Internal trace for detector variables and the quantum theory of measurement

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The wave-packet reduction in quantum measurements is investigated for quantum-mechanical treatments except for the macroscopic limit in the final step. We show that the internal trace represented by the integrations over the internal variables of detectors, which are independent of the measured physical quantity, leads to the disappearance of the interference in the density matrix and then a reduction of the wave packet. Through the evaluations of two simple examples we clarify the role of the internal trace in quantum measurements and point out the difference between our idea and the other.

I. INTRODUCTION

The theory of measurement is still one of the most controversial problems in quantum mechanics. We have many fundamental problems, that is, not only theoretical analyses of quantum-measurement processes but serious questions about the Copenhagen interpretation as well. Restricting ourselves to the theoretical analyses, some interesting approaches have been proposed in recent years. Machida and Namiki[1] tried solving the problem by taking an account of the macroscopic nature of the measurement apparatus (say detector) in terms of a continuous direct sum of many Hilbert spaces, which is called the continuous-superselection-rule space as was discussed by Araki.[2] Another interesting model was proposed by Fukuda,[3,4] who tried to interpret the problem by the separation of Hilbert spaces describing the detectors into different orthogonal spaces in the limit where the number of constituent particles in the detector goes to infinity (say the limit $N\rightarrow\infty$). These approaches play a very important role showing that the quantum-measurement process is a calculable problem as a physical process.

In this paper we would like to propose an idea to solve the quantum-measurement problem through the investigation of two examples which are completely describable and calculable as quantum-mechanical processes. We shall show that the so-called wave-packet reduction by the measurement can be derived from the trace with respect to the internal variables of the detector in the limit $N\rightarrow\infty$. Following the ordinary treatment of quantum mechanics, we describe the measurement process in terms of the transition from the initial state (Φ_I) for the total system including the quantum object and the detector to the final state (Φ_F) as

$$\Phi_I\equiv\phi\otimes\Psi_0=\sum_n c_n u_n\otimes\Psi_0 \rightarrow\Phi_F\equiv\sum_n c_n u_n\otimes\Psi_n\ , \tag{1.1}$$

where $\phi=\sum_n c_n u_n$ denotes the superposition of the state for the quantum object system in terms of the eigenfunctions of the measured physical quantity and Ψ stands for the detector state. The wave-packet reduction is represented by the change of the density matrix, that is,

$$\rho_F\equiv|\Phi_F\rangle\langle\Phi_F|=\sum_n\sum_m c_n c_m^*|u_n\rangle\langle u_m|\otimes|\Psi_n\rangle\langle\Psi_m| \rightarrow\hat{\rho}_F\equiv\sum_n|c_n|^2|u_n\rangle\langle u_n|\otimes|\Psi_n\rangle\langle\Psi_n|\ . \tag{1.2}$$

It is noted that in (1.2) the wave-packet reduction is expressed as the disappearance of the interference terms among the states with different eigenvalues. In this formalism for the wave-packet reduction our fundamental idea is represented as follows. Taking into account that most of the variables describing the internal state of the detector do not directly couple to the measured quantity and are deliberately ignored in the measurement process, those variables should be traced in the density matrix. That is, the state of the detector ($|\Psi_n\rangle$) may be described by the direct product of the state for the variables coupling to the measured quantity ($|\Psi_n^G\rangle$) and that for the variables not coupling to it ($|\Psi_n^U\rangle$). Then the density matrix in the measurement should be expressed as

$$\tilde{\rho}_F\equiv\iint|\Phi_F\rangle\langle\Phi_F| =\sum_n\sum_m c_n c_m^*|u_n\rangle\langle u_m|\otimes\langle\Psi_n^G\rangle\langle\Psi_m^G|\otimes\iint|\Psi_n^U\rangle\langle\Psi_m^U|\ , \tag{1.3}$$

where $\iint$ denotes the trace over all the variables describing $|\Psi_n^U\rangle$. We call this trace "the internal trace." In the off-diagonal terms (the interference terms) the states for those variables may be slightly different in each other for the detailed reaction triggered inside the detector. The trace over one of them, therefore, produces a factor (say Q) which is smaller than 1. (Image the overlapping integral between two slightly different normalized wave functions.) A macroscopic accumulation (proportional to N) of such a small difference can, however, induce a large difference as $|Q^N|\ll 1$. Actually, the interference terms vanish by the effect in the macroscopic limit $N\rightarrow\infty$. We may summarize the above considerations in two points.

(i) The interference terms of a density matrix partially traced over all variables not coupling to the measured

quantity (the internal trace) become a zero operator in the macroscopic limit $N \to \infty$.

(ii) The state vector of a quantum system coupled to a detector having macroscopic degrees of freedom is collapsed by taking the internal trace and then going to the macroscopic limit.

In Sec. II we present a model where a measurement process with a detector described by a stable internal state and having a strong electric external field is studied and the idea of the internal trace on the density matrix is proposed. The wave-packet reduction is led by the change of the internal state of the detector induced by the interaction of ionized atoms in the detector with the electric external field. A measurement by a detector with a quasistable internal state like the Wilson cloud chamber is proposed in Sec. III, where the observed quantity is the energy emitted from the detector. The wave-packet reduction is also realized by the internal trace corresponding to the change of the internal state of the detector. In Sec. IV we clarify the role of the internal trace through the discussions about the difference of our model from the others.[1-4] We summarize our idea for the quantum-measurement process with the internal trace and give the criteria for a good detector in Sec. V. In this paper we shall deal with the measurement on the quantum object with only two different eigenvalues like the Stern-Gerlach experiment and also describe the internal state of the detector in terms of harmonic oscillators.

II. MEASUREMENT BY DETECTORS WITH STABLE INTERNAL STATES (MODEL I)

When a detector has a stable internal state, a microscopic quantum object can not give any macroscopic and measurable change to the detector. In order to induce a macroscopic effect after the interaction of the object with the detector, an enhancement process is required. A simple example is presented by Fukuda,[3] where the center of mass of the detector is moved in a measurable order by the external electric field after many atoms in the detector are ionized by the interaction. We shall deal with this model here. In order to see the difference between Fukuda's consideration and ours, it is very instructive to study the measurement problem in the same process.

