Approximate model of soliton dynamics in all-optical couplers

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Using a variational method, we have developed an approximate model of the dynamics of soliton coupling described by coupled nonlinear Schrödinger equations. The results are analytical and can be recast into a suggestive particlelike description providing a physical insight into the problem. The particular case of the nonlinear coherent coupler is emphasized, and an estimate of the soliton switching peak power is obtained. A symmetry-breaking instability is also described, in a simple way, with the potential-well picture.

I. INTRODUCTION

Because of their unique property of propagating without distortion or spreading, optical solitons,[1] observed experimentally since 1980,[2] offer the possibility to achieve very high data transmission rates in optical-fiber communication systems. They are considered as the future optical bits. These prospects motivate important research efforts towards the development of all-optical components, such as the "all-optical switch." For example, the so-called nonlinear directional coupler[3–8] (NLDC) is a special device whose transmission characteristics depend sensitively on the light intensity. By increasing the power, one can pass from a high to a low transmission regime, then realizing the all-optical switch. The NLDC, as well as the nonlinear birefringent fiber (NLBF),[9–11] then represent good candidates as building blocks of future communication systems. For those considerations, it appears important to investigate the dynamics of soliton coupling.

Generally speaking, soliton coupling also occurs in many areas of nonlinear physics.[12] A large class of phenomena involving soliton interaction can be properly described by a pair of coupled nonlinear Schrödinger equations (NLSE's) [see Eqs. (1), Sec. II]. Unfortunately, except for a few particular cases,[13–18] such as solitary waves, there is no general analytical solution for coupled NLSE's. The usual tendency is then to investigate the dynamics by numerical simulations.[10,11,19,20] But, besides the problem of important computer time, the numerical approach is not very appealing, in the sense that it does not provide a global understanding of the underlying physics. In other words, it is not a simple task to get a physical insight from purely numerical experiments.

The idea is then to use approximate analytical methods in order to compensate for the lack of exact results. Considerable effort has been devoted, in the last few years, to developing such approximate tools. For the NLSE in general, this includes perturbation techniques,[21,22] adiabatic approximations,[23] semiheuristic models,[8] the method of moments[24,25] and invariants,[26] and particlelike descriptions.[27–30] Among the last category, the Lagrangian variational method has proven to be a successful approach for a variety of problems in nonlinear optics.[27,28]

In this paper, it is shown how the variational method can also be applied successfully to the system of coupled NLSE's. We will see that the main features of the complex dynamics can be anticipated by finding an equivalent potential-well description. In particular, the transformation of the shape of the potential, induced by increasing the nonlinearity above a characteristic parameter, is a quite appealing description of "critical" switching.

The paper is organized as follows. First, in Sec. II, the basic system of coupled NLSE's is briefly introduced. Section III describes the variational method and the importance of a suitable trial function is emphasized. The partial differential equations are then transformed into a set of coupled ordinary differential equations (ODE's), for which exact solutions are presented. The equivalent potential is then introduced. In Sec. IV we discuss the main results and check out their validity by comparing with exact numerical simulations. We indicate how the main characteristics can be anticipated from the potential description. Finally, in agreement with recent results of Wright *et al.*,[13] we, independently, also predict a "symmetry-breaking" instability for the case of identical solitonlike input pulses. The conclusions are given in the last section.

II. SYSTEM OF COUPLED NLSE's

Since the main results of the paper are valid for different problems of nonlinear physics, we will omit the details of the derivation of the coupled NLSE's. Let us simply mention that, in the context of nonlinear fiber optics, their use implies that the transverse distribution of the linear modes of the (identical) fibers is assumed to remain unaffected by the nonlinearity. The main effect of the nonlinearity is rather to induce phase shifts that will detune the linear system and modify its transmission characteristics. Such an approximation is always invoked for the investigation of soliton dynamics in fibers. We refer the reader to the literature for further details.[1,31,32]

Then, on a general level, we will consider the following basic system of coupled NLSE's:

$$-i\frac{\partial\psi_1}{\partial z}+\frac{1}{2}\frac{\partial^2\psi_1}{\partial\tau^2}+(|\psi_1|^2+\sigma|\psi_2|^2)\psi_1+\kappa\psi_2=0\ ,$$
$$-i\frac{\partial\psi_2}{\partial z}+\frac{1}{2}\frac{\partial^2\psi_2}{\partial\tau^2}+(|\psi_2|^2+\sigma|\psi_1|^2)\psi_2+\kappa\psi_1=0\ , \qquad (1)$$

where the complex functions $\psi_j(z,\tau)$ $(j=1,2)$ represent the slowly varying light-field envelopes. The independent variables z and τ correspond to propagation distance and retarded time, respectively. The constant κ stands for linear coupling, whereas cross-phase modulation is included through the parameter σ. The system (1) encompasses the particular case of NLDC,[19] where $\sigma=0$, as well as the NLBF problem $(\sigma\neq0)$.[9–11]

Here, we will be primarily concerned with the specific case $\sigma=0$, leaving the general case $\sigma\neq0$ for future work. We also notice that the particular case $\sigma=1$, $\kappa=0$ (no linear coupling) has been treated with the variational method by Anderson and Lisak[28] for the analysis of bandwidth limits in soliton-based communication systems.

III. THE VARIATIONAL MODEL

A. Lagrangian formulation

Our approximate analysis of soliton coupling involves a Rayleigh-Ritz optimization procedure. Since the details of the general method can be found elsewhere,[27] we will be brief and summarize the main steps.

