Squeezing via mixing of two modes in a system of driven two-level atoms

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The squeezing in a mixutre of two high-Q cavity modes interacting with strongly driven two-level atoms is discussed for the case in which these two modes are located near the two sidebands of collective resonance fluorescence. The collective effects, cavity damping, and thermal field are accounted for. For suitable values of the parameters of the system, as in recent experiments with rydberg atoms, a large squeezing can be obtained.

I. INTRODUCTION

In the last decade the problem of generation of squeezed light is the central one in quantum optics.[1-3] A number of nonlinear optical systems capable of producing a squeezed state have been analyzed theoretically. These include the parametric oscillator,[4-7] four-wave mixing,[8-13] resonance fluorescence,[14-22] optical bistability,[23-25] two-photon processes,[26-27] etc. Recently, squeezed light has been experimentally generated in various laboratories.[28-31,25]

In the works by Bogolubov, Shumovsky, and Quang[19,20] and Lawande,[21] a large amount of two-mode squeezing in the mixture of the two sidebands of collective (Dicke model) resonance fluorescence has been predicted for the case of a strong driving field, while squeezing is absent for all three spectrum components taken separately. The influence of the cavity (which has the resonant frequencies near the sidebands) has been calculated in the paper.[32] In all the works[19-21,32] the influence of the black-body radiation has been omitted. The effects of thermal fields are negligible for optical transitions at normal temperature. However, when atoms are Rydberg atoms and their transitions are microwave transitions (where the Dicke model is justified), the effects of thermal field become important at very low temperature ($T \leq 4$ K).[33,34] For example, the amount of squeezing in the Rydberg atom maser[34] is substantially reduced even for the case of a few black-body photons (mean number $\bar{n} \cong 1$).

In this paper we consider squeezing in the mixture of two high-Q cavity modes, interacting with strongly driven two-level atoms. The cavity modes are assumed to be located near two sidebands of resonance fluorescence. The collective effects, cavity damping, and thermal field are accounted for. It is shown that large squeezing can be obtained for realistic experimental parameters of the system.

II. THE BASIC EQUATION

The N two-level identical atoms interact with an intense driving (pumping) classical field E_L at frequency ω_L and with two modes E_1 and E_2 of double cavity[35] at frequencies ω_1 and ω_2 (Fig. 1). All N atoms are coupled to a common thermal-field reservoir at temperature T. The atoms are assumed to be concentrated in a region small compared to the wavelength of all the relevant radiation modes (Dicke model). The cavity modes E_1 and E_2 are located near two sidebands of resonance fluorescence, i.e., they are in resonance with transitions between "dressed" atomic states. Our aim is to study squeezing in a mixture of the generated modes E_1 and E_2 (steady-state limit). Our scheme is similar to the correlated emission laser[35] with coherent pumping in a system of two-level atoms.

The coherence part of the Hamiltonian in the rotating-wave approximation and interaction picture is (the system is taken with $\hbar \equiv 1$)

$$H_{\rm coh} = H_0 + H_1 \ , \tag{2.1}$$

where

$$H_0 = \tfrac{1}{2}\Delta_0 (J_{22} - J_{11}) + G(J_{21} + J_{12}) \ , \tag{2.2}$$

$$H_i = \Delta_1 a_1^\dagger a_1 + g_1 (a_1^\dagger J_{12} + J_{21} a_1) + \Delta_2 a_2^\dagger a_2 + g_2 (a_2^\dagger J_{12} + J_{21} a_2) \ , \tag{2.3}$$

where $\Delta_0 = \omega_{21} - \omega_L$ is the frequency detuning of the driving field frequency ω_L from the atomic resonance frequency ω_{21}; $\Delta_1 = \omega_1 - \omega_L$; $\Delta_2 = \omega_2 - \omega_L$; $G = -d_{21} E_L$ is the resonant Rabi frequency; $a_1^\dagger, a_1$ and $a_2^\dagger, a_2$ are the creation and annihilation operators of the signal modes E_1 and E_2, respectively;

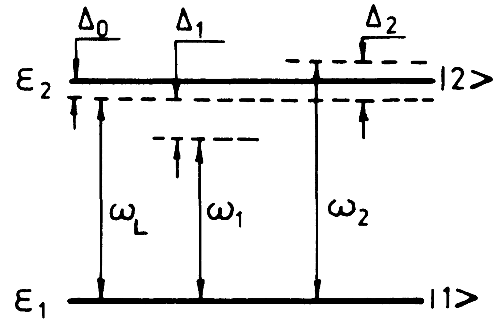

FIG. 1. Level scheme of the atomic system.

