Electron collision shift of the Lyman-α line of ionized helium

M. Blaha
Laboratory for Plasma Research, University of Maryland, College Park, Maryland 20742

J. Davis
Radiation Hydrodynamics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375-5000

(Received 22 January 1990)

The electron collision shift of the He^+ Lyman-α line emitted from a low-density plasma has been calculated in the distorted-wave approximation with the inclusion of second-order corrections to the diagonal elements of the scattering matrix. The effects of exchange, inelastic collisions, interference term, and second-order contributions to the line shift are compared. The present results are also compared with the semiclassical calculations, the Coulomb-Born approximation, the results based on the $\underline{R}$-matrix method, and the line shifts derived from the electrostatic interaction of the radiator with the surrounding plasma.

I. INTRODUCTION

The shift and shape of spectral lines caused by plasma interactions has remained a challenging problem since the experiments by Berg *et al.*[1] on the plasma polarization shift. Even though considerable theoretical progress has been made over the years, a satisfactory many-body treatment still remains elusive causing recourse to simpler approximate methods. However, in the low-density plasma regime (say, $N_e < 10^{18}$ cm^{-3}) the mutual interaction of perturbing particles can be neglected to first approximation and an intractable problem can be reduced to a manageable one. At these densities, the average separation distance of perturbing electrons is larger than 200-bohr radii and the Debye screening length at $kT > 2$ eV is of the same order of magnitude. Because the perturbations of the radiative process are primarily caused by electrons with small angular momenta, individual perturbing collisions are well separated in time. In addition, the motion of plasma electrons near the radiator is affected very little by distant ions and electrons. Under these circumstances, the spectral line profiles can be evaluated using the impact approximation of Baranger.[2]

In Baranger's formalism the initial perturber-radiator correlations are ignored resulting in negative (i.e., red) frequency shifts of the Lyman-α lines of hydrogenlike ions[3,4] contrary to experimental results[5-8] that yield blue shifts. Theimer and Kepple[9] pointed out that plasma polarization effects may cause blue shifts of hydrogenlike lines, and calculations for He^+ have been reported by several authors.[5-8,10,11] Blue shifts of Lyman lines have been obtained only if the assumption of charge neutrality outside the electron orbit was adopted.

In each method of calculation, the line shift is obtained as a sum of various contributions, and the importance of different terms may vary from one method to another. The basic motivation of this study is the comparison of different methods of calculation and clarification of their agreements or disagreements. In the present investigation, we have calculated the electron collision line shift using the distorted-wave method with exchange (DWX) and the $\underline{S}$-matrix formalism of Baranger. This method allows convenient separation of the various contributions to the total shift and an assessment of their importance. We have also compared the results to the shift derived from the electrostatic interaction. It can be shown that in the limit of weak interaction both results are identical, but in the case of ionized helium this limit has not been reached.

II. CALCULATION OF THE LYMAN-α LINE SHIFT USING THE SCATTERING MATRIX FORMALISM

In the present investigation, we assume that the density of plasma electrons is so low that we may adopt the following simplifications: (1) the mutual interaction and correlation of plasma electrons can be neglected; (2) the atomic wave functions are not affected by interaction with the surrounding plasma; (3) fine structure of atomic levels can be ignored; and (4) the electron collision shift and the shift produced by ions are additive so that the electron shift may be calculated separately. Therefore we ignore the effects of ions and neglect the positive background in our calculation. The mixing of the $2s$ and $2p$ levels due to ion fields is also omitted. All calculated shifts will be normalized to the electron density $N_e = 1$ cm^{-3}. Unless otherwise indicated, all quantities throughout the paper will be given in atomic units ($e = h/2\pi = m = 1$).

In the impact approximation, the electron collision profile of a spectral line is determined by electron scattering on the upper and the lower level of the line. Following Baranger,[2] the half-width w (at half-maximum) and the frequency shift $d = 2\pi\Delta\nu$ of an isolated spectral line can be expressed in terms of diagonal elements $S(\alpha,\alpha)$ of the scattering matrix. The scattering channel α associated with a hydrogenic atom and a colliding electron is defined by

$$\alpha \equiv n\bar{l}klL^T S^T ,$$

where $n\bar{l}$ denotes the principal and orbital quantum number of the bound electron, k and l are the momentum and the orbital angular momentum of the colliding electron, and L^T, S^T is the total orbital and spin angular momentum. For a spectral line with an upper level $n_p p$ and a lower level $n_s s$, the width and shift is given by[12]

$$w + id = \tfrac{1}{12}\pi N_e \int k^{-1} f(k) \sum_{l,S^T,L_p^T} (2L_p^T+1)(2S^T+1)[1-S(\alpha_p,\alpha_p)S^*(\alpha_s,\alpha_s)]dk ,$$

$$\alpha_p \equiv n_p p k_p l_p L_p^T S_p^T, \quad \alpha_s \equiv n_s s k_s l_s L_s^T S_s^T . \tag{1}$$