The following measurement process is provided. The quantum object has two different eigenvalues similar to the spin-up and spin-down of an electron, which is written in terms of the wave function

$$\phi(\mathbf{r},t)=\phi^{(1)}(\mathbf{r},t)+\phi^{(2)}(\mathbf{r},t) ,$$

where $\phi^{(1)}$ and $\phi^{(2)}$, respectively, stand for two states with the different eigenvalues and the normalization coefficients for ϕ are included in $\phi^{(1)}$ and $\phi^{(2)}$. We also provide two detectors, which, respectively, detect only one of the states $\phi^{(1)}$ (detector I) and $\phi^{(2)}$ (detector II). The quantum states of the two detectors are described in terms of the wave functions $\psi_0^{\mathrm{I}}(\mathbf{r}_1,\ldots,\mathbf{r}_{N_1};t)$ and $\psi_0^{\mathrm{II}}(\mathbf{s}_1,\ldots,\mathbf{s}_{N_2};t)$, where $\mathbf{r}_i$ and $\mathbf{s}_i$, respectively, denote the positions of the constituent particles of detectors I and II which are constructed by N_1 and N_2 particles with the same mass m. Then the wave function Ψ_0 for two detectors is written by the direct product of ψ_0^{I} and ψ_0^{II} as

$$\Psi_0(\mathbf{R},t)=\psi_0^{\mathrm{I}}(\mathbf{r}_1,\ldots,\mathbf{r}_{N_1};t)\otimes\psi_0^{\mathrm{II}}(\mathbf{s}_1,\ldots,\mathbf{s}_{N_2};t) ,$$

where $\mathbf{R}$ stands for N_1+N_2 coordinate variables corresponding to $\mathbf{r}_i$ and $\mathbf{s}_i$. Hereafter we put $N_1=N_2=N$ for simplicity.

The total wave function including the object and the detectors is represented with the direct product of ϕ and Ψ_0 as

$$\Phi_0=\phi\otimes\Psi_0$$

Following the ordinary quantum-mechanical treatment, we describe the change of the wave function during the interaction of the object with the detectors as

$$\begin{aligned}\Phi_I&=\phi(\mathbf{r},t_0)\otimes\Psi_0(\mathbf{R},t_0)\\ &\to\Phi_F=\phi^{(1)}(\mathbf{r},t_F)\otimes\Psi_F^{(1)}(\mathbf{R},t_F)\\ &\quad+\phi^{(2)}(\mathbf{r},t_F)\otimes\Psi_F^{(2)}(\mathbf{R},t_F) .\end{aligned} \tag{2.1}$$

Now we represent the interaction of the object with the detectors in terms of the ionization process of the constituent atoms in the detectors which are electrically neutral before the interaction. After the ionization takes place, the state of the detector changes by means of the interaction with the external electric field E. When n_1 atoms in detector I and n_2 in detector II are ionized, the Hamiltonian is expressed as

$$H^{(n_1,n_2)}=\frac{1}{2m}\sum_{i=1}^{N}(\mathbf{p}_{r_i}^2+\mathbf{p}_{s_i}^2)+e\sum_{i=1}^{n_1}\mathbf{r}_i\cdot\mathbf{E}+e\sum_{i=1}^{n_2}\mathbf{s}_i\cdot\mathbf{E} , \tag{2.2}$$

where $\mathbf{p}_{r_i}$ ($\mathbf{p}_{s_i}$) stands for the conjugate momentum of $\mathbf{r}_i$ ($\mathbf{s}_i$). Since the electric field is required to be strong enough to produce the macroscopic movement of the detector system, we postulate that E has a macroscopic order proportional to N, as was done by Fukuda.

After the interaction of the detectors with the object at $t=t_F$, the wave function of the detectors can be written in terms of a superposition of the states with the different number of ionized atoms, i.e.,

$$\begin{aligned}\Psi_F^{(1)}(\mathbf{R},t_F)&=\left[\sum_{n=0}^{N}c_n\psi_n^{\mathrm{I}}(\mathbf{r}_1,\ldots,\mathbf{r}_N;t_F)\right]\\ &\quad\otimes\psi_0^{\mathrm{II}}(\mathbf{s}_1,\ldots,\mathbf{s}_N;t_F) ,\\ \Psi_F^{(2)}(\mathbf{R},t_F)&=\psi_0^{\mathrm{I}}(\mathbf{r}_1,\ldots,\mathbf{r}_N;t_F)\\ &\quad\otimes\left[\sum_{n=0}^{N}c_n\psi_n^{\mathrm{II}}(\mathbf{s}_1,\ldots,\mathbf{s}_N;t_F)\right]\end{aligned} \tag{2.3}$$

In (2.3) the coefficients c_n should be determined in the quantum-mechanical process for the ionization and in the large-N limit they may be normalized as $\sum_{n=0}^{N}|c_n|^2=1$. Since the value of c_n is not important in the following discussions, the derivation of c_n is not discussed here. The detection is carried out by catching the movement of the

ionized system, that is, the movement of the center of mass of the detector. For this measurement process we may write the density matrix as

$$\tilde{\rho}_F \equiv \iint \Phi_F \Phi_F^* \tag{2.4}$$

where the integration $\iint$ should be carried out for all the internal relative coordinates for the two detectors, which are independent of the measured quantity, i.e., the motion of the center of mass. We call this integration "the internal trace." Explicitly it is written down as

$$\tilde{\rho}_F = \int \prod_{i=1}^{N} d\mathbf{r}_i \delta^{(3)}\left[\mathbf{G}_{\mathrm{I}} - \frac{1}{N}\sum_{i=1}^{N}\mathbf{r}_i\right] \int \prod_{j=1}^{N} d\mathbf{s}_j \delta^{(3)}\left[\mathbf{G}_{\mathrm{II}} - \frac{1}{N}\sum_{j=1}^{N}\mathbf{s}_j\right] [\phi^{(1)}\phi^{(1)*}\otimes\Psi_F^{(1)}\Psi_F^{(1)*} + \phi^{(2)}\phi^{(2)*}\otimes\Psi_F^{(2)}\Psi_F^{(2)*} + (\phi^{(1)}\phi^{(2)*}\otimes\Psi_F^{(1)}\Psi_F^{(2)*} + \text{c.c.})] \tag{2.5}$$

where $\mathbf{G}_{\mathrm{I}}$ and $\mathbf{G}_{\mathrm{II}}$, respectively, stand for the center-of-mass coordinates of the two detectors.

