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Sub-Poissonian statistics and squeezing in fluorescence from N atoms in a cavity

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The sub-Poissonian statistics and squeezing in the fluorescence field from N two-level atoms in a cavity are considered for the case for which the cavity mode is so intense that it can be treated as a classical field. It is shown that sub-Poissonian statistics of sidebands are present for a large number of atoms. Moreover, the sidebands become more sub-Poissonian with increasing number of atoms. The condition for obtaining a large degree of squeezing in the sidebands is discussed.

I. INTRODUCTION

The possibility of constructing high-Q cavities both in the optical and microwave regions has inaugurated the field of cavity quantum electrodynamics. The Jaynes-Cummings[1] and Tavis-Cummings models[2] of one and many two-level atoms interacting with a quantized cavity field mode have become experimentally realizable.[3–5] A number of interesting effects such as enhancement and suppression of spontaneous emission,[6–8] vacuum-field Rabi oscillations,[9,10] and the collapse and revival of Rabi oscillations[5] have been observed experimentally. In recent years a large number of publications have concentrated on nonclassical effects such as squeezing[11–13] and sub-Poissonian statistics.[14–20]

In this paper we discuss sub-Poissonian photon statistics and squeezing in the spectral components of fluoresence from N atoms in a cavity. By this we mean that the atoms are strongly excited by a single field mode. The subsequent atomic dipole and inversion evolution may be calculated directly. We then suppose that the atoms can in fact relax through the intervention of weak damping. This damping is considered to have a negligible effect on the atomic dynamics. However, the dipole correlation functions obtained by solving the undamped evolution are, to good approximation, responsible for the properties of the decay spectrum. In this, we are following the approach proposed by Eberly and co-workers[21] and Agarwal.[22] The purpose of this paper is to study the influence of the collective effects on the nonclassical properties of the fluorescence field in other modes. It is shown that for a suitable range of the system parameters the two sidebands have sub-Poissonian photon statistics even for the case of a large number of atoms. Moreover, when the number of atoms is increased the sidebands become more sub-Poissonian. We also consider the squeezing of the spectral components and show that in contrast with the collective resonance fluoresence in free space, where squeezing is absent in separate spectral components,[23] for the appropriate parameters of the system the squeezing is present only in one distinct sideband. The conditions for generating a large degree of squeezing are discussed.

II. SUB-POISSONIAN PHOTON STATISTICS

We consider the photon statistics of the component of the fluorescence spectrum from N two-level atoms interacting with a single cavity mode. The atoms are assumed to be placed in a cell of dimensions smaller than the wavelength of the field. This model is known to describe the interaction of Rydberg atoms in a high-Q microwave cavity.[3–5] The Hamiltonian of the system in the rotating-wave approximation and in the interaction representation has the following form:[2,24]

$$H=\frac{\delta}{2}(J_{22}-J_{11})+g(J_{12}b^{\dagger}+J_{21}b)\ , \tag{2.1}$$

where $\delta=\omega_{21}-\omega_L$ is the detuning of the cavity mode frequency ω_L from the atomic resonance frequency ω_{21}; g is the coupling constant; b and $b^{\dagger}$ are the annihilation and creation operators for the field mode obeying the commutation relation $[b,b^{\dagger}]=1$; J_{ij} $(i,j=1,2)$ are the collective operators for the atomic system which have the following form in the Schwinger representation:[25]

$$J_{ij}=a_i^{\dagger}a_j\quad (i,j=1,2)\ , \tag{2.2}$$

where the operators a_i and $a_i^{\dagger}$ obey the boson commutation relations

$$[a_i,a_j^{\dagger}]=\delta_{ij}$$

and can be regarded as the annihilation and creation operators for the atoms in the level $|i\rangle$.

We imagine that the field mode is sufficiently intense that the number of photons of the field mode is much larger than the number of atoms. Then the field mode can be considered as classical field,[1,21] i.e., one can write

$$b=(n_f)^{1/2}e^{-i\theta},\quad b^{\dagger}=(n_f)^{1/2}e^{i\theta}\ .$$

In treating the field mode as a classical field, one loses many interesting effects connected with the discrete nature of the quantum field such as vacuum-field Rabi oscillations, and collapse and revival phenomena. However, as was shown in previous papers in the case when the number of photons of the quantum cavity mode is large, the spectral properties[24] and photon statistics[26] of the fluorescence field from one atom are in agreement with the case when the field mode is treated classically.