N_e is the electron density, and $f(k)$ is the momentum distribution function of plasma electrons, which is represented by a Maxwellian distribution in the present calculation. The two matrix elements in (1) correspond to two different total energies which are such that $k_p = k_s \equiv k$, and the values of l_p, l_s, and S^T in (1) satisfy the conditions $l_p = l_s \equiv l$, $S_p^T = S_s^T \equiv S^T$.

We have calculated $\underline{S}$ in the distorted-wave approximation with exchange. Because we neglect mutual interaction of plasma electrons, we do not introduce any screening effects due to free electrons into the scattering potential, which consequently has the form of a Coulomb potential at large distances from the target ion. First we have evaluated the $\underline{\rho}$ matrix,[13] which is defined by the asymptotic form of the total wave function $\Psi_\alpha(\mathbf{r}_1,\mathbf{r}_2)$ describing the scattering. At large values of r_2, Ψ_α may be written in a form proportional to

$$Q_\alpha(\bar{\omega}_1,\omega_2) r_1^{-1} P(n\bar{l};r_1) r_2^{-1} k^{-1/2} \sin(x_2+\tau) + \sum_{\alpha'} \rho(\alpha',\alpha) Q_{\alpha'}(\bar{\omega}_1',\omega_2') r_1^{-1} P(n'\bar{l}';r_1) r_2^{-1} k'^{-1/2} \cos(x_2'+\tau') , \tag{2}$$

where $\bar{\omega}$ and ω represent the angular and spin coordinates of the bound and the free electron, respectively, and $Q_\alpha(\bar{\omega},\omega)$ is a function depending on the coupling of angular momenta in channel α. $r_1^{-1}P(n\bar{l};r_1)$ is the radial function of the bound electron, and $x_2 = kr_2 - \frac{1}{2}\pi l + Zk^{-1}\ln(2kr_2) + \arg\Gamma(l+1-iZk^{-1})$, where the symbols k,l correspond to the channel α. Z represents the asymptotic charge for the colliding electron. In our procedure, the phase τ ($\equiv\tau_\alpha$) for each channel α was determined by the solution of equation

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2(Z+1)}{r} - 2y_0(n\bar{l},n\bar{l};r) + k^2\right] F_{kl}(r) = CP(n\bar{l};r) , \tag{3}$$

which has asymptotic form

$$F_{kl}(r) \sim k^{-1/2}\sin(x+\tau) , \tag{4}$$

and consequently τ depends on $n\bar{l}kl$, but not on $S^T L^T$. $y_0(n\bar{l},n\bar{l};r)$ corresponds to the monopole part of interaction between the atomic and the colliding electron. The function y_λ is generally defined by

$$y_\lambda(n\bar{l},n'\bar{l}';r_2) = \int P(n\bar{l};r_1)(r_<^\lambda / r_>^{\lambda+1}) P(n'\bar{l}';r_1) dr_1 ,$$

$r_>$ being the greater and $r_<$ the smaller of r_1 and r_2. The Lagrange multiplier C is chosen to insure the orthogonality of F_{kl} and $P(n\bar{l})$ and it is zero unless $l = \bar{l}$, and $n\bar{l}$ is identical either to the upper or to the lower level of the line.

The $\underline{\rho}$ and $\underline{S}$ matrices are related by

$$\underline{S} = e^{i\tau}(1+i\underline{\rho})(1-i\underline{\rho})^{-1}e^{i\tau} , \tag{5}$$

where $e^{i\tau}$ are diagonal matrices with elements $e^{i\tau_\alpha}$.

