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Relativistic stimulated Brillouin and Raman scattering in a laser-produced plasma

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An investigation is made of the relativistic effect on the nonlinear stimulated scattering of large-amplitude laser radiation in the presence of the self-generated magnetic field in a laser-produced plasma. The relativistic Vlasov equation expressed in terms of the guiding-center coordinates has been solved to obtain the response of the magnetized plasma electrons. It is noted that the extreme relativistic effect increases the growth rates of the stimulated scattering by a factor of c/v_e, where c and v_e are the free-space speed of light and thermal velocity of the plasma electrons.

I. INTRODUCTION

Extensive investigations have been made on the stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS) of a considerably large-amplitude laser radiation with or without magnetic field in a plasma.[1–5] However, with the advent of the very-high-power sources of electromagnetic radiation, viz., high-power lasers, the directed component of the electron velocity in a plasma may become quite large, comparable to the free-space velocity of light. In such cases, the effects of relativistic electron mass must be taken into account. A limited number of attempts have been made using the relativistic fluid equations to investigate the relativistic effects in laser-plasma interactions.[6–8] In this paper we have made a rigorous investigation of the stimulated Brillouin and Raman scattering using the full relativistic kinetic equations.

The spontaneous generation of megagauss magnetic fields has been discovered in a laser-produced plasma due to a variety of reasons.[9–11] The magnetic field is self-generated in a direction transverse to the direction of propagation of the incident laser radiation and for resonance absorption perpendicular to the plane of polarization of the electric field of the incident laser radiation. Thus the incident laser wave may be considered to be propagating in the upper hybrid mode in the presence of the self-generated magnetic fields.[12–14]

In Sec. II we have solved the full relativistic Vlasov equation expressed in terms of relativistic gyrokinetic variables to obtain the relativistic nonlinear response of plasma electrons in the presence of the self-generated magnetic field. Since the electron plasma frequency is much greater than the electron cyclotron frequency in the laser-produced plasma, the high-frequency response of electrons is taken to be unmagnetized; only the low-frequency response being magnetized. Then, following references we obtain the growth rates for the SBS and SRS in the extreme relativistic and nonrelativistic limits in Sec. III. Finally, a brief discussion of the results is given in Sec. IV.

II. RELATIVISTIC VLASOV EQUATION IN GUIDING-CENTER COORDINATES

We consider the propagation of a large-amplitude upper hybrid laser radiation (pump) in a homogeneous, collisionless, and hot plasma along the x axis with its electric field polarized in the xy plane when the self-generated magnetic field $\mathbf{B}_s$ is taken in the z axis:[12–14]

$$\mathbf{E}_0=\mathbf{E}_0'\exp[-i(\omega_0 t-k_0 x)]\ ,\qquad \mathbf{B}_0=c\mathbf{k}_0\times\mathbf{E}_0/\omega_0\ , \tag{1}$$

where

$$E_{0x}=-i\beta_0 E_{0y}\ ,\qquad E_{0z}=0\ ,\qquad \beta_0=\frac{\omega_c}{\omega_0}\frac{\omega_p^2}{\omega_0^2-\omega_p^2-\omega_c^2}\ . \tag{2}$$

Here, $\omega_p=(4\pi e^2 n_0^0/m)^{1/2}$ and $\omega_c=eB_s/mc$ are the nonrelativistic electron plasma frequency and electron cyclotron frequency, respectively; $-e$, m, n_0^0, and c are the electronic charge, electron rest mass, unperturbed equilibrium electron density, and the velocity of light in a vacuum, respectively. On account of the nonlinear interaction of this electromagnetic pump wave $(\omega_0,\mathbf{k}_0)$ with an electrostatic perturbation $(\omega,\mathbf{k})$—an electron plasma wave for stimulated Raman scattering or an ion acoustic wave for stimulated Brillouin scattering—a scattered sideband is generated $(\omega_1,\mathbf{k}_1;\ \omega_1=\omega-\omega_0,\ \mathbf{k}_1=\mathbf{k}-\mathbf{k}_0)$. The upper sideband $(\omega+\omega_0)$ is neglected as it is considered to be off resonant for the nonlinear interactions under investigation.[5]

