

## Transverse coherence saturation: A method to enhance the coherence of x-ray beams

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A method is proposed to generate improved coherence in x-ray beams. The method leaves the photon source unchanged, and improves transverse coherence by reordering the emitted radiation through dynamical optical means (i.e., with at least one moving optical component). Since the method increases the transverse coherence of beams, it thus reduces the time needed to perform certain types of observations, such as in microscopy, and interference experiments such as holography. It does not increase the longitudinal coherence of beams. However, it can be supplemented by other means to increase longitudinal coherence. It does not increase the brightness, but allows one to more efficiently utilize beams of a given brightness. The use of this method with photons generated by pulsed electron beams, including those in storage rings, seems particularly appropriate.

### I. INTRODUCTION

So far the development of x-ray optics has been restricted to the domain of stationary instrumentation. An expansion of optics into the dynamical range would open up new possibilities. In particular, it has been shown that with this technique a rearrangement of photons is possible<sup>1</sup> thereby dramatically increasing the instantaneous intensity and reducing pulse lengths into the subpicosecond range. In the present paper it will be demonstrated that by using dynamical optics in a different manner, an alternative rearrangement of radiation is also possible, one which will increase the transverse coherence of a beam.

The availability of coherent optical photon beams caused significant advances in basic research and technology. It is generally expected that coherent x-ray beams will engender comparable advances, or even more significant ones, because of the orders of magnitude shorter wavelengths of x-ray beams compared to optical wavelengths.

To date two approaches have been proposed: to increase the number of coherent photons, bound-state lasers (BL's) and free-electron lasers (FEL's). Both rely on stimulated emission to ensure coherence between the emitted photons. As extension of these techniques to the x-ray regime represents a challenge: On the one hand, bound-state x-ray lasers (BXL's) have to overcome problems associated with the generally short lifetime of inverted bound states, as well as the associated high-energy densities and low repetition rates. On the other hand, if FEL's are to operate in the x-ray range (FEXL), very severe tolerance requirements will have to be met.

The method to be described is *not* based on stimulated emission. Therefore it is *not* subject to the same technological and cost considerations which have so far sharply limited the performance of BXL's and FEXL's in the short-wavelength range. The suggested method does not affect the process of photon emission at all. Instead it rearranges the radiation *after* it has been produced, to

make it more transversally coherent. Consequently, the method may be used in conjunction with most photon sources, including amplified ones. However, as discussed below, its use is more natural with certain types of sources than with others. Synchrotron radiation sources are among those for which the suggested method is particularly well suited.

The rearrangement of radiation is to be achieved by dynamic optical means, i.e., by a system composed of optical elements, at least one of which has to be nonstationary. The significant observation which results from the following argument, is this: The required speeds with which a nonstationary optical element has to move to accomplish such reordering, are not extravagant, and can be reached.

### II. COHERENCE CONDITIONS

In order that the radiation in a beam be coherent for purposes of interference and other similar experiments, it has to satisfy certain conditions. These can be most simply stated in terms of the physical requirements, as follows.

Denote by  $\lambda$  the wavelength of the radiation in the beam, by  $\Delta\lambda$  the full spread in  $\lambda$ , by  $\lambda_0$  the average value of  $\lambda$ , and by  $\Delta\theta_i$  the full angular width of the beam along the  $i$ th axis ( $i=x,y$ ). If one requires that the coherence length  $h_z$  be longer than a specified length,  $l_z$  then the beam must be sufficiently (a) monochromatic, and (b) collinear, as follows: (a),

$$\frac{\Delta\lambda}{\lambda_0} < f_1 \frac{1}{2} \frac{\lambda_0}{l_z}, \quad (1)$$

and (b),

$$\Delta\theta_i < f_2^{1/2} \left[ \frac{\lambda}{l_z} \right]^{1/2}, \quad i=x,y. \quad (2)$$

Here  $f_1$  and  $f_2$  are suitably chosen constants (whose

values will be fixed later).

(c) Further conditions are imposed if one requires that the coherence diameters  $h_x$  and  $h_y$  of the beam along the  $x$  and  $y$  axes be larger than some specified lengths  $l_x$  and  $l_y$ , respectively. Let  $D_{si}$  stand for the diameter of the photon beam source along the  $i=x,y$  axis, and assume that the beam axis is aligned with  $z$ . Consider coherence diameters at a distance  $L$  from the source, and assume that the beam is narrow, i.e.,  $L \gg D_{si}$ ,  $L \gg l_i$ ;  $i=x,y$ . Then the above requirements imply

$$L > f_3 \frac{D_{si} l_i}{\lambda_0}, \quad i=x,y. \quad (3)$$

The  $f_3$  is a constant to be specified.

