Continuous state reduction of correlated photon fields in photodetection processes

Masahito Ueda and Nobuyuki Imoto
NTT Basic Research Laboratories, 3-9-11 Midori-cho, Musashino-shi, Tokyo 180, Japan

Tetsuo Ogawa*
Department of Applied Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan
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A general theory is developed that describes *continuous* state reduction of two quantum-mechanically correlated photon fields by continuous photodetection of one of the fields. The effects of measurement back action and quantum correlation on the state reduction of the other field are highlighted from the viewpoint of continuous photodetection. Nonunitary time evolutions of the density operators and photon statistics are exactly described for both fields. The obtained formulas are applied to parametrically generated photon twins, i.e., signal and idler fields. The continuous state reduction of the signal field by photodetection of the idler field is examined. It is shown that the Manley-Rowe relation can be generalized to the intermediate state under the photodetection process; that is, the initial quantum correlation remains intact throughout the "destructive" photodetection process if we retain the information about the idler readout. The obtained results are compared to the case in which the idler readout is discarded. In this case, the Manley-Rowe relation can no longer hold. This fact demonstrates that retaining the readout information is essential for preserving the established quantum correlation.

I. INTRODUCTION

State reduction in quantum measurement process is usually classified into two categories. One category directly applies von Neumann's projection postulate to the observed system.[1] The other employs a model in which quantum correlation is first established between the observed system and measurement apparatus via a unitary interaction, and then this process is followed by the readout of the measurement apparatus using an operation-valued measure;[2] this measurement process causes a nonunitary state reduction of the observed system via the established quantum correlation. In either of these descriptions, state reduction is assumed to occur only at the last moment of the measurement process.

In photocounting experiments, however, the readout consists of a *real-time* sequence of photoelectric pulses, each of which corresponds to a single photodetection. A photocount registration indicates that one photon is annihilated from the field and that the field therefore experiences a nonunitary state evolution. Even when no photocount is registered, the field evolves in a nonunitary way because no photocount registration is also a result of measurement and requires a proper projection on the total system. Thus in the photodetection process, nonunitary state reduction occurs continuously throughout the measurement period. Von Neumann's projection postulate cannot be applied to the photodetection process[3,4] because it is not the first-kind measurement. In order to treat this problem, a quantum theory of the continuous photodetection process has been developed by many authors.[3-15] The theory enables us to trace the nonunitary state evolution of the photon field in continuous photodetection process.

In the photodetection process the photon field eventually reduces to the vacuum state regardless of the initial state because of the absorption of photons by a photodetector. However, if a certain quantum correlation has been established between two photon fields, one of the fields evolves differently when the other field is destructively measured by a photodetector and reduces towards the vacuum state. For parametrically generated photon twins, for example, it is known that the final state of the signal field that is not directly measured will reduce to a number state for each single-shot measurement,[10,16] where a perfect correlation is assumed between initial signal and idler states.

The purpose of the present article is to keep track of such a continuous state reduction of the correlated photon fields within the present framework of quantum mechanics. For a detailed description of the continuous state reduction, however, the conventional formalism which presupposes *instantaneous* state reduction (diagonalization) due to quantum measurement must be extended; this extension is the main subject of this paper. In particular, it is shown that the initial and final states alone cannot uniquely specify the nonunitary state reduction. That is, there exists infinitely many different intermediate paths which start from the same initial state and reduce to the same final state. This fact has not been manifest in the conventional framework of quantum measurement theory. In the present paper it will be shown that a path can be singled out by the real-time renormalization of the density operator according to the readout information. Next, we apply the general formalism to parametrically generated photon twins. We will calculate continuous state reduction for both signal and idler fields when only the idler field is being measured by photon counting. In this case, the usual Manley-Rowe relation no longer holds because the photon numbers become different for the signal and idler fields. It is shown, however, that the "lost" information concerning the photon-number correlation between the signal and idler fields is actually transferred to the observer's side, and therefore it can be completely recovered if the observer retains the number of detected photons. A generalized Manley-Rowe relation which incorporates this readout information is presented. A natural question then arises: what happens to the photon field if we discard the retained information? This question is answered by exactly describing nonunitary time evolution of the photon density operator for such a case.

This paper is organized as follows. Section II develops a general formalism that describes nonunitary state evolution of correlated photon fields under continuous photodetection of one of the fields for the referring measurement process (RMP). By the RMP we mean that we read out all available information concerning registrations of photocounts. The effects of measurement back action and quantum correlation on state reduction are highlighted from the viewpoint of continuous photodetection. To illustrate nonunitary time evolution of the photon field general formulas are developed for photon-number moments and variances. Section III applies the obtained general formulas to parametrically down-converted photon twins. The two correlated photon fields are referred to as the signal and idler fields. Time developments of signal and idler photon statistics are examined in detail when we perform continuous measurement of the idler photon number. In particular, the Manley-Rowe relation is generalized such that it incorporates the renormalization effect due to the real-time readout of registrations of photocounts. Section IV develops a general formalism for the nonreferring measurement process (NMP). By the NMP we mean that we certainly know that the detector performs a continuous measurement of the idler photon number but we discard all available information concerning the results of measurement. We develop general formulas for time evolutions of moments and variance of the signal field in the NMP and compare them with the corresponding quantities in the RMP. It is pointed out that the generalized Manley-Rowe relation cannot hold in the NMP any longer because quantum correlation between the signal and idler fields becomes less than perfect when we discard the readout information. It is shown that the lost information concerning the quantum correlation is precisely compensated by the readout information for the idler field. This fact explains why the Manley-Rowe relation holds in the RMP and why it does not hold in the NMP where the "transferred" information is discarded.

II. GENERAL FORMALISM OF THE REFERRING MEASUREMENT PROCESS

Suppose that quantum-mechanically correlated photon fields (signal and idler fields) are prepared and that the photodetection process for the idler field starts at $t=0$, as schematically illustrated in Fig. 1. Let us consider a regular point process in which the probability of more than one photocount being registered in an infinitesimal time interval is negligible. Then the one-count and no-count processes[12-15] form an exclusive exhaustive set of events in an infinitesimal time interval. The idler photons are destructively measured by a photodetector one by one. Therefore, the idler field reduces towards the vacuum state. The nonunitary state evolution of the idler field under the photodetection process is exactly the same as that described in a previous work;[14] it depends strongly on the initial photon statistics and the readout information concerning registrations of photocounts. It is the nonunitary state evolution of the signal field that is of interest here. The signal state evolution is determined by the back action of the idler measurement through the established quantum correlation between the signal and idler fields, as schematically illustrated in Fig. 2. That is, the density operator of the idler field at a time t_1 determines the photodetection probability $p(t_1)$ at the same time (measurement action). With this probability a photoelectric con-

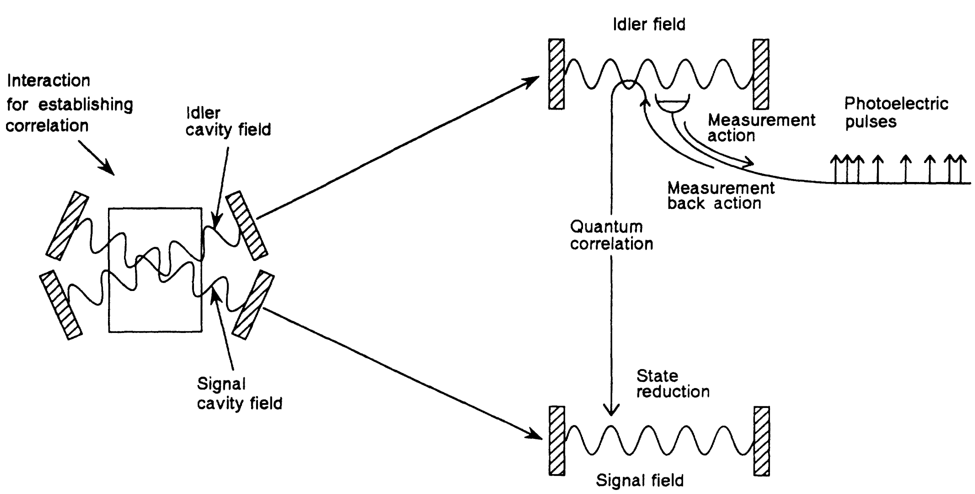

FIG. 1. Schematic illustration of continuous state reduction of quantum-mechanically correlated photon fields (signal and idler fields). Only the idler field is measured by the photodetector. State reduction of the idler field is caused by the measurement back action, while state reduction of the signal field is caused by the state reduction of the idler field via the established quantum correlation.