The three-wave nonlinear parametric interaction is described by the relativistic Vlasov equation. In the phase space of position vector $\mathbf{r}$, the world velocity $\mathbf{v}$, and the time t, a collisionless plasma can be described by[15]

$$\frac{\partial f}{\partial t}+\frac{\mathbf{v}}{\gamma}\cdot\nabla f-\frac{e}{m}\left[\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{\gamma c}\right]\cdot\nabla_v f=0\ , \tag{3}$$

where

$$\mathbf{v}=\gamma\dot{\mathbf{r}} ,\quad \gamma=(1-\dot{r}^2/c^2)^{-1/2}=(1+v^2/c^2)^{1/2} , \tag{4}$$

and the overdot denotes derivative with respect to time. The relativistic equation of motion in terms of the world velocity $\mathbf{v}$ is given by

$$m\dot{\mathbf{v}}=-e\mathbf{E}-e\mathbf{v}\times\mathbf{B}/\gamma c . \tag{5}$$

We express Eq. (3) in terms of the relativistic gyrokinetic variables,[3,16] i.e., the guiding-center coordinates $\mathbf{x}_g=\mathbf{x}-\mathbf{v}\times\boldsymbol{\omega}_c/\omega_c^2$, the magnetic moment $\mu=mv_\perp^2/2\omega_c$, the gyrophase angle θ of the perpendicular velocity (i.e., the angle $\mathbf{v}_\perp$ makes with the x axis), and the parallel momentum $p_z=mv_z$

$$\frac{\partial F}{\partial t}+\dot{\mathbf{x}}_g\cdot\frac{\partial F}{\partial \mathbf{x}_g}+\dot{\mu}\frac{\partial F}{\partial \mu}+\dot{\theta}\frac{\partial F}{\partial \theta}-eE_z'\frac{\partial F}{\partial p_z}=0 , \tag{6}$$

where we decompose the total distribution function as

$$F=f_0^0+f_0(\omega_0,\mathbf{k}_0)+f(\omega,\mathbf{k})+f_1(\omega_1,\mathbf{k}_1) , \tag{7}$$

f_0 and f_1 are the high-frequency distribution functions corresponding to the incident and the scattered laser waves, f is the low-frequency distribution function due to the electrostatic perturbation mode $(\omega,\mathbf{k})$, and f_0^0 is the equilibrium distribution function taken as Maxwellian[15] in the world velocity space $\mathbf{v}$ at temperature T_e:

$$f_0^0=n_0^0(m/2\pi T_e)^{3/2}\exp(-mv^2/2T_e) . \tag{8}$$

Here, we take f_0^0 as Maxwellian as $v_e \ll c$. The electrons acquire relativistic velocities due to the incident and scattered laser radiation only.

Using the identity

$$\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\equiv\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\times\sum_n \exp[in(\theta-\delta)]J_n(k_\perp\rho) , \tag{9}$$

where J_n is the Bessel function of order n and $\rho=v_\perp/\omega_c$ we can express

$$\begin{aligned}\mathbf{E}'=&\ \mathbf{E}_0'\exp[-i(\omega_0 t-k_0x_g)]\sum_n \exp(in\theta)J_n^0\\ &-i\mathbf{k}\phi\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]J_n\\ &+\mathbf{E}_1\exp[-i(\omega_1 t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1]J_n^1 ,\end{aligned} \tag{10}$$

$$\begin{aligned}F=&\ f_0^0+\exp[-i(\omega_0 t-k_0x_g)]\sum_n\exp(in\theta)f_n^0\\ &+\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]f_n\\ &+\exp[-i(\omega_1 t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]f_n^1 .\end{aligned} \tag{11}$$