The cross-sectional area  $A_\perp$ , corresponding to the two coherence diameters  $h_x$  and  $h_y$ , is customarily referred to as the coherence area of the beam<sup>2</sup> (at a distance  $L$  from the source), and the coherence volume of the beam there is  $h_z A_\perp$ . The number of photons in the coherence volume is the coherence number.

For purposes of this discussion, we introduce the following definitions. The "transversely coherent intensity"  $I_\perp$  is the number of photons passing through  $A_\perp$  per unit time. One can talk about instantaneous and average transversely coherent intensity. The transverse coherence number for a time interval  $\Delta t$  is the number of photons passing through  $A_\perp$ , during  $\Delta t$ , i.e.,  $I_\perp \Delta t$ . We denote the total photon beam intensity in the beam by  $I$ , and define the "degree of transverse coherence" or simply "transverse coherence," as

$$C_\perp = I_\perp / I. \quad (4)$$

Evidently,  $C_\perp \leq 1$ . The degree of transverse coherence is saturated when it reaches unity. At that point the transversely coherent intensity equals the total photon intensity, or, equivalently, the transverse coherence number for any  $\Delta t$  equals the total number of photons passing through a cross-sectional area of the beam, oriented normally to the beam axis, during  $\Delta t$ .

When conditions (1), (2), and (3) are satisfied, and when everywhere at the source the phase of the radiation is the same, then all radiation within a coherence volume  $l_z A_\perp$  will be coherent, the exact degree of coherence depending on the constants  $f_1$ ,  $f_2$ , and  $f_3$ . In particular, when the distributions in wavelength, angle, and point of emission are all uncorrelated, then the expected difference between the phases of the radiation at any two points within the coherence volume will satisfy

$$\Delta\phi < (f_1^2 + f_2^2 + f_3^2)^{1/2} \pi = f\pi. \quad (5)$$

For sufficient coherence in interference experiments, one usually requires  $f \leq 0.25$ .

Conversely, if radiation from various points within the source is emitted with mutually random phases, it follows from the symmetry of conditions (1)–(3) under the interchange  $l_i \leftrightarrow D_{si}$  ( $i=x,y$ ), that<sup>3</sup> inequality (5) will be satisfied by the phase difference between two branches of a wave emitted at any *one* source point, if the two branches lead to any two points on  $A_\perp$ , and  $A_\perp$  is within

a coherence volume containing the source. Therefore one can perform interference experiments with such a source by allowing the beam to pass through two openings in a screen located in  $A_\perp$ , and observing the interference pattern behind the screen.

Given any radiation beam, it is always possible to increase the coherence volume. Using monochromators one can increase monochromaticity, while passing the beam through appropriate slits can improve collinearity and decrease the effective source diameters. By contrast, neither the transversely coherent intensity nor the (transverse) coherence number can be increased in this manner. Monochromators and slits operate by discarding undesirable photons; they cannot increase the photon number within any volume. There would be no need for slits if the photon source itself had small enough emittance. For storage rings this would require the reduction of the electron beam emittances, but that cannot be carried beyond certain limits imposed by the present state of engineering technology.

Our purpose here is to suggest an alternative method to produce sufficiently coherent and intense photon beams. We are primarily interested in generating beams suitable for coherence experiments, including x-ray holography. The usefulness of photon beams for such experiments is determined by two parameters: the coherence length  $h_z$  and the degree of transverse coherence  $C_\perp$ .

The method to be described affects neither the brightness nor  $h_z$ . It can, however, increase the transverse coherence of the beam. In the limit it can saturate transverse beam coherence. In that case the degree of transverse coherence will be as high as it could ever have been, even if a beam of the given intensity had been produced by some other means such as a BXL or FEXL.

### III. DESCRIPTION OF THE METHOD

The principle of the method is illustrated in Fig. 1.

(a) First, the full photon beam is focused to have an angular divergence which does not contradict condition (2) and assume that the beam is sufficiently monochromatic. Assume<sup>4</sup> also that the cross section of the beam so obtained violates condition (3).