version occurs stochastically owing to the quantum-statistical nature of light. The actually obtained readout 0 or 1, where 0 stands for no count and 1 for one count, exerts measurement back action on the idler photon field, producing the density operator of the idler field at an infinitesimally later time t_2 via nonunitary state reduction. This measurement back action also causes a nonunitary state reduction of the signal field via the established quantum correlation. The crucial observation here is that the time-developed new density operator at time t_2 determines the photodetection probability at the same time $p(t_2)$ but that whether or not a photoelectric conversion actually occurs is again uncertain owing to the essentially quantum-statistical nature of light. Thus to completely determine the time evolution of the photon field we must specify the real-time readout 0 or 1 throughout a measurement period. In this section we develop a general formalism that describes continuous state reduction of the photon field and time development of photon statistics for signal and idler fields. Section II A describes the *discontinuous* evolution of the state at the moment one photon is detected (one-count process). Section II B describes the *continuous* reduction of the state when no photons are being detected (no-count process). Section II C describes state evolution in an arbitrary sequence of one-count and no-count processes (quantum photodetection process of forward recurrence times).

A. One-count process

The one-count process for the idler field is described by a superoperator $J^{(i)}$ as

$$J^{(i)}\rho(t) \equiv \lambda a_i \rho(t) a_i^\dagger , \tag{2.1}$$

where $\rho(t)$ is the total density operator of the signal and idler photon fields just before the one-count process, a_i ($a_i^\dagger$) is the annihilation (creation) operator of the idler field, and λ is a parameter which represents the probability of one idler photon being detected per unit time when the initial idler field is in a single-photon state. The probability $P(J^{(i)})dt$ that one idler photon is detected between t and $t+dt$ is given by

$$P(J^{(i)})dt \equiv \mathrm{Tr}[J^{(i)}\rho(t)]dt = \lambda \langle n_i(t) \rangle dt , \tag{2.2}$$

where

$$\langle n_i(t) \rangle \equiv \mathrm{Tr}[\rho(t) a_i^\dagger a_i] \tag{2.3}$$

is the average photon number of the idler field just before the one-count process, and it is understood that the trace is taken over both the signal and idler modes. When the trace is taken only over the signal (or idler) mode, we attach subscript s (or i) as Tr_s (or Tr_i). The density operator of the postmeasurement state is related to the premeasurement density operator by

$$\rho(t^+) = \frac{J^{(i)}\rho(t)}{\mathrm{Tr}[J^{(i)}\rho(t)]} = \frac{a_i \rho(t) a_i^\dagger}{\langle n_i(t) \rangle} , \tag{2.4}$$

where the symbol t^+ denotes a time infinitesimally later than t. We observe that the superoperator $J^{(i)}$ plays two distinct roles; it determines the photodetection probability (measurement action) according to Eq. (2.2) and produces a postmeasurement state via nonunitary state reduction (measurement back action) according to Eq. (2.4). Photon statistics of the signal and idler fields immediately after the one-count process is determined from Eq. (2.4), as shown below.

1. Time development of idler photon statistics in a one-count process

With respect to the idler field, all photon statistics such as moments of the photon number are the same as those

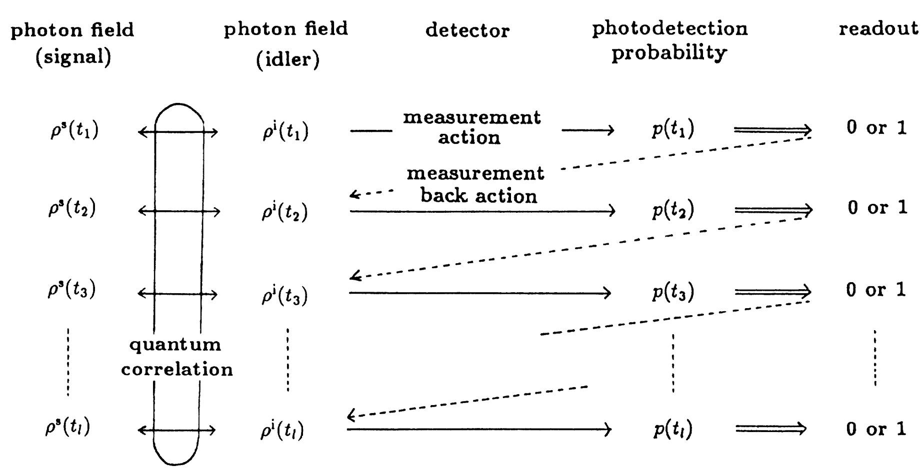

FIG. 2. Schematic illustration of nonunitary state evolution of the quantum-mechanically correlated photon field in continuous photodetection process. The photodetection probability at a time is determined by the density operator of the idler field at the same time (measurement action), while an actual readout, i.e., information concerning the no-count or one-count event, produces the density operator of the idler photon field at an infinitesimally later time via nonunitary state reduction (measurement back action). The state reduction of the idler field causes the state reduction of the signal field via the established quantum correlation.

obtained in a previous paper.[14] Here, however, we recapitulate them to compare the signal and idler state evolutions and to make the present paper self-contained.

The average photon number of the idler field for the postmeasurement state is obtained, from Eq. (2.4), as

$$\langle n_i(t^+)\rangle \equiv \mathrm{Tr}[\rho(t^+)a_i^\dagger a_i] = \langle n_i(t)\rangle - 1 + \frac{\langle[\Delta n_i(t)]^2\rangle}{\langle n_i(t)\rangle} , \qquad (2.5)$$

where $\Delta n_i(t) \equiv n_i(t) - \langle n_i(t)\rangle$. This result expresses the average photon number of the postmeasurement idler field in terms of the premeasurement photon statistics. We find that the difference between the average photon numbers before and after the one-count process does not exactly equal 1 but it has an additional term which is sometimes called the Fano factor:

$$F_i(t) \equiv \frac{\langle[\Delta n_i(t)]^2\rangle}{\langle n_i(t)\rangle} , \qquad (2.6)$$

where the subscript i in $F_i(t)$ refers to the idler mode. It is well known that the Fano factor takes values greater than unity for super-Poissonian states, less than unity for sub-Poissonian states, or equal to unity for Poissonian states. Thus we find that the average photon number of the idler field immediately after the one-count process increases, decreases, or remains unchanged according to whether the premeasurement idler photon statistics are super-Poissonian, sub-Poissonian, or Poissonian, respectively. In particular, for the thermal state the average photon number will be doubled by extracting one photon from the idler field. A physical interpretation of such a counterintuitive increase can be explained as follows.[14] The thermal state has a large probability of being in a vacuum state. However, as soon as one photon is detected, the probability of the vacuum state suddenly vanishes. Therefore by the renormalization according to Eq. (2.4) this vanishing probability is redistributed over the other number states, causing an increase in the average photon number.

In a similar way, it can be shown that the kth moment of the postmeasurement state is expressed in terms of up to the $(k+1)$th moments of the premeasurement state as

$$\langle n_i(t^+)^k\rangle = \frac{\mathrm{Tr}[\rho(t)a_i^\dagger(a_i^\dagger a_i)^k a_i]}{\mathrm{Tr}[\rho(t)a_i^\dagger a_i]} = \frac{1}{\langle n_i(t)\rangle}\sum_{m=0}^{k}\begin{bmatrix} k \\ m\end{bmatrix}(-1)^{k-m}\langle n_i(t)^{m+1}\rangle , \quad k=1,2,3,\ldots . \qquad (2.7)$$

From this result we find that the one-count process, in general, changes the statistics of the original photon field, which reflects the fact that continuous photon counting is a second-kind unsharp measurement of photon number.