In order to evaluate (2.5) exactly, we give a simple model for the detector. The internal structure of the detector is postulated to be represented by the ground state of the harmonic oscillator system with the following Hamiltonian before the ionization takes place:

$$H_0^D = \frac{1}{2m}\sum_{i=1}^{N}\mathbf{p}_i^2 + \frac{k}{2N}\sum_{j>l=1}^{N}\sum^{N-1}|\mathbf{r}_j - \mathbf{r}_l|^2 = \frac{Nm}{2}\mathbf{P}_G^2 + \sum_{l=2}^{N}\left[\frac{1}{2m}\mathbf{p}_l^2 + \frac{1}{2}k\boldsymbol{\rho}_l^2\right], \tag{2.6}$$

where $\mathbf{P}_G$ and $\mathbf{p}_l$ are, respectively, the conjugate momenta of $\mathbf{G}$ and $\boldsymbol{\rho}_l$ defined by

$$\mathbf{G} = \left[\frac{1}{N}\right]\sum_{i=1}^{N}\mathbf{r}_i$$

$$\boldsymbol{\rho}_l = \sqrt{(l-1)/l}\left[\mathbf{r}_l - \frac{1}{l-1}\sum_{i=1}^{l-1}\mathbf{r}_i\right].$$

Hereafter we always put $\hbar = 1$ and $e\mathbf{E} = N(0,0,g)$. Now we may write the process as a one-dimensional problem in the z direction. From now on we neglect the uninteresting two dimensions (x,y) in our discussions. The wave function for the detector is described by

$$\psi_0(Z,t) = \psi_0(Z_G,t)\tilde{\psi}_0(z,t), \tag{2.7}$$

where Z stands for the N-coordinate variables, $\psi(Z_G,t)$ denotes the wave function of the center of mass coordinate (Z_G), and the internal wave function for the relative coordinates $\tilde{\psi}_0(z)$ with $z = (z_2, \ldots, z_N)$ is evaluated as

$$\tilde{\psi}_0(z,t) = e^{-iE_0 t}\prod_{l=2}^{N}\left[\left[\frac{\alpha}{\pi}\right]^{1/4}\exp\left[-\frac{\alpha}{2}z_l^2\right]\right] \quad \text{with } E_0 = \tfrac{1}{2}(N-1)\sqrt{k/m}, \tag{2.8}$$

where $\alpha = \sqrt{mk}$ and z_l is the z component of $\boldsymbol{\rho}_l$.

Since the total structure of the detector should not be broken by the interaction with the object, we require that the harmonic oscillator is strong enough to keep the ground state even after the ionization of n atoms. The wave function with the n-ionized atoms is also separable into that of the center-of-mass variable $\psi_n(Z_G,T)$ and that of the internal variables $\tilde{\psi}_n(z_2,\ldots,z_N;T)$ as $\psi_n(Z,T) = \psi_n(Z_G,T)\tilde{\psi}_n(z_2,\ldots,z_N;T)$. From (2.2) the interaction of the detector with the external electric field including the harmonic oscillators is given by the following Hamiltonian for the internal variables:

$$\tilde{H}^n(z_2,\ldots,z_N) = \frac{1}{2m}\sum_{l=2}^{N}(p_l^2 + mkz_l^2) - Ng\sum_{l=n+1}^{N}\frac{1}{\sqrt{l(l-1)}}z_l = \frac{1}{2m}\sum_{l=2}^{n}(p_l^2 + mkz_l^2) + \sum_{l=n+1}^{N}\left\{\frac{1}{2m}\left[p_l^2 + mk\left[z_l - \frac{Ng}{k\sqrt{l(l-1)}}\right]^2 - \frac{(Ng)^2}{2kl(l-1)}\right]\right\}. \tag{2.9}$$

The ground state of $\tilde{H}^n$ is evaluated as

$$\tilde{\psi}_n(z_2,\ldots,z_N;T) = e^{-iE_0T}e^{i\tilde{\theta}_nT}\prod_{l=2}^{n}\left[\left[\frac{\alpha}{\pi}\right]^{1/4}e^{-(\alpha/2)z_l^2}\right]\prod_{l=n+1}^{N}\left[\left[\frac{\alpha}{\pi}\right]^{1/4}e^{(-\alpha/2)(z_l - Ng/k\sqrt{l(l-1)})^2}\right], \tag{2.10}$$

where

$$\tilde{\theta}_n = \frac{(Ng)^2}{2k}\sum_{l=n+1}^{N}\frac{1}{l(l-1)} \tag{2.11}$$

and T stands for the time interval for the interaction of the ionized detector with the electric field. As was noted in (2.4) and (2.5), in the measurement of the Z_G movement all the internal variables independent of the Z_G motion must be integrated and the density matrix with the internal trace should be defined as

$$\tilde{\rho}_F = \prod_{l=2}^{N}\int dz_l^{\mathrm{I}}\prod_{m=2}^{N}\int dz_m^{\mathrm{II}}|\phi^{(1)}\otimes\Psi_F^{(1)}(Z,T) + \phi^{(2)}\otimes\Psi_F^{(2)}(Z,T)|^2, \tag{2.12}$$

where Z stands for the $2N$ variable for the two detectors. Then $\tilde{\rho}_F$ is reduced to

$$\tilde{\rho}_F=\phi^{(1)}\phi^{(1)*}\otimes A_{11}+\phi^{(2)}\phi^{(2)*}\otimes A_{22}+\phi^{(1)}\phi^{(2)*}\otimes A_{12}+\phi^{(2)}\phi^{(1)*}\otimes A_{21}\ , \tag{2.13}$$

where

$$\begin{aligned}
A_{11}&=\sum_{m=0}^{N}\sum_{n=0}^{N}c_m c_n^{*}\psi_m^{\mathrm{I}}(Z_G^{\mathrm{I}},T)\psi_n^{\mathrm{I}*}(Z_G^{\mathrm{I}},T)\\
&\quad\times\prod_{l=2}^{N}\int dz_l^{\mathrm{I}}\tilde{\psi}_m^{\mathrm{I}}(z^{\mathrm{I}},T)\tilde{\psi}_n^{\mathrm{I}*}(z^{\mathrm{I}},T)\otimes\psi_0^{\mathrm{II}}(Z_G^{\mathrm{II}},T)\psi_0^{\mathrm{II}*}(Z_G^{\mathrm{II}},T)\prod_{l=2}^{N}\int dz_l^{\mathrm{II}}\tilde{\psi}_0^{\mathrm{II}}(z^{\mathrm{II}},T)\tilde{\psi}_0^{\mathrm{II}*}(z^{\mathrm{II}},T)\ ,\\
A_{22}&=A_{11}(\mathrm{I}\leftrightarrow\mathrm{II})\ ,\\
A_{12}&=\sum_{m=0}^{N}c_m\psi_m^{\mathrm{I}}(Z_G^{\mathrm{I}},T)\psi_0^{\mathrm{I}*}(Z_G^{\mathrm{I}},T)\\
&\quad\times\prod_{l=2}^{N}\int dz_l^{\mathrm{I}}\tilde{\psi}_m^{\mathrm{I}}(z^{\mathrm{I}},T)\tilde{\psi}_0^{\mathrm{I}*}(z^{\mathrm{I}},T)\otimes\sum_{n=0}^{N}c_n^{*}\psi_0^{\mathrm{II}}(Z_G^{\mathrm{II}},T)\psi_n^{\mathrm{II}*}(Z_G^{\mathrm{II}},T)\prod_{l=2}^{N}\int dz_l^{\mathrm{II}}\tilde{\psi}_0^{\mathrm{II}}(z^{\mathrm{II}},T)\tilde{\psi}_n^{\mathrm{II}*}(z^{\mathrm{II}},T)\ ,\\
A_{21}&=A_{12}^{*}\ ,
\end{aligned} \tag{2.14}$$

