Reply to "Exact solution of the Boltzmann equation for collision cascades produced by soft power-law potentials"

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I respond to the preceding Comment [Phys. Rev. A **41**, 7070 (1990)] by D. L. Maslov.

Both Maslov and I agree on the validity of the law of conservation of energy and on the curious behavior of collision cascades produced by soft power-law potentials. I pointed out that the apparent failure of the law—the nonconservation of measure for smooth solutions of the kinetic equation—could be understood in terms of anomalous currents at the boundary of the appropriate phase space. In our case the current is into the origin of the one-dimensional energy interval (θ, E_0). Whether one wishes to do the bookkeeping this way, or to add a term proportional to $\delta(E)$ to the smooth solution—as Maslov prefers—seems to me to be a matter of taste, with one portrayal being the integral of the other. (In fact, I used the δ-function representation in an earlier paper.[1])

I value Maslov's comments about the role of the threshold and about the properties of the screened Coulomb potential. The issue of the validity of the Boltzmann equation for the description of kinetic phenomena when the two-body potential is soft and not "cut off" is, I believe, a real one. The last words concerning it have not been uttered. Finally, it is interesting to find this issue of nonconservation of measure noted in the very last section of Vol. I of Feller's classic text on probability.[2] To quote: "At the time of its discovery, in 1940, this [phenomenon] came as a disturbing surprise. A huge literature has been devoted to it . . . ." In fact, it is interesting to survey some of the recent literature,[3] which describes phenomena ranging from fragmentation[4] to gelation, with some transport theory mentioned as well.

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[1]N. Corngold, Nucl. Sci. Eng. **102**, 114 (1989).
[2]W. Feller, *An Introduction to Probability Theory and its Applications*, 2nd ed. (Wiley, New York, 1957).
[3]N. Corngold and M. M. R. Williams, in *Modern Mathematical Methods in Transport Theory*, Proceedings of the 11th International Conference on Transport Theory, Blacksburg, 1989, edited by Wm. Greenberg (Birkhauser, Boston, in press).
[4]For example, E. D. McGrady and R. M. Ziff, Phys. Rev. Lett. **58**, 892 (1987).