First, it can easily be shown that the system of coupled NLSE's [Eqs. (1)] can be reformulated as a variational problem, namely,

$$\delta\int\int L\,dz\,d\tau=0\ , \qquad (2)$$

with the appropriate Lagrangian

$$L=L_1+L_2+L_{12}\ , \qquad (3)$$

where L_j $(j=1,2)$ corresponds to the single NLSE Lagrangian:[27]

$$L_j=\frac{i}{2}\left[\psi_j\frac{\partial\psi_j^*}{\partial z}-\psi_j^*\frac{\partial\psi_j}{\partial z}\right]-\frac{1}{2}\left|\frac{\partial\psi_j}{\partial\tau}\right|^2+\tfrac{1}{2}|\psi_j|^4\ , \qquad (4)$$

and L_{12} represents the interaction Lagrangian:

$$L_{12}=\kappa(\psi_1^*\psi_2+\psi_2^*\psi_1)+\sigma|\psi_1|^2|\psi_2|^2\ . \qquad (5)$$

The original system (1) is then derived from Euler-Lagrange equations:[27,33]

$$\frac{\delta L}{\delta\psi_j^*}=\frac{\partial}{\partial z}\frac{\partial L}{\partial(\partial\psi_j^*/\partial z)}+\frac{\partial}{\partial\tau}\frac{\partial L}{\partial(\partial\psi_j^*/\partial\tau)}-\frac{\partial L}{\partial\psi_j^*}=0\ . \qquad (6)$$

B. The trial functions

The Rayleigh-Ritz method is a constant-shape approximation. We have to choose an appropriate trial function describing the temporal form of the pulses. This is certainly the crucial step of the analysis. Accuracy and simplicity both depend on the ansatz and, therefore, a compromise has to be made between those two opposite requirements. For example, the need of accuracy would dictate an ansatz of the following form:

$$\psi_j(\tau,z)=A_j(z)f\left[\frac{\tau}{a_j(z)}\right]\exp[i\phi_j(\tau,z)]\ , \qquad (7)$$

with f being a Gaussian or a hyperbolic secant function, for example. The trial function (7) also includes the possibility of a chirp (time-varying phase ϕ_j) resulting from self-phase modulation.

The shortcoming of this—probably accurate—ansatz is that it leads (after using Euler-Lagrange equations) to a set of coupled nonlinear equations for the six real variables A_j, a_j, and ϕ_j, necessitating the recourse to numerical tools. Although less demanding in computer time than the original system (1), we are not really satisfied by this "achievement," as we aim to a simpler and analytical description. In other words, we prefer to sacrifice some (but not too much) accuracy in order to get a more appealing result in terms of physical insight.

In this paper we are mostly concerned by two important cases: switching and symmetry breaking. Then, as will be seen below, we can get a satisfying model by using the following simple trial function:

$$\psi_j(\tau,z)=F_j(z)\mathrm{sech}(\rho\tau)\exp[i\theta_j(z)]\ , \qquad (8)$$

where the real functions F_j and θ_j depend only on z variable, and ρ is constant. Obviously, the simplicity of this ansatz calls for further justification. First, the trend in all-optical communication systems is toward the use of fundamental soliton pulses as optical bits. Under ap-

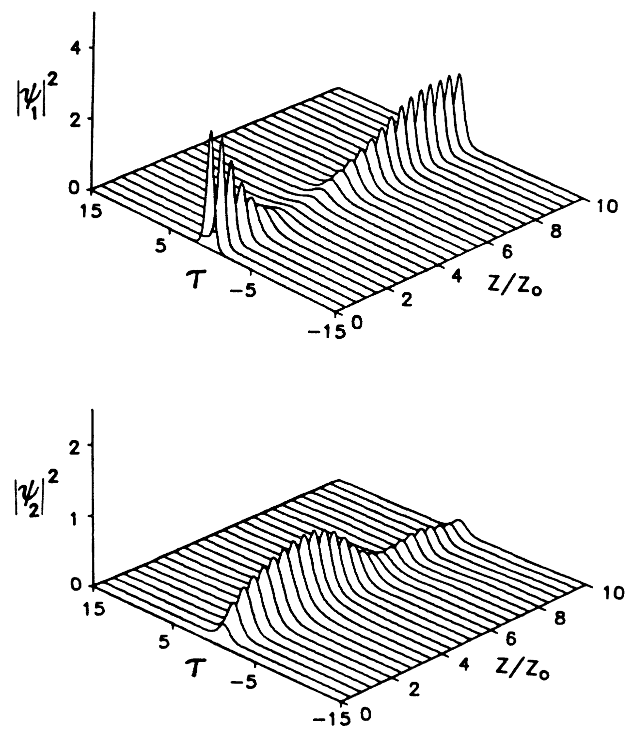

FIG. 1. Typical evolution of the intensity profiles $|\psi_j|^2$ in a NLDC ($\kappa=0.5$, $F_{01}=1.8$, $F_{02}=0$). The distance z is normalized to the "multisoliton period" $z_0=\pi/2$.

propriate launching conditions, the pulse shape $\rho\,\mathrm{sech}(\rho\tau)$ represents the asymptotic distribution inside a single fiber and would correspond to the input of a coupling device. Hereafter, we consider the interaction between input pulses corresponding to fundamental soliton of a single NLSE. We then use the terms "soliton" and "soliton coupling" in that sense, without meaning a soliton solution of the coupled system (1).

However, on the basis of quasi-cw arguments, the constant-shape approximation itself is apparently difficult to defend. Such arguments would refer to the cw [when $\psi_{\tau\tau}=0$ in Eq. (1)] results[3,4] and predict a pulse breakup[34] when time-varying inputs are considered. However, as will be seen in Sec. III (Fig. 1) (and noticed before[10,19,20]), when solitons are launched at the input, it appears that the pulses are essentially unperturbed, in shape, during the switching process and no pulse breakup occurs. Once again, solitons prove to be robust entities. An appealing explanation of that behavior has been given by Blow *et al.*[8] Basically a nonlinear coherent coupler is a phase-sensitive device. The use of pulses yields nonconstant phase distribution (chirp), as a result of self-phase modulation, and a pulse breakup can be anticipated. Fortunately, the soliton is a peculiar type of pulse as it is characterized by a constant-phase distribution. This comes from a perfect balance between dispersive and nonlinear effects. Then, as numerical simulations indicate, this inhibits the breakup phenomenon.

