One-dimensional kinetic Ising model with competing spin-flip and spin-exchange dynamics: Ordering in the case of long-range exchanges

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Ordering of a one-dimensional stochastic Ising model evolving by a combination of spin flips and spin exchanges is investigated. The spin-flip rates satisfy detailed balance for the equilibrium state of the Ising model at temperature T, while the spin exchanges are random and of arbitrary range. Analytical methods and Monte Carlo simulations are used to show that, depending on the details of the spin-flip rate, finite-temperature phase transition may or may not occur in the system. When ordering occurs, it is of mean-field type and the scaling function describing the finite-size scaling of the magnetization fluctuations is found to be indistinguishable from that of an equilibrium Ising model with infinite-range interaction.

I. INTRODUCTION

Kinetic Ising models such as the one-spin-flip Glauber model[1] and the spin-exchange Kawasaki model[2,3] were designed to study relaxational processes near equilibrium states. Their simplicity notwithstanding these models played an important role in understanding the dynamics of second-order phase transitions[4] and it appears that their generalizations[5-6] may be equally instrumental in sorting out questions about nonequilibrium phase transitions. Although the last few years saw vigorous activity in this field,[5-28] problems such as the existence of broken symmetry in steady states, the relevance of upper and lower critical dimensions, and the characterization of universality classes have remained largely unsolved.[26]

One of the interesting features of nonequilibrium steady states is that symmetry breaking may occur even in a one-dimensional ($d=1$) system with short-range interactions. An example is the kinetic Ising model[6] in which spin flips at temperature T compete with nearest-neighbor spin exchanges at $T=\infty$. Using appropriate spin-flip rates,[6] one finds that the system orders ferromagnetically in $d=1$ provided the temperature is low enough and the ratio of the rate of exchanges to the rate of flips is taken to infinity.[29] The presence and the mean-field nature of the phase transition in this case can be partially understood by noting that the random exchanges mix the spins completely between rare events of spin flips. As a consequence, the ordering process (spin flip) takes place in an average, local equilibrium-type environment which is the condition for the mean-field approaches to be valid.[30] This argument, however, originates from equilibrium theories and it is not obvious that local equilibrium caused by the random exchanges promote cooperative behavior in a nonequilibrium steady state. Note, for example, that mixing fails to produce a phase transition[6,26] in the above flip-and-exchange model if the spin-flips obey Glauber or Metropolis dynamics. It is also clear that the cooperative behavior is not helped if the ordering effect of the exchanges is enhanced by making them satisfy detailed balance at the same temperature as the spin flips occur. The system then relaxes to the equilibrium state of the Ising model and, consequently, does not order in $d=1$.

In order to examine the effect of mixing, in Sec. II we introduce a kinetic Ising model in which spin flips at temperature T compete with random spin exchanges of arbitrary range. The long-range exchanges are expected to increase the ordering tendencies by increasing the effectiveness of creating a mean-field-type environment for the spin flips. Nevertheless, an exact calculation shows that long-range order is absent in $d=1$ if Glauber's form of spin-flip rates is used (Sec. III). The effectiveness of the long-range exchanges becomes apparent, however, if the spin-flip rate employed generates correlations between the average magnetization and the higher-order correlation functions. Our Monte Carlo (MC) simulations demonstrate that a finite-temperature phase transition occurs in the $d=1$ system even when the ratio of exchange-to-flip rate is finite (Sec. IV). The critical exponents of both the magnetization and the fluctuations of the magnetization are determined and they are found to have mean-field values. We also find that the finite-size scaling function of magnetization fluctuations follows closely the corresponding scaling function of an Ising model with infinite-range interaction.

II. FLIP-AND-EXCHANGE MODEL WITH LONG-RANGE EXCHANGE

Kinetic Ising models with long-range spin exchanges which satisfy a detailed balance at a given temperature have been used both as "effective" models in a renormalization-group approach to the Kawasaki model[31] and as nonlocal acceleration algorithms[32] that eliminate or reduce critical slowing down. Here we omit the requirement of detailed balance, and assume that the spin exchanges take place between randomly chosen pairs with a rate which is independent of the state of the system. As an "ordering" process we add spin flips which are assumed to satisfy the detailed balance condition following from nearest-neighbor Ising interactions at temperature T.

