Delta Kz}\,, \tag{66}$$

where the complex-absorption coefficient of Eq. (59) can be linearized as $\alpha\cong\alpha_2\gamma\mathcal{D}(\delta)$, and the limiting value of the field amplitude is

$$\mathcal{E}_f=i\frac{2\beta_0}{\alpha_0\gamma\mathcal{D}+i\Delta K}B_t\lambda_{ab}\,. \tag{67}$$

We concern ourselves with the propagation of a weak field for the special case of central tuning and phase matching in Sec. VII.

VII. PHASE-MATCHED WEAK-FIELD PROPAGATION

The normal mode (unperturbed) electric field has an associated wave number that must satisfy the free-space dispersion relation $K=\nu/c$. The dispersion relation requires a fixed relationship between the magnitude of the wave vector and the oscillation frequency of an electromagnetic disturbance. There is no restriction on the spatial distribution of atoms comprising the medium, unless we impose one of our own choosing. As a result, the

medium oscillates in time with a frequency determined by the difference in the unperturbed energies of the levels involved in the transition (Bohr condition), but the spatial variation is arbitrary. Therefore, there is no dispersion relation constraining K_Λ and ω.

The relative phase variations in time and space between the field and injected coherence determine the effectiveness of the coupling as they propagate. In order to achieve maximum coupling between the electric wave and the polarization wave, the total relative phase (space and time) must be constant as the wave propagate. This corresponds to the case of perfect phase matching in which the field is tuned to the atomic line center ($\delta=0$) and the spatial variation of the injected coherence satisfies $\mathbf{K}_\Lambda\cdot\hat{\mathbf{K}}=\mathbf{K}$. We assume negligible spatial variations transverse to the direction of propagation so let us choose the z axis to be along the propagation direction and $K_\Lambda=K$ for spatial phase matching. The phase-matched weak-field solution is readily determined from Eqs. (66) and (67) as

$$\mathcal{E}(z)=[\mathcal{E}(0)-\mathcal{E}_f]e^{-\alpha_0 z}+\mathcal{E}_f\ , \tag{68}$$

with

$$\mathcal{E}_f=i\frac{2\beta_0}{\alpha_0}\lambda_{ab}\ . \tag{69}$$

We take the transition dipole-matrix element $\wp$ to be real so that β_0 may be considered real valued. The weak-field solution shows that the exponential dependence of the complex field amplitude on the propagation distance is not affected by the injected atomic coherence. The absorption coefficient α is simply the standard single-mode linear absorption coefficient α_0. The injected coherence λ_{ab} can directly pump the field amplitude in this case since it represents a polarization in the medium the same as the electric-field mode $\mathcal{E}$ induces.

For purposes of demonstration let us choose a convenient set of parameter values and investigate how the electric field is affected by changing particular values. We choose the level-pumping properties so that we may achieve the maximum available coherence. The maximum coherence for a single atom occurs when the moduli of the probability amplitudes of the two levels are equal. Expressing the pumping mechanisms in units of the product of the number density and atomic injection rate leads us to set $\lambda_a=\lambda_b=\frac{1}{2}$. There are actually two ways of achieving these values for an ensemble of atoms: (i) injecting an inhomogeneous collection of atoms, one-half of which are in the upper energy eigenstate and the other half in the lower eigenstate, and (ii) injecting a homogeneous collection of atoms such that an energy measurement for any given atom is equally likely to yield the upper-state value as it is the lower-state value. We are naturally interested in the latter case since the first case, by definition, injects no atomic coherence into the system. In addition to the strength of the coherence, it also possesses a phase for each atom. The phases of the atoms may range from completely uncorrelated (no net coherence) to perfectly correlated for which all phases are identical (maximum coherence). This information is contained in the system density operator and reflects the purity of the statistical mixture. Let ϵ denote the purity of the statistical mixture we pump into the system. The resulting strength of the injected atomic coherence becomes $|\lambda_{ab}|=\frac{1}{2}\epsilon$.

There is still freedom in choosing the level decay rates. Choosing equal decay rates $\gamma_a=\gamma_b$ wipes out the unsaturated population difference ($N_-=0$), causing the medium to become transparent (noninteracting) in so far as its level populations are concerned. The existing two-level coherence is not inert since it gives rise to an electric field or couples to an existing one. We prefer to keep all effects present so we choose $\gamma_a\neq\gamma_b$, in particular, $\gamma_a=\frac{5}{2}$ and $\gamma_b=\frac{5}{8}$. This yields $N_-=-0.6$ and $T_1=1$. The coherence lifetime we choose is $T_2=1$. We express the field amplitude $\mathcal{E}$ in units of the saturation amplitude $|\mathcal{E}_s|$ determined from Eq. (36). Any phases are given in degrees. The propagation distance is given in units of $(\alpha_0')^{-1}$, where $\alpha_0'=-\alpha_0/N_-$ from Eq. (62). We estimate $\wp/\hbar\sim 10^{-2}$ (in mks units) and consider fields to be weak when $|\mathcal{E}|\ll 10^{-2}$ (in units of $|\mathcal{E}_s|$). In summary, the starting values of interest are

$$\begin{aligned}&\lambda_a=\lambda_b=\tfrac{1}{2},\quad |\lambda_{ab}|=\tfrac{1}{2}\epsilon,\quad \phi_\lambda=0\ ,\\ &\gamma_a=\tfrac{5}{2},\quad \gamma_b=\tfrac{5}{8},\quad N_-=-0.60,\quad T_1=1=T_2\ .\end{aligned} \tag{70}$$

We now illustrate the effects of the injected atomic coherence on the propagation of a weak single-mode electric field. The linearized relation of Eq. (68) implies that the initial value of the field becomes less important as the mode propagates and attains its limiting value given by $\mathcal{E}_f$. This effect is shown in Fig. 5 (for absorbers), which plots the field modulus against distance for increasing strengths of the injected coherence. The limiting value of the field strength grows with $|\lambda_{ab}|$. The field quickly approaches its final value which, for the maximum allowed coherence, begins to violate the weak-field assumption.