The proposed ansatz (7) implies that one also neglects the possible appearance of a chirp as a result of the interaction. A more sophisticated trial function would certainly improve our model but, again, this would be at the expense of its simplicity and we then disregard this "improvement."

Finally, we assume pulses of constant width. This is certainly a crude approximation for our system. However, it can be shown[35] that the system (1) (with $\sigma=1$) has an exact solution describing periodic soliton energy transfer with such a constant width (that exact solution has been a useful test for our computer code). In our case, the above solution does not remain valid. But, for the variational model, it appears to be a quite fruitful approximation to neglect the width variation.

In spite of all those shortcomings, the present model gives useful approximate analytical expressions for, among others, the transmission properties of the NLDC and the maximum energy transfer achievable. More generally, it provides a physically suggestive description of the complex interplay between the nonlinear index, the group velocity dispersion, and the linear coupling.

C. Euler-Lagrange equations

Using the ansatz (8) and the Lagrangian (3)–(6) (with $\sigma=0$), we can perform the τ integration in (2) and reduce the variational problem to a simpler form:

$$\delta\int\mathcal{L}\,dz=0\ , \tag{9}$$

where

$$\mathcal{L}=\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_{12}\ , \tag{10a}$$

$$\mathcal{L}_j=\frac{2}{\rho}F_j^2\frac{d\theta_j}{dz}-\frac{\rho}{3}F_j^2+\frac{2}{3\rho}F_j^4\ , \tag{10b}$$

and

$$\mathcal{L}_{12}=(4\kappa/\rho)F_1F_2\cos(\theta_1-\theta_2)\ . \tag{10c}$$

The Euler-Lagrange equations

$$\frac{\partial\mathcal{L}}{\partial y_j}-\frac{d}{dz}\frac{\partial\mathcal{L}}{\partial(dy_j/dz)}=0\ , \tag{11}$$

with $y_j=F_j$, θ_j ($j=1,2$), then lead to a set of four coupled nonlinear ODE's, namely,

$$\frac{d\theta_j}{dz}=\frac{\rho^2}{6}-\frac{2F_j^2}{3}-\kappa\frac{F_{3-j}}{F_j}\cos(\theta_1-\theta_2)\ , \tag{12}$$

$$\frac{dF_j}{dz}=(-1)^j\kappa F_{3-j}\sin(\theta_1-\theta_2)\ . \tag{13}$$

Equation (13) satisfies the law of energy conservation

$$F_1^2+F_2^2=E=\text{const}\ . \tag{14}$$

The system (12) and (13) is also consistent with the particular case of exact solitary-wave solution to Eq. (1). The reader can easily check that Eq. (1) admits the following exact solutions:[13]

$$\psi_j(\tau,z)=\eta_j\rho\,\mathrm{sech}(\rho\tau)\exp[i\beta(z)]\ , \tag{15}$$

with $\eta_j=\pm1$ and

$$\beta(z)=(-\rho^2/2-\eta_2\kappa/\eta_1)z\ . \tag{16}$$

This corresponds to $\theta_1-\theta_2=0$ or $\pm\pi$ and $F_j=\rho$ in (12) and (13). To proceed, we define a new phase variable

$$\phi(z)=\theta_1(z)-\theta_2(z)\ , \tag{17}$$

and introduce another variable for the energy difference

$$U(z)=F_1^2(z)-F_2^2(z)\ . \tag{18}$$

Using Eq. (14), the system (12) and (13) can then be reduced to only two coupled equations:

$$\frac{d\phi}{dz}=-\frac{2U}{3}+\frac{2\kappa U}{(E^2-U^2)^{1/2}}\cos\phi\ , \tag{19}$$

$$\frac{dU}{dz}=-2\kappa(E^2-U^2)^{1/2}\sin\phi\ . \tag{20}$$

D. Analytic solution

We find that this new system has another constant of motion, namely,

$$G=\tfrac{1}{3}U^2(z)+2\kappa(E^2-U^2)^{1/2}\cos\phi(z)\ . \tag{21}$$

This allows us to derive a single equation for U:

$$\frac{dU}{dz}=\pm[4\kappa^2(E^2-U^2)-(U^2/3-G)^2]^{1/2}\ , \tag{22}$$

which can be solved exactly in terms of elliptic functions.[36] Here, we limit ourselves to the specific, but important, case of switching. The initial conditions then read $F_1(0)=F_0=\sqrt{E}$ and $F_2(0)=0$. If we define the parameter γ, characterizing the importance of the nonlinearity, as

$$\gamma=E/(6\kappa) , \tag{23}$$

then, for $\gamma \le 1$,

$$u(z)=\mathrm{cn}(2\kappa z|\gamma^2) , \tag{24a}$$

and, for $\gamma \ge 1$,

$$u(z)=[1-(1/\gamma^2)\mathrm{sn}^2(2\gamma\kappa z|1/\gamma^2)]^{1/2} , \tag{24b}$$

where $u(z)=U(z)/E$ and sn and cn are Jacobian elliptic functions.[36]

In the limit of low intensities ($\gamma \ll 1$), we recover, from (24a), the exact solution of the linear case, describing the sinusoidal variation of energy in each channel:

$$U(z)=\cos(2\kappa z) . \tag{25}$$

Increasing the power does not affect the periodic nature of the interaction (periodicity of elliptic functions), but modifies the period. This will be discussed in more detail in Sec. IV.

E. Particlelike description

The essential information concerning the switching process is contained in Eqs. (23) and (24). However, as was done by Anderson for the single NLSE,[27] we find it worthwhile to recast the general results into a suggestive potential formulation. Then the evolution of the energy distribution in each guide can be associated to the time-varying position of a particle in an equivalent potential well. Such an analogy is particularly appealing as it gives a much better physical insight of the dynamics.