$$g_1=(\omega_1 d_{21}^2/2\epsilon_0 V_1)^2 \text{ and } g_2=(\omega_2 d_{21}^2/2\epsilon_0 V_2)^2 \tag{2.4}$$

are the coupling constants where V_1, V_2 are the cavity-mode volumes, d_{21} is the atomic dipole matrix element between states $|2\rangle$ and $|1\rangle$, and ϵ_0 is the permeability of free space. The operators

$$J_{ij}=\sum_{k=1}^{N}|i\rangle_{k\,k}\langle j| \quad (i,j=1,2) , \tag{2.5}$$

are the collective angular operators of the atoms. They satisfy the commutation relation

$$[J_{ij},J_{i'j'}]=J_{ij'}\delta_{ji'}-J_{i'j}\delta_{ij'} . \tag{2.6}$$

Using the Markov approximation we employ the master-equation technique described in the works[36,37,43] for the case of a vacuum-field reservoir and in the work[38] for the case of thermal-field rerservoir to derive the master equation for the reduced density operator ρ of the atom-field system comprised of N atoms and two modes E_1 and E_2. ρ is obtained from the density operator of the entire system by tracing over the thermal-field reservoirs at temperature T. The equation of motion in the interaction picture is given by

$$\frac{\delta\rho}{\delta t}=-i(L_0+L_1)\rho+\Lambda_a\rho+\Lambda_f\rho , \tag{2.7}$$

where

$$L_0\rho\equiv[H_0,\rho] , \tag{2.8}$$

$$L_1\rho\equiv[H_1,\rho] , \tag{2.9}$$

$$\begin{aligned}\Lambda_a\rho=&\tfrac{1}{2}\gamma(\bar n_0+1)(2J_{12}\rho J_{21}-J_{21}J_{12}\rho-\rho J_{21}J_{12})\\&+\tfrac{1}{2}\gamma\bar n_0(2J_{21}\rho J_{12}-J_{12}J_{21}\rho-\rho J_{12}J_{21}) ,\end{aligned} \tag{2.10}$$

$$\begin{aligned}\Lambda_f\rho=&\chi_1(1+\bar n_1)(2a_1\rho a_1^\dagger-a_1^\dagger a_1\rho-\rho a_1^\dagger a_1)\\&+\chi_1\bar n_1(2a_1^\dagger\rho a_1-a_1a_1^\dagger\rho-\rho a_1a_1^\dagger)\\&+\chi_2(1+\bar n_2)(2a_2\rho a_2^\dagger-a_2^\dagger a_2\rho-\rho a_2^\dagger a_2)\\&+\chi_2\bar n_2(2a_2^\dagger\rho a_2-a_2a_2^\dagger\rho-\rho a_2a_2^\dagger) .\end{aligned} \tag{2.11}$$

In Eqs. (2.9) and (2.10) γ is Einstein A coefficient;

$$\bar n_0=\bar n(\omega_{21})=\frac{1}{\exp(\omega_{21}/kT)-1} ,$$

$$\bar n_1=\bar n(\omega_1)=\frac{1}{\exp(\omega_1/kT)-1} ,$$

$$\bar n_2=\bar n(\omega_2)=\frac{1}{\exp(\omega_2/kT)-1}$$

are the mean photon numbers at the frequencies ω_{21}, ω_1, and ω_2, respectively; $\chi_1=\omega_1/Q_1$, $\chi_2=\omega_2/Q_2$ are the cavity field decay constants of the modes E_1 and E_2, respectively. Following Refs. 19, 20, and 39 we introduce the Schwinger representation for the collective angular operators J_{ij}:

$$J_{ij}=C_i^\dagger C_j \quad (i,j=1,2) ,$$

where C_i obeys the boson commutation relation

$$[C_i,C_j^\dagger]=\delta_{ij} .$$

It can be shown that[36] $\Lambda_a\cong N\gamma$; $\Lambda_f\cong\chi_1,\chi_2$; $L_{af}\cong g_{1,2}\sqrt{N}$. Further, we shall consider only the case of an intense pumping field so that

$$\Omega=(\tfrac{1}{4}\Delta_0^2+G)^{1/2}\gg N\gamma, g_{1,2}\sqrt{N} . \tag{2.12}$$

We also assume that (for good cavities)

$$\chi_1,\chi_2\ll N\gamma , \tag{2.13}$$

then Λ_f can be neglected in the maser equation for the reduced density matrix of the atomic system. Perform the canonical (dressing) transformation

$$\begin{aligned}C_1&=S_1\cos\varphi+S_2\sin\varphi ,\\C_2&=-S_1\sin\varphi+S_2\cos\varphi ,\end{aligned} \tag{2.14}$$

where

$$\tan(2\varphi)=2G/\Delta_0 .$$

In the case when conditions (2.12) and (2.13) are satisfied one can use the secular approximation, i.e., neglect the terms rapidly oscillating with frequencies $2\Omega, 4\Omega$ (Refs. 40, 19, and 20), and neglect Λ_f. After that one can find the master equation for the reduced density matrix ρ_a of the atomic system in the form

$$\begin{aligned}\frac{\delta\rho_a}{\delta t}=&-i\Omega[R_3,\rho]+B(2R_3\rho_aR_3-R_3^2\rho_a-\rho_aR_3^2)\\&+X_1(2R_{12}\rho_aR_{21}-R_{21}R_{12}\rho_a-\rho_aR_{21}R_{12})\\&+X_2(2R_{21}\rho_aR_{12}-R_{12}R_{21}\rho_a-\rho_aR_{12}R_{21}) ,\end{aligned} \tag{2.15}$$

where

$$R_3=R_{22}-R_{11} , \tag{2.16}$$

$$B=\frac{\gamma}{2}(\bar n_0+1)\sin^2\varphi\cos^2\varphi , \tag{2.17}$$

$$X_1=\frac{\gamma}{2}(\bar n_0+1)\cos^4\varphi+\frac{\gamma}{2}\bar n_0\sin^4\varphi , \tag{2.18}$$

$$X_2=\frac{\gamma}{2}(\bar n_0+1)\sin^4\varphi+\frac{\gamma}{2}\bar n_0\cos^4\varphi . \tag{2.19}$$