More specifically, we consider a one-dimensional system whose state $\{\sigma\}\equiv\{\cdots,\sigma_i,\sigma_{i+1},\cdots\}$ at time t is given by stochastic Ising variables $\sigma_i(t)=\pm 1$ assigned to lattice sites $i=1,2,\ldots,N$. Periodic boundary conditions imply $\sigma_{N+1}=\sigma_1$. The time evolution of the system is described in terms of the probability distribution $P(\{\sigma\},t)$ which satisfies the following master equation:

$$\frac{\partial P(\{\sigma\},t)}{\partial t}=\sum_i [w_i(\{\sigma\}_i)P(\{\sigma\}_i,t)-w_i(\{\sigma\})P(\{\sigma\},t)] + \frac{1}{2N\tau_2}\sum_{i,j}[P(\{\sigma\}_{i,j},t)-P(\{\sigma\},t)] \ . \qquad (1)$$

Here the first sum describes the spin-flip processes. The state $\{\sigma\}_i$ differs from $\{\sigma\}$ by flipping the ith spin and the flip rate is given by

$$\omega_i(\sigma)=\frac{1}{2\tau_1}\left[1-\frac{\overline{\gamma}}{2}\sigma_i(\sigma_{i+1}+\sigma_{i-1})\right](1+\delta\sigma_{i+1}\sigma_{i-1}) \ . \qquad (2)$$

Without the second sum in the master equation, Eqs. (1) and (2) define a generalized version of the Glauber model[1] which relaxes to the equilibrium state of the Ising model at temperature T provided $\overline{\gamma}=\tanh(2J/kT)$ and J is the strength of the nearest-neighbor interaction. The parameter δ is arbitrary apart from the restriction that it must lie in the interval [-1, 1]. The two choices we consider below are $\delta=0$ which defines the exactly solvable[1,33] Glauber model, and $\delta=\tanh^2(J/kT)$ which has been used[6,14,27] in connection with phase transitions in nonequilibrium steady states.

The process of random exchanges of spins is described by the second sum in Eq. (1) where the state denoted by $\{\sigma\}_{i,j}$ is obtained from $\{\sigma\}$ by exchanging spins at sites i and j. The exchanges are independent of the state of the system and their rate is $1/(2N\tau_2)$. Note that the factor N in the definition of the rate of exchanges is needed to ensure that the rate of exchange for a given spin remains finite in the thermodynamic limit ($N\to\infty$).

III. ABSENCE OF ORDERING IN THE CASE OF $\delta=0$ (GLAUBER FLIPS)

The absence of ordering for $\delta=0$ is shown below by demonstrating that the average magnetization

$$\langle m(t)\rangle=\frac{1}{N}\sum_i\langle\sigma_i\rangle=\frac{1}{N}\sum_i\sum_{\{\sigma\}}\sigma_i P(\{\sigma\},t) \qquad (3)$$

relaxes to zero exponentially for any nonzero temperature. To do this, first, we derive the equation of motion for $\langle\sigma_i\rangle$ by multiplying both sides of (1) by σ_i and summing over all configurations $\{\sigma\}$. The result can be written in the following form:

$$\frac{\partial\langle\sigma_i\rangle}{\partial t}=-\frac{1}{\tau_1}\left[\langle\sigma_i\rangle-\frac{\overline{\gamma}}{2}(\langle\sigma_{i+1}\rangle+\langle\sigma_{i-1}\rangle)\right] -\frac{1}{\tau_2}(\langle\sigma_i\rangle-m) \ . \qquad (4)$$