In Eqs. (10) and (11) ϕ is the amplitude of the electrostatic low-frequency mode, $J_n^0=J_n^0(k_0\rho)$, $J_n^1=J_n^1(k_{1\perp}\rho)$, and δ and δ_1 are the angles that $\mathbf{k}_\perp$ and $\mathbf{k}_{1\perp}$ make with the x axis. Using the relativistic equation of motion, Eq. (5), we obtain

$$\begin{aligned}\dot{\mu}=&-\frac{eE_{0y}v_\perp}{\omega_c}(-i\beta_0\cos\theta+\sin\theta)\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)J_n^0\\ &-\frac{eE_{1y}v_\perp}{\omega_c}(-i\beta_1\cos\theta+\sin\theta)\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]J_n^1\\ &+ie\phi\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n n\exp[in(\theta-\delta)]J_n ,\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{\theta}=&\ \frac{\omega_c}{\gamma}+\frac{eE_{0y}}{mv_\perp}\left[-i\beta_0\sin\theta-\cos\theta+\frac{k_0v_\perp}{\gamma\omega_0}\right]\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)J_n^0\\ &+\frac{eE_{1y}}{mv_\perp}\left[-i\beta_1\sin\theta-\cos\theta+\frac{(k_{1x}+i\beta_1k_{1y})v_\perp}{\gamma\omega_1}\right]\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]J_n^1\\ &-\frac{e\phi k_\perp}{mv_\perp}\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]J_n' ,\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{x}_g=&\ \frac{eE_{0y}}{m\omega_c}\left[1-\frac{k_0v_\perp\cos\theta}{\gamma\omega_0}\right]\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)J_n^0\\ &+\frac{eE_{1y}}{m\omega_c}\left[1-\frac{(k_{1x}+i\beta_1k_{1y})v_\perp\cos\theta}{\gamma\omega_1}\right]\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]J_n^1\\ &-\frac{ie\phi k_\perp\sin\delta}{m\omega_c}\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]J_n ,\end{aligned} \tag{14}$$

$$\dot{y}_g=\frac{eE_{0y}}{m\omega_c}\left[i\beta_0-\frac{k_0v_\perp\sin\theta}{\gamma\omega_0}\right]\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)J_n^0$$
$$+\frac{eE_{1y}}{m\omega_c}\left[i\beta_1-\frac{v_\perp\sin\theta(k_{1x}+i\beta_1k_{1y})}{\gamma\omega_1}\right]\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]J_n^1$$
$$+\frac{ie\phi k_\perp\cos\delta}{m\omega_c}\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]J_n\ , \tag{15}$$

$$\dot{z}_g=\dot{z}=p_z/m\gamma\ . \tag{16}$$

In deriving Eqs. (12)–(16) the electric field of the scattered wave is assumed to be similarly polarized as the incident laser wave with $\beta_1=\beta_0(0\rightarrow1)$ and the prime on the Bessel function J denotes derivative with respect to its arguments. Substituting Eqs. (12)–(16) in the relativistic Vlasov equation, Eq. (6), we obtain the following relativistic linear response of electrons:

$$f_n^{0L}=-\frac{eE_{0y}}{T_e}\frac{\beta_0\cos\theta+i\sin\theta}{\omega_0-n\omega_c/\gamma}v_\perp J_n^0f_0^0\ , \tag{17}$$

$$f_1^{1L}=-\frac{eE_{1y}}{T_e}\frac{\beta_1\cos\theta+i\sin\theta}{\omega_1-n\omega_c/\gamma}v_\perp J_n^1f_0^0\ , \tag{18}$$

$$f_n^{L}=-\frac{e\phi}{T_e}\frac{n\omega_c}{\omega-n\omega_c/\gamma}J_nf_0^0\ . \tag{19}$$