$R_{ij}=S_i^\dagger S_j$ $(i,j=1,2)$ are the collective operators of the "dressed" atoms. The operators S_i and $S_i^\dagger$ satisfy the boson commutation relation

$$[S_i,S_j^\dagger]=\delta_{ij} , \tag{2.20}$$

so that

$$[R_{ij},R_{i'j'}]=R_{ij'}\delta_{i'j}-R_{i'j}\delta_{ij'} . \tag{2.21}$$

The exact stationary solution of Eq. (2.15) takes the form

$$\rho_{a,\mathrm{st}}=Z^{-1}\sum_{n_1=0}^{N}X^{n_1}|n_1\rangle\langle n_1| , \tag{2.22}$$

where

$$X=\frac{X_1}{X_2}=\frac{(\bar{n}_0+1)\cos^4\varphi+\bar{n}_0\sin^4\varphi}{(\bar{n}_0+1)\sin^4\varphi+\bar{n}_0\cos^4\varphi}\ , \tag{2.23}$$

$$Z=\frac{X^{N+1}-1}{X-1}\ . \tag{2.24}$$

The state $|n_1\rangle$ is the eigenstate of the operators R_{11} and $R_{11}+R_{22}$. The solution (2.22) allows one to calculate all the one-time stationary expectation values of the atomic observable. The correlation functions such as $\langle R_{12}(t)R_{21}\rangle_s$ and $\langle R_{21}(t)R_{12}\rangle_s$, where $\langle\ \rangle_s$ denotes an expectation value over the steady-state (2.22), can be found using the equations of motion for $\langle R_{ij}(t)\rangle$ and the quantum regression theorem.

By using the master equation (2.15) one finds

$$\frac{d}{dt}\langle R_{12}(t)\rangle=-2i\Omega\langle R_{12}(t)\rangle-\tilde{\gamma}\langle R_{12}(t)\rangle -\frac{\gamma}{4}(\sin^2\varphi-\cos^2\varphi)\langle\{R_3(t),R_{12}(t)\}\rangle\ , \tag{2.25}$$

$$\frac{d}{dt}\langle R_{21}(t)\rangle=\left[\frac{d}{dt}\langle R_{12}(t)\rangle\right]^*\ , \tag{2.26}$$

where

$$\{R_{ij},R_3\}\equiv R_{ij}R_3+R_3R_{ij}\ , \tag{2.27}$$

$$\tilde{\gamma}=4B+X_1+X_2=\gamma(\bar{n}_0+\tfrac{1}{2})(1+2\cos^2\varphi\sin^2\varphi)\ . \tag{2.28}$$

Equations (2.25) and (2.26) are so far exact. They contain, however, term with the products of operators which make them unsolvable in the general case.

For the one-atom case one can use the operator relation

$$R_{ij}R_{i'j'}=R_{ij'}\delta_{i'j}\quad (i,j,i',j'=1,2)\ ,$$

then Eqs. (2.25) and (2.26) reduce to the linear, exact solvable equations

$$\frac{d}{dt}\langle R_{12}(t)\rangle=-2i\Omega\langle R_{12}(t)\rangle-\tilde{\gamma}\langle R_{12}(t)\rangle\ , \tag{2.29}$$

$$\frac{d}{dt}\langle R_{21}(t)\rangle=\left[\frac{d}{dt}\langle R_{12}(t)\rangle\right]^*\ . \tag{2.30}$$

The quantity $\tilde{\gamma}$ found according to Eq. (2.28) is the linewidth of two sidebands of the single-atom fluorescence field.

For the case of exact resonance $\cos^2\varphi=\frac{1}{2}$ we have $X_1=X_2$ and the term with the products of operators vanish; then both Eqs. (2.25) and (2.26) reduce to the exact solvable linear differential equations.

For the off-resonance case, analogously to the Refs. 20 and 41, we use the decorrelation scheme

$$\langle\{R_3,R_{ij}\}\rangle=2\langle R_3\rangle_s\langle R_{ij}\rangle\ . \tag{2.31}$$