In the evaluation of diagonal elements of the scattering matrix, it is generally necessary to take into account the coupling of several channels. This effect is partially accounted for by transformation (5). However, in the case of the Lyman-α line, the energy difference between the $1s$ and $2p$ levels of ionized helium is much larger than the average kinetic energy of plasma electrons at temperatures considered in our calculation, so that only a very small fraction of electrons scattered from the $1s$ level is affected by coupling to other channels. Therefore, in the evaluation of matrix elements $S(\alpha_s,\alpha_s)$ we ignored all other channels and set

$$S(\alpha_s,\alpha_s) = e^{2i\tau_s}[1+i\rho(\alpha_s,\alpha_s)][1-i\rho(\alpha_s,\alpha_s)]^{-1} .$$

On the other hand, we have included levels $1s$, $2s$, $2p$, $3s$, $3p$, and $3d$ in the $\underline{\rho}$ matrix for the evaluation of $S(\alpha_p,\alpha_p)$ according to (5), but we have ignored all elements that do not involve the $2p$ level and have only a small effect on $S(\alpha_p,\alpha_p)$.

The elements of the $\underline{\rho}$ matrix were calculated using approximate functions Ψ_α^t with the asymptotic form proportional to (2) containing approximate matrices $\underline{\rho}^t$. From the Kohn's variation principle it follows

$$\rho(\alpha',\alpha) \simeq \rho^t(\alpha',\alpha) - 2\langle \Psi_{\alpha'}^t | H - E | \Psi_\alpha^t \rangle , \tag{6}$$

where H and E are the Hamiltonian and the energy of the total system.

In our procedure, we calculated the off-diagonal elements $\rho(\alpha',\alpha)$ and the diagonal elements $\rho(\alpha_s,\alpha_s)$ using the antisymmetric functions

$$\Psi^t_\alpha=\sqrt{\tfrac{1}{2}}r_1^{-1}r_2^{-2}[\Phi^t_\alpha(r_1,r_2)+(-1)^{S^T}\Phi^t_\alpha(r_2,r_1)]\ , \tag{7a}$$

with

$$\Phi^t_\alpha(r_1,r_2)=Q_\alpha(\bar{\omega}_1,\omega_2)P(n\bar{l};r_1)F_{kl}(r_2)\ , \tag{7b}$$

where F_{kl} satisfies Eq. (3) with the following modification: for a matrix element associated with channels α and α', we used the Lagrange multipliers in all cases where l (or l') was equal to $\bar{l}$ or $\bar{l}'$ or both. With the approximate functions (7), $\underline{\rho}^t=0$, and $\rho(\alpha,\alpha')$ can be expressed in terms of coefficients and $f_\lambda(\bar{l}\ l\ \bar{l}'\ l';L^T)$ and $g_\lambda(\bar{l}\ l\ \bar{l}'\ l';L^T)$ defined by Percival and Seaton[14] and integrals involving functions $P(n\bar{l})$, $P(n',\bar{l}')$, F_{kl}, and $F_{k'l'}$.

In the evaluation of diagonal elements $\rho(\alpha_p,\alpha_p)$, we also took into account the coupling to other channels by including scattered waves into the approximate function Ψ^t_α defined by (7a). Instead of (7b), we have set

$$\Phi^t_\alpha(r_1,r_2)=Q_\alpha(\bar{\omega}_1,\omega_2)P(n\bar{l};r_1)F_{kl}(r_2)+\sum_{\alpha'\neq\alpha}Q_\alpha,(\bar{\omega}'_1,\omega'_2)P(n',\bar{l}';r_1)G_{k'l'}(r_2)\ . \tag{8}$$

This procedure introduces second-order corrections to the diagonal elements of ρ and $\underline{S}$. We also made the following adjustments: (a) functions F_{kl}, which are solutions of (3), were Schmidt orthogonalized to all atomic functions $P(n\bar{l})$ with $\bar{l}=l$ (this procedure does not change the phase τ of F_{kl}), (b) terms containing integrals with products of two $G_{k'l'}$ functions in the expression for $\rho(\alpha_p,\alpha_p)$ were neglected.

Functions $G_{k'l'}$ were obtained as solutions of the equation

$$\left[\frac{d^2}{dr^2}-\frac{l'(l'+1)}{r^2}+\frac{2(Z+1)}{r}-2y_0(n'\bar{l},n'\bar{l}';r)+k'^2\right]G_{k'l'}(r)=2U_{\alpha'\alpha}(r)F_{kl}(r)\ , \tag{9}$$

with the asymptotic form proportional to $\cos(x'+\tau')$, x' and τ' corresponding to the channel α'.