2. *Time development of signal photon statistics in a one-count process*

The average photon number of the signal mode just after the one-count process is given by

$$\langle n_s(t^+)\rangle \equiv \mathrm{Tr}[\rho(t^+)a_s^\dagger a_s] = \frac{\langle n_s(t)n_i(t)\rangle}{\langle n_i(t)\rangle} , \qquad (2.8)$$

where

$$\langle n_s(t)n_i(t)\rangle = \mathrm{Tr}[\rho(t)a_s^\dagger a_s a_i^\dagger a_i] . \qquad (2.9)$$

Equation (2.8) shows that the time development of the signal photon number depends on the quantum correlation between the signal and idler photon numbers of the premeasurement state. In general, the kth moment of the postmeasurement signal photon number is given by

$$\langle[n_s(t^+)]^k\rangle = \frac{\langle[n_s(t)]^k n_i(t)\rangle}{\langle n_i(t)\rangle} . \qquad (2.10)$$

When there is no correlation between signal and idler photon numbers, then Eq. (2.10) reduces to $\langle[n_s(t^+)]^k\rangle = \langle[n_s(t)]^k\rangle$, that is, signal photon statistics do not change at all. This fact clearly demonstrates that the signal photon statistics is changed only through quantum correlation.

B. No-count process

The no-count process of photodetection of the idler field is described by a superoperator $S_\tau^{(i)}$ as

$$S_\tau^{(i)}\rho(t) = e^{Y\tau}\rho(t)e^{Y^\dagger\tau} . \qquad (2.11)$$

The generator Y of this superoperator should be determined to meet the following two requirements: (i) the no-count process and one-count process form an exclusive exhaustive set of events in an infinitesimal time interval, and (ii) the operator $S_\tau^{(i)}$ describes the free motion of the total field when the detector is switched off ($\lambda=0$). Therefore we obtain

$$Y = -\left[i\omega_i + \frac{\lambda}{2}\right]a_i^\dagger a_i - i\omega_s a_s^\dagger a_s , \qquad (2.12)$$

where ω_i and ω_s refer to the frequencies of the idler and signal modes, respectively. The probability, $P(S_\tau^{(i)})$, that no photons are detected between t and $t+\tau$ is given by

$$P(S_\tau^{(i)}) = \mathrm{Tr}[S_\tau^{(i)}\rho(t)] = \mathrm{Tr}[\rho(t)\exp(-\lambda a_i^\dagger a_i \tau)] . \qquad (2.13)$$

The density operator of the postmeasurement state is related to that of the premeasurement state by

$$\rho(t+\tau) = \frac{S_\tau^{(i)}\rho(t)}{\mathrm{Tr}[S_\tau^{(i)}\rho(t)]} . \qquad (2.14)$$

We observe that the superoperator $S_\tau^{(i)}$ plays two distinct roles; it determines the probability of no photocount being registered during τ (measurement action) according to Eq. (2.13) and produces a postmeasurement state via nonunitary state reduction (measurement back action) according to Eq. (2.14). Photon statistics of the signal and idler fields immediately after the no-count process is determined from Eq. (2.14), as shown below.

1. *Time development of idler photon statistics in a no-count process*

With respect to the idler field, photon statistics are the same as those obtained in a previous paper.[14] Here, again, we review the main results. With the help of Eq. (2.14), we can evaluate the photon-number moments of the idler field immediately after the no-count process:

$$\langle [n_i(t+\tau)]^k\rangle = \mathrm{Tr}[\rho(t+\tau)(a_i^\dagger a_i)^k] = \frac{\mathrm{Tr}[\rho(t)\exp(-\lambda a_i^\dagger a_i\tau)(a_i^\dagger a_i)^k]}{\mathrm{Tr}[\rho(t)\exp(-\lambda a_i^\dagger a_i \tau)]}, \quad k=1,2,3,\ldots . \tag{2.15}$$

We consider the time development of the average photon number as an example. Using Eq. (2.15), we find that the average photon number of the postmeasurement idler state, $\langle n_i(t+\tau)\rangle$, satisfies the following differential equation:

$$\frac{d}{dt}\langle n_i(t)\rangle = -\lambda\langle[\Delta n_i(t)]^2\rangle . \tag{2.16}$$

This equation can be immediately integrated to give

$$\langle n_i(t+\tau)\rangle = \langle n_i(t)\rangle - \lambda\int_t^{t+\tau}\langle[\Delta n_i(t')]^2\rangle dt' . \tag{2.17}$$

In the no-count process the detector absorbs no photons from the photon field. Nevertheless, Eq. (2.16) shows that the average photon number of the idler field decreases monotonically at a rate proportional to the photon-number variance. This is because the readout information of no photocount being registered requires us to modify the knowledge about the original photon statistics.[12,14]

2. *Time development of signal photon statistics in a no-count process*

The photon-number moments of the postmeasurement signal state are given by

$$\langle [n_s(t+\tau)]^k\rangle = \mathrm{Tr}[\rho(t+\tau)(a_s^\dagger a_s)^k] = \frac{\mathrm{Tr}[\rho(t)\exp(-\lambda a_i^\dagger a_i\tau)(a_s^\dagger a_s)^k]}{\mathrm{Tr}[\rho(t)\exp(-\lambda a_i^\dagger a_i\tau)]} = \langle[n_s(t)]^k\rangle - \lambda\int_t^{t+\tau}[\langle[n_s(t')]^k n_i(t')\rangle - \langle[n_s(t')]^k\rangle\langle n_i(t')\rangle]dt' , \tag{2.18}$$

where $k=1,2,3,\ldots$. For the first moment (the average photon number), we have

$$\frac{d}{dt}\langle n_s(t)\rangle = -\lambda\langle\Delta n_i(t)\Delta n_s(t)\rangle . \tag{2.19}$$

This equation can be immediately integrated to give

$$\langle n_s(t+\tau)\rangle = \langle n_s(t)\rangle - \lambda\int_t^{t+\tau}\langle\Delta n_i(t')\Delta n_s(t')\rangle dt' . \tag{2.20}$$

In contrast to Eq. (2.17), the time development of the signal photon number depends on the covariance between the signal and idler photon numbers. When there is no correlation between signal and idler photon numbers, then Eq. (2.20) reduces $\langle n_s(t+\tau)\rangle = \langle n_s(t)\rangle$, that is, signal photon statistics does not change at all. This fact again demonstrates that the signal photon statistics is changed only through quantum correlation.

C. Quantum photodetection process of forward recurrence times

Suppose we read out all information concerning registrations of photocounts in real time throughout the measurement period. We refer to such a process as the quantum photodetection process of forward recurrence times (QPF).[12] Suppose that the measurement process starts at $t=0$ and ends at $t=T$, and that m idler photons were registered at times τ_j ($j=1,2,\ldots, m$, $0\le\tau_j\le T$) with no further photons registered in the measurement period. The density operator of the total photon field immediately after the QPF is given by

$$\rho_m^{\mathrm{QPF}}(\tau_1,\tau_2,\ldots,\tau_m;0,T) = \frac{S_{T-\tau_m}^{(i)}J^{(i)}S_{\tau_m-\tau_{m-1}}^{(i)}\cdots J^{(i)}S_{\tau_1}^{(i)}\rho(0)}{\mathrm{Tr}[S_{T-\tau_m}^{(i)}J(i)S_{\tau_m-\tau_{m-1}}^{(i)}\cdots J^{(i)}S_{\tau_1}^{(i)}\rho(0)]} . \tag{2.21}$$

The denominator on the right-hand side (rhs) of Eq. (2.21) is called the probability distribution of forward recurrence times (PDF):[12,17]

$$P_m^{\mathrm{forward}}(\tau_1,\tau_2,\ldots,\tau_m;0,T) = \mathrm{Tr}[S_{T-\tau_m}^{(i)}J^{(i)}S_{\tau_m-\tau_{m-1}}^{(i)}\cdots J^{(i)}S_{\tau_1}^{(i)}\rho(0)] , \tag{2.22}$$

which gives the probability per (unit time)^m that m idler photons are registered at m distinct times $\tau_1,\tau_2,\ldots,\tau_m$ with no further photons detected between 0 and T. It can be shown that

$$S_{T-\tau_m}^{(i)}J^{(i)}S_{\tau_m-\tau_{m-1}}^{(i)}\cdots J^{(i)}S_{\tau_1}^{(i)}\rho(0) = \lambda^m\exp\left[-\lambda\sum_{j=1}^m\tau_j\right]\exp\left\{-\left[\left[i\omega_i+\frac{\lambda}{2}\right]a_i^\dagger a_i + i\omega_s a_s^\dagger a_s\right]T\right\} \times a_i^m\rho(0)(a_i^\dagger)^m\exp\left\{\left[\left[i\omega_i-\frac{\lambda}{2}\right]a_i^\dagger a_i + i\omega_s a_s^\dagger a_s\right]T\right\} . \tag{2.23}$$

