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Convection in ^{3}He-superfluid-^{4}He mixtures: Measurement of the superfluid effects

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Dilute superfluid mixtures bridge the Prandtl-number range between liquid metals and water: $0.04 < \mathcal{N}_{\rm Pr} < 2$. The convective equations of motion for superfluid mixtures are the equations for Rayleigh-Bénard convection in a normal single-component fluid plus additional superfluid terms. We have measured the latter through their effect on the critical Rayleigh number R_c. The corrections can be only a few percent for $1.0 < T < 2.0$ K, implying an accessible Prandtl-number range near onset of at least $0.04 < \mathcal{N}_{\rm Pr} < 1.5$.

Rayleigh-Bénard convection (RBC) has been the subject of much recent study.[1] An interesting variation is convection in ^{3}He-superfluid-^{4}He mixtures. Of interest here is the fact that the equations of motion for superfluid mixture convection (SMC) resemble those for ordinary RBC but with additional terms representing superfluid corrections. To the extent that the latter are small, SMC should behave like standard RBC but with a remarkable Prandtl-number range ($\mathcal{N}_{\rm Pr}$ defined below), one which is unattainable with any other fluid: $0.04 < \mathcal{N}_{\rm Pr} < 2$. The instabilities of the convective state, which strongly affect the transition to turbulence, change significantly over this Prandtl-number range.[2] SMC has the potential to be an extremely valuable tool for understanding convective flows, but the extent of the superfluid corrections must be determined first.

Previously, the size of the superfluid corrections was not well known; we have made the first direct determination. The experiments were carried out on a mixture of ^{3}He molar concentration $X=0.014$ (mass concentration $c=0.011$) and over a temperature range $1.0 < T < 1.9$ K.

Experiments on SMC were pioneered by Wheatley and co-workers,[3,4] and have been pursued more recently by Mainieri, Sullivan, and Ecke.[5] Theoretical work has been carried out by Steinberg,[6,7] Fetter,[8,9] and Steinberg and Brand.[10]

The primary dimensionless parameters for SMC are the Rayleigh number $R=|\alpha_{p,\mu_4}|gd^3\Delta T/\nu\chi_{\rm eff}$ and the Prandtl number $\mathcal{N}_{\rm Pr}=\nu_n/\chi_{\rm eff}$. Here, ΔT is the temperature difference across a fluid layer of height d. The fluid parameters, α_{p,μ_4}, $\chi_{\rm eff}$, and ν are, respectively, the expansion coefficient at constant pressure and ^{4}He chemical potential μ_4, an effective thermal diffusivity, and the kinematic viscosity; g is the acceleration of gravity. Also, $\nu_n=(\rho/\rho_n)\nu$, where ρ and ρ_n are, respectively, the total and normal fluid densities.

The superfluid corrections can be obtained in terms of three parameters, ϵ_1, ϵ_2, and ϵ_3 in Fetter's notation.[8] The first two are proportional to ΔT, which, in turn, at the onset of convection is proportional to $1/d^3$ (see the definition of R). The third parameter is proportional to d, independent of ΔT, and a function of several thermohydrodynamic quantities, including the normal viscosity and the second viscosities.[11]

In this paper, we will focus on the effect which these corrections have on the onset of convection. Fetter has shown for a horizontally infinite layer that up to second order in the ϵ_i, the critical Rayleigh number has the form

$$R_c \approx R_{c0}+24.6\epsilon_1\epsilon_2+10.2(\epsilon_1-\epsilon_2)^2 - 19.9\epsilon_2(\epsilon_1\rho/\rho_s-\epsilon_3)\,, \quad (1)$$

where R_c is the Rayleigh number at the onset of convection and $R_{c0}=1707\ldots$ is the critical Rayleigh number for classical RBC.

Equation (1) implies that $R_c \to R_{c0}$ as $d^{-2}\to 0$; thus, the correction terms can be determined by measuring the variation of R_c with d. From this viewpoint, R_c is obtained as a series in inverse powers of d:

$$R_c=R_{c0}+A(\lambda_0/d)^2+B(\lambda_0/d)^4+O(d^{-6})\,. \quad (2)$$

The first term comes from Eq. (1), as calculated by Fetter (specifically from the product $\epsilon_2\epsilon_3$). An extension of Fetter's perturbation analysis to include terms in $(\epsilon_2\epsilon_3)^2$ shows the $O(d^{-4})$ perturbation to be negligible. The sixth-order term includes products of the ϵ_i of the form ϵ_1^2, ϵ_2^2, and $\epsilon_1\epsilon_2$, as well as $(\epsilon_2\epsilon_3)^3$. The constants A and B of Eq. (2) are numerical (specifically, $A=19.9\ldots$); however, λ_0 is a function of c and T:

$$\lambda_0^2=\frac{\beta_c\chi_{\rm eff}(\zeta_1-\rho\zeta_3)(\partial c/\partial T)_{p,\mu_4}}{c\,|\alpha_{p,\mu_4}(\partial\mu_4/\partial c)_{T,P}|}\,, \quad (3)$$

where $\beta_c=-(1/\rho)\partial\rho/\partial c$. Both the effective thermal diffusion coefficient $\chi_{\rm eff}$ and the second viscosity coefficients ζ_i are poorly determined in the superfluid phase. Particularly the latter are poorly characterized, since they are measured by first- and second-sound attenuation where they appear only in combination with other considerably uncertain coefficients.