where $\mathrm{I}\leftrightarrow\mathrm{II}$ in A_{11} indicates that terms with I and II are to be interchanged. The integrations over the internal variables are performed as follows for $n\geq m$

$$\begin{aligned}
I_{mn}&\equiv\prod_{l=2}^{N}\int dz_l\tilde{\psi}_m(z,T)\tilde{\psi}_n^{*}(z,T)\\
&=e^{i(\tilde{\theta}_m-\tilde{\theta}_n)T}\exp\left[-\frac{\alpha(Ng)^2}{4k^2}\left[m^2\sum_{l=m+1}^{n}\frac{1}{l(l-1)}+(m-n)^2\sum_{l=n+1}^{N}\frac{1}{l(l-1)}\right]\right].
\end{aligned} \tag{2.15}$$

We immediately see that in the macroscopic limit $N\rightarrow\infty$

$$\lim_{N\rightarrow\infty}I_{m,n}=\delta_{m,n}\ . \tag{2.16}$$

It should be stressed that the internal trace described by the integrations over all the internal coordinates produces the nonoverlapping property among the states with the different number of ionized atoms. Note also that the macroscopic phase factor $(\tilde{\theta}_m-\tilde{\theta}_n)T$ in (2.15) has no essential role to derive (2.16). In the limit we have

$$\begin{aligned}
\bar{A}_{11}&=\left[\sum_{m=0}^{\infty}|c_m|^2|\psi_m^{\mathrm{I}}(Z_G^{\mathrm{I}},T)|^2\right]|\psi_0^{\mathrm{II}}(Z_G^{\mathrm{II}},T)|^2\ ,\\
\bar{A}_{22}&=|\psi_0^{\mathrm{I}}(Z_G^{\mathrm{I}},T)|^2\left[\sum_{n=0}^{\infty}|c_n|^2|\psi_n^{\mathrm{II}}(Z_G,T)|^2\right]\ ,\\
\bar{A}_{12}&=\bar{A}_{21}=|c_0|^2|\psi_0^{\mathrm{I}}(Z_G^{\mathrm{I}},T)|^2|\psi_0^{\mathrm{II}}(Z_G^{\mathrm{II}},T)|^2\ ,
\end{aligned}$$

where

$$\bar{A}_{ij}\equiv\lim_{N\rightarrow\infty}A_{ij}\ . \tag{2.17}$$

The density matrix is written down in the limit as

$$\begin{aligned}
\bar{\rho}_F&\equiv\lim_{N\rightarrow\infty}\tilde{\rho}_F=\phi^{(1)}\phi^{(1)*}\bar{A}_{11}+\phi^{(2)}\phi^{(2)*}\bar{A}_{22}\\
&\quad+(\phi^{(1)}\phi^{(2)*}+\phi^{(2)}\phi^{(1)*})\bar{A}_{12}\ .
\end{aligned} \tag{2.18}$$

Finally the trace with respect to the center of mass gives the density matrix only for the quantum object, that is,

$$\begin{aligned}
\rho_F&\equiv\mathrm{Tr}_G\bar{\rho}_F\equiv\int\int_{-\infty}^{\infty}dZ_G^{\mathrm{I}}dZ_G^{\mathrm{II}}\bar{\rho}_F\\
&=\phi^{(1)}\phi^{(1)*}+\phi^{(2)}\phi^{(2)*}+(\phi^{(1)}\phi^{(2)*}+\phi^{(2)}\phi^{(1)*})|c_0|^2\\
&=(1-|c_0|^2)(\phi^{(1)}\phi^{(1)*}+\phi^{(2)}\phi^{(2)*})+|c_0|^2(\phi^{(1)}+\phi^{(2)})(\phi^{(1)*}+\phi^{(2)*})\ ,
\end{aligned} \tag{2.19}$$

where $\sum_{n=0}^{\infty}|c_n|^2=1$ is used. In (2.19) ρ_F still has the interference term proportional to $|c_0|^2$. Then we may say that $|c_0|^2$ is the inefficiency of the detector as was mentioned by Fukuda.[3] The existence of the interference term is quite natural because the elastic forward scattering represented by the $|c_0|^2$ term does not change the detector state at all and then the measurement is not carried out. For the complete measurement the $\lim_{N\rightarrow\infty}|c_0|^2\rightarrow 0$ is also required.

Here we briefly comment on the nonoverlapping property derived in (2.16). The disappearance of the integration given in (2.15) can generally be proved in the case where the integrant of (2.15) includes any polynomial functions of $z_1,\ldots,z_l$. This property can be generalized for all the eigenfunctions of $\tilde{H}^{n}$ given in (2.9). This fact means that the Hilbert spaces labeled with the number of ionized atoms n become orthogonal with each other in the marcoscopic limit $N\rightarrow\infty$.

Our idea for the disappearance of the interference terms is quite different from Fukuda's idea presented in Secs. 2–4 of Ref. 3, where he tried to interpret it in terms of decreasing property of the overlapping of the wave functions for the center-of-mass variable $[\psi_m(Z_G,T)\psi_0^*(Z_G,T)$ in (2.14)]. Before going to the details of the discussion about the difference which will be carried out in Sec. IV, we would like to present the next model in order to make our idea clearer.

Note that in the above argument we do not need two detectors I and II in order to vanish the interference. Actually the disappearance of the interference terms except the $|c_0|^2$ term can easily be shown even if one of them is taken off from our measurement system. Then it is trivial that our model leads the wave-packet reduction in the so-called negative-result measurement.

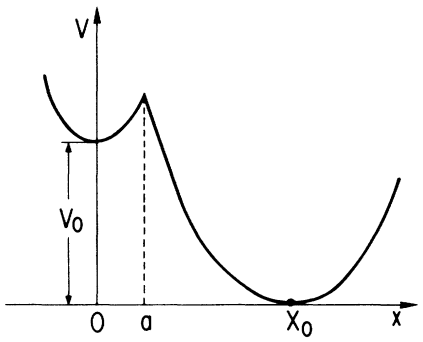

FIG. 1. Potential with two minima represented by two harmonic oscillators.