Substituting Eq. (2.23) into Eq. (2.22) yields

$$P_m^{(\text{forward})}(\tau_1,\tau_2,\ldots,\tau_m,0,T)=\lambda^m \exp\left[-\lambda\sum_{j=1}^{m}\tau_j\right]\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m] . \quad (2.24)$$

Substituting Eq. (2.23) into Eq. (2.21) yields

$$\rho_m^{\text{QPF}}(\tau_1,\tau_2,\ldots,\tau_m;0,T)=\frac{\exp\{-[(i\omega_i+\lambda/2)a_i^\dagger a_i+i\omega_s a_s^\dagger a_s]T\}a_i^m\rho(0)(a_i^\dagger)^m\exp\{[(i\omega_i-\lambda/2)a_i^\dagger a_i+i\omega_s a_s^\dagger a_s]T\}}{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m]} . \quad (2.25)$$

It is interesting to note that the rhs of this equation no longer depends on the times of photodetection τ_j ($j=1,2,\ldots,m$) (henceforth we denote this quantity as $\rho_m^{\text{QPF}}(T)$ for simplicity). This is because the prefactor, $\exp(-\lambda\sum_{j=1}^m\tau_j)$, which indicates the times of photocount registrations, is canceled out between the denominator and the numerator of the rhs in Eq. (2.21). This means that any two different paths to time T give the same final state so long as the numbers of photocount registrations are the same. Thus we find that the initial and final states alone do not uniquely specify the continuous state reduction, and that there exist infinitely many different intermediate paths which start from the same initial state and reduce to the same final state. Investigating such an intermediate time development in quantum photodetection processes was the main subject of our previous paper.[14] There we found that it is uniquely determined by the readout information concerning registrations of photocounts. This complete predictability is remarkable if we recall that photocounting experiments are second-kind measurements. In the photodetection process, nevertheless, it is predictable because the readout information and the associated nonunitary state evolution are uniquely related by the measurement action and back action.[12] Our motivation in the present paper is to answer the following question: How does the real-time readout information cause the continuous state reduction of the correlated photon fields via the established quantum correlation?

The kth moment of the signal photon number immediately after the QPF is defined by

$$\langle[n_s(T)]^k\rangle_m\equiv[\rho_m^{\text{QPF}}(T)(a_s^\dagger a_s)^k],\quad k=1,2,3,\ldots \quad (2.26)$$

where the subscript m in $\langle n_s(T)^k\rangle_m$ denotes the number of detected idler photons in the QPF. Substituting Eq. (2.25) into the rhs of Eq. (2.26) yields

$$\langle[n_s(T)]^k\rangle_m=\frac{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m(a_s^\dagger a_s)^k]}{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m]} . \quad (2.27)$$

In a similar way, the moments of the idler photon number immediately after the QPF are given by

$$\langle[n_i(T)]^k\rangle_m=\frac{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)(a_i^\dagger a_i)^k a_i^m]}{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m]} . \quad (2.28)$$

Using the identity

$$(a_i^\dagger a_i)^k a_i^m=a_i^m(a_i^\dagger a_i-m)^k , \quad (2.29)$$

Eq. (2.28) is rewritten as

$$\langle[n_i(T)]^k\rangle_m=\frac{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m(a_i^\dagger a_i-m)^k]}{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m]} . \quad (2.30)$$

The rhs of this equation has a functional form similar to Eq. (2.27) except for a term $-m$ in the numerator. This term arises from the fact that m idler photons are absorbed by time T. The photon-number correlation between the signal and idler fields is defined by

$$\langle n_s(T)n_i(T)\rangle_m\equiv\text{Tr}[\rho_m^{\text{QPF}}(T)a_s^\dagger a_s a_i^\dagger a_i] . \quad (2.31)$$

Substituting Eq. (2.25) into the rhs of Eq. (2.31) yields

$$\langle n_s(T)n_i(T)\rangle_m=\frac{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m(a_i^\dagger a_i-m)a_s^\dagger a_s]}{\text{Tr}[\rho(0)(a_i^\dagger)^m\exp(-\lambda a_i^\dagger a_i T)a_i^m]} . \quad (2.32)$$

Equations (2.25), (2.27), (2.30), and (2.32) express the postmeasurement photon statistics and photon-number correlation between the signal and idler fields in terms of the initial density operator and the readout information (results of measurement).

III. APPLICATION TO PHOTON FIELDS GENERATED BY PARAMETRIC DOWN-CONVERSION

We now apply the general formalism developed in Sec. II to discuss the nonunitarity time evolution of two quantum-mechanically correlated photon fields generated by parametric frequency down conversion. In the present analysis, two fields in the optical cavities, namely the signal field and the idler field, are assumed to be generated from the vacuum state via the parametric amplification. These two fields satisfy the Manley-Rowe relation, i.e., there is a complete correlation in photon numbers for the two fields. This complete correlation is gradually deteriorated as the photodetection process for the idler field proceeds because idler photons are absorbed one by one. The Manley-Rowe relation, however, can be generalized so that it is always satisfied during the measurement process by renormalizing the density operator of the photon field according to the real-time readout information.

A. Preparation of quantum mechanically correlated photon fields

A nondegenerate parametric process can establish quantum-mechanical correlation between signal and idler fields via the interaction Hamiltonian[18]

$$H_{\rm int}=\hbar(\kappa a_s^\dagger a_i^\dagger+\kappa^* a_s a_i)=\hbar\kappa_0(e^{i\theta}a_s^\dagger a_i^\dagger+e^{-i\theta}a_s a_i)\ , \tag{3.1}$$

where $\kappa=|\kappa|e^{i\theta}\equiv\kappa_0 e^{i\theta}$ and $\hbar$ is the Planck constant divided by 2π. The time development operator $U(t,0)$ obeys the equation

$$i\hbar\frac{\partial U(t,0)}{\partial t}=H_{\rm int}U(t,0) \tag{3.2}$$

with the initial condition $U(0,0)=1$. The solution of this equation is given by[19]

$$\begin{aligned}U(t,0)=&\exp[-ie^{i\theta}\tanh(\kappa_0 t)a_i^\dagger a_s^\dagger]\\ &\times\exp[-(a_s^\dagger a_s+a_i^\dagger a_i+1)\ln\cosh(\kappa_0 t)]\\ &\times\exp[ie^{-i\theta}\tanh(\kappa_0 t)a_i a_s]\ .\end{aligned} \tag{3.3}$$

The time development of the density operator in a nondegenerate parametric process is given by

$$\rho(t)=U(t,0)\rho(0)U(0,t)\ . \tag{3.4}$$

Suppose that input signal and idler fields are in the vacuum states, i.e.,

$$\rho(0)=|0\rangle_{s\,s}\langle 0|\otimes|0\rangle_{i\,i}\langle 0|\ . \tag{3.5}$$

Then the total density operator of the signal and idler fields at $t=t_0$ is given by

$$\begin{aligned}\rho(t_0)=\frac{1}{\cosh^2(\kappa_0 t_0)}\sum_{k,l=0}^{\infty}&[-ie^{i\theta}\tanh(\kappa_0 t_0)]^k\\ &\times[ie^{-i\theta}\tanh(\kappa_0 t_0)]^l\\ &\times|k\rangle_{s\,s}\langle l|\otimes|k\rangle_{i\,i}\langle l|\ .\end{aligned} \tag{3.6}$$

This equation shows that the output signal and idler fields are perfectly correlated including the off-diagonal elements. If we take the trace of Eq. (3.6) only over the idler mode, we obtain the density operator of the signal field before the measurement process starts:

$$\begin{aligned}\rho_s(t_0)&\equiv{\rm Tr}_i[\rho(t_0)]\\ &=\frac{1}{1+\sinh^2(\kappa_0 t_0)}\\ &\quad\times\sum_{k=0}^{\infty}\left[\frac{\sinh^2(\kappa_0 t_0)}{1+\sinh^2(\kappa_0 t_0)}\right]^k|k\rangle_{s\,s}\langle k|\ ,\end{aligned} \tag{3.7}$$

which represents a thermal state with average photon number $\sinh^2(\kappa_0 t_0)$. Similarly the premeasurement density operator of the idler field is obtained by taking the partial trace for the signal mode as

$$\begin{aligned}\rho_i(t_0)&\equiv{\rm Tr}_s[\rho(t_0)]\\ &=\frac{1}{1+\sinh^2(\kappa_0 t_0)}\sum_{k=0}^{\infty}\left[\frac{\sinh^2(\kappa_0 t_0)}{1+\sinh^2(\kappa_0 t_0)}\right]^k|k\rangle_{i\,i}\langle k|,\end{aligned} \tag{3.8}$$

which is also in the thermal state with the same average photon number as the signal field.