From Eq. (22), we then get, for arbitrary initial conditions,

$$\frac{d^2u}{dt^2}=-\frac{\partial V(u)}{\partial u} , \tag{26}$$

where we have introduced the "time" variable

$$t=(2)^{1/2}\kappa z , \tag{27}$$

and the anharmonic potential

$$V(u)=[1-2\gamma^2u_0^2-2\gamma(1-u_0^2)^{1/2}\cos\phi_0]u^2+\gamma^2u^4 , \tag{28}$$

where $u_0=u(z=0)$ and $\phi_0=\phi(z=0)$.

In the next section, it will be shown how this potential description can be useful for interpreting, and also for predicting, the main features of the soliton dynamics.

IV. DISCUSSION OF RESULTS

In order to assess the ability of our simplified model to describe the essential characteristics of the soliton coupling, we have also made a series of computer simulations. The original system [Eq. (1) with $\sigma=0$] has been solved numerically by adapting the well-known standard beam propagation method in order to include the linear coupling. The vast parameter space associated with Eq. (1) can be reduced significantly through a rescaling of the equation. Defining $z'=\kappa z$ and $\tau'=\sqrt{\kappa}\tau$, it can be shown that for input pulses with a width inversely proportional to the amplitude (e.g., solitons), then the important parameters are γ [Eq. (23)], the ratio of the input amplitudes $\psi_2(0,0)/\psi_1(0,0)$, and the initial phase difference ϕ_0.

A. Switching case

Since we were mostly interested in the switching problem, we used the fundamental soliton as input pulse:

$$\begin{aligned}\psi_1(\tau,z=0)&=F_0\mathrm{sech}(F_0t) ,\\ \psi_2(\tau,z=0)&=0 .\end{aligned} \tag{29}$$

Figure 1 illustrates a typical result indicating, as mentioned above, that the pulses do not suffer serious distortion of their shape during the process. This observation is the basis of our model. In the context of NLDC, a practical device consists, for example, in a half-beat length coupler, for which total energy transfer (from channel 1 to channel 2) occurs in the linear regime. This is when $u(z=L_c)=-1$, i.e., [from (25)] at the exit of a coupler of length L_c given by

$$L_c=\pi/(2\kappa) . \tag{30}$$

The nonlinear device is interesting for all-optical communications because an increase of the nonlinearity will change the effective refractive index and then detune the system. The transmission is reduced and can eventually be nearly zero ("switched off").

From Eq. (24), a simple analytical expression can be obtained for the peak-power dependence of the transmission characteristics:

$$T=\begin{cases}[1-\mathrm{cn}(\pi|\gamma^2)]/2, & \gamma\le 1\\ \{1-[1-(1/\gamma^2)\mathrm{sn}^2(\gamma\pi|1/\gamma^2)]^{1/2}\}/2, & \gamma>1\end{cases} \tag{31}$$

where $T=F_2^2(L_c)/F_0^2=[1-u(L_c)]/2$. This function is drawn in Fig. 2 (solid line). One can notice the similarity

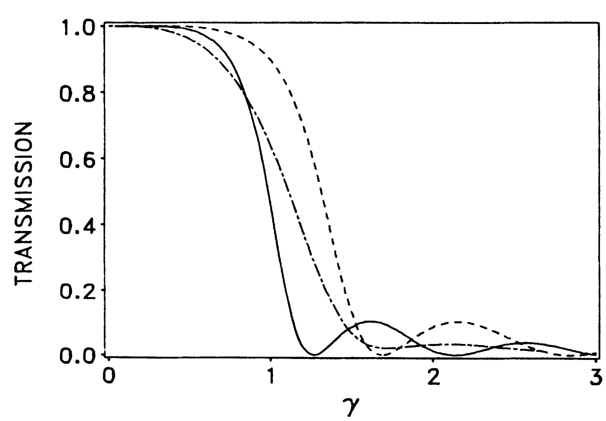

FIG. 2. Energy transmission characteristics of a half-beat length coupler. The curves represent the exact numerical results (—·—·—·), the present model (——), and the soliton-phase model (— — —).

of the cw results[3,4] and ours. The essential difference is for the switching peak-power value (the value for which $\gamma=1$). According to the cw results, this value should be $P_{sw}=4\kappa$, whereas we predict [Eq. (23)] $P_{sw}=F_0^2=6\kappa$. This correction gives a much better estimate if we compare with the exact numerical results shown by the dashed-dotted line in Fig. 2. If we loosely define the switching peak power as the one for which 50% transmission is achieved [this corresponds to $\gamma=1$ in Eq. (31)], then from the numerics we infer $P_{sw}\approx 6.7\kappa$. This represents an interesting result for the model.

Following the soliton-phase argument of Blow *et al.*,[8] another model (hereafter called "soliton-phase model"), can be built. In the absence of coupling ($\kappa=0$), in contrast to the cw case for which self-phase modulation leads to a nonlinear phase shift given by $\theta(z)=F_0^2 z$ [Eq. (1) with $\psi_{\tau\tau}=0$, $\sigma=\kappa=0$], a fundamental soliton will rather be phase shifted by $\theta(z)=\frac{1}{2}F_0^2 z$ [Eq. (16) with $\kappa=0$]. Then, in a first approximation (see Ref. 8 for more details), one can assume that the cw results will remain valid as long as one defines an effective intensity $F_{0\text{eff}}^2=\frac{1}{2}F_0^2$, so that a rough estimate of the switching peak power would be $P_{sw}=8\kappa$. For completeness, we also present the transmission curve predicted by this other model (dashed line in Fig. 2).

We will not insist on a detailed comparison of the two approximate models. Each of them has its own advantages, as well as inconveniences. We simply want to stress the fact that in our model we have not included any adjustable parameter which could be introduced in order to get a better fit. The main advantage of the present method is that the ingredients are there for improvement (in terms of accuracy): by choosing a different trial function and following the lines given in Sec. II.