III. RELATIVISTIC GROWTH RATES

Using Eqs. (12)–(16) the nonlinear relativistic Vlasov equation for electrons for the low-frequency electrostatic mode can be written as

$$\frac{\partial f^{\rm NL}}{\partial t}+\tfrac{1}{2}(\dot{\mathbf{x}}_g)^0\cdot\left[\frac{\partial f}{\partial\mathbf{x}_g}\right]^1+\tfrac{1}{2}(\dot{\mathbf{x}}_g)^1\cdot\left[\frac{\partial f}{\partial\mathbf{x}_g}\right]^0+\tfrac{1}{2}(\dot{\mu})^0\left[\frac{\partial f}{\partial\mu}\right]^1+\tfrac{1}{2}(\dot{\mu})^1\left[\frac{\partial f}{\partial\mu}\right]^0+\tfrac{1}{2}(\dot{\theta})^0\left[\frac{\partial f}{\partial\theta}\right]^1+\tfrac{1}{2}(\dot{\theta})^1\left[\frac{\partial f}{\partial\theta}\right]^0=0\ , \tag{20}$$

where the superscripts 0 and 1 indicate the incident and scattered laser waves. Using Eqs. (12)–(19) in Eq. (20) and on simplification we obtain the relativistic nonlinear distribution function for electrons for the low-frequency mode due to the beating of the incident and scattered laser waves as

$$f_n^{\rm NL}=\frac{\exp(in\delta)}{i(\omega-n\omega_c/\gamma)}\sum_l\exp[il(\theta-\delta_1)]$$
$$\times\left\{\left[\frac{ik_{1x}eE_{0y}}{2m\omega_c}\left[1-\frac{k_0v_\perp\cos\theta}{\gamma\omega_0}\right]\right]J_n^0f_l^1+\left[\frac{ik_0eE_{1y}}{2m\omega_c}\left[1-\frac{k_{1x}+i\beta_1k_{1y}}{\gamma\omega_1}v_\perp\cos\theta\right]\right]J_l^1f_n^0\right.$$
$$+\left[\frac{ik_{1y}eE_{0y}}{2m\omega_c}\left[i\beta_0-\frac{k_0v_\perp\sin\theta}{\gamma\omega_0}\right]\right]J_n^0f_l^1+\left[-\frac{eE_{0y}}{2\omega_c}(-i\beta_0\cos\theta+\sin\theta)\right]v_\perp J_n^0\frac{\partial f_l^1}{\partial\mu}$$
$$+\left[-\frac{eE_{1y}}{2\omega_c}(-i\beta_1\cos\theta+\sin\theta)\right]v_\perp J_l^1\frac{\partial f_n^0}{\partial\mu}+\left[\frac{eE_{0y}}{2mv_\perp}\left[-i\beta_0\sin\theta-\cos\theta+\frac{k_0v_\perp}{\gamma\omega_0}\right]\right]ilJ_n^0f_l^1$$
$$\left.+\left[\frac{eE_{1y}}{2mv_\perp}\left[-i\beta_1\sin\theta-\cos\theta+\frac{k_{1x}+i\beta_1k_{1y}}{\gamma\omega_1}v_\perp\right]\right]inJ_l^1f_n^0\right\}\ . \tag{21}$$

In deriving Eq. (21) we have assumed that $\mathbf{k}$ and $\mathbf{k}_1$ lie in the plane perpendicular to the magnetic field, i.e., the XY plane. This is possible as the maximum growing modes propagate in a plane transverse to the direction of the self-generated magnetic field.[12,13,17]

Now we obtain the nonlinear density fluctuation associated with the low-frequency electrostatic mode from the relation

$$n^{\rm NL}=\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{\infty}\sum_n\exp[in(\theta-\delta)]\exp[-ik_\perp v_\perp\sin(\theta-\delta)/\omega_c)]f_n^{\rm NL}v_\perp dv_\perp d\theta\, d\theta_z\ . \tag{22}$$