B. Nonunitary time evolution of the photon fields by continuous measurement of idler photon number

Suppose that the continuous photodetection process for the idler field starts at $t=t_0$ and ends at $t=T$. Substituting Eq. (3.6) into the rhs of Eq. (2.23), we obtain the unnormalized density operator after the QPF:

$$\begin{aligned}\rho^{(\rm unnormalized)}(T)&\equiv S_{T-\tau_m}^{(i)}J^{(i)}S_{\tau_m-\tau_{m-1}}^{(i)}\cdots J^{(i)}S_{\tau_1-t_0}^{(i)}\rho(t_0)\\ &=N\sum_{k,l=0}^{\infty}\alpha^{k+m}(\alpha^*)^{l+m}e^{\beta_s(k-l)}e^{\beta_i(k-l)}e^{-(\lambda/2)(T-t_0)(k+l)}\\ &\qquad\times\left[\frac{(k+m)!(l+m)!}{k!l!}\right]^{1/2}|k+m\rangle_{s\,s}\langle l+m|\otimes|k\rangle_{i\,i}\langle l|\ ,\end{aligned} \tag{3.9}$$

where

$$\alpha\equiv-ie^{i\theta}\tanh(\kappa_0 t_0)\ , \tag{3.10}$$

$$\beta_s\equiv-i\omega_s(T-t_0)\ , \tag{3.11}$$

$$\beta_i\equiv-i\omega_i(T-t_0)\ , \tag{3.12}$$

and

$$N\equiv\frac{\lambda^m\exp\left[\lambda\sum_{j=1}^{m}(\tau_j-t_0)\right]}{\cosh^2(\kappa_0 t_0)}\ . \tag{3.13}$$

From Eq. (3.9) we can calculate the total density operator immediately after the QPF as

$$\rho_m^{\mathrm{QPF}}(T)=\frac{\rho^{(\mathrm{unnormalized})}(T)}{\mathrm{Tr}[\rho^{(\mathrm{unnormalized})}(T)]}$$
$$=[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^{m+1}\sum_{k,l=0}^{\infty}\left[\begin{bmatrix}k+m\\k\end{bmatrix}\begin{bmatrix}l+m\\l\end{bmatrix}\right]^{1/2}\alpha^k(\alpha^*)^l$$
$$\times e^{(\beta_s+\beta_i)(k-l)-(\lambda/2)(T-t_0)(k+l)}|k+m\rangle_{s\,s}\langle l+m|\otimes|k\rangle_{i\,i}\langle l| \ . \quad (3.14)$$

We note that before the QPF there is a perfect quantum correlation between the signal and idler fields, as evident from Eq. (3.6). A remarkable feature is that such a perfect correlation remains intact during the QPF. After this process, the matrix elements of the signal state are shifted relatively to those of the idler state exactly by the number of detected photons m. Since we know the precise value of this number in the RMP, we can uniquely determine the signal field from the knowledge about the idler field by shifting the matrix element by this value.

The reduced density operator of the signal field is given by

$$\rho_m^{\mathrm{QPF},s}(T)=\mathrm{Tr}_i[\rho_m^{\mathrm{QPF}}(T)]=[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^{m+1}\sum_{k=0}^{\infty}\begin{bmatrix}k+m\\k\end{bmatrix}|\alpha|^{2k}e^{-\lambda(T-t_0)k}|k+m\rangle_{s\,s}\langle k+m| \ . \quad (3.15)$$

Similarly, the reduced density operator of the idler field is given by

$$\rho_m^{\mathrm{QPF},i}(T)=\mathrm{Tr}_s[\rho_m^{\mathrm{QPF}}(T)]$$
$$=[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^{m+1}$$
$$\times\sum_{k=0}^{\infty}\begin{bmatrix}k+m\\k\end{bmatrix}|\alpha|^{2k}e^{-\lambda(T-t_0)k}|k\rangle_{i\,i}\langle k| \ . \quad (3.16)$$

Thus we find that the density operator of the signal field is exactly the same as that of the idler field except that the matrix elements are shifted by m. We also note that the reduced density operators are both diagonalized. It can be shown that the photocount distribution for the idler field retains the Bose-Einstein character, even though the photon statistics develops into different statistics as shown in Eq. (3.16).

C. Time development of signal photon statistics in QPF

Next let us evaluate the moments for the postmeasurement signal photon number after the QPF. They are defined by

$$\langle n_s(T)^k\rangle_m\equiv\mathrm{Tr}[\rho_m^{\mathrm{QPF}}(T)(a_s^\dagger a_s)^k], \quad T>\tau_m \ . \quad (3.17)$$

Substituting Eq. (3.14) into Eq. (3.17), we obtain

$$\langle n_s(T)^k\rangle_m=\frac{(1-x)^{m+1}}{x^m}\left[x\frac{d}{dx}\right]^k\frac{x^m}{(1-x)^{m+1}} , \quad (3.18)$$

where

$$x\equiv e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0) \ . \quad (3.19)$$

The first two moments are written down as

$$\langle n_s(T)\rangle_m=\frac{m+e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)}{1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)} \quad (3.20)$$

and

$$\langle n_s(T)^2\rangle_m=\frac{[m+e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^2+(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)}{[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^2} \ . \quad (3.21)$$

Hence we obtain the photon-number variance of the signal field as

$$\langle[\Delta n_s(T)]^2\rangle_m=\frac{(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)}{[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]^2} \ . \quad (3.22)$$

Figure 3(a) schematically illustrates the time development of the average photon number of the signal fields, where one-count processes are assumed to occur at $\tau_1,\tau_2,\ldots$. This figure shows that the average photon number monotonically decreases during the no-count process, while it jumps upwards when the detector registers one photocount. The upward jump is a result of the super-Poissonian character of the observed field as discussed in Ref. 14. A new feature, however, arises here due to quantum correlation; that is, if some idler photons are detected, the signal photon number no longer reduces to zero but to the number state whose eigenstate is the same as the number of detected idler photons. The dashed curves show the time development of the average photon number if no further photons are registered. This fact can also be seen from Eqs. (3.20) and (3.22) by taking the limit $T\to\infty$.

The time development of photon statistics can be most conveniently characterized with the Fano factor, which is defined as the ratio of the variance to the average value. The Fano factor of the signal photon number is given, from Eq. (3.20) and Eq. (3.22), as

$$F_s(T)\equiv\frac{\langle[\Delta n_s(T)]^2\rangle_m}{\langle n_s(T)\rangle_m}=\frac{(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)}{[m+e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)][1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)]}\ . \tag{3.23}$$

The signal Fano factor F_s depends on the number of photocounts m. Therefore, it yields discontinuous jumps in the one-count ($J^{(i)}$) process at times τ_j as shown in Fig. 4(a). In addition, it is easily shown that F_s always decreases in the one-count process. It is interesting to note that

$$\lim_{\lambda(T-t_0)\gg 1} F_s(T)=\begin{cases}0 & \text{if } m\neq 0\\ 1 & \text{if } m=0\ .\end{cases} \tag{3.24}$$

The physical interpretation of this equation is as follows. When some (nonzero) idler photons are detected ($m\neq 0$) in the RMP, the signal state reduces to the number state $\rho_s(T\gg 1/\lambda)=|m\rangle_{s\,s}\langle m|$ owing to both the quantum correlation (the Manley-Rowe relation) and the field attenuation (measurement back action). When no idler photons are detected, the signal state reduces to the Poissonian (vacuum) state, since in this case only the effect of field attenuation works.