(b) Next, optical means are used to split the beam into several component beams, altogether  $N_c$  of them. These beams all have cross sections consistent with inequality (3). If the full beam has radii  $\sigma_{yi}$  ( $i=x,y$ ) then the  $n$ th

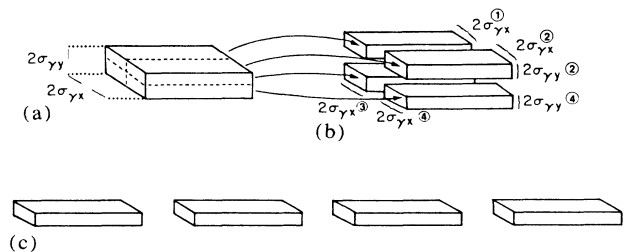


FIG. 1. Schematic illustration of decomposing a beam into component beams, and subsequently reconstituting from those components a new beam to increase transverse coherence.

component beam has radii  $\sigma_{\gamma i}^{(n)} < \sigma_{\gamma i}$ ,  $n = 1, \dots, N_c$ .

(c) These component beams are allowed to travel along paths of different lengths to a common collection point  $P_c$ , so that they arrive there in sequence, "stacked" one after the other.

Finally, at point  $P_c$  a rotating mirror directs all beam sections through a port to the user. The reconstituted beam emerging through the port will thus have not only the required angular divergence, but also the required cross section. If needed, adequate monochromatization (at any stage of the process) will then lead to appropriate coherence.

As a result of this procedure, the photon beam will be transformed into a longer, but narrower one. When the beam cross section does not exceed the coherence area, then transverse coherence will be saturated,  $C_1 = 1$ . Although the length of a pulse will increase, the longitudinal coherence length  $h_z$  will not be increased by the method. On the other hand, there are techniques by which the method can be supplemented to increase  $h_z$ . For example, longer undulators will cause an increase in  $h_z$ , if the electron beam quality is good enough. Particularly impressive  $\lambda/\Delta\lambda$  values are expected with micro-pole undulators.<sup>5</sup>

Figure 2 illustrates various ways in which a beam cross section can be split into component beams. In Figs. 2(a) and 2(b) the decomposition is two dimensional, while in Fig. 2(c) it is one dimensional.

A practical realization is shown in Fig. 3. In the case chosen here the decomposition pattern is one dimensional. There is no difference in principle between one- and two-dimensional decompositions, but the one-dimensional case is easier to illustrate in a figure such as this one. Furthermore, in many important cases<sup>6</sup> one can reach complete coherence saturation by a one-dimensional decomposition alone, and therefore, it is likely that this type of device will be realized first. Figure 3(a) illustrates a general outlay of optical elements which will increase the transverse coherence, Fig. 3(b) shows a one-dimensional beam multisplitter, while in Fig. 3(c) additional details are given.

One can prove, in general,<sup>7</sup> that by static optical means alone, one can never achieve an increase in the transverse coherence of an entire photon beam (as opposed to only a segment of it). Therefore, we have the following statement.

(i)  $C_1$  can be increased only if at least one optical element is nonstationary.

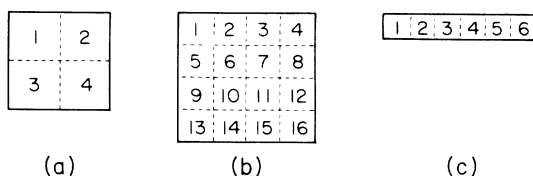


FIG. 2. Schematic illustration of (a) and (b) two-dimensional and (c) one-dimensional decomposition schemes.

In Fig. 3(a) that element is a rotating mirror. We will designate by  $M_r$  this rotating mirror.

After the beam multisplitter at least one system of reflectors is needed to direct all beam components to  $P_c$ . It can be shown<sup>8</sup> that the following statement is true.