Treating the field mode classically enables us to solve directly for the atomic evolution. We can then describe the quantitative properties of the fluorescence field and especially the dependence of the collective radiation of the number of atoms.

By using the unitary transformation

$$U=\exp\left[-\frac{i}{2}\theta(a_2^\dagger a_2-a_1^\dagger a_1)\right],$$

the Hamiltonian (2.1) reduces to

$$H=\frac{\delta}{2}(a_2^\dagger a_2-a_1^\dagger a_1)+G(a_1a_2^\dagger+a_2^\dagger a_1), \tag{2.3}$$

where $G=g(n_f)^{1/2}$ is one-half of the resonant one-atom Rabi frequency describing the interaction of the intense cavity mode with the atomic system.

After performing the canonical (dressing) transformation

$$\begin{aligned} a_1&=c_1\cos\varphi+c_2\sin\varphi, \\ a_2&=-c_1\sin\varphi+c_2\cos\varphi, \end{aligned} \tag{2.4}$$

where

$$\tan 2\varphi=2G/\delta$$

one finds the Hamiltonian (2.4) in the simple form

$$H=\Omega(c_2^\dagger c_2-c_1^\dagger c_1), \tag{2.5}$$

where

$$\Omega=(\tfrac{1}{4}\delta^2+G^2)^{1/2}.$$

One can find the exact form for the operators $c_1(t)$, $c_2(t)$ very easily using Eq. (2.5). Using the canonical transformation (2.5) and the Schwinger definition Eq. (2.2), the collective atomic operators $J_{ij}(t)$ can be written in the following form:

$$J_{21}(t)=S_1^\dagger e^{2i\Omega t}+S_{-1}^\dagger e^{-2i\Omega t}+S_0, \tag{2.6}$$

$$J_{12}(t)=[J_{21}(t)]^\dagger, \tag{2.7}$$

where

$$S_0=\sin\varphi\cos\varphi[J_s\cos 2\varphi+(J_{12}+J_{21})\sin 2\varphi]=S_0^\dagger, \tag{2.8}$$

$$S_1^\dagger=\cos^2\varphi(-\tfrac{1}{2}J_3\sin 2\varphi+J_{21}\cos^2\varphi-J_{12}\sin^2\varphi), \tag{2.9}$$

$$S_{-1}^\dagger=\sin^2\varphi(\tfrac{1}{2}J_3\sin 2\varphi+J_{21}\sin^2\varphi-J_{12}\cos^2\varphi), \tag{2.10}$$

$$J_3=J_{22}-J_{11}. \tag{2.11}$$

In the following we shall investigate the photon statistics and squeezing of the fluoresence field in other modes. Such a fluoresence can be observed in a direction perpendicular to the cavity axis and its spectrum will be[21,22,27,28]

$$\begin{aligned} S_F(\nu,T)=2\Gamma\int_0^T dt_1\int_0^T dt_2 \exp[&-(\Gamma-i\nu)(T-t_1) \\ &-(\Gamma+i\nu)(T-t_2)] \\ &\times\langle J_{21}(t_1)J_{12}(t_2)\rangle \\ &\times\exp[-i\omega_L(t_2-t_1)], \end{aligned} \tag{2.12}$$

where Γ is the bandwidth of the detector, T is the time at which the spectrum is evaluated, and $\langle\ \rangle$ stands for the average over the initial state of the atomic system. The assumption of the presence of a fluoresence field in other modes means that a cavity relaxation is present. However, we will also assume the Q factor of the cavity to be high enough so that the influence of the cavity losses on the system is negligible due to the short duration of the spectral observations.[21,22,27,24]

By using the definition of fluoresence spectrum (2.12) and Eqs. (2.6) and (2.7) one can show that for the case of the intense cavity mode, the terms in the fluoresence spectrum (2.12) which are proportional to Ω^{-1} can be ignored (secular approximation) and the operators $S_1^\dagger$, $S_0^\dagger$, and $S_{-1}^\dagger$ can be considered as the source operators of the spectrum components of the fluoresence field at frequencies $\omega_L+2\Omega$, ω_L, and $\Omega_L-2\Omega$, respectively. The linewidths of all the spectral components S_0, $S_{\pm 1}$ are equal to 2Γ.