D. Time development of idler photon statistics in QPF

The moments for the postmeasurement idler photon number are the same as those for the thermal state obtained in a previous paper.[14] The moments are defined by

$$\langle n_i(T)^k\rangle_m\equiv\mathrm{Tr}[\rho_m^{\mathrm{QPF}}(T)(a_i^\dagger a_i)^k],\quad T>\tau_m\ . \tag{3.25}$$

Substituting Eq. (3.14) into Eq. (3.25) yields

$$\langle n_i(T)^k\rangle_m=(1-x)^{m+1}\left[x\frac{d}{dx}\right]^k\frac{1}{(1-x)^{m+1}}\ , \tag{3.26}$$

where x is defined in Eq. (3.19). For the first two moments we have

$$\langle n_i(T)\rangle_m=\frac{(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)}{1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0 t_0)} \tag{3.27}$$

and

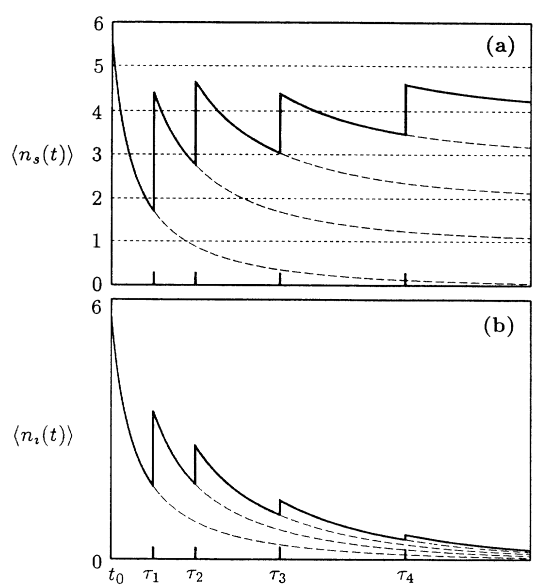

FIG. 3. Time development of the average photon number in the referring measurement process (RMP): (a) signal field $\langle n_s(t)\rangle$, and (b) idler field $\langle n_i(t)\rangle$. One-count processes are assumed to occur at $\tau_1,\tau_2,\ldots$. The dashed curves show that the signal field approaches the number state if no further counts are registered and that the idler field approaches the vacuum state.

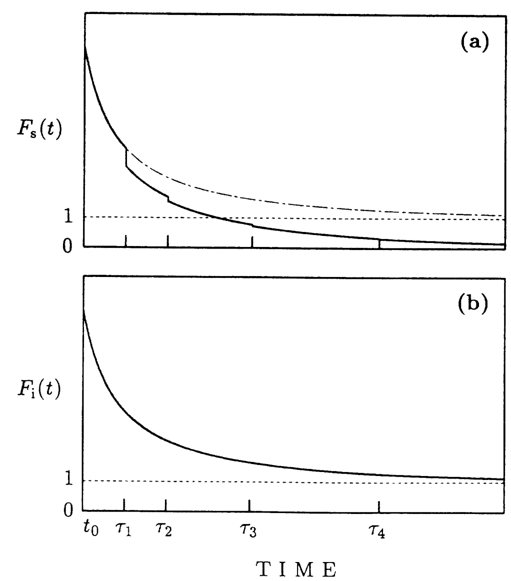

FIG. 4. Time evolution of the Fano factor in the referring measurement process (RMP): (a) signal field $F_s(t)$, and (b) idler field $F_i(t)$. One-count processes are assumed to occur at $\tau_1,\tau_2,\ldots$. In the RMP, the idler photon statistics approach the Poissonian $F_i(t)\to 1$, but the signal photon statistics become the sub-Poissonian and finally the number state $F_s(t)\to 0$. If no photons are registered, the signal field, however, approaches the Poissonian $F_s(t)\to 1$ [see dash-dotted curve and Eq. (3.24)].

$$\langle n_i(T)^2\rangle_m=\frac{[(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)]^2+(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)}{[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)]^2} . \tag{3.28}$$

Hence we obtain the photon number variance of the idler field as

$$\langle[\Delta n_i(T)]^2\rangle_m=\frac{(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)}{[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)]^2} . \tag{3.29}$$

From Eqs. (3.20) and (3.27), we find that

$$\langle n_s(T)\rangle_m-\langle n_i(T)\rangle_m=m , \tag{3.30}$$

that is, the average photon number in the signal field is larger than that in the idler field by m because we know that m idler photons were absorbed by the detector. Nevertheless, the variances in the signal and idler photon numbers remain the same. In fact, from Eqs. (3.22) and (3.29), we find

$$\langle[\Delta n_s(T)]^2\rangle_m=\langle[\Delta n_i(T)]^2\rangle_m . \tag{3.31}$$

Time evolution of the average idler photon number is shown in Fig. 3(b). The Fano factor for the idler photon number $F_i(T)\equiv\langle[\Delta n_i(T)]^2\rangle_m/\langle n_i(T)\rangle_m$ is given from Eqs. (3.27) and (3.29) as

$$F_i(T)=\frac{1}{1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)} . \tag{3.32}$$

Here we note that F_i does not depend on m in contrast to the signal Fano factor [see Eq. (3.23)]. The time development of the Fano factor for the idler field shown in Fig. 4(b) decreases monotonically and has no discontinuities, although both the average and variance of the photon number change discontinuously in the one-count process. This is a special feature of the state reduction of an initially thermal state.[14] The idler Fano factor always decays and approaches unity as time progresses, as shown in Fig. 4(b). This is caused by the measurement back action on the idler photon field; if we neglect this effect ($\lambda=0$), the Fano factor remains constant:

$$F_i(T)=F_i(t_0)=\cosh^2(\kappa_0t_0)$$
$$=1+\langle n_i(t_0)\rangle \quad \text{for } \lambda=0 , \tag{3.33}$$

which is characteristic of the thermal state. Lastly we mention that if there is no photocount, the signal and idler Fano factors coincide with each other for example, for $t\in[t_0,\tau_1)$ in Fig. 4. Information on whether or not *at least* one idler photon is detected plays a crucial role in the state reduction of the signal field.

E. Generalized Manley-Rowe relation

An operator Manley-Rowe relation is usually expressed as[20]

$$n_s(t_0)-n_i(t_0)=n_s(0)-n_i(0) . \tag{3.34}$$

When both input signal and idler fields are in the vacuum states, we obtain

$$n_s(t_0)-n_i(t_0)=0 . \tag{3.35}$$

This operator equation leads to

$$\langle n_s(t_0)\rangle-\langle n_i(t_0)\rangle=0 , \tag{3.36}$$

and

$$\langle\{\Delta[n_s(t_0)-n_i(t_0)]\}^2\rangle=0 . \tag{3.37}$$

That is, the signal and idler photon fields generated in the process of parametric down conversion have a perfect photon number correlation if the input signal and idler fields are in the vacuum state. A question arises whether or not this perfect correlation is deteriorated by the measurement back action of the idler photon number. In the QPF, Eq. (3.36) corresponds to Eq. (3.30). On the other hand, how is Eq. (3.37) modified in the QPF? To obtain the corresponding equation to Eq. (3.37), we must calculate the cross-correlation function $\langle n_s(T)n_i(T)\rangle_m$. This can be obtained by substituting Eq. (3.14) into Eq. (2.31):

$$\langle n_s(T)n_i(T)\rangle_m=\frac{(m+1)e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)[m+1+e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)]}{[1-e^{-\lambda(T-t_0)}\tanh^2(\kappa_0t_0)]^2} . \tag{3.38}$$

Using Eqs. (3.20), (3.21), (3.27), (3.28), and (3.38), we obtain

$$\langle\{\Delta[n_s(T)-n_i(T)]\}^2\rangle_m=0 , \tag{3.39}$$

where

$$\Delta[n_s(T)-n_i(T)]\equiv n_s(T)-n_i(T)-[\langle n_s(T)\rangle_m-\langle n_i(T)\rangle_m] .$$

Thus we find that in the QPF the average photon numbers in the signal and idler modes differ by exactly the number of detected idler photons in this process. Nevertheless, the photon-number noises are still correlated perfectly. In other words, the Manley-Rowe relation is preserved in the QPF if we retain the information about the number of detected idler photons. Thus a new operator representation of the Manley-Rowe relation which applies to the QPF can be written as

$$n_s(T)-n_i(T)-m=n_s(0)-n_i(0) , \tag{3.40}$$

where m is the number of detected photons in an observation time T.