One can see that the contribution from the spin flips ($1/\tau_1$ term on the right-hand side) is the same as in case of the Glauber model, while the effect of spin exchanges ($1/\tau_2$ term) is not an effective diffusion of the magnetization, but a uniform relaxation towards the average value. The latter term does not influence the time evolution of the total magnetization and so Eq. (4) yields

$$\langle \dot{m}(t)\rangle=-\frac{1}{\tau}\langle m(t)\rangle \qquad (5)$$

with $\tau=\tau_1/(1-\overline{\gamma})$ being finite for any nonzero temperature. This proves that the magnetization in the stationary state of the system is zero, i.e., homogeneous ordering may not occur in the system. Actually, the decay time of inhomogeneous fluctuations can also be calculated. Equation (4) is linear, and so its Fourier transform yields the relaxation times τ_q of perturbations of wave number q ($q=2\pi n/N$, $n=1,\ldots,N-1$) in the following form:

$$\frac{1}{\tau_q}=\frac{1}{\tau_1}(1-\overline{\gamma}\cos q)+\frac{1}{\tau_2} \ . \qquad (6)$$

As expected, the long-range random exchanges strongly enhance the relaxation of inhomogeneous perturbations. It can be seen from (6) that arbitrary long-wavelength perturbations decay in finite time even at $T=0$.

The reason for the lack of ordering in the above model can be traced back to the special form of the spin-flip rate for $\delta=0$. In this case, the magnetizations $\langle\sigma_i\rangle$ satisfy a closed set of linear differential equations, i.e., they are independent of the correlations between σ_i's. As a consequence, no amount of mixing can induce cooperative behavior among them. We believe, this result is related to the general rule[26] that finite-temperature ferromagnetic transition cannot occur in a system in which the average magnetization at a site $\langle\sigma_i\rangle$ is independent of the correlations between the "input" sites $\langle\sigma_{i+a'}\sigma_{i+a}\rangle$ (sites which influence the dynamics at i) but depends only on their average magnetization $\langle\sigma_{i+a}\rangle$ and, furthermore, the number of "input" sites is not more than 2. The applicability of this rule to our case is not obvious since the exchange dynamics makes the number of input sites equal to N. Note, however, that the exchanges conserve the total magnetization and thus they do not affect the time evolution of magnetization in a translationally invariant state. The effective number of input sites is then determined by the spin-flip process. Since this number is 2, one expects that the exclusion of ordering is a consequence of the above rule.

IV. ORDERING IN THE CASE OF $\delta=\tanh^2(J/kT)$

The rule discussed above about the exclusion of ordering does not apply for $\delta\neq 0$. In this case, the average magnetization $\langle\sigma_i\rangle$ does depend on the correlations between the input sites as can be seen from the equation of motion given below for the particular case of $\delta=\tanh^2(J/kT)$:

$$\frac{\partial\langle\sigma_i\rangle}{\partial t}=-\frac{1}{\tau_1}[\langle\sigma_i\rangle-v(\langle\sigma_{i+1}\rangle+\langle\sigma_{i-1}\rangle)+v^2\langle\sigma_{i+1}\sigma_i\sigma_{i-1}\rangle]-\frac{1}{\tau_2}(\langle\sigma_i\rangle-m)\ , \tag{7}$$

where we introduced $v=\tanh(J/kT)$. The equation for $\partial\langle\sigma_{i+1}\sigma_i\sigma_{i-1}\rangle/\partial t$ can also be derived and one finds that (7) is just the first equation in an infinite hierarchy of equations for the higher-order correlation functions. This coupling of the correlations makes ordering a possibility. Note, for example, that if exchanges provided a mean-field environment and thus a simple decoupling approximation $\langle\sigma_{i+1}\sigma_i\sigma_{i-1}\rangle=m^3$ could be used then Eq. (7) would predict a ferromagnetic ordering in the steady state at $v_c=\frac{1}{2}$.