We consider very-high-power level of the pump wave, so that the electrons of the plasma acquire high relativistic quivering velocities. Therefore we may write $\gamma \gg 1$ and $\gamma=(1+v^2/c^2)^{1/2}\simeq v/c$. On carrying out the integration with the approximation $kv_e < \omega_c$ we obtain

$$n^{\mathrm{NL}}=n_0^0 e^2 E_{0y} E_{1y} X / m^2 \omega_c \omega_0 \omega \,, \tag{23}$$

where

$$X=-\frac{2k_0 k_\perp}{\omega_c}\left[1+\frac{\omega_0 k_{1x}}{\omega_1 k_0}\right]\cos\delta+\frac{ik_0 k_{1y} c\sqrt{\pi}}{4\omega_1 v_e} -\frac{\omega_c}{v_e^2}\left[1+\frac{\omega_0}{\omega_1}\right] . \tag{24}$$

We have retained $n=0$ terms only in Eq. (23) as $kv_e \ll \omega_c$ has been assumed. The nonlinear response of ions has been neglected as it is smaller than that of electrons by the mass ratio m/m_i, where m_i is the mass of an ion in the plasma. However, their effect may be included in the linear dielectric function of the low-frequency electrostatic mode.

Following the same procedures we obtain the nonlinear density perturbation for the nonrelativistic consideration ($\gamma=1$) as

$$n^{\mathrm{NL}}(\text{nonrel.})=n_0^0 e^2 E_{0y} E_{1y} X' / m^2 \omega_c \omega_0 \omega \,, \tag{25}$$

where

$$X'=-\frac{2k_0 k_\perp}{\omega_c}\left[1+\frac{\omega_0 k_{1x}}{\omega_1 k_0}\right]\cos\delta+\frac{ik_0 k_{1y}}{2\omega_1} -\frac{\omega_c}{v_e^2}\left[1+\frac{\omega_0}{\omega_1}\right] . \tag{26}$$

Since we restrict ourselves to the laser-produced plasmas ($\omega_p^2 \gg \omega_c^2$) we may consider the high-frequency response of electrons to be unmagnetized.[12–14] From Eq. (3) the nonlinear distribution function for the scattered sideband may be obtained from

$$f_1^{\mathrm{NL}}=\frac{ie(1+\mathbf{k}_1\cdot\mathbf{v}/\gamma\omega_1)}{2m\omega_1} \times\left[\mathbf{E}\cdot\nabla_v f_0^{\mathrm{L}*}+\mathbf{E}_0^*\cdot\nabla_v f^{\mathrm{L}}+\frac{\mathbf{v}\times\mathbf{B}_0^*}{\gamma c}\cdot\nabla_v f^{\mathrm{L}}\right] , \tag{27}$$

where the symbol $*$ denotes complex conjugate of the quantities involved and

$$f_0^{\mathrm{L}}\simeq-\frac{ief_0^0}{T_e\omega_0}(\mathbf{E}_0\cdot\mathbf{v})\left[1+\frac{\mathbf{k}_0\cdot\mathbf{v}}{\gamma\omega_0}\right] , \tag{28}$$

$$f^{\mathrm{L}}\simeq-\frac{e\Phi f_0^0}{T_e\omega}(\mathbf{k}\cdot\mathbf{v})\left[1+\frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right] . \tag{29}$$

We have neglected the relativistic effect in the linear response, Eq. (29), of the low-amplitude low-frequency electrostatic mode. On simplification we thus obtain

$$f_1^{\mathrm{NL}}=\left[\frac{ie^2\Phi f_0^0 E_0^*}{2mT_e\omega_0\omega_1}\right]\left[1+\frac{\mathbf{k}_1\cdot\mathbf{v}}{\gamma\omega_1}\right] \times\left\{\left[1+\frac{\mathbf{k}_0\cdot\mathbf{v}}{\gamma\omega_0}\right]\left[k_y-\frac{m}{T_e}(\mathbf{k}\cdot\mathbf{v})v_y\right]-\frac{k_0 c v_y}{\omega_0}\left[\frac{k_y v_x v_z}{v^3}-\frac{k_x(v_y^2+v_z^2)}{v^3}\right] -\left[1+\frac{2\mathbf{k}\cdot\mathbf{v}}{\omega}\right]\left[\frac{k_0}{\omega\gamma}(k_x v_y-k_y v_x)+\frac{\omega_0}{\omega}k_y+\frac{m\omega_0}{T_e\omega}\left[1+\frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right](\mathbf{k}\cdot\mathbf{v})v_y\right]\right\} . \tag{30}$$