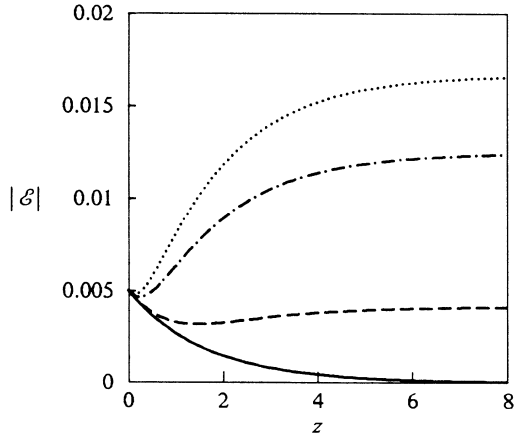

FIG. 5. The modulus of the weak-electric-field amplitude $\mathcal{E}$ of Eq. (68) as a function of distance traveled in the medium z for $|\lambda_{ab}|=0$, 0.125, 0.25, and 0.5. We set $\delta=0$, $\phi_\lambda=0$, and $\mathcal{E}(0)=(0.005,0)$. The field is in units of the saturation amplitude $|\mathcal{E}_s|$ and z in units of $(\alpha_0')^{-1}$. Other values of interest are listed in Eq. (70).

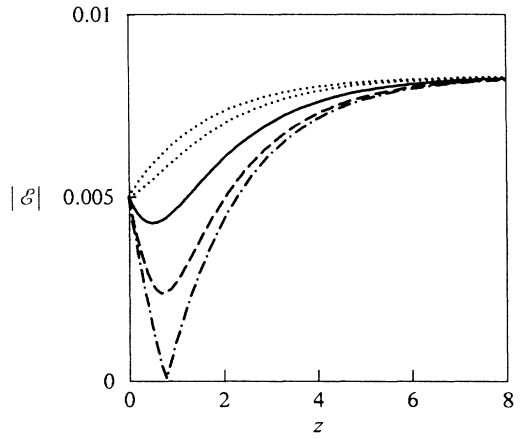

FIG. 6. The modulus of the weak-field amplitude $\mathcal{E}$ as a function of propagation distance z. The variation of the field modulus with the phase ϕ_λ of the injected coherence is illustrated for $\phi_\lambda = 0°$, 45°, 90°, 225°, and 270° (uppermost curve). The initial field value $\mathcal{E}(0) = (0.005, 0)$ and the strength of the injected coherence $|\lambda_{ab}| = \frac{1}{2}$.

For that case a better approximation can be obtained by numerically integrating the nonlinear propagation equation. Since the detuning has vanished, the degenerate absorption coefficient is real. The field only experiences phase shifts due to λ_{ab}. For the case that λ_{ab} vanishes the solid curve in this figure shows a simple Beer's law decay of the field. Figure 6 demonstrates the effect of the phase of the injected coherence on the modulus of the electric field for fixed $|\lambda_{ab}|$. The modulus shows an initial variation for $z \lesssim 1$ that indicates whether the interference between the injected field and the injected coherence is constructive or destructive. The field phase again approaches the final value of $\phi_\lambda + 90°$.

VIII. CONCLUSIONS

In this paper we examined the effects of an injected atomic coherence on the polarization, propagation equation, and weak electric field for a medium consisting of a homogeneous collection of two-level atoms with a dipole-allowed transition. Some interesting results emerge. The polarization of the medium acquires a direct (field-independent) contribution and frequency-symmetric sidebands about the field-induced polarization. For the propagation problem we find that the standard single-mode complex-absorption coefficient is not affected by the injected coherence, but additional terms in the slowly varying propagation equation indicate that the net effect on the electric field behaves as a linear superposition of processes: one process arising from the phase-independent part of the pumping (the standard result due to the injected population difference), a contribution resulting directly from the input coherence, and two scattering processes which are nonlinear in the electric field.

The coupling of the new polarization contributions to the electric field depends on the phase mismatch in both space and time between the electric field and the injected coherence. For poorly phase-matched interactions, the field and injected coherence are uncoupled. If the phase mismatch is in time, the injected field does not couple strongly, if at all, to the transition of interest. The injected coherence eventually gives rise to a resonant electric field on its own, and its spatial variation is determined by that of the injected coherence. For this mode to propagate it must satisfy the dispersion relation imposed by the Maxwell equations. In general, this leads to a spatial phase mismatch and so the spatial structure of the injected coherence is crucial in determining whether an electric field can develop. If the original injected-coherence-field phase mismatch is only in space, the injected field couples to the transition but not to the injected coherence. For the completely phase-matched case the resulting electric field in the weak-field limit is a linear superposition of the exponential damping of the injected field and a constant field driven directly by the injected coherence.

The presence of an injected atomic coherence provides a means of studying phase matching in a simple context. The freedom in the coherence we inject allows us to control the degree of phase matching, and so study its effects in detail. In addition, the semiclassical treatment presented in this work enables us to investigate the main features of interactions of fields with atomic coherences, without becoming overly involved with the demands of a fully quantum-mechanical description. After becoming familiar with the simpler semiclassical results, one may further the study by quantizing the electric field.

ACKNOWLEDGMENTS

We thank M. O. Scully for helpful discussions. This work was supported in part by the U.S. Office of Naval Research, in part by the U.S. Army Research Office, and in part by the U.S. Air Force Office of Scientific Research.

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