III. MEASUREMENT BY DETECTORS WITH QUASISTABLE INTERNAL STATES (MODEL II)

In this section we study the case where detectors are set up to be in a quasistable state before the interaction with the quantum object like the Wilson chamber or the bubble chamber. In such detectors the interaction of the detector with the quantum object plays the role of a trigger for inducing the macroscopic physical quantity of the detector.

As a simple example we provide the following detector in one dimension (say x). The detector is constructed by N atoms with the same mass m, which are bounded by a harmonic oscillator potential with two minima as shown in Fig. 1. We approximate the potential for all atoms as

$$V(x)=\begin{cases}\frac{1}{2}kx^2+V_0 & \text{for } x<a\\ \frac{1}{2}K(x-X_0)^2 & \text{for } x>a\ ,\end{cases} \tag{3.1}$$

where

$$V_0=\tfrac{1}{2}K(a-X_0)^2-\tfrac{1}{2}ka^2>0$$

is required and the interaction among the atoms are neglected. The relation

$$V_0\gg\tfrac{1}{2}ka^2$$

is postulated in the following discussions. As an ideal example all atoms are set up to be in the ground state of the potential $\frac{1}{2}kx^2+V_0$ corresponding to the local minimum at $x=0$, that is, the wave function of the detector before the interaction with the object is written approximately as

$$\psi_0(x_1,\ldots,x_N;t)=\left[\frac{\alpha}{\pi}\right]^{N/4}\exp\left[-\frac{\alpha}{2}\sum_{i=1}^{N}x_i^2\right]e^{-iE_0t}\ , \tag{3.2}$$

where $E_0=N(V_0+\frac{1}{2}\omega_k)$ with $\omega_k=\sqrt{k/m}$ and $\alpha=\sqrt{mk}$. In order that the detector is stable in a certain time scale before the measurement, the condition

$$\alpha a^2\gg 1$$

should be satisfied.

The measurement process is figured out as follows. By the interaction with the object a certain number of the atoms are excited. Some of them, which are given energies enough to overcome the potential barrier $\frac{1}{2}ka^2$, transit to excited states of the potential $\frac{1}{2}K(x-X_0)^2$ for the real minimum at $x=X_0$. They again transit themselves to the ground state of the potential $\frac{1}{2}K(x-X_0)^2$, losing the energy $\sim V_0$, that is, partially giving the energy to the other atoms and partially emitting the energy to the outside of the detector. Such interactions and transitions successively take place like a snowslide phenomena, until a new stable state is realized in the detector. In the new stable state a certain large number of atoms (say n) is in the ground state for the real minimum. The detector in the state is described as

$$\psi_n(x_1,\ldots,x_N;t)=e^{-iE_nt}\prod_{i=1}^{n}\left[\left[\frac{\beta}{\pi}\right]^{1/4}e^{-(\beta/2)(x_i-X_0)^2}\right]\prod_{j=n+1}^{N}\left[\left[\frac{\alpha}{\pi}\right]^{1/4}e^{-(\alpha/2)x_j^2}\right], \tag{3.3}$$

where $E_n=(N-n)(V_0+\frac{1}{2}\omega_k)+\frac{1}{2}n\omega_K$ with $\omega_K=\sqrt{K/m}$ and $\beta=\sqrt{mK}$. The total energy emitted in the above process is estimated by

$$E_e=n(V_0+\tfrac{1}{2}\omega_k-\tfrac{1}{2}\omega_K)\ , \tag{3.4}$$

where we may put

$$V_0+\tfrac{1}{2}(\omega_k-\omega_K)\simeq V_0\ .$$

Provided that the detector is set up to have a property

$$n\sim qN \quad \text{with } 0<q\leq 1\ , \tag{3.5}$$

the above energy E_e becomes a macroscopic energy in the limit $N\to\infty$ and can play a role for the physical quantity

to be measured in this process.

We put the above detector into the measurement of the same quantum object ϕ presented in Sec. II. In this model the wave functions of the detector are, respectively, represented by ψ_0 in Eq. (3.2) before the interaction and by

$$\psi_F=\sum_{n=0}^{N} c_n\psi_n(x_1,\ldots,x_N;t) \tag{3.6}$$

after the interaction, where ψ_n is given in (3.3) and c_n are the coefficients determined in the quantum-mechanical process. It is noticed that the normalization $\sum_n|c_n|^2=1$ given in model I of Sec. II should not be imposed here, because not all states with different n become orthogonal even in the limit $N\to\infty$ as will be seen later in this section. The normalization for ψ_F is evaluated as follows:

$$\prod_{i=1}^{N}\int_{-\infty}^{\infty}dx_i|\psi_F|^2 = \sum_{l=0}^{N}\sum_{n=0}^{N} c_l^* c_n e^{-(A/2)|n-l|}e^{i(E_l-E_n)t}, \tag{3.7}$$

where

$$A=\frac{\alpha^2\beta^2}{\alpha^2+\beta^2}+\ln\frac{\alpha^2+\beta^2}{2\alpha\beta}\ .$$

Since the evaluation of c_n, which is basically calculable, is not the main theme of this paper, we postulate the following form for c_n for simplicity:

$$c_n=e^{i\theta n}\frac{B(t)}{(\sqrt{2\pi}\sigma)^{1/2}}e^{-(1/4\sigma^2)(n-\langle n\rangle)^2}, \tag{3.8}$$

where c_n is normalized as $\int_{-\infty}^{\infty}dn|c_n|^2=|B|^2$

$$\langle n\rangle=qN \quad \text{with } 0<q\le 1\ [q\sim O(1)]\ ,$$
$$\sigma=\sigma_0 N \quad \text{with } \sigma_0=\text{const}\sim O(1)\ , \tag{3.9}$$
$$\theta_n=\omega_0 n \quad \text{with } \omega_0=\text{const}\ ,$$

and B is the normalization coefficient for ψ_F. The relation for $\langle n\rangle$ and σ in (3.9) must be required to produce the macroscopic energy emission proportional to N and also to guarantee that the fluctuation of the energy ΔE_e is of the order of $\sqrt{N}$. For the large-N limit the summation over l and n in (3.7) may be replaced by the integrations and we obtain

$$|B(t)|^2=(A+4\tilde{\omega}^2)/4A \quad \text{with} \tag{3.10}$$
$$\tilde{\omega}=\omega_0+[V_0+\tfrac{1}{2}(\omega_k-\omega_K)]t\simeq\omega_0+V_0 t\ .$$

In (3.10) we see that, since $A\neq 0$, the value of $\tilde{\omega}$ is not important, that is, the following discussion does not change at all, even if we put $\tilde{\omega}=0$. This fact that the phase factor does not play any important role, even if it is a macroscopic number, is the same as the situation for the phase $\tilde{\theta}_n T$ noted in model I.