By using the density matrix (2.22) one shows that in the case of large N the decorrelation (2.31) yields a small error (of an order of $N^{-1/2}$) in the calculation of the correlation functions $\langle R_{12}(t)R_{21}\rangle_s$, $\langle R_{21}(t)R_{12}\rangle_s$ and of the steady-state fluorescent spectrum.[20,41] With the approximation (2.31), Eqs. (2.25) and (2.26) have simple exponential solutions and applying the quantum regression theorem[42] one obtains the following expressions for the correlation functions $\langle R_{21}(t)R_{12}\rangle_s$ and $\langle R_{12}(t)R_{21}\rangle_s$:

$$\langle R_{21}(t)R_{12}\rangle_s=\langle R_{21}R_{12}\rangle_s\exp(2i\Omega t-i\Gamma t)\ , \tag{2.32}$$

$$\langle R_{12}(t)R_{21}\rangle_s=\langle R_{12}R_{21}\rangle_s\exp(-2i\Omega t-i\Gamma t)\ , \tag{2.33}$$

where

$$\langle R_{21}R_{12}\rangle_s=-\langle R_{11}^2\rangle_s+(N-1)\langle R_{11}\rangle_s+N\ , \tag{2.34}$$

$$\langle R_{12}R_{21}\rangle_s=-\langle R_{11}^2\rangle_s+(N+1)\langle R_{11}\rangle_s\ , \tag{2.35}$$

$$\Gamma=\tilde{\gamma}+\frac{\gamma}{2}(\sin^2\varphi-\cos^2\varphi)\langle R_3\rangle_s\ , \tag{2.36}$$

$$\langle R_3\rangle_s=N-2\langle R_{11}\rangle_s\ , \tag{2.37}$$

$$\langle R_{11}\rangle_s=\frac{NX^{N+2}-(N+1)X^{N+1}+X}{Z^{-1}(X-1)^2}\ , \tag{2.38}$$

$$\langle R_{11}^2\rangle_s=\frac{N^2X^{N+3}-(2N^2+2N-1)X^{N+2}+(N+1)^2X^{N+1}-X^2-X}{Z^{-1}(X-1)^3}\ . \tag{2.39}$$

The quantity Γ in Eq. (2.36) is the linewidth of the two sidebands of collective resonance fluorescence in the coherent and black-body radiation fields.

III. LIGHT SQUEEZING

In this section we investigate the squeezing in a mixture of the two modes a_1 and a_2. The Langevin equations for the operators a_1 and a_2 have the following terms:[44]

$$\dot{a}_1(t)=(-i\Delta_1-\chi_1)a_1(t)-ig_1J_{12}(t)+F_1(t)\ , \tag{3.1}$$

$$\dot{a}_2(t)=(-i\Delta_2-\chi_2)a_2(t)-ig_2J_{12}(t)+F_2(t)\ , \tag{3.2}$$

where

$$J_{12}(t)=\cos^2\varphi R_{12}(t)-\sin^2\varphi R_{21}(t)+\sin\varphi\cos\varphi R_3(t)\ . \tag{3.3}$$

The operators $F_\lambda(t)$ $(\lambda=1,2)$ are the noise operators for the modes E_1 and E_2, respectively, and obey the relations $(\lambda,\lambda'=1,2)$

$$\begin{aligned}&\langle F_\lambda(t)\rangle_H=\langle F_\lambda^\dagger(t)\rangle_H=0\ ,\\&\langle F_\lambda^\dagger(t)F_{\lambda''}^\dagger(t)\rangle_H=\langle F_\lambda(t)F_{\lambda''}(t')\rangle_H=0\ ,\\&\langle F_\lambda^\dagger(t)F_{\lambda''}(t)\rangle_H=\bar{n}_\lambda 2\chi_\lambda\delta(t-t')\delta_{\lambda,\lambda''}\ ,\\&\langle F_\lambda(t)F_{\lambda''}^\dagger(t)\rangle_H=(\bar{n}_\lambda+1)2\chi_\lambda\delta(t-t')\delta_{\lambda,\lambda''}\ ,\end{aligned}\tag{3.4}$$

where $\langle\ \rangle_H$ indicates the thermal average over the state of a heat bath.

It is easy to see that in the secular approximation the operators $R_{12}(t)$ and $R_{21}(t)$ are rapidly oscillating terms as $\exp(-2i\Omega t)$ and $\exp(2i\Omega t)$ where the operator $R_3(t)$ is slowly varying in time. Further, we shall discuss only the case when the cavity modes a_1 and a_2 are located near the two sidebands of the collective resonance fluorescence, i.e.,

$$|\delta_1|,|\delta_2|\ll\Omega\ ,\tag{3.5}$$

where

$$\delta_1=\Delta_1-2\Omega,\quad \delta_2=\Delta_2+2\Omega\ .$$

Under the transformation

$$\begin{aligned}&a_1(t)\rightarrow\exp(-2i\Omega t)\tilde{a}_1(t),\\&a_2(t)\rightarrow\exp(2i\Omega t)\tilde{a}_2(t)\ ;\\&R_{12}(t)\rightarrow\exp(-2i\Omega t)\tilde{R}_{12}(t)\ ,\\&R_{21}(t)\rightarrow\exp(2i\Omega t)\tilde{R}_{21}(t)\ ;\\&F_1(t)\rightarrow\exp(-2i\Omega t)\tilde{F}_1(t)\ ,\\&F_2(t)\rightarrow\exp(2i\Omega t)\tilde{F}_2(t)\end{aligned}$$

and with the use of the secular approximation Eqs. (3.1) and (3.2) reduce to

$$\dot{\tilde{a}}_1(t)=(-i\delta_1-\chi_1)\tilde{a}_1(t)-iG_1\tilde{R}_{12}(t)+\tilde{F}_1(t)\ ,\tag{3.6}$$