(ii) *The system of reflectors cannot consist of only a single continuous mirror surface.*

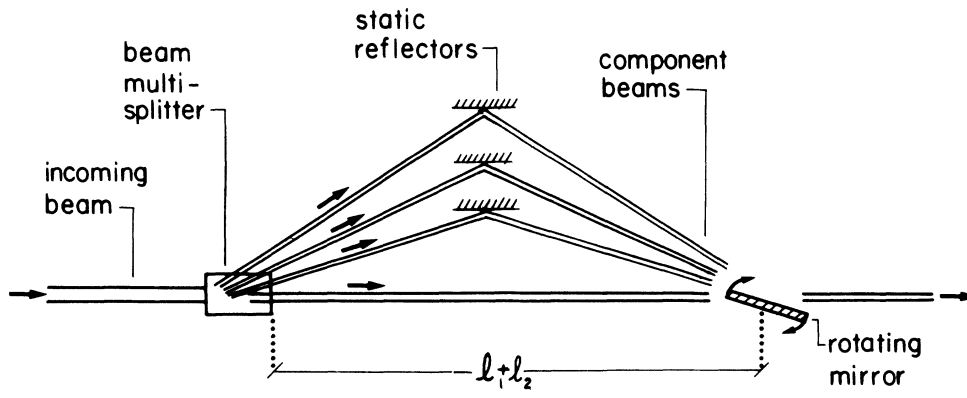
In Fig. 3. the reflector system  $R$  consists of a sequence of disjointed static mirrors  $R_i$ ,  $i = 1, 2, \dots, N_c$ . Alternatively it may contain a grating structure<sup>1</sup> or other equivalent discontinuous components.

Let us denote the length of a component beam by  $h_i$  ( $i = 1, 2, \dots, N_c$ ) so that it takes  $\Delta t_i = (1/c)h_i$  time for it to pass through any stationary optical element. To ensure that each component beam will be clearly distinguished from every other one, it is necessary that the angular frequency of the rotating mirror satisfy the condition.

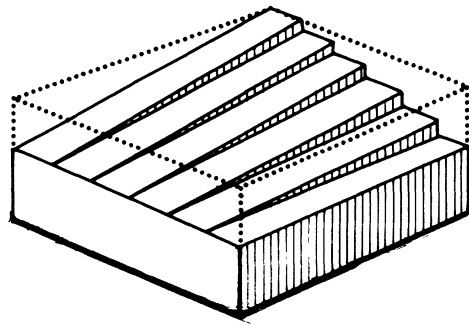
$$\omega_M \geq \frac{1}{\Delta t_i} \left[ \frac{\Delta}{2} \theta_\gamma + \frac{1}{l_{i1}} (r_M \cos \phi + r_{R_i} \cos \phi_{R_i}) \right], \quad i = 1, \dots, N_c. \quad (6)$$

Here  $l_{i1}$  is the optical path length between  $R_i$  and  $M_r$ . The  $2r_M \cos \phi$  is the diameter of  $M_r$  projected onto the unit vector  $\hat{m}$ . By definition,  $\hat{m}$  lies in a plane perpendicular to the axis of rotation of  $M_r$ , and is also perpendicular to the component parallel to this plane of the axis of the photon beam reflected from  $M_r$  (Fig. 4). The  $2r_{R_i} \cos \phi_{R_i}$  is the diameter, projected onto  $\hat{m}$ , of  $R_i$  when the beam reflected from  $M_r$  impacts on  $R_i$ ; the  $\cos \phi_{R_i}$  is the angle of incidence of the photons on  $R_i$ , and the components of the reflector system  $R$  (Fig. 4) are denoted by  $R_i$ ,  $i = 1, \dots, N_c$ . The  $\Delta \theta_\gamma$  represents the full divergence of the reflected photon beam at  $M_r$ , in the plane containing  $\hat{m}$ . In Eq. (6)  $l_{i1} \gg r_M, r_{R_i}$  is assumed. Inequality (6) can be proven by considerations similar to those which led to inequality (9) in Ref. 1.

Let us denote by  $f_c$  the factor by which the transverse coherence is increased as a result of coherence saturation. Then the time required to perform a certain interference experiment, e.g., holography, will be reduced by this same factor. Clearly, large values of  $f_c$  are desirable. It must be emphasized that, of course, not only interference experiments benefit from large  $f_c$ . For example, in x-ray microscopy, too, the limiting factor is frequently the transverse coherence number, although it is often expressed in terms of other (equivalent) quantities. Consider the example of diffraction-limited imaging, e.g., in scanning x-ray microscopy. If blurring is to be avoided, the instrument can accept only photons within a transverse phase-space volume of order  $\lambda^2$ . Therefore the exposure time is inversely proportional to the number of transversely coherent photons within that volume, i.e., proportional to the transverse coherence number. Since usually the required exposure time limits the performance of the microscope, the limitation can just as well be ex-



(a)



(b)