Furthermore,

$$U_{\alpha'\alpha}(r)=\sum_\lambda f_\lambda(\bar{l}\ l\ \bar{l}'\ l';L^T)y_\lambda(n\bar{l},n'\bar{l}';r)\ ,$$

and in the expansion over λ only the dominant term was retained. The functions $G_{k'l'}$ were then Schmidt orthogonalized to all atomic functions $P(n\bar{l})$ with $\bar{l}=l'$. The second-order contributions to the diagonal elements $\rho(\alpha_p,\alpha_p)$ can again be expressed in terms of coefficients f_λ,g_λ, and radial integrals involving atomic wave functions and functions F_{kl} and $G_{k'l'}$. All multipole orders λ that correspond to nonvanishing coefficients f_λ and g_λ were included in the expansion.

In the summation over l in Eq. (1), sufficient convergence was achieved by including values up to $l=13$.

The matrix elements $S(\alpha_p,\alpha_p)$ and $S(\alpha_s,\alpha_s)$ were calculated for ten different energies of the colliding electron. The energy mesh was chosen so that it would give sufficient accuracy of the integral over the velocity distribution, and at the same time take into account the discontinuity of $S(\alpha_p,\alpha_p)$ at the excitation threshold of the $n=3$ levels. No resonance effects below the $n=3$ threshold were taken into account in our procedure.

The shifts of the Lyman-α line were calculated for electron temperatures corresponding to 2.0, 2.5, 3.0, and 4.0 eV.

It is convenient to rewrite (1) in terms of transmission matrices $\underline{T}$. From the relation $\underline{T}=1-\underline{S}$ it follows

$$1-S(\alpha_p,\alpha_p)S^*(\alpha_s,\alpha_s)=T(\alpha_p,\alpha_p)+T^*(\alpha_s,\alpha_s)-T(\alpha_p,\alpha_p)T^*(\alpha_s,\alpha_s)\ , \tag{10}$$

and the total shift is thus decomposed into two parts: contributions from direct terms $T(\alpha_p,\alpha_p)+T^*(\alpha_s,\alpha_s)$ and from the interference term $-T(\alpha_p,\alpha_p)T^*(\alpha_s,\alpha_s)$.

III. DISCUSSION OF RESULTS

The major contributions to the line shift come from the first four lowest total angular momenta L^T of the system, as shown in Fig. 1 for $kT=2$ and 4 eV. With increasing temperature, the contributions from low values of L^T decrease and those from higher L^T increase due to changes in the electron velocity distribution.

Curve 1 on Figs. 2, 3, and 4 represents the final result of our calculation for the electron collision shift of the Lyman-α line of He^+. The line shift is to the red and its magnitude slowly decreases with temperature.

If exchange terms in the expression for d are neglected, the result is represented by curve 2 (Fig. 2) indicating

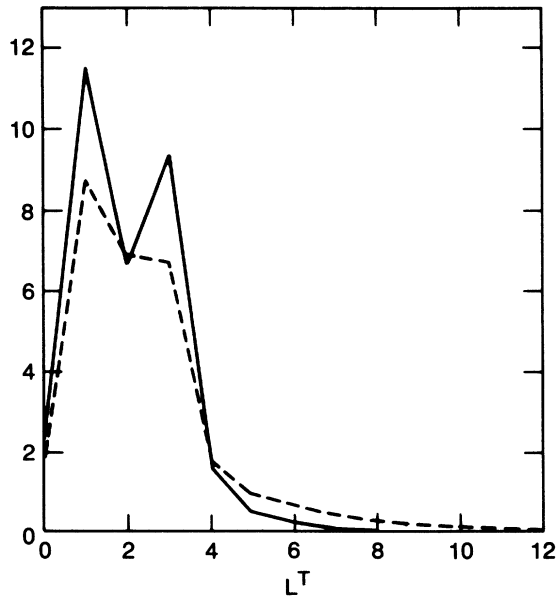

FIG. 1. Comparison of partial contributions (arbitrary units) to the Lyman-α shift of He^+ from total angular momentum $L^T\equiv L_p^T$. Solid line: $kT=2$ eV; dashed line: $kT=4$ eV.