Since we were unable to solve the infinite set of equations even for the steady-state correlations, the possibility of ordering was investigated by Monte Carlo simulations. Systems containing $N=75$, 150, 300, . . . , 9600 spins were studied and we set $\tau_1=\tau_2$, i.e., the probabilities of attempting a spin flip or a spin exchange in a given Monte Carlo step were equal. In case of spin exchange two arbitrarily chosen spins were exchanged while in case of spin flip the spin at a randomly chosen site i was flipped with probability $p=1-v\sigma_i(\sigma_{i+1}+\sigma_{i-1})+v^2\sigma_{i+1}\sigma_{i-1}$. The magnetization and the energy of the system was monitored and the disappearance of drift in these quantities was considered to be the sign of reaching the steady state. Preliminary runs on small systems indicated that ferromagnetic ordering occurred at $v\approx 0.6$ thus we chose to investigate the system in the parameter range $0.5<v<0.65$. The relaxation time for the magnetization in the largest system of 9600 spins at the temperture closest to the critical point ($v=0.6$) was found to be of the order of 50 MC steps/spin, thus reaching the steady state and gathering enough data for satisfactory statistics did not pose a problem.

The location of the critical point (v_c) and the critical exponents β and γ defined as $\langle m\rangle\approx(v-v_c)^\beta$ for $v_c\le v$ and $\langle m^2\rangle\approx(v_c-v)^{-\gamma}$ for $v\le v_c$ were determined by analyzing the magnetization fluctuations ($\langle M^2\rangle=N^2\langle m^2\rangle$) in the steady state. We assumed that the finite-size scaling theory[34] could be generalized to nonequilibrium phase transitions and thus $\langle M^2\rangle/N$ was written as

$$\frac{\langle M^2\rangle}{N}\approx N^{\gamma/d\nu}\Phi_\pm(\epsilon N^{1/d\nu})\ , \tag{8}$$

where $\epsilon=|v_c-v|/v_c$, Φ_- and Φ_+ were the scaling functions for $v_c-v>0$ and $v_c-v<0$, respectively, and ν was the critical index of the correlation length. The parameters v_c, γ, and $d\nu$ were fixed by trying to achieve best collapse of data when $\langle M^2\rangle/N^{1+\gamma/d\nu}$ was plotted against $\epsilon N^{1/d\nu}$. Consistency tests of the fit were the conditions that for large argument the scaling function $\Phi_+(x)$ behaved as $\Phi_+(x)\approx x^{-\gamma}$ and, furthermore, $\Phi_-(x)=\Phi_+(x)$ for $x\to 0$. Finally, the large-x limit of the scaling function $\Phi_-(x)$ provided the exponent β since $\Phi_-(x)\approx x^{2\beta}$ for $x\to\infty$.

The scaling plot obtained for $v_c=0.601$, $\gamma=1$, and $d\nu=2$ is displayed in Fig. 1. One can see that the collapse of data is excellent over three decades of the scaling variable $\epsilon N^{1/d\nu}$. Noticeable deterioration in the quality of data collapse occurs if v_c is shifted more than 0.3% or the exponents are changed by more than 3%. The large-x behavior of $\Phi_\pm(x)$ is consistent with $\Phi_+\sim x^{-\gamma}\sim x^{-1}$ and $\Phi_-\sim x^{2\beta}\sim x$. Thus $\beta=\frac{1}{2}$, and all the measured exponents have mean-field values (note that $\nu=\frac{1}{2}$ and $d\nu=2$ in $d=4$ where the mean-field theory is valid for equilibrium ferromagnets).

It is not entirely surprising that long-range mixing of spins produces a mean field transition. Similar transition is observed in the short-range exchange model[6,14] in the limit when the exchange frequency to flip frequency ratio goes to infinity. More interesting is the result that, as far as the critical fluctuations of the magnetization are con-