$$\dot{\tilde{a}}_2(t)=(-i\delta_2-\chi_2)\tilde{a}_2(t)-iG_2\tilde{R}_{21}(t)+\tilde{F}_2(t)\ ,\tag{3.7}$$

where

$$G_1=g_1\cos^2\varphi,\quad G_2=-g_2\sin^2\varphi\ .$$

The solutions of Eqs. (3.6) and (3.7) can be written in the form

$$\tilde{a}_1(t)=\tilde{a}_{1,s}(t)+\tilde{a}_{1,n}(t)\ ,\tag{3.8}$$

$$\tilde{a}_2(t)=\tilde{a}_{2,s}(t)+\tilde{a}_{2,n}(t)\ ,\tag{3.9}$$

where

$$\begin{aligned}\tilde{a}_{1,n}(t)=&\,a_1(0)\exp[(-i\delta_1-\chi_1)t]\\&+\int_0^t\tilde{F}_1(t')\exp[(-i\delta_1-\chi_1)(t-t')]dt'\ ,\end{aligned}\tag{3.10}$$

$$\begin{aligned}\tilde{a}_{2,n}(t)=&\,a_2(0)\exp[(-i\delta_2-\chi_2)t]\\&+\int_0^t\tilde{F}_2(t')\exp[(-i\delta_2-\chi_2)(t-t')]dt'\ ,\end{aligned}\tag{3.11}$$

$$\tilde{a}_{1,s}(t)=-iG_1\int_0^t\tilde{R}_{12}(t')\exp[(-i\delta_1-\chi_1)(t-t')]dt'\ ,\tag{3.12}$$

$$\tilde{a}_{2,s}(t)=-iG_2\int_0^t\tilde{R}_{21}(t')\exp[(-i\delta_2-\chi_2)(t-t')]dt'\ .\tag{3.13}$$

In the steady-state limit $t\rightarrow\infty$ by using Eqs. (2.32), (2.33), and (3.4) one finds

$$\langle a_1^\dagger a_1\rangle=\bar{n}_1+G_1^2\langle R_{21}R_{12}\rangle_s\frac{1}{\chi_1(\chi_1+\tilde{\Gamma})}\ ,\tag{3.14}$$

$$\langle a_2^\dagger a_2\rangle=\bar{n}_2+G_2^2\langle R_{12}R_{21}\rangle_s\frac{1}{\chi_2(\chi_2+\tilde{\Gamma})}\ ,\tag{3.15}$$

$$\begin{aligned}\langle a_1a_2\rangle=\langle a_2^\dagger a_1^\dagger\rangle=&-G_1G_2\langle R_{12}R_{21}\rangle_s\frac{1}{\chi_1+\chi_2}\\&\times\left[\frac{1}{\chi_1+\tilde{\Gamma}}+\frac{1}{\chi_2+\tilde{\Gamma}}\right]\ ,\end{aligned}\tag{3.16}$$

$$\begin{aligned}\langle a_2a_1\rangle=\langle a_1^\dagger a_2^\dagger\rangle=&-G_1G_2\langle R_{21}R_{12}\rangle_s\frac{1}{\chi_1+\chi_2}\\&\times\left[\frac{1}{\chi_1+\tilde{\Gamma}}+\frac{1}{\chi_2+\tilde{\Gamma}}\right]\ ,\end{aligned}\tag{3.17}$$

where

$$\tilde{\Gamma}=\begin{cases}\tilde{\gamma} & \text{if } N=1\\ \Gamma & \text{if } N\gg1\ .\end{cases}\tag{3.18}$$

In Eqs. (3.14)–(3.18), for simplicity we take $\delta_1=\delta_2=0$, i.e., $\omega_1=\omega_L-2\Omega$ and $\omega_2=\omega_1+2\Omega$. The symbol $\langle\ \rangle$ indicates the expectation value over the states of thermal-field reservoir and atomic steady state (2.22). The quantities $\tilde{\gamma}$, $\langle R_{21}R_{12}\rangle_s$, $\langle R_{12}R_{21}\rangle_s$ and Γ can be found in the Eqs. (2.28) and (2.34)–(2.36), respectively. The quadrature phase components of the mixture of the two modes a_1 and a_2 are defined as

$$b_\theta=\tfrac{1}{2}(b^+e^{i\theta}+be^{-i\theta})\ ,\tag{3.19}$$

where

$$b=\frac{1}{\sqrt{2}}(a_1+a_2),\quad b^\dagger=\frac{1}{\sqrt{2}}(a_1^\dagger+a_2^\dagger)\ .\tag{3.20}$$

For $\theta=0$ and $\theta=\frac{1}{2}$ the quadrature phase components b_θ coincide with the in-phase (b_1) and out-of-phase (b_2) components, respectively.