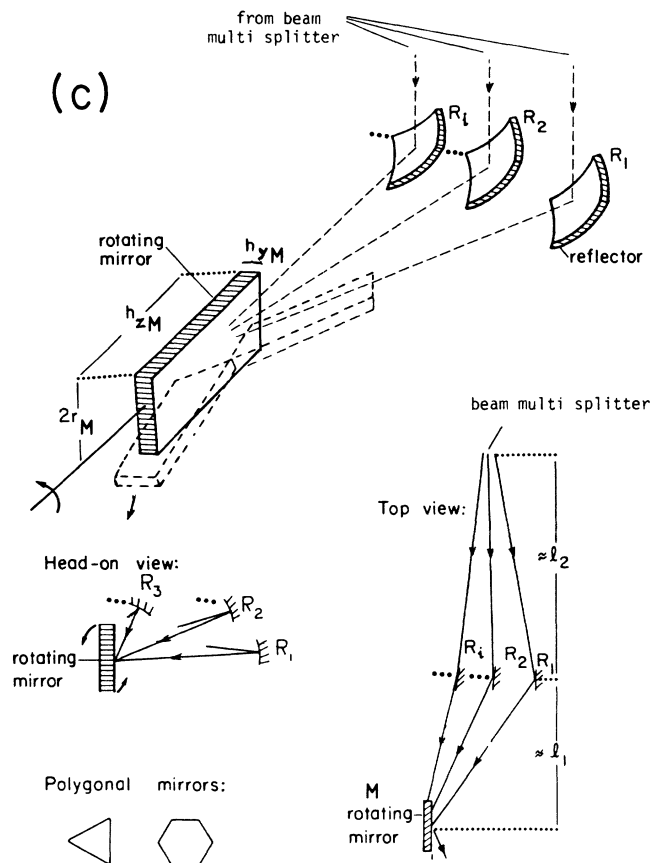


FIG. 3. Illustration of a practical realization of beam decomposition and reconstitution: (a) a simplified view, (b) a (one-dimensional) beam multisplitter, (c) details of the rotating mirror geometry.

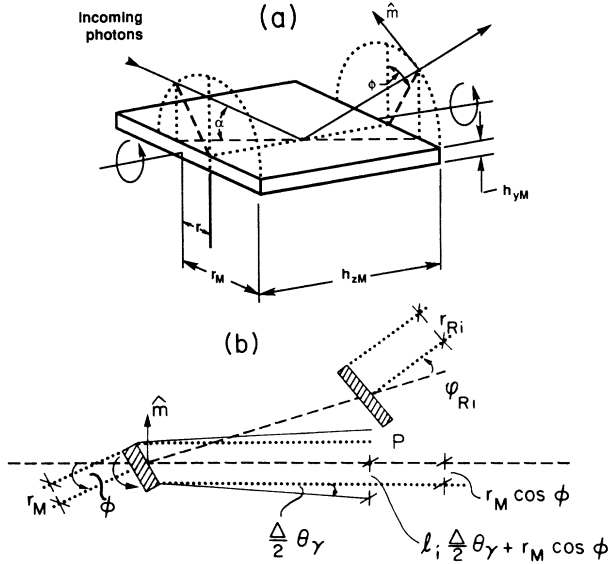


FIG. 4. Rotating mirror geometry: (a) general view, (b) side view.

pressed in terms of the degree of transverse coherence, once the total photon intensity is given. Any procedure which increases  $C_{\perp}$  will then extend the microscope's operational range. Therefore, for these experiments, too large  $f_c$  is desirable.

In practice the maximum value of  $f_c$  will be limited. One limitation on  $f_c$  is related to the duty cycle  $D$  of the source. In designs such as the one shown in Fig. 1, one has to have

$$f_c \leq 1/D. \quad (7)$$

Since for high-energy synchrotron sources  $D \leq 10^{-3}$ , very significant  $f_c$  values can be achieved before one has to deal with this constraint.