The essential features of the dynamics are well described by the present version. It comes out that, as in the cw regime, there is a sharp transition from high to low transmission as one goes from $\gamma<1$ to $\gamma>1$. This is not so true when the input pulse is not a soliton.[5]

The approximate formula (31) is useful, but the potential formulation appears more fruitful if the objective is to improve our physical insight. For the case of switching, the equivalent particle is subject to the following initial conditions: $u(t=0)=+1$, $du/dt|_{u=0}=0$, and the potential $V(u)$ is such that [see Eq. (28)] $V(u=0)=0$, $dV/du|_{u=0}=0$, $V(\pm1)=(1-\gamma^2)$, and $dV/du|_{u=\pm1}=\pm2$. Thus the particle is initially at rest at $u=+1$, the slope of the potential, there, is independent of γ, and the effect of increasing the input power is to reduce the initial height of the particle inside the well (Fig. 3). Complete energy transfer occurs when the particle reaches the position $u=-1$. The essential features of the dynamics can then be anticipated from the physical intuition provided by classical mechanics.

The qualitative aspect of the potential (and then the dynamics) is drastically different as we increase the nonlinearity from $\gamma<1$ to $\gamma>1$, as depicted in Fig. 3. At low input power [$\gamma\ll1$, Fig. 3(a)], the potential is quasiharmonic; a complete and periodic energy transfer is then predicted [in agreement with Eq. (25)]. If we increase the nonlinearity [e.g., $\gamma^2=\frac{1}{2}$, Fig. 3(b)], the anhar-

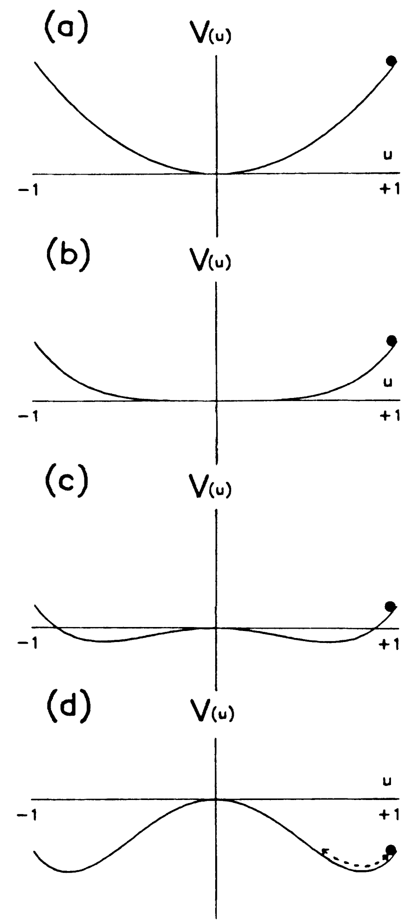

FIG. 3. Transformation of the equivalent potential as the nonlinearity is increased: (a) $\gamma^2\ll1$; (b) $\gamma^2=0.5$; (c) $0.5<\gamma^2<1$; (d) $\gamma^2>1$. The particle (●) is initially at rest at $u=+1$.

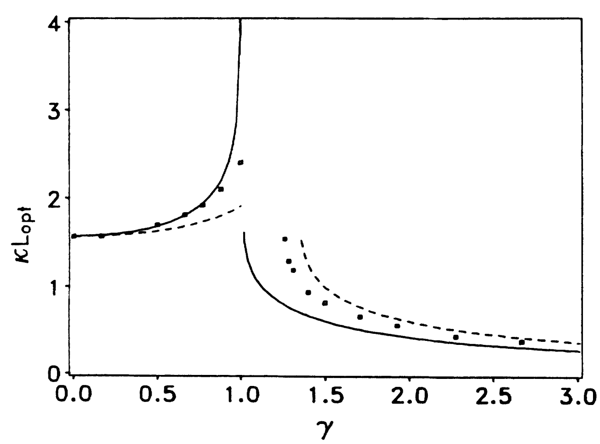

FIG. 4. Influence of the parameter γ on the optimum coupling length. The squares (□) represent exact numerical results and the curves correspond to the predictions of the present model (——) and the soliton-phase model (— — —).

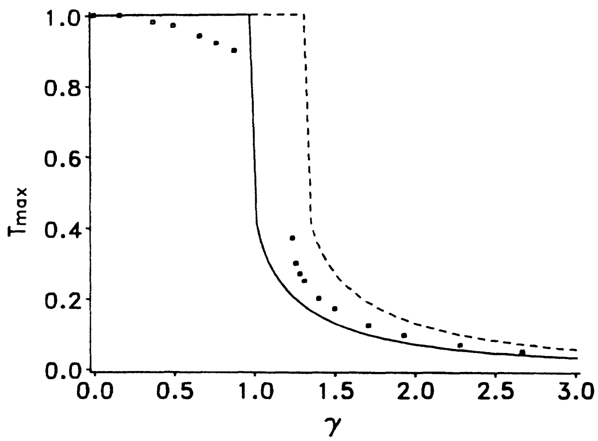

FIG. 5. Maximum energy transfer achievable as a function of γ. The curves are identified as in Fig. 4.

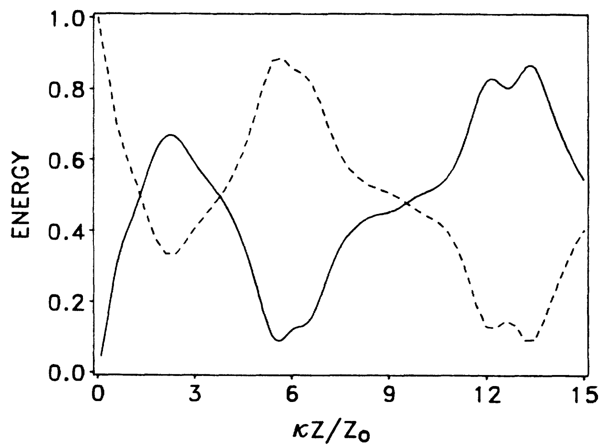

FIG. 7. Same as Fig. 6 but $\gamma = 1.17$.

monicity becomes more important and this flattens the potential well. Moreover, the initial height is lower. The particle will then take a longer time to reach the position $u = -1$, which means that the period is increased. For a half-beat length coupler (which corresponds to a fixed time interval), this implies a decrease in transmission.