IV. GENERAL FORMALISM OF THE NONREFERRING MEASUREMENT PROCESS

Thus far we have examined the nonunitary time evolution of the total system when we read out all available information concerning registrations of photocounts. In an actual photocount experiment, however, some of the available information is usually discarded. In photoelectric correlation experiments, for example, only photocounts which occur at some fixed time points are registered and all the other information is discarded. In this section we will investigate the effect of discarding photocounting information on state reduction. To simplify matters, we would like to examine the simplest case in which we discard all available information concerning the registrations of photocounts. This process is referred to as the nonreferring measurement process since we do not refer to the results of measurement. Let us introduce a superoperator $T_\tau^{(i)}$ that gives the time development of the total density operator in the NMP. Since we do not refer to the results of measurement, it must satisfy

$$\mathrm{Tr}[T_\tau^{(i)}\rho(t)]=1 \ . \tag{4.1}$$

The action of $T_\tau^{(i)}$ is determined as follows. Since the one-count and no-count processes form an exclusive exhaustive set of events, we have

$$T_{dt}^{(i)}\rho(t)=J^{(i)}\rho(t)dt+S_{dt}^{(i)}\rho(t) \ . \tag{4.2}$$

Using Eqs. (2.1), (2.11), and (2.12), we obtain the following differential equation for $\rho(t)$:

$$\begin{aligned}\frac{d\rho(t)}{dt}=&\lambda a_i\rho(t)a_i^\dagger-\left[i\omega_i+\frac{\lambda}{2}\right]a_i^\dagger a_i\rho(t)\\&+\left[i\omega_i-\frac{\lambda}{2}\right]\rho(t)a_i^\dagger a_i-i\omega_s a_s^\dagger a_s\rho(t)\\&+i\omega_s\rho(t)a_s^\dagger a_s \ .\end{aligned}\tag{4.3}$$

This differential equation can be solved to give[21]

$$\begin{aligned}T_{T-t_0}^{(i)}\rho(t_0)=&\sum_{m=0}^{\infty}\frac{[1-e^{-\lambda(T-t_0)}]^m}{m!}\exp\left\{-\left[\left[i\omega_i+\frac{\lambda}{2}\right]a_i^\dagger a_i+i\omega_s a_s^\dagger a_s\right](T-t_0)\right\}\\&\times a_i^m\rho(t_0)(a_i^\dagger)^m\exp\left\{\left[\left[i\omega_i-\frac{\lambda}{2}\right]a_i^\dagger a_i+i\omega_s a_s^\dagger a_s\right](T-t_0)\right\} \ .\end{aligned}\tag{4.4}$$

From Eqs. (4.1) and (4.4), we obtain the density operator after the NMP as

$$\begin{aligned}\rho^{\mathrm{NMP}}(T)=&\frac{T_{T-t_0}^{(i)}\rho(t_0)}{\mathrm{Tr}[T_{T-t_0}^{(i)}\rho(t_0)]}\\=&\sum_{m=0}^{\infty}\frac{[1-e^{-\lambda(T-t_0)}]^m}{m!}\exp\left\{-\left[\left[i\omega_i+\frac{\lambda}{2}\right]a_i^\dagger a_i+i\omega_s a_s^\dagger a_s\right](T-t_0)\right\}\\&\times a_i^m\rho(t_0)(a_i^\dagger)^m\exp\left\{\left[\left[i\omega_i-\frac{\lambda}{2}\right]a_i^\dagger a_i+i\omega_s a_s^\dagger a_s\right](T-t_0)\right\} \ .\end{aligned}\tag{4.5}$$

By taking trace of Eq. (4.5) over the idler Hilbert space, we obtain the density operator for the signal field after the NMP as

$$\begin{aligned}\rho_s^{\mathrm{NMP}}(T)\equiv&\mathrm{Tr}_i[\rho^{\mathrm{NMP}}(T)]\\=&\sum_{m=0}^{\infty}\frac{[1-e^{-\lambda(T-t_0)}]^m}{m!}\mathrm{Tr}_i\{\rho(t_0)(a_i^\dagger)^m\exp[-\lambda a_i^\dagger a_i(T-t_0)]a_i^m\}\\=&\sum_{m=0}^{\infty}\frac{[1-e^{-\lambda(T-t_0)}]^m}{m!}\sum_{k=m}^{\infty}{}_i\langle k|\rho(t_0)|k\rangle_i\frac{k!}{(k-m)!}e^{-\lambda(k-m)(T-t_0)} \ ,\end{aligned}\tag{4.6}$$

where $|k\rangle_i$ denotes the number state for the idler mode. Exchanging the order of summations we find

$$\begin{aligned}\rho_s^{\mathrm{NMP}}(T)=&\sum_{k=0}^{\infty}{}_i\langle k|\rho(t_0)|k\rangle_i\sum_{m=0}^{k}\binom{k}{m}[e^{-\lambda(T-t_0)}]^{k-m}[1-e^{-\lambda(T-t_0)}]^m\\=&\sum_{k=0}^{\infty}{}_i\langle k|\rho(t_0)|k\rangle_i=\mathrm{Tr}_i[\rho(t_0)]=\rho_s(t_0) \ .\end{aligned}\tag{4.7}$$

Thus we find that the density operator for the signal state after the NMP is the same as that of the initial state. In other words, the density operator for the signal field is not changed by the measurement process as long as we discard all information which can be read out from the photodetector. This result presents a sharp contrast to that for the RMP, where the signal photon field reduces to a number state whose eigenvalue is the same as the number of detected idler photon number.

On the other hand, the idler field suffers a substantial change by the NMP. In fact, taking the trace of Eq. (4.5) over the signal field, we find the density operator for the idler field after the NMP.

$$\rho_i^{\mathrm{NMP}}(T)=\mathrm{Tr}_s[\rho^{\mathrm{NMP}}(T)]$$
$$=\sum_{m=0}^{\infty}\frac{[1-e^{-\lambda(T-t_0)}]^m}{m!}\exp\left[-\left(i\omega_i+\frac{\lambda}{2}\right)a_i^\dagger a_i\right]a_i^m\rho_i(t_0)(a_i^\dagger)^m\exp\left[\left(i\omega_i-\frac{\lambda}{2}\right)a_i^\dagger a_i\right], \quad (4.8)$$

where we set

$$\rho_i(t_0)\equiv\mathrm{Tr}_s[\rho(t_0)] \ . \quad (4.9)$$

It follows from Eq. (4.8) that the average photon number and photon number variance of the idler field changes in time as

$$\langle n_i(t_0+T)\rangle=e^{-\lambda T}\langle n_i(t_0)\rangle \ , \quad (4.10)$$

$$\langle[\Delta n_i(t_0+T)]^2\rangle=e^{-2\lambda T}\langle[\Delta n_i(t_0)]^2\rangle + e^{-\lambda T}(1-e^{-\lambda T})\langle n_i(t_0)\rangle \ , \quad (4.11)$$

as shown in Figs. 5(b) and 6(b), respectively. This is how the Fano factor is obtained:

$$F_i(t_0+T)=e^{-\lambda T}F_i(t_0)+1-e^{-\lambda T} \ . \quad (4.12)$$

Thus we find that the average photon number decreases exponentially in time, and the statistical properties of the original field, which is represented by $F_i(t_0)$, lose their characteristics and approach the Poissonian statistics $F_i(t)\rightarrow 1$ as shown in Fig. 7(b). The corresponding time development of the Fano factor in the RMP is superimposed. We observed that the Fano factor approaches unity more rapidly in the RMP than in the NMP owing to the real-time renormalization of the density operator according to the readout information.