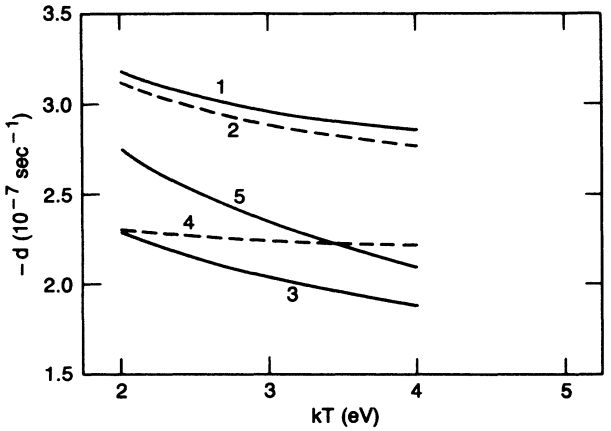

FIG. 2. Lyman-α shift of He^+. 1, present distorted-wave method with exchange, levels 1s-3d included in the scattering matrix; 2, same as 1 without exchange; 3, same as 1 without second-order contributions; 4, same as 1 without the interference term; 5, shift calculated only from elastic monopole contributions from Eq. (11).

that the inclusion of exchange is not important in this particular case. On the other hand, omission of second-order contributions to the matrix elements $S(\alpha_p,\alpha_p)$, described in the preceding paragraph, results in a substantial reduction of the shift as demonstrated by curve 3 (Fig. 2). (The present result is slightly different from analogous calculations in Ref. 15 due to differences in the orthogonalization procedure.) The omission of the interference term has a similar effect. The result is shown as curve 4.

The dominant contribution to the total line shift consists of direct elastic monopole terms. If all other terms are ignored, $\underline{\rho}=0$, $\underline{S}=e^{2i\tau}$, and from (1) one obtains

$$d=-\pi N_e\int k^{-1}f(k)\sum_l(2l+1)\sin 2(\tau_p-\tau_s)dk\ , \qquad (11)$$

where τ_p and τ_s are phase shifts of functions F_{kl} defined by (4). The result is shown as curve 5. The contribution of all other terms is therefore smaller than 30%.

The cumulative effect of other than diagonal elements in the $\underline{\rho}$ matrix on the line shift is small. It has been studied in detail in Ref. 15 (however, no second-order contributions have been included in the calculations described in Ref. 15).

Figure 3 shows a comparison of our results (curve 1) with the semiclassical (SC) calculations of Griem,[3,16] with quantum calculations of Nguyen *et al.*,[17] and with the $\underline{R}$-matrix method of Yamamoto and Narumi.[4]

The original SC results of Griem[3] (circles) contain only inelastic contributions from second-order terms without exchange (with adjustment for effects of higher orders) and no interference term. The effect of 2p-2s interaction on the line shift was not included, and the result contains no monopole contributions, but the estimated "strong collision term" representing contributions from low angular momenta is expected to simulate to a certain degree the effect of monopole terms in the DWX method. Griem has also included contributions from transitions to levels higher than 3d, and effects of resonances below inelastic thresholds, which are omitted in the present calculation. Griem's original line shifts are almost 50% smaller than our values, but if we ignore exchange, the interference term, and contributions from the 2p-2s transition to the second-order terms in the DWX approximation, we obtain curve 6, which is in a good agreement with the semiclassical results. In a later paper,[16] Griem has added contributions from the 2p-2s interaction obtained according to the kinetic theory approach of Boercker and Iglesias.[18] For ionized helium, these contributions are proportional to the electron density. The result is shown as triangles on Fig. 3. The corresponding DWX result (i.e., curve 1 without exchange and interference term) is represented by curve 7, giving an agreement within 15% with Griem's calculations.[16] The contributions from the 2p-2s interaction to the line shift in the