Further we shall discuss the photon statistics of spectral components S_0, $S_{\pm 1}$. For this purpose we consider the relative intensity fluctuations of the spectral components S_i ($i=0,\pm 1$):

$$Q_i=\frac{\langle S_i^{\dagger 2}S_i^2\rangle-\langle S_i^\dagger S_i\rangle^2}{\langle S_i^\dagger S_i\rangle}. \tag{2.13}$$

The factors Q_i for the atomic operator S_i are equal (up to constant terms) to the Mandel Q parameter[14] and describe the photon statistics of the spectal component S_i. The necessary condition for sub-Poissonian photon statistics to be generated in the spectral component S_i ($i=0,\pm 1$) is given by[14–18]

$$Q_i<0.$$

We take the atoms to be initially prepared in the state

$$\psi_\alpha=|n_1,n_2\rangle \tag{2.14}$$

where

$$J_{22}|n_1,n_2\rangle=n_2|n_1,n_2\rangle;\quad J_{11}|n_1,n_2\rangle=n_1|n_1,n_2\rangle.$$

Using the expression (2.8) for the operator S_0, and the atomic initial state (2.14) one can show that the central component has super-Poissonian photon statistics. Unlike the central spectral component S_0, the sideband components $S_{\pm 1}$ have sub-Poissonian photon statistics for a suitable choice of the parameters of the system. The implicit expressions for $Q_{\pm 1}$ can be found using Eqs. (2.9), (2.10), and (2.14):

$$Q_{-1}=\frac{\sin^4\varphi(Y_1+Y_2+Y_3)}{[\frac{1}{4}(\sin^2 2\varphi)D^2+(\sin^4\varphi)B_{+}+(\cos^4\varphi)B_{-}]}\ , \tag{2.15}$$

where

$$Y_1=\tfrac{1}{16}\sin^4 2\varphi(B_{+}^2+B_{-}^2-2D^2B)\ , \tag{2.16}$$

$$Y_2=\sin^8\varphi(DB_{+}-2B_{+})-\cos^8\varphi(DB_{-}+2B_{-})\ , \tag{2.17}$$

$$\begin{aligned}Y_3=&\sin^2 2\varphi\sin^4\varphi(\tfrac{1}{2}D^2B_{+}-2DB_{+}+B_{+})\\&+\sin^2 2\varphi\cos^4\varphi(\tfrac{1}{2}D^2B_{-}+2DB_{-}+B_{-})\ ,\end{aligned} \tag{2.18}$$

$$D=\langle J_3\rangle=n_2-n_1\ , \tag{2.19}$$

$$B_{+}=\langle J_{21}J_{12}\rangle=\tfrac{1}{4}(N^2-D^2+2N+2D)\ , \tag{2.20}$$

$$B_{-}=\langle J_{12}J_{21}\rangle=\tfrac{1}{4}(N^2-D^2+2N-2D)\ , \tag{2.21}$$

$$B=B_{+}+B_{-}=\tfrac{1}{2}(N^2-D^2+2N)\ , \tag{2.22}$$

and similarly, Q_{+1} can be obtained by the substitution

$$Q_{+1}=Q_{-1}(\cos^2\varphi\leftrightarrow\sin^2\varphi)\ . \tag{2.23}$$

The behavior of the factor Q_{-1} as a function of the parameter $\cos^2\varphi$ for various values of the atomic number N and the inversion D is plotted in Figs. 1 and 2. In Fig. 1 one sees the dependence of the factor Q_{-1} on the number of atoms for the case when all the atoms are initially in the excited state $|2\rangle$. As is shown in Fig. 1 the sidebands have sub-Poissonian photon statistics even in the case of a large number of atoms N. This result is quite different from the steady-state collective resonance fluorescence in the free space[29] where the sidebands only have sub-Poissonian photon statistics for the case of a few atoms (up to three). For the case of a large number atoms, the sidebands exhibit super-Poissonian photon statistics. Moreover, in Fig. 1 one sees that, when the number of atoms is increased the photon statistics of the sidebands becomes more sub-Poissonian. Because the intensity of the sidebands is proportional to the square of the number of atoms this result may be used to generate an intense field which remains sub-Poissonian.