By using Eqs. (3.14)–(3.17) one can find the normally ordered variance of the phase quadrature components b_θ in the form

$$\langle :(\Delta b_\theta)^2: \rangle = \frac{1}{4}\left[\bar{n}_1+\bar{n}_2+g_1^2(\cos^4\varphi)\langle R_{21}R_{12}\rangle_s \frac{1}{\chi_1(\chi_1+\tilde{\Gamma})} + g_2^2(\sin^4\varphi)\langle R_{12}R_{21}\rangle_s \frac{1}{\chi_2(\chi_2+\tilde{\Gamma})} + \cos(2\theta) g_1 g_2 \sin^2\varphi\cos^2\varphi \frac{1}{\chi_1+\chi_2}\left[\frac{1}{\chi_1+\tilde{\Gamma}}+\frac{1}{\chi_2+\tilde{\Gamma}}\right](\langle R_{12}R_{21}\rangle + \langle R_{21}R_{12}\rangle)\right] . \quad (3.21)$$

For simplicity, we shall consider only the case of $\chi_1=\chi_2=\chi$, $\bar{n}_1 \cong \bar{n}_2 \cong \bar{n}_0$. By using the condition (2.13), Eq. (3.21) reduces to

$$\langle :(\Delta b_\theta)^2: \rangle = \frac{1}{4}\left[2\bar{n}_0 + \frac{g_1^2}{\chi\tilde{\Gamma}}\left[(\cos^4\varphi)\langle R_{21}R_{12}\rangle_s + \eta^2(\sin^4\varphi)\langle R_{12}R_{21}\rangle_s + \cos(2\theta)\eta\sin^2\varphi\cos^2\varphi(\langle R_{12}R_{21}\rangle_s + \langle R_{21}R_{12}\rangle_s)\right]\right] , \quad (3.22)$$

where $\eta=g_2/g_1$. It is clear from Eqs. (3.21) and (3.22) that the largest amount of squeezing is present for the case of $\theta=\pi/2$, i.e., in the out-of-phase component b_2.

Further, we consider the degree of squeezing in the out-of-phase component b_2. By using Eqs. (3.14) and (3.15) one finds the commutator of the operators b_1 and b_2 in the form

$$\tfrac{1}{2}|\langle [b_1,b_2]\rangle| = \frac{1}{4}\left|1+\frac{1}{2}\langle R_3\rangle_s \frac{g_1^2}{\chi\tilde{\Gamma}}(\eta^2\sin^4\varphi - \cos^4\varphi)\right| . \quad (3.23)$$

The factor of squeezing in the phase component b_2 can be defined as

$$\beta_2 = \frac{\langle :(\Delta b_2)^2: \rangle}{\frac{1}{2}|\langle [b_1,b_2]\rangle|} . \quad (3.24)$$

Squeezing is present in the mixture of the two modes a_1 and a_2 if the factor β_2 is less than zero. For the exact resonance case $\cos^2\varphi=\frac{1}{2}$ we have $\langle R_{21}R_{12}\rangle_s = \langle R_{12}R_{21}\rangle_s = N(N+2)/6$ resulting in $\langle :(\Delta b_2)^2: \rangle \geq 0$; thus, squeezing is absent in this case. One can also show that $\langle :(\Delta b_2)^2: \rangle \geq 0$ for the case of $g_1 \to 0$ or $g_2 \to 0$; thus, squeezing is absent for separate modes a_1 or a_2.

It can be seen from Eq. (3.22) that for the case $g_1^2/\chi\gamma \ll 1$ (bad cavity limit) squeezing will disappear for a small value of the thermal field intensity $\bar{n}_0$. For high-Q cavity such that $g_1^2/\chi\gamma \gg 1$, a large amount of squeezing may be present for suitable values of the parameters of the system. The behavior of the factor of squeezing β_2 as a function of the parameters $\cos^2\varphi$ for $g_1^2/\chi\gamma=10^3$, $\eta=0.68$ and for various values of the thermal field intensity $\bar{n}_0$ is plotted in Fig. 2 (for $N=1$) and Fig. 3 (for $N=1000$). The behavior of the factor of squeezing β_2 as a function of the parameter η for $g_1^2/\chi\gamma=10^3$, $\cos^2\varphi=0.4$ and for various values of the thermal field intensity $\bar{n}_0$ is plotted in Fig. 4 (for $N=1$) and Fig. 5 (for $N=1000$). In experiments, the value $\cos^2\varphi$ may be altered with alteration of the detuning Δ_0 of a driving field frequency ω_L from the atomic resonance frequency ω_{21} and the value $\eta=g_2/g_1$ may be altered with the change of optical path lengths [see Eq. (2.4)] of the modes E_1 and E_2 of the doubly cavity. It is seen from Figs. 2 and 4 that

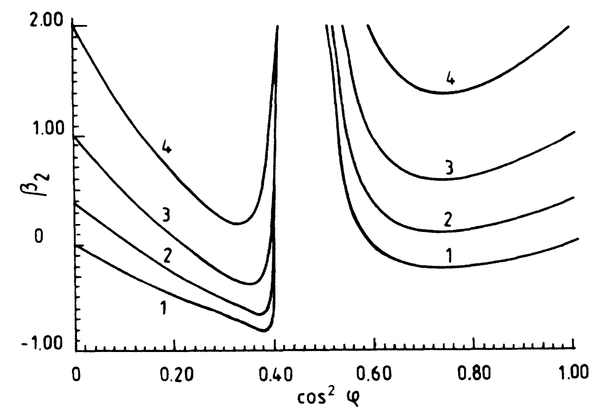

FIG. 2. Factor of squeezing β_2 as a function of $\cos^2\varphi$ for $N=1$, $g_1^2/\chi\gamma=10^3$, $\eta=0.68$; curves 1–4 correspond to $\bar{n}_0=0$, 0.2, 0.5, and 1, respectively.