The sharp transition in the transmission curve, at $\gamma \approx 1$, can also be easily interpreted. For $\gamma > 1$ [Fig. 3(d)], $V(+1) < V(0)$ and, from energy conservation, the particle will not be able to cross the origin and will remain on the right side. Complete energy transfer is impossible. From Eq. (24), an estimate can be given for the maximum energy transfer achievable (T_{max}) and the corresponding optimum coupling length (L_{opt}):

$$T_{max} = \begin{cases} 1, & \gamma < 1 \\ [1-(1-1/\gamma^2)^{1/2}]/2, & \gamma \geq 1 \end{cases} \tag{32a}$$

$$L_{opt} = \begin{cases} (1/\kappa)K(\gamma^2), & \gamma < 1 \\ [1/(2\kappa\gamma)]K(1/\gamma^2), & \gamma \geq 1 \end{cases} \tag{32b}$$

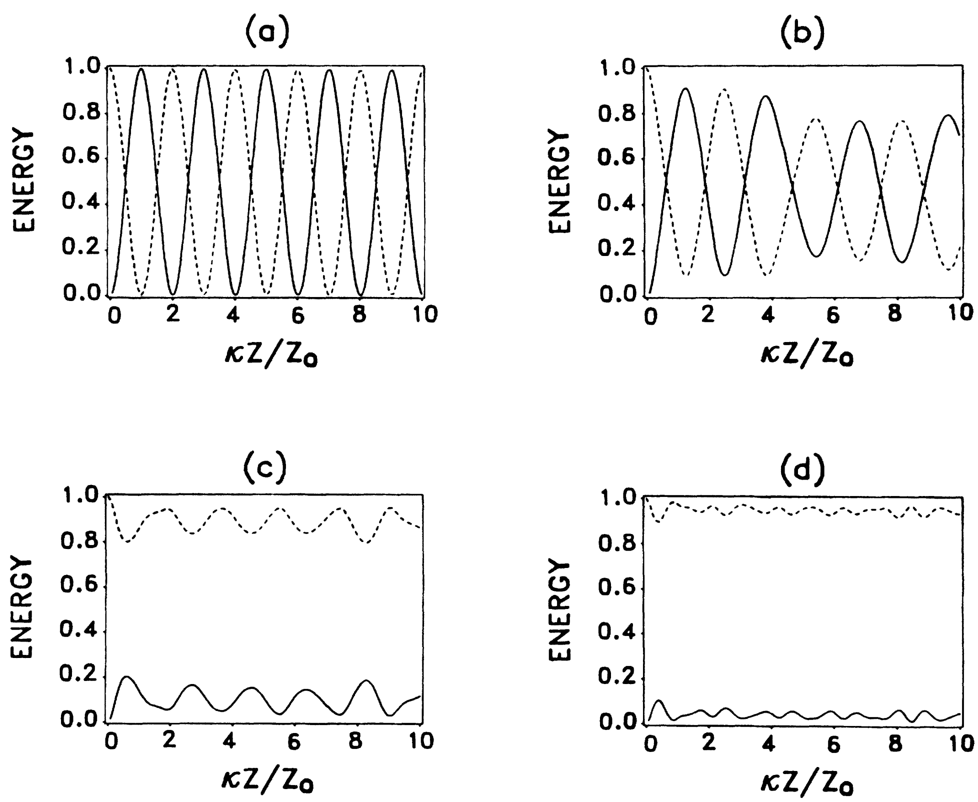

FIG. 6. Numerical results showing the evolution of the normalized energy of the input pulse ψ_1 (— — —) and the transmitted pulse ψ_2 (——): (a) $\gamma = 0.17$; (b) $\gamma = 0.80$; (c) $\gamma = 1.40$; (d) $\gamma = 1.82$.

where K is the complete elliptic integral of the first kind.[36] Figures 4 and 5 indicate that for $\gamma < 1$ and $\gamma \gtrsim 1.3$ the simple model gives a reasonably good account of the intensity dependence of $T_{\max}$ and L_{opt}. As for the transmission curve (Fig. 2), the numerical results lie between the predictions of the two models.

The results of a few numerical simulations are also presented in Fig. 6. For $\gamma < 1$ the optimum length is greater when the amplitude of the input pulse F_0 is increased [Figs. 4, 6(a), and 6(b)]. This comes from the flattening of the potential discussed above [Fig. 3(b)]. The opposite occurs when $\gamma \gtrsim 1.3$. Figures 6(c) and 6(d) show the energy exchange is not really periodic but remains oscillatory and it is correct to predict a decrease of $T_{\max}$ in that region (Fig. 5) and shorter optimum lengths as well (Fig. 4). This is associated with the more concave potential well depicted in Fig. 3(d).

As an approximate rule of thumb, the model fails when it predicts an optimum length which is too large for the restrictive basic assumptions of the present trial function (Sec. II A) to remain valid. This is the case near $\gamma \approx 1$ (or, more generally, near the switching peak power). Then, reality turns out to be more complex than expected, as seen in Fig. 7. A general ansatz of the form (7) would be more appropriate for that region if we are interested in propagation distances longer than the half-beat length of the NLDC. As mentioned in Sec. II, this would require the use of numerical techniques. Clearly, a detailed investigation of the complex behavior in the region $1 < \gamma \lesssim 1.3$ is beyond the scope of the present analysis.