Since $\beta_0 \ll 1$ for the parameters in the laser-produced plasmas we assume that the large-amplitude incident laser wave is almost polarized in the y direction and $\gamma\simeq v_y/c$. This is assumed to avoid the mathematical complexities in the integration of f_1^{NL} to obtain the current density. With this assumption we obtain the nonlinear relativistic current density at the frequency of the sideband as[15]

$$\mathbf{J}_1^{\mathrm{NL}}=-e\int(\mathbf{v}/c)f_1^{\mathrm{NL}}d\mathbf{v} \simeq\hat{\mathbf{y}}\left[\frac{in_0^0 e^3 c\Phi E_0^*}{m^2 v_e^2\omega_0\omega_1\omega}\left[1+\frac{k_{1y}c}{\omega_1}\right](2\omega_0 k_y+k_0 k_x c)\right] . \tag{31}$$

Again, following the same procedures we obtain the nonlinear and nonrelativistic current density at the scattered sideband as

$$\mathbf{J}_1^{\mathrm{NL}}(\text{nonrel.})\simeq\hat{\mathbf{y}}\left[\frac{-in_0^0 e^3\Phi E_0^*}{2m^2\omega_0\omega_1\omega}\right] \times\left[\left[\frac{\omega_1}{\omega_0}k_0 k_x+k_y k_{1y}-\frac{2\omega_0}{\omega}k_y^2\right] +3k_y\left[\frac{\omega_0}{\omega}k_y-k_{1y}\right]\right] . \tag{32}$$

Without loss of any generality we consider a specific situation in which all the waves involved propagate in one dimension, i.e., the x axis. Using the nonlinear relativistic density fluctuation at $(\omega,\mathbf{k})$, Eq. (23) in the Poisson's equation, and the nonlinear relativistic current density at $(\omega_1,\mathbf{k}_1)$, Eq. (31) in the wave equation, and following the procedures of earlier works[12–14] we finally obtain the growth rates of the relativistic Brillouin and Raman scatterings as

$$\gamma_0(\text{rel.}) = \left\{ -\frac{|V_{0y}/c|^2 \omega_p^2 \omega^2 c^4 k_0}{4\omega_1^2 k v_e^4 DF} \left[-\frac{\omega_1^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega_1^2} \right] \right] \times \left[k_1^2 - \frac{\omega_1^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega_1^2} \right] \right] \right\}^{1/2}, \quad (33)$$

where

$$D = \left[k_1^2 - \frac{\omega_1^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega_1^2} \right] \right] \times \left[k_1^2 - \frac{3\omega_1^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega_1^2} \right] - \frac{\omega_p^4 \omega_c^2}{\omega_1^4 c^2} \right] - \frac{\omega_p^4 \omega_c^2}{c^4 \omega_1^2},$$

$$F = \frac{m}{m_i} + \frac{1}{[1-(\omega_c^2 + k^2 v_e^2)/\omega^2]^2}. \quad (34)$$

In Eq. (33) the angular frequency of the low-frequency mode is to be calculated from $\omega^2 = \omega_p^2 + \omega_c^2 + 3k^2 v_e^2/2$ for the stimulated Raman scattering and $\omega = kC_s/(1 + k^2\lambda_D^2)^{1/2}$ where $\lambda_D = v_e/\sqrt{2}\omega_p$ and $C_s = (2T_e/m_i)^{1/2}$ for the stimulated Brillouin scattering.