Another limitation is imposed by the values of the angular velocity,  $d\phi/dt = \omega$ , that the rotating reflector element can achieve. Indeed, to reach a certain  $f_c$  value, the device must stack the  $i$ th component beam within the time  $\Delta t_i$ . Assuming that all component beams have equal length, i.e.,  $\Delta t_1 = \Delta t_2 = \dots = \Delta t_{N_c} = \Delta t$ , one finds  $\Delta t = T_p/f_c$ , where  $T_p$  is the time which elapses between the onset of any two successive photon pulses generated by the source. Denote by  $r_M$  the radius of the rotating mirror (Fig. 4) and by  $h_{zM}$  its length. Let  $\sigma_{\gamma y}$  and  $\sigma_{\gamma x}$  be the radius of the photon beam along the direction  $\hat{m}$  and perpendicular to it, respectively. Assume that  $\phi = 0$ . If the rotating mirror is large enough to intercept the entire photon beam incident on it then

$$r_M \geq \sigma_{\gamma y}, \quad (8a)$$

$$h_{zM} \geq 2\sigma_{\gamma x} / \sin \alpha_{\min}. \quad (8b)$$

From Eq. (6) one then finds that the rotating mirror perimeter moves with a velocity

$$v(r_M) = r_M \omega_M > \frac{f_c}{T_p} r \frac{\Delta}{2} \theta_{\gamma y} \geq \epsilon_{\gamma y} f_c / T_p. \quad (9)$$

Here  $\epsilon_{\gamma y}$  is the emittance of the photon beam perpendicular to the direction  $\hat{m}$  (Fig. 4). On the other hand, the highest values  $v(r_M)$  can reach are determined by the properties of the mirror material. For example,<sup>1</sup> for uniform composition, denoting by  $\rho$  and  $Y$  the density and tensile strength,

$$v(r_M) < F_v \sqrt{Y/\delta}, \quad (10)$$

where  $F_v$  is close to  $\sqrt{2}$  when  $h_y \ll r_M$  (see Fig. 4). Therefore

$$f_c < \frac{T_p}{\epsilon_{\gamma y}} F_v (Y/\delta)^{1/2}. \quad (11)$$

For a high grade steel mirror as shown in Fig. 4(a), one finds  $v(r_M) \leq 7 \times 10^4$  cm/s, so that when  $\epsilon_{\gamma y} = 10^{-9}$  rad m, and  $T_p = 7.33 \times 10^{-6}$  s (similar to the values prevailing in the electron positron storage ring at Stanford),  $f_c \leq 5 \times 10^6$ . This limit's being even more remote than the previous one is no cause for concern.

A third restriction on  $f_c$  derives from the fact that the maximum difference in path length traveled by the various component beams  $\Delta l_{\max}$  is related to the total optical path length across the instrument. For example, in the geometry illustrated in Fig. 3(c), one has

$$\Delta l_{\max} \approx \frac{1}{2} l \alpha_{\max}^2, \quad l = l_1 + l_2 \quad (12)$$

where  $\alpha_{\max} \ll 1$  is the maximum grazing angle of incidence. When one must have  $\alpha_{\max} \ll 1$  (as when the photon energy is high, and no multilayers are used), one is restricted to  $\Delta l_{\max} \ll l$ . To achieve any particular  $f_c$  value, one obviously needs

$$\Delta l_{\max} \geq f_c T_p D c, \quad (13)$$

if the lengths of the individual component beams are assumed to be all equal, i.e., have the value  $c T_p D$ . On the other hand, the optics must be so designed that the effective phase space occupied by the photons is not significantly increased by random errors, mirror motion, various irregularities, or diffraction. In particular, the effect of random errors in angle,  $\delta\theta$ , due to mirror surface irregularities, should be small compared with the beam diameter. These effects have a value approximately equal to  $l\delta\theta$ , which requires

$$l \leq \sigma / \delta\theta, \quad (14)$$

and limits  $\delta l_{\max}$ . This limitation can be significant. If so, it can be dealt with as described below. If  $\alpha_{\max}$  need not be  $\ll 1$ , this restriction is far less severe, and at the same time  $h_{zM}$  as given in Eq. (8b) can be reduced. From this point of view, multilayer and crystal reflectors are preferred.

#### IV. PRACTICAL CONSIDERATIONS

To evaluate the capabilities of the suggested approach, consider the SPEAR and PEP electron rings at Stanford. We assume that for SPEAR operating at circulating electron energy  $E_e = 1.5$  GeV, the emittances are  $\epsilon_x = 1.125 \times 10^{-7}$  rad m, and  $\epsilon_y = 1.125 \times 10^{-9}$  rad m, and that in the region of photon generation the beta func-

TABLE I. The coherence saturation factor  $f_c$  and related parameters for photon beams generated by the SPEAR and PEP electron rings at Stanford. For  $E_\gamma = 60$  eV, transverse coherence is saturated at  $f_c = 25$  in SPEAR and at  $f_c = 5$  in PEP, reflecting the smaller emittance of the latter. At  $E_\gamma = 10$  keV, saturation occurs in PEP at  $f_c = 550$  (and at even higher  $f_c$  in SPEAR, not shown).