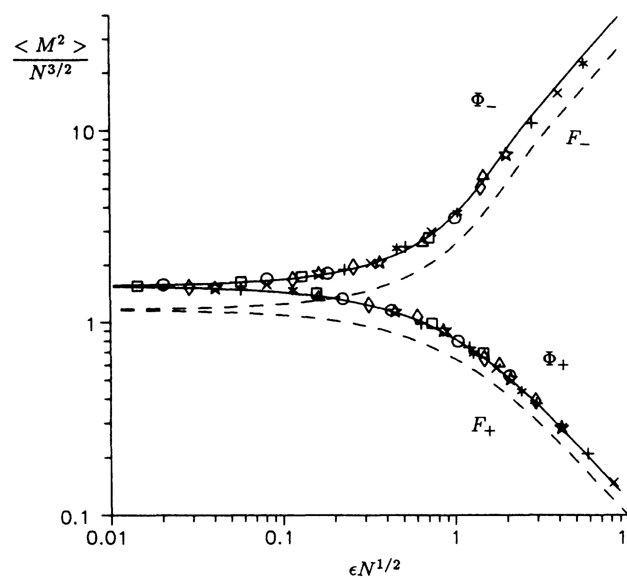

FIG. 1. Finite-size scaling of the magnetization fluctuations $\langle M^2\rangle$ in the flip-and-exchange model discussed in Sec. IV. The Monte Carlo data were obtained for systems containing N = 75 (□), 150 (○), 300 (◇), 600 (★), 1200 (+), 2400 (×), 4800 (*), and 9600 (△) spins. The deviation from the critical point (ϵ) is given by $|v_c-v|/v_c$. The dashed lines ($F_\pm$) are the results for the Ising model with infinite-range interactions ($\epsilon=|T-T_c|/T_c$) while the solid lines ($\Phi_\pm$) are scaled versions of ($F_\pm$) as described by Eq. (11).

cerned, our model is equivalent to an equilibrium Ising model with infinite-range interaction described by the Hamiltonian $(-J/2N)\sum_{i,j}\sigma_i\sigma j$. This Ising model undergoes a continuous transition at $J/kT_c=1$, and near the critical point ($\epsilon=|T-T_c|/T_c \ll 1$), the fluctuations of the magnetization obey scaling

$$\frac{\langle M^2\rangle}{N}\approx N^{1/2}F_{\pm}(\epsilon N^{1/2}) \tag{9}$$

with the scaling function $F_{\pm}(x)$ given by

$$F_{\pm}(x)=2\sqrt{3}\frac{\int_0^{\infty}dy\, y^2\exp(\mp\sqrt{3}xy^2-y^4)}{\int_0^{\infty}dy\,\exp(\mp\sqrt{3}xy^2-y^4)} . \tag{10}$$

This scaling function is plotted by dashed line on Fig. 1. The solid line which goes through the Monte Carlo data and can be considered as the scaling function $\Phi_{\pm}(x)$ is obtained from $F_{\pm}(x)$ by the following rescaling:

$$\Phi_{\pm}(x)=\lambda^{3/2}F_{\pm}(\lambda^{1/2}x) , \tag{11}$$

where $\lambda=0.83$. Comparing Eqs. (8), (9), and (11), we can now state more precisely what is the connection between the nonequilibrium flip-and-exchange model and the infinite-range Ising model in equilibrium: the critical fluctuations of the magnetization in the flip-and-exchange model of N spins are equal to those in the equilibrium Ising model containing λN spins.

It should be noted that we arrived to Eq. (11) by using "temperature" fields $\epsilon=|T-T_c|/T_c$ and $\epsilon=|v_c-v|/v_c$, expressed through the natural variables in the corresponding models. In principle, there is a freedom of scaling these fields into each other $|T-T_c|/T_c=\kappa|v_c-v|/v_c$ and there would be correspondence between the critical fluctuations even if a more general form of scaling $\Phi_{\pm}(x)=\lambda^{3/2}F_{\pm}(\kappa\lambda^{1/2}x)$ would be needed to fit the Monte Carlo data. It is just a coincidence that the choice of natural variables eliminates the need for fitting κ.

The parameters λ and κ as well as the location of the critical point v_c are expected to be nonuniversal. Investigating their dependence on τ_1/τ_2 should be instrumental in understanding the disappearance of finite-temperature phase transition in the limit of slow exchanges ($\tau_1/\tau_2\to 0$). This problem, however, is beyond the scope of the present paper.

Finally, we note that the long-range exchanges are expected to generate long-range interactions in a higher dimension as well. For $d\geq 2$, the question of interplay between Ising and mean-field ordering tendencies arises.[13,14] A question presently investigated by us is whether the Ising transition survives for finite τ_1/τ_2 as in the case of short-range exchanges[14] or the long-range random exchanges are effective enough to bring about a mean-field transition for any nonzero τ_1/τ_2.

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