Again, using the nonrelativistic nonlinear density perturbation, Eq. (25) in the Poisson's equation, and the nonrelativistic current density Eq. (32) in the wave equation, and following the same procedures[12–14] we obtain the nonrelativistic growth rates of the stimulated Brillouin and Raman scattering as follows:

$$\gamma_0(\text{nonrel.}) = (-v_e^2 \omega_1 / 2c^2 \omega_0)^{1/2} \gamma_0(\text{rel.}) . \quad (35)$$

This expression for the nonrelativistic growth rates, Eq. (35), differs from Eq. (27) of our earlier work[14] as we assume $kv_e < \omega_c$ in the present study. The earlier theory[14] was valid only in the high-temperature regime where $kv_e \gg \omega_c$. Thus, since for the low-frequency mode $\omega \ll \omega_0$, and $\omega_1 = \omega - \omega_0 \simeq -\omega_0$ we note that the relativistic growth rates of the SBS and SRS are greater than those for the nonrelativistic instabilities by a factor of $\sim c/v_e$. The self-generated magnetic field does not appreciably affect the growth rates of the SBS and SRS in the laser-produced plasma. However, the growth rates of these parametric instabilities decrease appreciably with the rise of the plasma temperature.

IV. DISCUSSION

The general solution of the relativistic Vlasov equation has been obtained for the three-wave parametric interaction in a collisionless plasma in the presence of a magnetic field. The relativistic linear response so obtained has been employed to find the nonlinear response of plasma electrons in the study of the stimulated Brillouin and Raman scattering in a laser-produced plasma. In the extreme relativistic consideration the relativistic effects increase the growth rates of the stimulated scatterings by a factor of $\sim c/v_e$. However, the growth rates of these instabilities decrease with increase of temperature. It is also noticed that the self-generated magnetic field does not have appreciable effects on these stimulated scatterings.

It may be added here that the effects of the inhomogeneities in the plasma and the self-generated magnetic field, and the nonlinear saturation of the stimulated scatterings at high pump wave power are also problems of considerable importance in the high-energy laser-plasma interactions.

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[1]J. F. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, and M. N. Rosenbluth, Phys. Fluids **17**, 778 (1974).

[2]C. S. Liu and P. K. Kaw, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Wiley, New York, 1976), Vol. VI, p. 83.

[3]C. S. Liu and V. K. Tripathi, Phys. Rep. **130**, 143 (1986).

[4]T. J. M. Boyd, Plasma Phys. Contr. Fusion **28**, 1887 (1986).

[5]W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, New York, 1988).

[6]S. E. Bodner and J. L. Eddleman, Phys. Rev. A **5**, 355 (1975).

[7]P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintzsadze, Phys. Rep. **138**, 1 (1986).

[8]P. K. Shukla, R. Bharuthram, and N. L. Tsintzsadze, Phys. Scr. **38**, 578 (1988).

[9]J. A. Stamper, K. Papadopoulos, S. O. Dean, E. A. Lindman, and J. M. Dawson, Phys. Rev. Lett. **26**, 1012 (1971).

[10]C. E. Max, W. M. Manheimer, and J. J. Thompson, Phys. Fluids **21**, 128 (1978).

[11]M. Ogasawara, A. Hirao, and H. Ohkubo, J. Phys. Soc. Jpn. **49**, 322 (1980), and the references therein.

[12]V. K. Tripathi and R. R. Sharma, Phys. Fluids **22**, 1799 (1979).

[13]C. Grebogi and C. S. Liu, Phys. Fluids **23**, 1330 (1980).

[14]M. Salimullah, Y. G. Liu, and M. G. Haines, Phys. Rev. A **30**, 3235 (1984).

[15]D. E. Baldwin, I. B. Bernstein, and M. P. H. Weenink, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Wiley, New York, 1969), Vol. III, p. 1.

[16]R. G. Littlejohn, Phys. Fluids **27**, 976 (1984).

[17]C. Grebogi and C. S. Liu, J. Plasma Phys. **23**, 147 (1980).