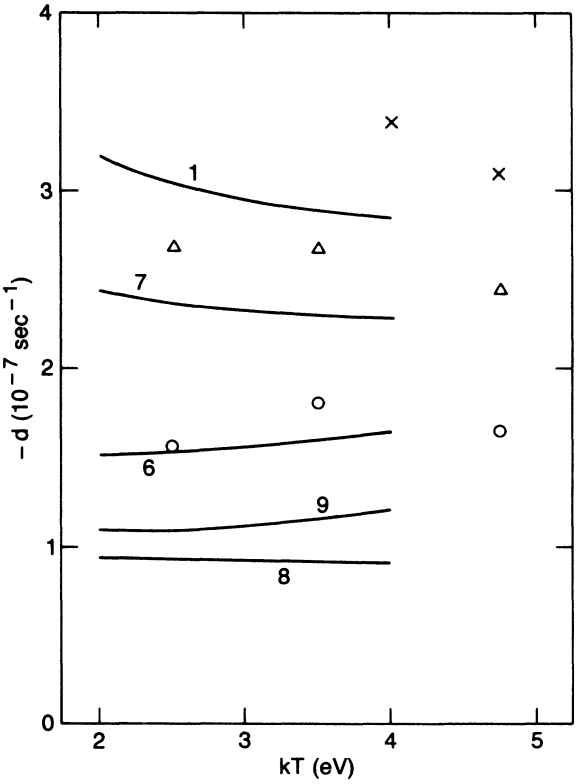

FIG. 3. Lyman-α shift of He^+. 1, same as in Fig. 2; 6, same as 1 without exchange, interference term, and the 2p-2s contributions to the second-order terms; 7, same as 1 without exchange and the interference term; 8, $\underline{R}$-matrix method by Yamamoto and Narumi (Ref. 4); 9, same as 1 with all phase shifts τ_p for the scattering on the upper level 2p set equal to the corresponding phase shifts τ_s for the lower level 1s. The circles denote SC calculations by Griem (Ref. 3) (without contributions from the 2p-2s interaction); triangles denote SC calculations by Griem (Ref. 16) (with contributions from the 2p-2s interaction); crosses denote Coulomb-Born-Oppenheimer approximation by Nguyen *et al.* (Ref. 17).

DWX method are about 30% smaller than those obtained from the kinetic theory.

In spite of differences in the methods, the SC values (triangles) compare favorably with our final result (curve 1). It appears that omission of the interference term and of monopole contributions in the SC method is largely compensated by the inclusion of transitions to levels higher than $n=3$, by the strong collision term, and by contributions from resonances below inelastic threshold. On the other hand, the inclusion of these contributions to DWX shifts (except for the strong collision term) would further increase the result represented by curve 1.

The result of quantum-mechanical calculations by Nguyen *et al.*[17] is shown on Fig. 3 as crosses. The authors used the Coulomb-Born-Oppenheimer approximation and did not include any second-order corrections to the elements of the scattering matrix. In agreement with our calculations, they claim that the monopole interaction plays a leading part in the red shift of the Lyman-α line. Nevertheless, their value of the shift for He^+ is larger by 20% than our result (curve 1), and larger by 60% than curve 5 (Fig. 2), which represents our calculations with only monopole terms included according to Eq. (11). The difference may be attributed to the fact that the CBO method overestimates the value of elastic monopole terms.

It is more difficult to understand the difference between the present result and the $\underline{R}$-matrix calculation of Yamamoto and Narumi,[4] represented by curve 8 on Fig. 3. These authors included all atomic levels from $1s$ to $3d$ in accordance with our procedure, and also contributions from resonances below the $n=3$ threshold, and yet their result is more than three time smaller than ours. We suspect that the discrepancy may be caused by elastic terms in the $\underline{R}$-matrix method associated with phase shifts τ. In support of this hypothesis, we repeated our DWX calculations, but we arbitrarily set all phase shifts τ_p for the upper level equal to phase shifts τ_s for the lower level in Eq. (5). This procedure eliminates the effect of phase-shift differences and the result (curve 9 on Fig. 3) comes close to the line shift reported by Yamamoto and Narumi.

IV. THE SHIFT OF THE LYMAN-α LINE DERIVED FROM THE ELECTROSTATIC INTERACTION

Several authors have studied the Lyman-α shifts from the point of view of electrostatic interaction of the radiator and plasma.[19–22] The status of the theory prior to 1978 has been reviewed by Volonté.[19] The basic assumption of this approach is that each atomic level is independently shifted by plasma interaction and that the frequency shift of the spectral line is equal to the difference of level shifts.

The static (or "polarization") line shift associated with the interaction of plasma electrons and the radiator is simply related to the shift given by formulas (1) and (11). We can separate the direct terms and the interference term in expression (11) by using the expansion

$$\sin 2(\tau_p-\tau_s)=\sin 2\tau_p-\sin 2\tau_s +4\sin\tau_p\sin\tau_s(\sin\tau_p-\sin\tau_s)+\mathcal{N} ,$$

where $\mathcal{N}$ represents higher-order terms. The first two terms on the right-hand side of this equation correspond to the direct terms defined by (10). If the interaction is weak, values of τ are small and then, assuming a spherical charge distribution corresponding to the bound electron and using the integral expression for phase shifts,[23] we obtain