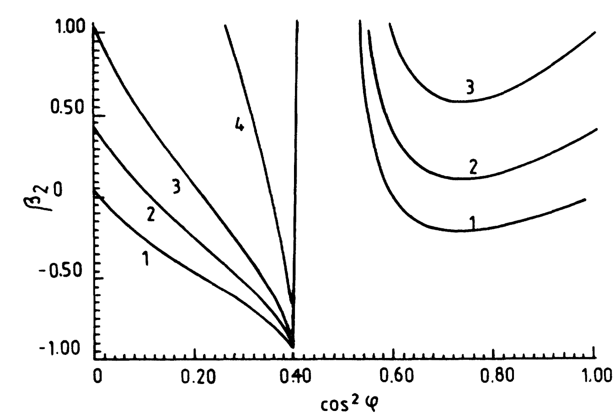

FIG. 3. Factor of squeezing β_2 as a function of $\cos^2\varphi$ for $N=10^3$, $g_1^2/\chi\gamma=10^3$, $\eta=0.68$; curves 1–4 correspond to $\bar{n}_0=0$, 0.5, 1, and 2, respectively.

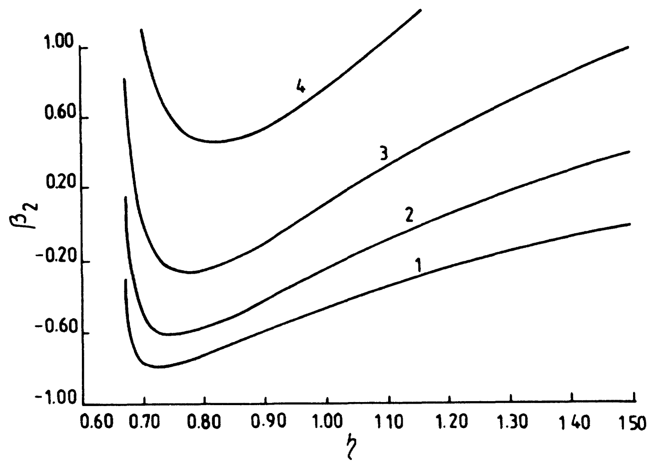

FIG. 4. Factor of squeezing β_2 as a function of η for $N=1$, $g_1^2/\chi\gamma=10^3$, $\cos^2\varphi=0.4$; curves 1–4 correspond to $\bar{n}_0=0$, 0.2, 0.5, and 1, respectively.

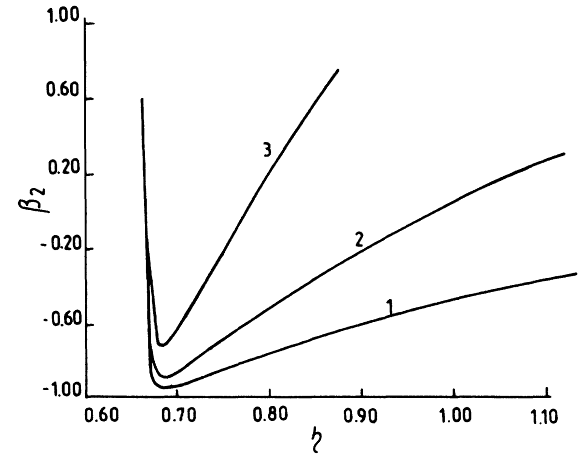

FIG. 5. Factor of squeezing β_2 as a function of η for $N=10^3$, $g_1^2/\chi\gamma=10^3$, $\cos^2\varphi=0.4$; curves 1–3 correspond to $\bar{n}_0=0$, 1, and 2, respectively.

a thermal field strongly reduces the degree of squeezing in the one-atom case which vanishes for $\bar{n}_0 \geq 1$. For the many-atom case $N=1000$ and for high-Q cavity ($g_1^2/\chi\gamma=10^3$) substantial squeezing is present for $\bar{n}_0 \cong 1$. This value for $\bar{n}_0$ is of direct experimental relevance[45–47] since temperature in experiments with Rydberg atoms changes from 2 to 4 K. We note that the amount of squeezing in the mixture of the two sidebands of collective resonance fluorescence strongly reduces in the presence of the thermal black-body field and vanishes for the case of $\bar{n}_0 \geq 0.5$.

IV. CONCLUSION

In this paper, we have studied the generation of squeezing in the mixture of two high-Q cavity modes interacting with a collection of N two-level atoms coherently driving by external classical field. We have investigated the influence of the thermal-field reservoir, cavity damping as well as the collective effects on the degree of squeezing. The frequencies of the cavity modes are assumed to be located near two sidebands of the resonance fluorescence. We have shown that in the presence of thermal-field reservoir the cavity enhances the degree of squeezing in the mixture of the two sidebands of collective resonance fluorescence. For the many atoms and high-Q cavity case one can obtain large degree of squeezing for the case of few black-body photons. This is of direct experimental relevance with recent experiments using Rydberg atoms.