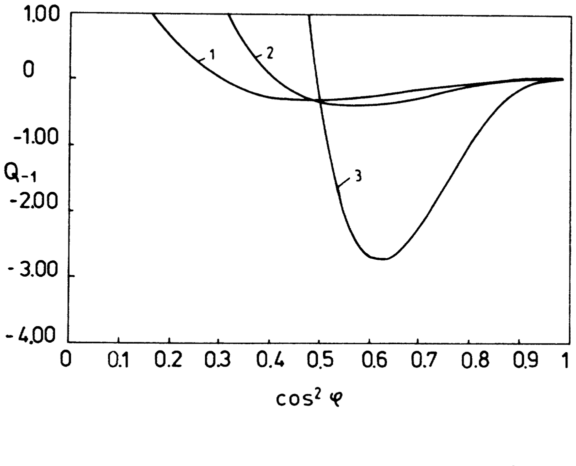

FIG. 1. Mandel parameter Q_{-1} as function of $\cos^2\varphi$ for the case of $D=N$. The lines 1–3 correspond to $N=5$, 10, 100.

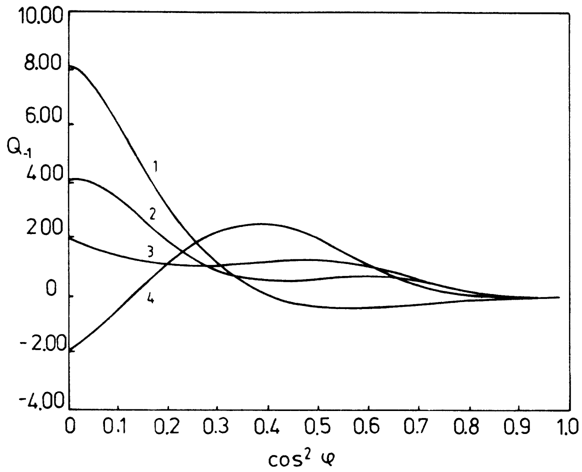

FIG. 2. Mandel parameter Q_{-1} as function of $\cos^2\varphi$ for the case of $N=10$. The lines 1–4 correspond to $D=10,6,4,0$.

In Fig. 2 we see that the initial condition strongly affects the sub-Poissonian photon statistics of the sidebands. When the initial inversion is varied from positive through to zero values the Q parameter completely changes in its dependence on $\cos^2\varphi$. We also note using Eqs. (2.15)–(2.22) that the factors $Q_{\pm 1}$ are unchanged if we change the sign of the atomic inversion D together with the exchange of $\cos^2\varphi$ by $\sin^2\varphi$ and conversely.

III. SQUEEZING IN THE FLUORESCENCE FIELD

The light squeezing by two-level atoms in a cavity near a bistability threshold[30,31] and squeezing in collective resonance fluoresence in a free space[32–38,23] have been considered in recent papers. It has been shown in the works of Bogolubov *et al.*[23] and Lawande *et al.*[37] that in the steady-state collective resonance fluoresence (intense-laser-field case) in free space the squeezing of the two sidebands is absent in all three spectral components taken separately and is present only in the mixture of the two sidebands as a result of the quantum correlation between these components and is related to the class of two-mode squeezing. In this section we discuss the squeezing in the spectral component of the fluoresence field from atoms in a cavity. We show that, contrary to collective resonance fluoresence in free space, squeezing is present in one separate sideband and the degree of squeezing can be large. As is easily shown using the relation (2.8), the operator S_0 is a Hermitian operator ($S_0^{\dagger}=S_0$) and squeezing is absent for the central spectral component S_0. The squeezing properties of the two sidebands S_{-1} and S_{+1} may be studied by introducing the Hermitian operators

$$S_{-1}=L_1+iL_2\ , \tag{3.1}$$

$$S_{+1}=R_1+iR_2\ . \tag{3.2}$$

In the case when the atoms are initially in the state (2.14) one find the normally ordered variances of the operators

L_1, L_2, R_1, and R_2 in the form

$$\langle :(\Delta L_1)^2: \rangle = \tfrac{1}{4}\sin^4\varphi(\tfrac{1}{2}N^2 - \tfrac{1}{2}D^2 + N - D\cos2\varphi) , \tag{3.3}$$

$$\langle :(\Delta L_2)^2: \rangle = -\tfrac{1}{8}\sin^4\varphi\cos^2 2\varphi \times (D^2 + 2D\cos^{-1}2\varphi - N^2 - 2N) , \tag{3.4}$$

$$\langle :(\Delta R_1)^2: \rangle = \tfrac{1}{4}\cos^4\varphi[\tfrac{1}{2}(N^2 - D^2) + N + D\cos^2\varphi] , \tag{3.5}$$

$$\begin{aligned}\langle :(\Delta R_2)^2: \rangle &= -\tfrac{1}{8}\cos^4\varphi\cos^2 2\varphi \times (D^2 - 2D\cos^{-1}2\varphi - N^2 - 2N) \\ &= \langle :(\Delta L_2)^2: \rangle(\cos^2\varphi \leftrightarrow \sin^2\varphi) .\end{aligned} \tag{3.6}$$