$$\sin 2(\tau_p-\tau_s)\simeq 2(\sin\tau_p-\sin\tau_s) \simeq 4\int [y_0(2p,2p;r) -y_0(1s,1s;r)][F^c_{kl}(r)]^2 dr , \quad (12)$$

where $F^c_{kl}(r)$ is a Coulomb function that has asymptotic form $k^{-1/2}\sin x$ corresponding to charge Z. A spherically symmetric density distribution $n_e(r)$ of mutually noninteracting free electrons moving in the field of a positive point charge Z can be written in the form

$$n_e(r)=N_e\int k^{-1}f(k)r^{-2}\sum_l (2l+1)[F^c_{kl}(r)]^2 dk , \quad (13)$$

where $N_e=\lim_{r\to\infty} n_e$ and $f(k)$ is the momentum distribution function. If $W(n\bar{l})$ is the energy of electrostatic interaction of a bound electron in the $n\bar{l}$ state and free electrons with density distribution $n_e(r)$, then the difference $W(2p)$-$W(1s)$ is given by

$$W(2p)-W(1s)=4\pi\int r^2 n_e(r)[y_0(2p,2p;r) -y_0(1s,1s;r)]dr . \quad (14)$$

This quantity can be interpreted as the polarization shift of the Lyman-α line due to interaction with plasma electrons. (Note that we do not introduce any neutralizing positive background in accordance with the calculation described in Sec. II.) Substituting (13) into (14), and assuming the validity of (12), (14) becomes identical to (11). Therefore, if all inelastic collisions can be neglected, and if the monopole part of the interaction is small, the static shift of the line calculated from unperturbed atomic and continuum functions is equal to the shift obtained from the Baranger's formula, and both methods become equivalent.

Figure 4 shows a comparison of our DWX calculation (curve 1) with the static shift $W(2p)$-$W(1s)$ calculated from expressions (14) and (13), represented by curve 10. At low temperature, both results are in very good agreement, but this appears to be rather fortuitous. To properly assess the accuracy of the static approach and of the validity of (12), curve 10 should be compared with the shift obtained from Eq. (11), in which contributions from the interference term are neglected. The result is shown as curve 11. There is a substantial difference between the static shift (curve 10) and curve 11, and it is caused predominantly by the fact that phase shifts for low angular momenta l are not small and the integral expression (12) for phase shifts is not valid.

Figure 4 also shows results of more elaborate models for static shifts of the Lyman-α line in ionized helium. Squares correspond to the self-consistent calculation by Benredjem *et al.*[22] based on the ion sphere model. This model gives results very similar to the simple approach

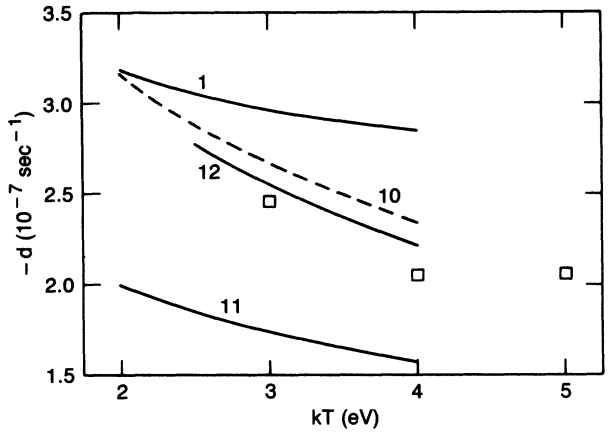

FIG. 4. Lyman-α shift of He^+. 1, same as in Fig. 2; 10, static shift from Eq. (14); 11, shift calculated only from monopole elastic contributions from Eq. (11) without the interference term; 12, static shift calculated with the inclusion of ion distribution (Ref. 24). The squares denote the self-consistent calculation of Benredjem *et al.* (Ref. 22).

described earlier, represented by curve 10. Another model, which takes into account the spherically symmetric ion distribution, was described in Ref. 24. The result is shown as curve 12.

In all static models the red line shift of Lyman-α decreases with temperature more rapidly than the DWX result, because they do not include contributions from inelastic collisions that are more pronounced at higher temperatures.