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[1]D. F. Walls, Nature (London) **30**, 141 (1983).
[2]H. P. Yuen, Phys. Rev. A **13**, 2226 (1976).
[3]R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).
[4]G. J. Milburn and D. F. Walls, Opt. Commun. **39**, 401 (1981).
[5]L. A. Lugiato and G. S. Strini, Opt. Commun. **41**, 67 (1982).
[6]M. J. Collet and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[7]Ling-An Wu, Min Xiao, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1468 (1987).
[8]H. P. Yuen and J. H. Shapiro, Opt. Lett. **4**, 334 (1979).
[9]R. S. Bondurant, P. Kumar, J. H. Shapiro, and M. Maeda, Phys. Rev. A **30**, 343 (1984).
[10]M. D. Reid and D. F. Walls, Phys. Rev. A **31**, 1622 (1985).
[11]J. R. Klauder, S. L. McCall, and B. Yurke, Phys. Rev. A **33**, 3204 (1986).
[12]N. N. Bogolubov, Jr., A. S. Shumovsky, and Tran Quang, J. Phys. B **20**, 1447 (1987).
[13]G. S. Agarwal and R. W. Boyd, Phys. Rev. A **38**, 4019 (1988).
[14]L. Mandel, Phys. Rev. Lett. **49**, 136 (1982).
[15]D. F. Walls and P. Zoller, Phys. Rev. Lett. **47**, 709 (1981).
[16]R. Loudon, Opt. Commun. **49**, 24 (1984).
[17]M. J. Collet, D. F. Walls, and P. Zoller, Opt. Commun. **52**, 425 (1984).
[18]P. A. Lakshmi and G. S. Agarwal, Opt. Commun. **51**, 425 (1984).
[19]N. N. Bogolubov, Jr., A. S. Shumovsky, and Tran Quang, Phys. Lett. A **116A**, 175 (1986).
[20]N. N. Bogolubov, Jr., A. S. Shumovsky, and Tran Quang, Opt. Commun. **64**, 351 (1987).
[21]Q. V. Lawande and S. V. Lawande, Phys. Rev. A **38**, 800 (1988).
[22]A. Heidmann and S. Reynaud, J. Phys. (Paris) **46**, 1937 (1985).
[23]L. A. Lugiato and G. Strini, Opt. Commun. **41**, 374 (1982).
[24]M. D. Reid and D. F. Walls, Phys. Rev. A **32**, 396 (1985).
[25]L. A. Orozco, M. G. Raizen, Min Xiao, R. J. Brecha, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1490 (1987).
[26]R. Loudon, Opt. Commun. **49**, 67 (1984).
[27]C. M. Savage and D. F. Walls, Phys. Rev. A **33**, 3282 (1986).
[28]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[29]L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[30]B. L. Schumaker, S. H. Perlmultter, R. M. Shelby, and M. D. Levenson, Phys. Rev. Lett. **58**, 357 (1987).
[31]M. Maeda, P. Kumar, and J. Shapiro, Opt. Lett. **12**, 161

(1987).

[32]N. N. Bogolubov, Jr., A. S. Shumovsky, and Tran Quang, Opt. Commun. **62**, 49 (1987).

[33]G. S. Agarwal, R. K. Bullough, and G. P. Hildred, Opt. Commun. **59**, 23 (1986).

[34]A. Heidmann, J. M. Raimond, S. Reynaud, and N. Zagyry, Opt. Commun. **54**, 54 (1985).

[35]M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985).

[36]R. Bonifacio, P. Schwendimann, and F. Haake, Phys. Rev. A **4**, 302, (1971); **4**, 854 (1971).

[37]W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[38]Seng-Tiong Ho, P. Kumar, and J. H. Shapiro, Phys. Rev. A **37**, 2017 (1988).

[39]J. Schwinger, *Quantum Theory of Angular Momentum,* edited by L. C. Biederharn and H. Van Dam (Academic, New York, 1965).

[40]G. S. Agarwal, L. M. Narducci, D. H. Feng, and R. Gilmol, Phys. Rev. Lett. **42**, 1260 (1978).

[41]G. Compagno and F. Persico, Phys. Rev. A **25**, 3138 (1982).

[42]M. Lax, Phys. Rev. **172**, 350 (1968).

[43]F. Haake, in *Quantum Statistics in Optics and Solid-State Physics,* Vol. 66 of *Springer Tracts in Modern Physics,* edited by G. Höhler (Springer-Verlag, Berlin, 1973).

[44]M. Sargent, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974).

[45]S. Haroche and J. M. Raimond in *Advances in Atomic and Molecular Physics,* edited by D. R. Bates and B. Bederson (Academic, London, 1985), Vol. 20, p. 350.

[46]J. A. C. Gallas, G. Leuchs, H. Walther, and H. Figger, *Advances in Atomic and Molecular Physics,* edited by D. R. Bates and B. Bederson (Academic, London, 1985), Vol. 20, p. 414.

[47]H. I. Yoo and J. H. Eberly, Phys. Rev. **118**, 240 (1985).