Now we evaluate the density matrix corresponding to (2.4) by

$$\tilde{\rho}_F=\prod_{i=1}^{N_1}\int_{-\infty}^{\infty}dx_i^{\mathrm{I}}\prod_{j=1}^{N_2}\int_{-\infty}^{\infty}dx_j^{\mathrm{II}}\Phi_F\Phi_F^*\ , \tag{3.11}$$

where the internal trace is represented by the integrations over all internal variables. We immediately see that the integrations for $\Psi^{(1)}\Psi^{(1)*}$ and $\Psi^{(2)}\Psi^{(2)*}$ are the same as that of the normalization, where

$$\Psi^{(1)}(x^{\mathrm{I}},x^{\mathrm{II}})=\left[\sum_{n=0}^{N}c_n\psi_n^{\mathrm{I}}(x^{\mathrm{I}})\right]\psi_0^{\mathrm{II}}(x^{\mathrm{II}})$$

and

$$\Psi^{(2)}(x^{\mathrm{I}},x^{\mathrm{II}})=\psi_0^{\mathrm{I}}(x^{\mathrm{I}})\left[\sum_{n=0}^{N}c_n\psi_n^{\mathrm{II}}(x^{\mathrm{II}})\right]$$

with $x^{\mathrm{I}}\equiv(x_1^{\mathrm{I}},\ldots,x_N^{\mathrm{I}})$ and $x^{\mathrm{II}}\equiv(x_1^{\mathrm{II}},\ldots,x_N^{\mathrm{II}})$. The integrations for the interference terms with $\Psi^{(1)}\Psi^{(2)*}$ and $\Psi^{(2)}\Psi^{(1)*}$ can be performed as follows in the limit $N_1=N_2=N\to$large:

$$\left|\prod_{i=1}^{N}\prod_{j=1}^{N}\int\int_{-\infty}^{\infty}dx_i^{\mathrm{I}}dx_j^{\mathrm{II}}\Psi_F^{(1)}\Psi_F^{(2)*}\right|\le\sum_{l=0}^{N}|c_l e^{-(A/2)l}|\sum_{n=0}^{N}|c_n^* e^{-(A/2)n}|\propto\begin{cases}\sqrt{N}\,e^{-(A/2)(2q-A\sigma_0^2)N} & \text{for } A\sigma_0^2\le q\\ \dfrac{1}{\sqrt{N}}e^{-(q^2/2\sigma_0^2)N} & \text{for } A\sigma_0^2>q\ ,\end{cases} \tag{3.12}$$

where the summations over l and n are carried out after the replacement with the integrations. In both cases the integration goes to zero in the limit $N\to\infty$, that is, the interference vanishes in the macroscopic limit $N\to\infty$.

The above argument is not enough to show the disappearance of the interference terms, because the observed quantity is the mean value of n. In the large-N limit we can easily derive the expectation value of n (say $\langle\langle n\rangle\rangle$) and the fluctuation of n (say Δn) as follows:

$$\langle\langle n\rangle\rangle\equiv\lim_{N\to\text{large}}\prod_{i=1}^{N}\int\int_{-\infty}^{\infty}dx_i^{\mathrm{I}}dx_j^{\mathrm{II}}\sum_{l=0}^{N}\sum_{n=0}^{N}nc_n^*c_l\psi_n^*\psi_l = \langle n\rangle=qN\ , \tag{3.13}$$

$$\Delta n\equiv[\langle\langle(n-\langle\langle n\rangle\rangle)^2\rangle\rangle]^{1/2}=\sigma=\sigma_0 N\ . \tag{3.14}$$

The observed energy in the measurement is expected to be

$$E_{\exp}\equiv\langle\langle n\rangle\rangle V_0\pm\Delta n V_0=\langle n\rangle V_0\pm\sigma V_0\ , \tag{3.15}$$

where $\pm\Delta n V_0=\pm\sigma V_0$ is the experimental error. As far as the on-off measurement discussed here is concerned, the relation

$$\lim_{N\to\infty}\Delta n V_0/\langle\langle n\rangle\rangle V_0=0 \tag{3.16}$$

is enough to detect the object. We shall discuss the cri-

terion for the good detector in the measurement of the energy value in Sec. V. For the interference terms we can easily prove that the expectation values for any polynomials of n vanish in the limit $N\to\infty$ because of the exponentially dumping property for N as was given in (3.12).

Here we comment on the interesting difference between the models I and II. As was seen in (3.10), $|B|^2\neq 1$ even in the case $k=K$ and $\omega_0=0$. This fact indicates that even in the limit $N\to\infty$ the wave functions with different numbers of n are not always orthogonal with each other. We can actually show

$$\lim_{N\to\infty} I_{n,l}\equiv \lim_{N\to\infty}\prod_{i=1}^{N}\int_{-\infty}^{\infty} dx_i\psi_n^*\psi_l \propto \lim_{N\to\infty} e^{-(A/2)|n-l|}\neq 0 \quad \text{for } |n-l|\sim O(1)\ . \tag{3.17}$$

In the limit $N\to\infty$, however, those contribution does vanish in the summations over l and n in the interference terms given in (3.12). This situation in model II is quite different from that of model I, where all different wave functions become orthogonal with each other in the limit $N\to\infty$ and therefore we do not require any constraints for c_n.

IV. COMPARISON WITH THE OTHER MODELS

Our basic idea is similar to Fukuda's,[3] that is, in the macroscopic limit $N\to\infty$ where the observed quantity becomes a macroscopic variable, the interference vanishes. His idea for the disappearance of the interference is, however, quite different from ours. The difference is clearly seen in model I. Following his discussion given in Secs. 2–4 of Ref. 3, the disappearance is interpreted in terms of the decreasing property of the overlapping between the wave packets for the center-of-mass variable of the detector. That is, when the wave packet for the center-of-mass variable Z_G is given by

$$\psi_0(Z_G)=\left[\frac{1}{\sqrt{\pi}a}\right]^{1/2}\exp\left[-\frac{Z_G^2}{2a^2}\right] \tag{4.1}$$

before the interaction with the quantum object, the overlap function for the wave packets with the different number of the ionized atoms after the interaction, i.e., the factors $\psi_n^{\mathrm{I}}(Z_G^{(1)},T)\psi_0^{\mathrm{I}*}(Z_G^{(1)},T)$ and $\psi_0^{\mathrm{II}}(Z_G^{(2)},T)\psi_n^{\mathrm{II}*}(Z_G^{(2)},T)$ of A_{12} in (2.14), is evaluated as follows:

$$\frac{1}{1+T^2/(a^2Nm)^2}\left[\frac{1}{\sqrt{\pi}a}\right]^{1/2}\exp\left[-\frac{\Delta Z_G^2}{4a^2(1+\rho)}\right], \tag{4.2}$$

where T, N, and m have the same meanings as those in our case, $\rho=(T/aNm)^2$, and

$$\Delta Z_G=\frac{ng}{2m}T^2\ . \tag{4.3}$$

In his argument, therefore, the ideal limit to vanish the interference is represented by

$$N\to\infty,\quad \Delta Z_G/a\to\infty\quad (\text{then } T\to\infty) \tag{4.4}$$

as was given in Eq. (2.53) of his paper.[3] He needs not only the limit $N\to\infty$ but also the limit $T\to\infty$, that is, a macroscopic time scale, while such a time scale is not required in our model. The difference arises from the fact that the contribution of the variables of the detector except the center-of-mass variable are completely neglected in his argument for the disappearance of the interference, whereas those variables play an essential role for vanishing the interference in our model. His idea, that is, the wave functions of the detector labeled with $n(\psi_n)$ belong to the different Hilbert spaces which are orthogonal with each other in the limit $N\to\infty$, is right. The orthogonality among the Hilbert spaces is, however, realized by the internal trace of the detector defined in (2.5) of Sec. II, not by the decreasing property of the overlap function between the wave packets for the center-of-mass variable of the detector. We can never understand the separation of the detector wave functions into different Hilbert spaces in the limit $N\to\infty$ alone unless the internal trace is correctly introduced.

The Machida-Namiki theory[1] has a quite different idea from ours, that is, in their theory the Hilbert space describing the detector is represented by a continuous direct sum of Hilbert spaces such as

$$\int dl\, w(l)\mathcal{H}(l)\ , \tag{4.5}$$

where $w(l)$ denotes a weight function normalized as $\int dl\, w(l)=1$ and $\mathcal{H}(l)$ is the Hilbert space of systems with sharp size l. The density matrix for the detector space is given by

$$\sigma^D=\int dl\, w(l)\rho^D(l)\ , \tag{4.6}$$

where $\rho^D(l)$ stands for the density matrix of a local system with size l. After the interaction with the quantum object the integration over l vanishes the interference. We can not say whether their idea for the direct sum of Hilbert spaces is really understandable in a certain limit of the quantum theory. The difference of our model from their theory is, however, very clear, because no quantity and no process not well describable and not calculable in quantum mechanics are introduced in our model except for the macroscopic limit $N\to\infty$ in the final step, while the sum over macroscopic variable l is introduced *ad hoc* in their theory. That is to say, the internal trace in our model is exactly defined within quantum mechanics, while the sum over l in their model is introduced as the superselection rule.[2] An interesting difference is also seen in the mechanism for the disappearance of the interference, that is, phases proportional to the macroscopic variable l due to the interaction of the object with the detector is important to realize the disappearance in their theory, whereas such phases do not play any essential role in our models though the phase proportional to the macroscopic variable N does appear in our evaluations [see (2.15) and (3.8)].

We cannot refer all models for the quantum measurement here. We do, however, want to stress that the advantage of our models is the fact that all processes and states relating to the measurement, such as spectral decomposition of the quantum object, interaction of the

object with detectors, change of quantum states of the detector after the interaction with the object, etc. can be defined and described in terms of the words of the ordinary quantum mechanics before taking the macroscopic limit $N\to\infty$ which is always carried out after all the evaluations are finished according to the quantum-mechanical treatment.

V. REMARKS ON DETECTORS AND THE INTERNAL TRACE

As was noted in Sec. IV our model for the quantum measurement is completely definable and calculable in the ordinary quantum mechanics. Therefore we can fundamentally estimate whether a certain process plays a role of a good detector or not. Such criteria for the good detector may be summarized as the following two points.

1. Criterion A. We have shown that a macroscopic change in the detector, whose effect is taken in as the internal trace in the density matrix, induces the wave-packet reduction. Then the first criterion is that a measurement apparatus must have a mechanism for inducing a macroscopic change described as the internal trace in the density matrix.

We should, however, pay attention to the important difference between the internal traces given in the two models I and II. In model I the macroscopic change is induced by the external field, while such a change is initially provided in the detector itself in model II. The difference is clearly seen in the comparison of (2.16) and (3.17), which are given by $\lim_{N\to\infty} I_{m,n}=\delta_{m,n}$ in Eq. (2.16), and $\lim_{N\to\infty} I_{n,l}\neq 0$ for $|n-l|\sim O(1)$ in Eq. (3.17). Namely, (2.16) for model I tells us that the wave-packet reduction is realized even in the case where only one atom is ionized in the detector. (Such a situation is described by $c_n=\delta_{n,1}$.) On the other hand the wave-packet reduction does not take place, unless a macroscopic number of constituent particles expressed as $\langle n\rangle \propto N$ in (3.8) and (3.9) change their states in the detector in model II. That is, the detector must be set up so that the contribution arising from the region with nonzero limits for $I_{0,l}$ to the interference terms may vanish in the macroscopic limit. (This is realized by the choice of c_n given in (3.8) in model II.)

We may understand that detectors should be grouped into the following two types. One of them is characterized by the feature that the macroscopic change in the detector is due to the external force in the enhancement mechanism (the model I type). Another type is characterized by the change of the macroscopic number of constituent particles in the detector (the model II type). Then the model II type detector must be set up to be in a quasistable internal state before the measurement.

2. Criterion B. Though criterion *A* is enough to realize the wave-packet reduction, it is not enough to say whether the detector is good or not. In the case of the model I type detectors no more constraint is required for the wave-packet reduction except the inefficiency arising from the elastic forward scattering [see (2.19)]. For the good detector, therefore, one more constraint, that is, $\lim_{N\to\infty}|c_0|^2\to 0$, is required. This may be realized in the case where the number of constituent particles of the detector which change their states during the interaction with the quantum object becomes a macroscopic number in the limit $N\to\infty$.

For the model II type detectors we need a little more precise treatment for the coefficients c_n. As was mentioned in (3.16), experiments have no meaning if the error, i.e., the fluctuation of the observed quantity, has the same order of magnitude for N as the experimental value. We can not provide any accurate experiment. A simple example of a bad detector for model II is described by the case with $c_n=B/\sqrt{N}$, which guarantees the normalization of ψ_F and the disappearance of the interference. In such a choice, however, we obtain

$$\langle\langle n\rangle\rangle \propto N,\quad \Delta n \propto N\ ,$$

which are defined in (3.13) and (3.14). It means that the ratio given in (3.16) does not vanish even in the limit $N\to\infty$ and we can not make the experimental error small enough in comparison with the experimental value. In order to make a good detector of the model II type, we must design the detector so as to have a process inducing the macroscopic change which produces the coefficient c_n having the following property: c_n has a narrow peak at $n=\langle n\rangle$ proportional to N so as to satisfy the relation $\Delta n/\langle\langle n\rangle\rangle\to 0$ in the limit $N\to\infty$. This is the criterion for a good detector of the model II type.