	$E_e$ (GeV)	$E_\gamma$ (eV)	$\epsilon_{yx}$ (rad m)	$\epsilon_{yy}$ (rad m)	$L_p$ (m)	$r_M \geq$ (mm)	$\frac{\Delta}{2}\theta_{yy}$ (mrad)	$v(r_M)$ (cm/sec)	$\alpha$ (rad)	$\frac{\Delta}{2}\theta_{yx}$ (mrad)	$h_{zM} \geq$ (mm)	$f_c$
SPEAR	1.5	60	$2.54 \times 10^{-7}$	$1.03 \times 10^{-8}$	2.5	0.21	0.05	$3 \times 10^{+4}$	0.105	1.5	3.2	25
									$\pi/2$	0.25	2.0	25
PEP	4.5	60	$5.8 \times 10^{-8}$	$1.0 \times 10^{-8}$	0.15	0.21	$5 \times 10^{-3}$	$9.4 \times 10^4$	0.105	1.5	0.74	5
		$10^4$	$1.1 \times 10^{-6}$	$1.9 \times 10^{-8}$	16.5	0.039	$5 \times 10^{-3}$	$2.2 \times 10^2$	$\pi/2$	0.15	0.77	5
									$\pi/4$	0.025	0.91	550

tions are  $\beta_x^* = 90$  cm and  $\beta_y^* = 8$  cm; while for PEP operating at 4.5 GeV,  $\epsilon_x = 1.05 \times 10^{-8}$  rad m,  $\epsilon_y = 1.125 \times 10^{-10}$  rad m,  $\beta_x^* = 300$  cm, and  $\beta_y^* = 40$  cm. From these photon beam emittances  $\epsilon_{yx}$  and  $\epsilon_{yy}$  can be calculated at the source for both machines. The half length of the electron bunches will be taken to be  $\sigma_{z0} = 5$  cm for SPEAR, and 1.5 cm for PEP. These then are also the half lengths,  $\sigma_{\gamma z0}$ , of the respective photon beam pulses generated.

Table I lists the calculated values  $\epsilon_{yx}$  and  $\epsilon_{yy}$  for both machines for photons with energy  $E_\gamma$ , the coherence enhancement factor  $f_c$ , the total length of the reconstructed resultant photon pulse  $L_p$ , the perimeter velocity of the rotating mirror  $v(r_M)$ , as well as  $\frac{1}{2}\Delta\theta_{yy}$ ,  $r_M$ ,  $h_{zM}$ , and  $\alpha$  (see Fig. 4). The approximate length<sup>9</sup> of the total optical path through the device can be estimated from  $l \geq 2L_p/\alpha^2$  when  $\alpha \ll 1$ , while for  $\alpha = \pi/2$ , only the trivial condition  $l \geq L_p$  remains.

For soft x rays one may use grazing incidence mirrors or multilayered surfaces. At photon energy  $E_\gamma = 60$  eV, the reflectivity of the former at 0.105 rad angle of incidence is  $> 90\%$  for most mirror materials,<sup>10</sup> while for normal incidence on multilayered mirrors the reflectivity is about 30%, and maybe  $\geq 60\%$ .<sup>10,11</sup> At  $E_\gamma = 10$  keV, for grazing incidence at 0.05 rad the reflectivity on platinum is typically<sup>12</sup> 85%, while at large angles crystal reflectors may be used with high reflectivities if the wavelength and angle range are properly matched.