B. Symmetry-breaking instability

The particlelike description is interesting not only for interpreting results, but also for predicting special behaviors. Here, we give an example by looking at the stability of the solitary-wave solution [Eqs. (15) and (16)]. For that case, $F_2(0)=F_1(0)=F_0$, $u(0)=0$, $du/dt(t=0)=0$,

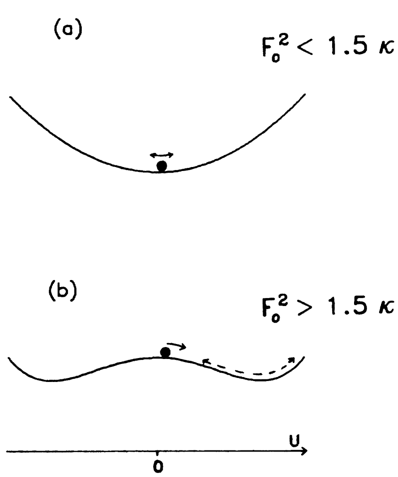

FIG. 8. Transformation of the shape of the potential leading to a symmetry-breaking instability. The equivalent particle (●) is initially at rest at $u=0$ but becomes unstable when $F_0^2 > 1.5\kappa$ ($\gamma > 0.5$).

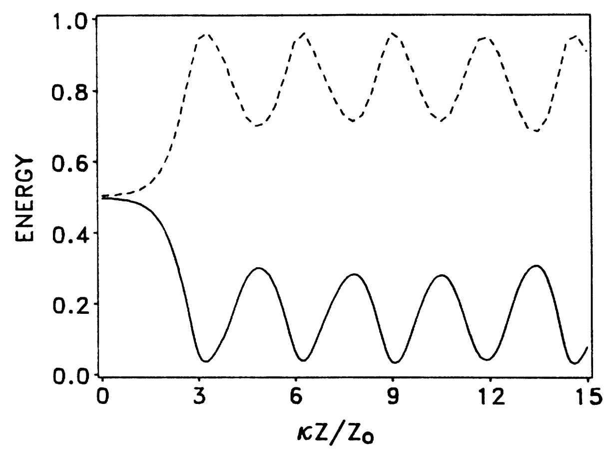

FIG. 9. Quasiperiodic energy exchange following the symmetry-breaking instability ($\gamma=0.61$). The dashed (solid) line represents the normalized energy of the pulse ψ_1 (ψ_2).

and the potential is given by [Eq. (28)]

$$V(u)=(1-2\gamma\cos\phi_0)u^2+\gamma^2u^4\ , \tag{33}$$

with

$$\gamma=E/(6\kappa)=F_0^2/(3\kappa)\ , \tag{34}$$

and $\phi_0=0$ or π.

At low intensities ($\gamma \ll 1$), the equivalent particle is initially at rest, at the bottom of the quasiharmonic potential well [Fig. 8(a)]. From the shape of the potential, we expect a stable system. However, in the case $\phi_0=0$, there is a topological transformation of the potential occurring at $\gamma > \frac{1}{2}$ [Fig. 8(b)], and a symmetry-breaking instability can easily be anticipated. A small initial fluctuation in power corresponds to a slight displacement of the particle from the origin and the particle will accelerate down the bump. A periodic oscillation will follow, and the particle remains on one side. In terms of energy repartition, this implies a periodic, but unequal, energy sharing be-

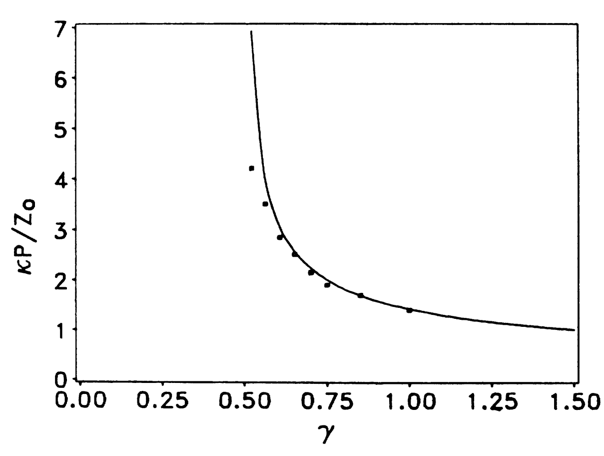

FIG. 10. Period of the oscillations following the instability. The squares refer to numerical results and the solid curve corresponds to the predictions of the model [Eq. (37)].

tween the pulses. This is well confirmed by the computer simulations, as shown in Fig. 9.

We must point out that, independently, this instability was also predicted by Wright *et al.*, in a recent paper.[13] These authors used the linear stability analysis (LSA). Quantitatively, both predictions are similar. From Eq. (34) and the condition mentioned above ($\gamma > \frac{1}{2}$), we predict instability when

$$F_0^2 > 1.5\kappa \ , \tag{35}$$

whereas linear stability analysis leads to[13]

$$F_0^2 > 1.33\kappa \ . \tag{36}$$

Let us also mention that the soliton-phase model would predict $F_0^2 \geq 2\kappa$, which represents a too high estimate.

An advantage of the potential approach is that it can predict the long-time behavior (the unequal and quasiperiodic energy transfer) whereas the LSA implies a weak perturbation and can then hardly predict what happens once the instability has settled. Above threshold ($\gamma > \frac{1}{2}$), a simple expression for the oscillation period P can be obtained by approximating the new local minimum of the potential [Fig. 8(b)] as a parabola. From (33) and after straightforward steps, we get

$$P \approx (\gamma - \tfrac{1}{2})^{-1/2}\pi/(2\kappa) \ . \tag{37}$$

This formula turns out to be quite good, as illustrated in Fig. 10. For $\gamma > 1$, the behavior is still oscillatory but more complex and subharmonics appear. The detailed scenario is not a simple period-doubling process and will not be discussed here in further detail. For γ close to 0.5, the parabolic approximation is not valid and, then, the agreement is less good. In addition, our estimate of the threshold as $\gamma = 0.5$ is only approximative.