Here we would like to comment on the description of the internal states in terms of harmonic oscillators which is used in both models. It is easily seen that the expression by harmonic oscillators is not important. The important point is that each change of the states for the constituent particles in the detector brings a coefficient (say Q) evaluated as the overlap integral whose modulus is always smaller than 1 because of the normalization of the wave functions. In model I the interaction of the ionized atoms with the electric external field is proportional to N as $Ng\sum_{i=1}^{n} z_i$. For simplicity let us discuss the case where only one atom in the detector is ionized, whose coordinate is denoted by z. The original potential $V_0(z)$ before the ionization has a minimum at a finite position of z, which corresponds to the peak of the wave function with no ionized atom ($\tilde{\psi}_0$). On the other hand the minimum of the potential with one ionized atom given by $V_0(z)+Ngz$ moves with N and its position goes to infinity in the limit $N\to\infty$. This indicates that the peak of the wave function with one ionized atom ($\tilde{\psi}_1$) also becomes infinite with $N\to\infty$ [as an example, see (2.10)]. Now it is trivial that the internal trace described by the overlap integral for $\tilde{\psi}_0$ and $\tilde{\psi}_1$ vanishes in the limit $N\to\infty$ [see $\lim_{N\to\infty} I_{0,1}=0$ in (2.16)]. We may say that the harmonic oscillator expression is not important for the derivation of the wave-packet reduction in model I.

In the case of model II the situation is a little different from that of model I, as it was seen that the overlap integral Q in model II whose modulus is evaluated as

$$|Q| = \left| \prod_{i=1}^{N} \int_{-\infty}^{\infty} dx_i \psi_1^*(x_1, \ldots, x_N) \psi_0(x_1, \ldots, x_N) \right| = e^{-A/2} < 1$$

does not vanish in the limit $N \to \infty$. This situation is general in the model II type, because Q describes the change of only one particle, i.e., a microscopic transition of one particle, and does not include any macroscopic change like that induced by the macroscopic external field in model I. A macroscopic number (say n) of such changes in the detector, that is, $n = qN$ with $q \sim 0(1)$, produces the factor $|Q|^n = e^{-an}$ $(a > 0)$ induced by the internal trace in the interference terms, which vanishes in the macroscopic limit $N \to \infty$ as $\lim_{N\to\infty} |Q|^n \to 0$ and brings the disappearance of the interference terms. In the model II type detectors it is clear that the fact that the macroscopic number of the constituent particles in the detector change their states is essential to derive the wave-packet reduction, but the harmonic oscillator expression is not important. Now we may conclude that the harmonic oscillator expression is not important in both models.

Finally we comment on an interesting role of the internal trace. The density matrix ρ_F given in (1.2) can be decomposed into the diagonal term $\rho_{F,D}$ and the off-diagonal term $\rho_{F,\mathrm{OD}}$ as $\rho_F = \rho_{F,D} + \rho_{F,\mathrm{OD}}$, where

$$\rho_{F,D} = \sum_n |c_n|^2 |u_n\rangle\langle u_n| \otimes |\Phi_n\rangle\langle\Phi_n| ,$$

$$\rho_{F,\mathrm{OD}} = \sum_{\substack{m \\ m \neq n}} \sum_n c_m c_n^* |u_m\rangle\langle u_n| \otimes |\Phi_m\rangle\langle\Phi_n| .$$

In order to vanish the interference terms, the trace of ρ_F for the detector variables

$$\mathrm{Tr}_\Phi \rho_{F,D} = \sum_n |c_n|^2 |u_n\rangle\langle u_n| ,$$

$$\mathrm{Tr}_\Phi \rho_{F,\mathrm{OD}} = \epsilon(N) \to 0$$

is required in the limit $N \to \infty$. We, however, see that the trace of $(\rho_{F,\mathrm{OD}})^2$

$$\mathrm{Tr}_\Phi[(\rho_{F,\mathrm{OD}})^2] = \sum_n |c_n|^2 (1 - |c_n|^2) |u_n\rangle\langle u_n| + \epsilon(N)$$

does not vanish in the limit $N \to \infty$. It is clear that $\rho_{F,\mathrm{OD}}$ is not a zero operator. In our idea the density matrix should be defined by the form having the internal trace $\tilde{\rho}_F$ given in Eq. (1.3). That is, it is written as

$$\tilde{\rho}_F = \sum_m \sum_n c_m c_n^* |u_m\rangle\langle u_n| \otimes |\Phi_m^G\rangle\langle\Phi_n^G| A_{mn} ,$$

where Φ_k^G is the wave function for the variables relating to the observed quantity [$\psi_n(Z_G, T)$ in model I] and A_{mn} is the coefficient induced by the internal trace. The coefficient A_{mn} has always the property

$$|A_{mn}| \leq 1$$

because of the normalization of the wave functions. The difference of $\tilde{\rho}_F$ from ρ_F is the presence of the factor A_{mn}. Considering that the disappearance of $\tilde{\rho}_{F,\mathrm{OD}}$ is derived from the fact that

$$\lim_{N\to\infty} A_{mn} = \delta_{m,n} ,$$

we can easily show

$$\lim_{N\to\infty} \mathrm{Tr}_\Phi[(\tilde{\rho}_{F,\mathrm{OD}})^m) = 0 \quad \text{for arbitrary integer } m .$$

We may say that $\tilde{\rho}_{F,\mathrm{OD}}$ is a zero operator in the macroscopic limit. We would like again to insist that the quantum measurement should be defined by our form $\tilde{\rho}_F$, not by ρ_F. Taking into account that only a measured quantity should be left in a measurement, the integrations over the other variables independent of the measured quantity should be included in the description of the quantum theory of measurement.

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[1] S. Machida and M. Namiki, Prog. Theor. Phys. **63**, 1457 (1980); **63**, 1833 (1980); M. Namiki, Found. Phys. **18**, 29 (1988).

[2] H. Araki, Prog. Theor. Phys. **64**, 719 (1980).

[3] R. Fukuda (unpublished).

[4] R. Fukuda, Phys. Rev. A **35**, 8 (1987); **36**, 3023 (1987).