V. DISCUSSION AND CONCLUSIONS

In photocounting experiments, we usually prepare the experimental situation so that photoelectric conversion occurs one by one. Therefore state reduction of the observed system occurs continuously, and the associated

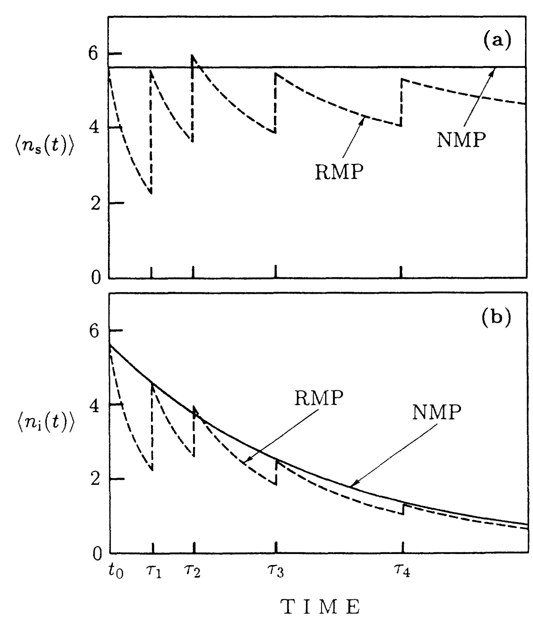

FIG. 5. Time development of the average photon number in the nonreferring measurement process (NMP): (a) signal field $\langle n_s(t)\rangle$, and (b) idler field $\langle n_i(t)\rangle$. One-count processes are assumed to occur at $\tau_1,\tau_2,\ldots$. The dashed curves corresponding to the RMP are superimposed for comparison.

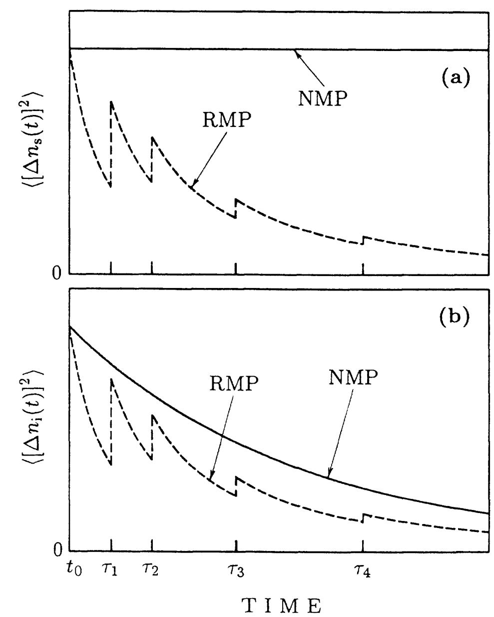

FIG. 6. Time development of the photon-number variance in the referring (RMP, dashed curves) and nonreferring (NMP, solid curves) measurement processes: (a) signal field $\langle[\Delta n_s(t)]^2\rangle$, and (b) idler field $\langle[\Delta n_i(t)]^2\rangle$. One-count processes are assumed to occur at $\tau_1,\tau_2,\ldots$. The dashed curves show the corresponding time developments of the signal and idler variances in the RMP.

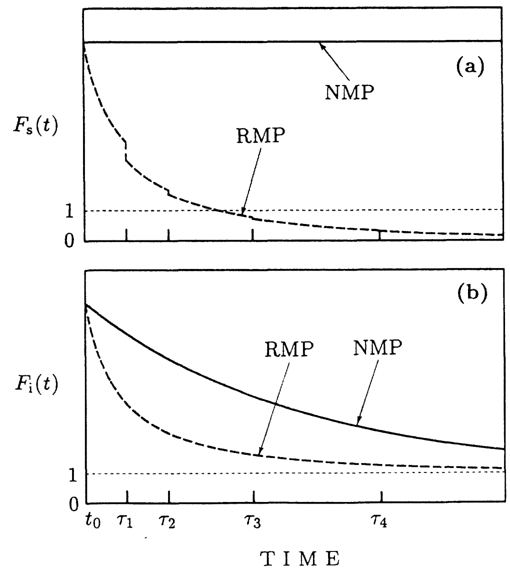

FIG. 7. Time evolution of the Fano factor in the nonreferring measurement process (NMP): (a) signal field $F_s(t)$, and (b) idler field $F_i(t)$. One-count processes are assumed to occur at $\tau_1, \tau_2, \ldots$. The dashed curves corresponding to the RMP are superimposed for comparison.

state evolution of the system becomes nonunitary. This nonunitary state evolution is uniquely determined by the readout information concerning registrations of photocounts.[12,14] When some other system is quantum-mechanically correlated to the observed system, the measurement back action will extend to the other system, causing state reduction of this system. The conventional treatment of this problem assumes the *instantaneous* state reduction and uses an operation-valued measure to calculate the postmeasurement state. We have discussed this problem in continuous photodetection context by extending the conventional quantum theory of measurement to continuous measurement in which state reduction occurs *continuously* according to the readout information, and the operation-valued measure is replaced by superoperators $J^{(i)}$, $S^{(i)}$, and $T^{(i)}$.

We have shown in Eq. (2.25) that in the general case, the density operator after the QPF no longer depends on the times of photocount registrations; it depends only on the measurement time interval and on the total number of detected photons. It implies that the initial and final states alone cannot uniquely specify the quantum measurement process. That is, there exist infinitely many different intermediate paths which start from the same initial state and reduce to the same final state. We have demonstrated some examples of the intermediate state evolution by giving specific readout information.

The parametric frequency down-conversion process has been employed here as a typical example in order to clarify the nonunitary state evolution of signal and idler fields due to continuous readout of the idler photocounts. Measurement back action and quantum correlation have been shown to play crucial roles in the continuous state reduction of the signal field. In particular, the Manley-Rowe relation in a quantum-mechanically correlated system have been extended to apply during the continuous measurement process. Here we note that the generalized Manley-Rowe relation (3.40) is a particular characteristic of the quantum-mechanically correlated fields prepared by the parametric process which yields a complete correlation in the photon number (i.e., diagonal elements of the density matrices). If we consider another quantum-mechanically correlated system, correlation between diagonal matrix elements is not always preserved. For example, the signal photon number and the idler photon *phase* are correlated in the optical Kerr effect.[22] It can be shown that the method developed in the present method is generalized also to such a case.[23]

Recently, a theory of continuous measurement of photon number has been applied to the system of parametrically down-converted photon twins[10] focusing on the postmeasurement signal state which depends on the result of the measurement of idler photon number. In the present paper, on the other hand, we focused on the nonunitary, intermediate time development of the fields during the continuous measurement process. We have demonstrated how the initial wave function of the quantum-mechanically correlated systems collapses to the number (sub-Poissonian) or the vacuum states owing to the real-time readout information. The Manley-Rowe relation has been generalized in the referring measurement process (RMP). Furthermore, we have examined the nonunitary state evolution in the nonreferring measurement process (NMP) and clarified the effect of discarding the readout information on the continuous state reduction. We find that the NMP does nothing for the signal field, while for the idler field it plays the simple role as a linear loss. In other words, if we do not use the readout information concerning registrations of photocounts to renormalize the initial density operator, there is no difference between photon counting (nonunitary process even if the detector is included) and linear dissipation (unitary if the reservoir is included). Although the measurement back action of the idler photon counting does not directly act on the signal field, it changes the signal state via quantum correlation. Accordingly, the signal state changes in the RMP but not in the NMP.

It is pointed out that the Manley-Rowe relation cannot hold in the NMP any longer because perfect quantum correlation between the signal and idler fields is deteriorated by our discarding the readout information. It is shown that the lost information concerning the quantum correlation is precisely compensated by the readout information for the idler field during the photodetection process. This fact explains why the generalized Manley-Rowe relation holds in the RMP and why it does not hold in the NMP where the "transferred" information is discarded.

*Present address: NTT Basic Research Laboratories, 3-9-11, Midori-cho, Musashino-shi, Tokyo 180, Japan.

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