For $\phi_0 = \pi$, the potential remains concave and our model predicts stability against unequal initial intensities. We cannot address the question of stability against desynchronization (i.e., when the pulses are slightly delayed) on the basis of the present model. But this has been done elsewhere[21] by using perturbative methods and the conclusions are that the system is then unstable against this type of perturbation. Those remarks are also in agreement with the results of Wright *et al.*[13]

V. CONCLUSIONS

Admittedly, the model of the dynamics of soliton coupling presented in this paper is simplified. But the first objective was to improve our physical insight. Then, by insisting on preserving the simplicity at the expense of some quantitative accuracy, we have been able to get analytical results. Although approximate, the results are satisfying and the analogy with classical mechanics is particularly enlightening.

If more precision is desired, then the recipe consists of choosing a more sophisticated trial function and following the lines given here. This may be necessary when long propagation distances are considered or for different initial conditions. Numerical simulations are helpful when the time comes to choose an appropriate ansatz for describing the evolution of the pulses. Conversely, an approximate model can be of great help by locating the regions of the vast parameter space which could be of particular interest and deserve further examination via numerical simulations. This combination of both approaches could reduce significantly the otherwise prohibitive computer time necessary.

In order to check the validity of the model, we have presented the results in the particular context of nonlinear fiber optics. However, as mentioned at the beginning, the system of coupled NLSE's is encountered in other areas of nonlinear physics. The present approach is then of general interest.

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[1]For introduction, see G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, Boston, 1989).
[2]L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[3]S. M. Jensen, IEEE Trans. Microwave Theory Tech. **MTT-30**, 1568 (1982).
[4]A. A. Maier, Kvant. Elektron. (Moscow) **9**, 2296 (1982) [Sov. J. Quantum Electron. **12**, 1490 (1982)].
[5]D. D. Gusovskii, E. M. Dianov, A. A. Maier, V. B. Nestruev, V. V. Osiko, A. M. Prokhorov, K. Yu. Sitarskii, and I. A. Shcherbakov, Kvant. Elektron. (Moscow) **14**, 1144 (1987) [Sov. J. Quantum Electron. **17**, 724 (1987)].
[6]S. R. Friberg, A. M. Weiner, Y. Silberberg, B. G. Sfez, and P. S. Smith, Opt. Lett. **13**, 904 (1988).
[7]R. Jin, C. L. Chuang, H. M. Gibbs, S. W. Koch, J. N. Polky, and G. A. Pubanz, Appl. Phys. Lett. **53**, 1791 (1988).
[8]K. J. Blow, N. J. Doran, and B. K. Nayar, Opt. Lett. **14**, 754 (1989).
[9]C. R. Menyuk, IEEE J. Quantum Electron. **QE-23**, 174 (1987).
[10]K. J. Blow, N. J. Doran, and D. Wood, Opt. Lett. **12**, 202 (1987).
[11]S. Trillo, S. Wabnitz, E. M. Wright, and G. I. Stegeman, Opt. Commun. **70**, 166 (1989).
[12]R. K. Dodd, J. C. Eilbeck, J. D. Gibbon, and H. C. Morris, *Solitons and Nonlinear Wave Equations* (Academic, New York, 1982).
[13]E. M. Wright, G. I. Stegeman, and S. Wabnitz, Phys. Rev. A **40**, 4455 (1989).
[14]S. Trillo, S. Wabnitz, E. M. Wright, and G. I. Stegeman, Opt. Lett. **13**, 871 (1988).
[15]M. V. Tratnik and J. Sipe, Phys. Rev. A **38**, 2011 (1988).
[16]Y. Inoue, J. Plasma Phys. **16**, 439 (1976).
[17]D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 53 (1988); D. N. Christodoulides, Phys. Lett. A **132**, 451 (1988).

[18]M. Florjańczyk and R. Tremblay, Phys. Lett. A **141**, 34 (1989).
[19]S. Trillo, S. Wabnitz, E. M. Wright, and G. I. Stegeman, Opt. Lett. **13**, 672 (1988).
[20]C. R. Menyuk, J. Opt. Soc. Am. B **5**, 392 (1988).
[21]F. Kh. Abdullayev, R. M. Abrarov, and S. A. Darmanyan, Opt. Lett. **14**, 131 (1989).
[22]V. V. Afanasyev, Yu. S. Kivshar, V. V. Konotop, and V. N. Serkin, Opt. Lett. **14**, 805 (1989).
[23]K. J. Blow, N. J. Doran, and D. Wood, Opt. Soc. Am. B **6**, 1301 (1988).
[24]F. H. Berkshire and J. D. Gibbon, Stud. Appl. Math. **69**, 229 (1983).
[25]E. Caglioti, B.Crosigniani, and P. Di Porto, Phys. Rev. A **38**, 4036 (1988).
[26]A. Kumar and M. S. Sodha, Electron. Lett. **23**, 275 (1987).
[27]D. Anderson, Phys. Rev. A **27**, 3135 (1983).
[28]D. Anderson and M. Lisak, Phys. Rev. A **32**, 2270 (1985).
[29]D. Anderson, M. Lisak, and T. Reichel, Phys. Rev. A **38**, 1618 (1988).
[30]A. B. Aceves, J. V. Moloney, and A. C. Newell, Phys. Rev. A **39**, 1809 (1989).
[31]B. Crosigniani and A. Yariv, J. Opt. Soc. Am. B **5**, 507 (1988).
[32]B. Crosigniani, A. Cutolo, and P. Di Porto, J. Opt. Soc. Am. **72**, 1136 (1982).
[33]S. Trillo and S. Wabnitz, J. Opt. Soc. Am. B **6**, 238 (1989).
[34]A. A. Maier, Yu. N. Serdyuchenko, K. Yu. Sitarskii, M. Ya. Shchelev, and I. A. Shcherbakov, Kvant. Elektron. (Moscow) **14**, 1157 (1987) [Sov. J. Quantum Electron. **17**, 735 (1987)].
[35]P. A. Bélanger and C. Paré, Phys. Rev. A **41**, 5254 (1990).
[36]*Table of Integrals, Series and Products,* edited by I. S. Gradstein and I. M. Ryzhik (Academic, New York, 1980); *Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965).