The procedure described in Sec. III and illustrated in Fig. 1 is well suited to explain the principle of coherence saturation. However, if  $\alpha \ll 1$ , and the limitation on  $l$  as discussed in connection with Eqs. (12)–(14) presents a problem, the design should be modified. In that case, rather than starting with a small  $(\Delta/2)\theta_{yy}$ , it is better to first focus the beam with  $(\Delta/2)\theta_{yy}$  sufficiently large compared to the random  $\delta\theta$ , so that the effect of the latter should become negligible. One pays for that either by having to deal with a significantly larger diameter photon beam later on, or by having to refocus the beam at least once before it reaches the rotating mirror. With a subsequent refocusing  $\frac{1}{2}\Delta\theta_{yy}$  can eventually be reduced to its desired value. An alternative strategy (which may be used in combination with the one just described) consists of decomposing the original beam in more than one step. In the first step each of the component beams is allowed

to occupy a relatively large transverse phase space, large enough so that the relative increase caused by the random  $\delta\theta$  is negligible. In this step large  $\Delta l_1$  can be induced, and, in addition, a certain  $\Delta l_2$  space is left between successive component beams to allow the second step to take place. In the second step each component beam is considered to be the original beam, and further decomposed into subcomponent beams. In the second step it is sufficient to introduce  $\Delta l \leq \Delta l_2$  difference in the optical path. When  $\Delta l_2 \ll \Delta l_1$ , the benefits of this strategy become significant. There is no reason why one cannot decompose the beam in more than two steps, but of course each additional step introduces an added degree of design complexity.

It should be noted that, in principle, the technique just described can be employed to increase the transverse coherence of beams of particles other than photons, provided that the necessary optical elements (e.g., reflectors) are available (e.g., crystals or electromagnetic fields), and that the interaction of particles within the beam can be dealt with.

## V. SUMMARY AND CONCLUSIONS

The method described in this paper was designed to facilitate the performance of certain kinds of interference experiments (e.g., x-ray holography) with existing photon beams. However, it can also be used to increase the range of certain other techniques, such as scanning x-ray microscopy. In general, the method can be used to increase, and saturate, the transverse coherence  $C_1$ . It does not affect the coherence length  $h_z$  (but that can be increased for synchrotron emitters by appropriate undulators, especially micropole undulators). It does not increase the photon brightness, but makes it possible to more effectively utilize beams of a given brightness.

In principle, the method may be used in conjunction with any photon source, amplified or nonamplified, whenever  $C_1 < 1$ . It should prove most immediately helpful when (a) the photons are expensive to generate, (b) the photon duty cycle is low, (c) the photon intensity is one of the principal limiting factors in the experiment. For high-energy electron synchrotron radiation sources both (a) and (b) hold, and for interference experiments (c) is also true. Therefore coherence saturation should prove

to be a particularly valuable technique for such interference x-ray experiments.

The proposed method increases  $C_1$  without relying on stimulated photon emission. Therefore it is not subject to the same conditions and cost considerations which

sharply limit the performance of amplified x-ray sources (BXL's) and (FEXL's) at short wavelengths. There seems to be no fundamental reason why the method should not be effective over a range reacting well into the hard x-ray region.

<sup>1</sup>(a) Paul L. Csonka, Proc. Int. Soc. Opt. Eng. **582**, 298 (1985);  
(b) J. Appl. Phys. **64**, 967 (1988).

<sup>2</sup>For example, when the beam cross section is rectangular  $A_1 = h_x h_y$ .

<sup>3</sup>Reversing all optical paths, and considering pairs of them originally emerging from any point chosen on  $A_1$ .

<sup>4</sup>If condition (3) is not violated, then  $C = 1$ , and there is no need to increase the transverse coherence.

<sup>5</sup>Paul L. Csonka, Proc. Int. Soc. Opt. Eng. **582**, 298 (1985); Roman O. Tatchyn and Paul L. Csonka, J. Appl. Phys. Lett. **50**, 377 (1987); Proc. Int. Soc. Opt. Eng. **733**, 106 (1987).

<sup>6</sup>For example, such is the case at both the SPEAR and PEP electron rings at Stanford, for the soft x-ray range.

<sup>7</sup>The proof is straightforward; it parallels the proof of statement

(S-1) in Ref. 1(b). It hinges on the observation that  $N_c$  different beam components will arrive at  $P_c$  at different times from different directions, and a nonstationary component is required to forward all of them from  $P_c$  to the same port of exit.

<sup>8</sup>The proof is identical to that of statement (S-2) in Ref. 1(b).

<sup>9</sup>The precise value depends on the details of the optics, as discussed below.

<sup>10</sup>J. Kirz *et al.*, Center for X-ray Optics, Lawrence Berkeley Laboratory, Report No. PUB-490.

<sup>11</sup>T. Barbie, S. Mrowka and M. Hettrick, Appl. Opt. **24**, 883 (1985).

<sup>12</sup>Martin V. Zambeck (unpublished).