V. SUMMARY OF CONCLUSIONS

The frequency shift of the Lyman-α line of ionized helium due to collisions with electrons was calculated in the DWX approximation with the inclusion of atomic level $1s$ to $3d$. The result is applicable to low-density plasma ($N_e < 10^{18}$ cm^{-3}), where mutual interaction of perturbing electrons may be neglected. For temperatures between 2 and 4 eV, frequency shifts are negative. The main contributions to the line shift are due to elastic scattering on the $2p$ level, but the shift is substantially affected by the inclusion of inelastic contributions to second-order terms in the scattering matrix, representing coupling of different channels. The most important inelastic process according to the DWX calculation corresponds to the $2p$-$2s$ transition.

Major contributions to the shift come from the lowest angular momenta of the colliding electrons, and the interference term increases the absolute value of the shift.

The static shift of the line, defined as a difference of electrostatic interactions of the bound electron with plasma electrons moving in the potential of a point charge Z, agrees with the DWX approximation at low temperature, but it decreases with T more rapidly than the DWX result. However, the agreement appears to be largely the result of two compensating effects: overestimation of the monopole part of the interaction by the static approach and omission of inelastic processes and of the interference term.

The present DWX result for the line shift of ionized helium agrees within 15% with semiclassical calculations of Griem,[16] but disagrees with results of the $\underline{R}$-matrix method of Yamamoto and Narumi.[4] A possible explanation of the disagreement is the difference of elastic contributions in the two methods.

Experimentally found positive shifts of the Lyman-α line of He^+ (Refs. 5–8) are in sharp contrast to theoretical results that yield negative shifts of much smaller magnitude. However, for a comparison with measured shifts, the present results should be modified by the inclusion of gradrupole interaction with ions that produce a shift in the opposite direction[3] and will therefore decrease the total red shift of the line. It is difficult to measure small shifts predicted by the theory, and additional investigations, experimental as well as theoretical, still remain to be done in order to resolve the discrepancy.

A further improvement of the present approach should include effects of resonances below inelastic thresholds and of transitions to $n > 3$ levels.

ACKNOWLEDGMENTS

This work was supported by the Office of Naval Research. The authors are indebted to Professor Hans R. Griem for valuable discussions and comments.

[1]H. F. Berg, A. W. Ali, R. Lincke, and H. R. Griem, Phys. Rev. **125**, 199 (1962).
[2]M. Baranger, Phys. Rev. **111**, 481 (1958).
[3]H. R. Griem, Phys. Rev. A **27**, 2566 (1983).
[4]K. Yamamoto and H. Narumi, Progr. Theor. Phys. **64**, 436 (1980).
[5]J. R. Greig, H. R. Griem, L. A. Jones, and T. Oda, Phys. Rev. Lett. **24**, 3 (1970).
[6]A. H. Gabriel and S. Volonté, J. Phys. B **6**, 2684 (1973).
[7]T. Goto and D. D. Burgess, J. Phys. B **7**, 857 (1974).
[8]M. Neiger and H. R. Griem, Phys. Rev. A **14**, 291 (1976).
[9]O. Theimer and P. Kepple, Phys. Rev. A **1**, 957 (1970).
[10]S. Volonté, J. Phys. B **8**, 1170 (1975).
[11]R. Cauble, J. Quant. Spectros. Radiat. Transfer **28**, 41 (1982).
[12]K. S. Barnes and G. Peach, J. Phys. B **3**, 350 (1970).
[13]H. E. Saraph, M. J. Seaton, and J. Shemming, Philos. Trans. R. Soc. London Ser. A **264**, 77 (1969).
[14]I. C. Percival, and M. J. Seaton, Proc. Cambridge Philos. Soc. **53**, 654 (1957).
[15]M. Blaha and J. Davis, Naval Research Laboratory Memorandum Report No. 6294, 1988 (unpublished).
[16]H. R. Griem, Phys. Rev. A **38**, 2943 (1988).
[17]H. Nguyen, M. Koenig, D. Benredjem, M. Caby, and G. Coulaud, Phys. Rev. A **33**, 1279 (1986).

[18]D. B. Boercker and C. A. Iglesias, Phys. Rev. A **30**, 2771 (1984).
[19]S. Volonté, J. Phys. D **11**, 1615 (1978).
[20]J. C. Adcock and H. R. Griem, Phys. Rev. Lett. **50**, 1369 (1983).
[21]J. Davis and M. Blaha, J. Quant. Spectros. Radiat. Transfer **27**, 307 (1982).
[22]D. E. Benredjem, H. Nguyen, and G. Coulaud (unpublished).
[23]N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1965).
[24]M. Blaha and J. Davis, Naval Research Laboratory Memorandum Report No. 5155, 1983 (unpublished).