It is easy to show[32–38] that in the case when the fluorescence field is initially in the vacuum state (temperature $T=0$) one can ignore the influence of the free part of the fluoresence field on the normally ordered variances of the field operators. The normally ordered variances of the atomic operators $\langle :(\Delta L_{1,2})^2: \rangle$ and $\langle :(\Delta R_{1,2})^2: \rangle$ are equal (up to constant terms) to the normally ordered variances of the corresponding field operators and describe the squeezing of the left (S_{-1}) and right (S_{+1}) sidebands of the fluoresence field. Further we concentrate only in a discussion of the influence of the number of atoms N, initial population inversion D, and of the parameter $\cos^2\varphi$ (detuning δ) on the squeezing of the sidebands.

The necessary condition for squeezing to be present in the spectral component S_{-1} is given by

$$\langle :(\Delta L_1)^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta L_2)^2: \rangle < 0 , \tag{3.7}$$

and in the spectral component S_{+1} is given by

$$\langle :(\Delta R_1)^2: \rangle < 0 \quad \text{or} \quad \langle :(\Delta R_2)^2: \rangle < 0 . \tag{3.8}$$

From Eqs. (3.3)–(3.8) one sees that the squeezing may be present only in the out-of-phase components L_2 and R_2 given the initial phase choice of the cavity field. The normal-ordered variances $\langle :(\Delta L_2)^2: \rangle$ and $\langle :(\Delta R_2)^2: \rangle$ are dependent on the number of atoms N, the detuning δ, and the amplitude of the intense cavity mode G (via the parameter $\cos^2\varphi$), and of the initial condition, i.e., of the initial population inversion of the atomic system D.

For the case of exact resonance ($\cos^2\varphi = \sin^2\varphi = \frac{1}{2}$) and for the case when the initial atomic population inversion is equal to zero ($D=0$), as is seen from Eqs. (3.3)–(3.8), the squeezing is absent for all spectral components of the fluoresence field.

For the case when all the atoms are initially in the ground state (or excited state), i.e., $D=-N$ ($D=+N$) the relations (3.4) and (3.6) take the form

$$\langle :(\Delta L_2)^2: \rangle = \begin{cases} \dfrac{N}{4}\sin^4\varphi\cos2\varphi(1+\cos2\varphi) & \text{if } D=-N \\ -\dfrac{N}{4}\sin^4\varphi\cos2\varphi(1-\cos2\varphi) & \text{if } D=N , \end{cases} \tag{3.9}$$

$$\langle :(\Delta R_2)^2 \rangle = \begin{cases} -\dfrac{N}{4}\cos^4\varphi\cos2\varphi(1-\cos2\varphi) & \text{if } D=-N \\ \dfrac{N}{4}\cos^4\varphi\cos2\varphi(1+\cos2\varphi) & \text{if } D=N . \end{cases} \tag{3.10}$$

Thus the normal-ordered variances $\langle :(\Delta L_2)^2: \rangle$ and $\langle :(\Delta R_2)^2: \rangle$ are proportional to N.

In the general case the conditions for squeezing in the left sideband S_{-1} can be found from the relation (3.6) in the following form.

(i) $\langle :(\Delta L_2)^2: \rangle < 0$ if

$$\cos^2\varphi > \tfrac{1}{2} \quad \text{and} \quad D > -\cos^{-1}2\varphi + D' , \tag{3.11}$$

or

$$0 < \cos^2\varphi < \tfrac{1}{2} \quad \text{and} \quad D < -\cos^{-1}2\varphi - D' , \tag{3.12}$$

where

$$D' = (\cos^{-2}2\varphi + N^2 + 2N)^{1/2} . \tag{3.13}$$

Similarly, the conditions for the squeezing in the right sideband S_{+1} can be found from the relation (3.8) in the form (ii) $\langle :(\Delta R_2)^2: \rangle$ if

$$\cos^2\varphi > \tfrac{1}{2} \quad \text{and} \quad D < \cos^{-1}2\varphi - D' , \tag{3.14}$$

or

$$0 < \cos^2\varphi < \tfrac{1}{2} \quad \text{and} \quad D > \cos^{-2}2\varphi + D' , \tag{3.15}$$

where D' is found from Eq. (3.13). Comparing Eq. (3.11) with (3.14) and Eq. (3.12) with (3.15) one can see that for the appropriate system parameters $\cos^2\varphi$ and D the squeezing is present only in one of the sidebands but not simultaneously in both of the two sidebands.

The dependence of the relative variance $V_L = \langle :(\Delta L_2)^2: \rangle / N$ on the system parameter $\cos^2\varphi$ and on the initial atomic condition is shown in Fig. 3. As can be seen from Fig. 3 the squeezing is largest if all the atoms are initially in the ground (or excited) state. The reduction of the value $|D|/N$ results in a reduction of the squeezing.

Finally, we discuss the degree of squeezing, for example, for the atomic operator L_2 which has the following form:

$$F_L = \frac{\langle :(\Delta L_2)^2: \rangle}{\frac{1}{2}|\langle [L_1, L_2] \rangle|} , \tag{3.16}$$

where

$$\tfrac{1}{2}|\langle [L_1, L_2] \rangle| = \tfrac{1}{4}\sin^4\varphi|D||\cos2\varphi| . \tag{3.17}$$

For the case $D=-N$, using Eq. (3.9) one finds

$$F_L = \begin{cases} 1+\cos2\varphi & \text{if } \cos^2\varphi > \frac{1}{2} \\ -(1+\cos2\varphi) & \text{if } 0 < \cos^2\varphi < \frac{1}{2} . \end{cases} \tag{3.18}$$

Thus the degree of squeezing F_L can tend to the limiting value $F_L = -1$ (i.e., 100% squeezing) in the small region given by $\cos^2\varphi \le \frac{1}{2}$. We note that in the case of exact res-

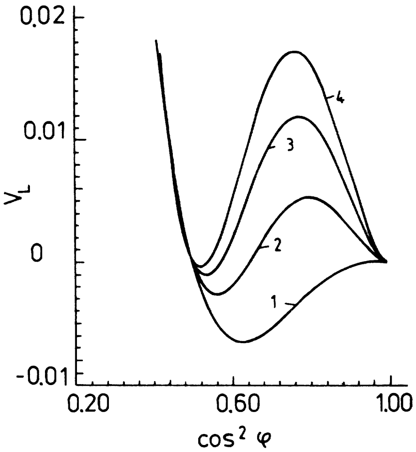

FIG. 3. Relative normally ordered variance as function of $\cos^2\varphi$ for the case of $N=10$. The lines 1–4 correspond to $D=10,8,6,4$.

onance $\cos^2\varphi=\frac{1}{2}$ it is straightforward to show that $\langle [L_1,L_2] \rangle = 0$; and in this case the concept of squeezing for the commuting operators L_1 and L_2 loses meaning. One can show that the factors F_L for the atomic operators L_1 and L_2 is not coincident with the factors of squeezing for the corresponding field operator, due to the influence of the free part of the fluorescence field on the commutator of the field operators. However, for the off-resonance case $\cos^2\varphi \neq \frac{1}{2}$, $N \simeq |D| \gg 1$ one can see from Eq. (3.17) that the value $\frac{1}{2} |\langle [L_1,L_2] \rangle|$ is very large and in this case the influence of the free part of the fluorescence field on the commutator of the field operators can be ignored. In this case the large degree of squeezing of the atomic operators L_2 and R_2 means that a large degree of field squeezing can be obtained.

IV. CONCLUSIONS

We have considered sub-Poissonian photon statistics and squeezing in the fluorescence field from N two-level atoms in a cavity for the case when the cavity mode is so intense that it can be considered as a classical field. We have shown that the two sidebands have sub-Poissonian photon statistics for the case of a large number of atoms. This result is drastically different from steady-state collective resonance fluorescence in free space where sub-Poissonian statistics are present only for small numbers of atoms. Moreover the sidebands become more sub-Poissonian with an increasing number of atoms. The influence of the initial condition of the atomic system on the photon statistics has been investigated.

We have also discussed squeezing in the fluorescence field. Unlike the steady-state collective resonance fluorescence in free space, the squeezing is present in separate sideband components of the fluorescence and not only in superpositions of these sidebands. The condition for obtaining a large degree of squeezing is considered in this paper.

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