We have carried out extensive measurements to determine $R_c(d)$ using an apparatus which allowed the *in situ* variation of d. The horizontal geometry of the layer was rectangular with a length of 2.28 cm and a width of 1.01 cm. A similar arrangement was used by Gao and coworkers[12] to study convection in normal liquid ^{4}He. The present apparatus differs from that used by Gao *et al.* since the present experiment requires the use of a ^{3}He refrigerator, and since with SMC the expansion coefficient α_{p,μ_4} is negative (i.e., convection is driven by heating from above), necessitating additional complexity in the design of a variable height apparatus.

The experiments were made by fixing the temperature at the bottom boundary of the fluid layer and applying a succession of closely spaced steady heat currents Q to the top plate fluid boundary. We then determined the steady state ΔT for each Q. For each bottom plate temperature, we carried out measurements on Q and ΔT for a number of heights. Temperatures were measured using bridge circuits and germanium resistance thermometry with a temperature resolution of 0.3 μK. The height of the layer was determined by a capacitative technique[12] and indepen-

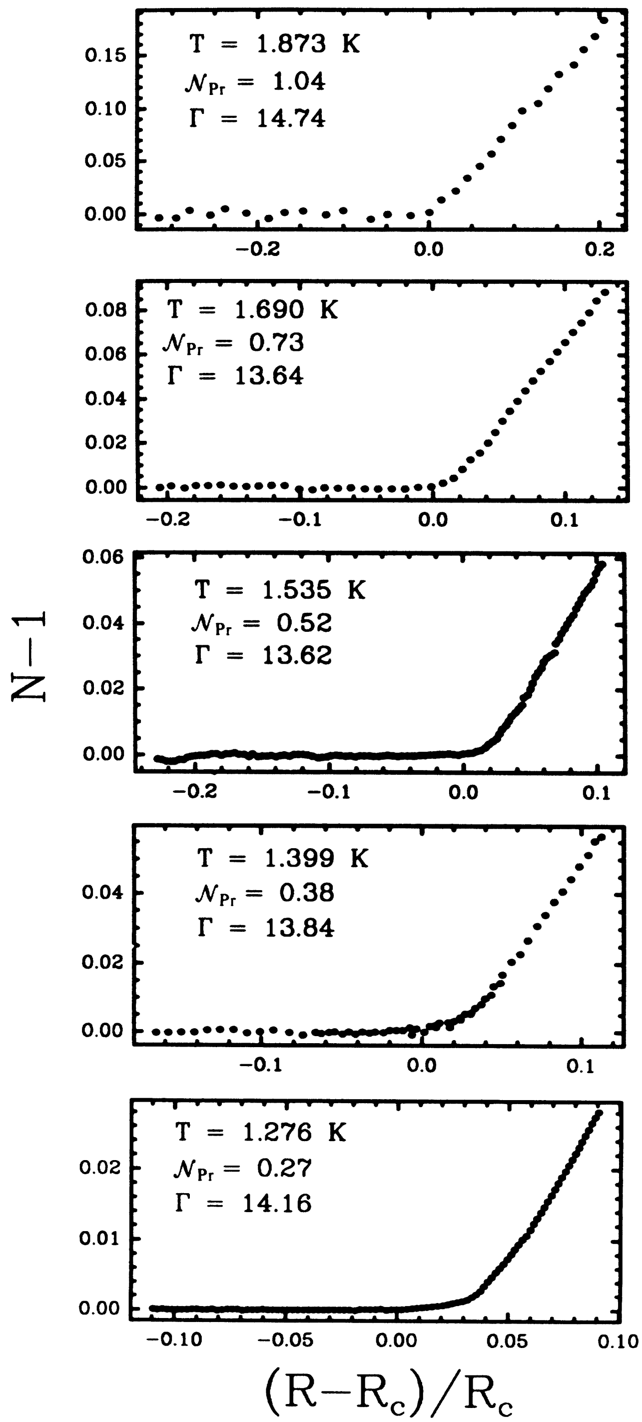

FIG. 1. Data for $N-1$ vs $r=(R-R_c)/R_c$ for five temperatures spanning most of the temperature range covered by the present experiments. These data were obtained for essentially the same height ($d=0.1633$ cm, within 0.0083 cm).

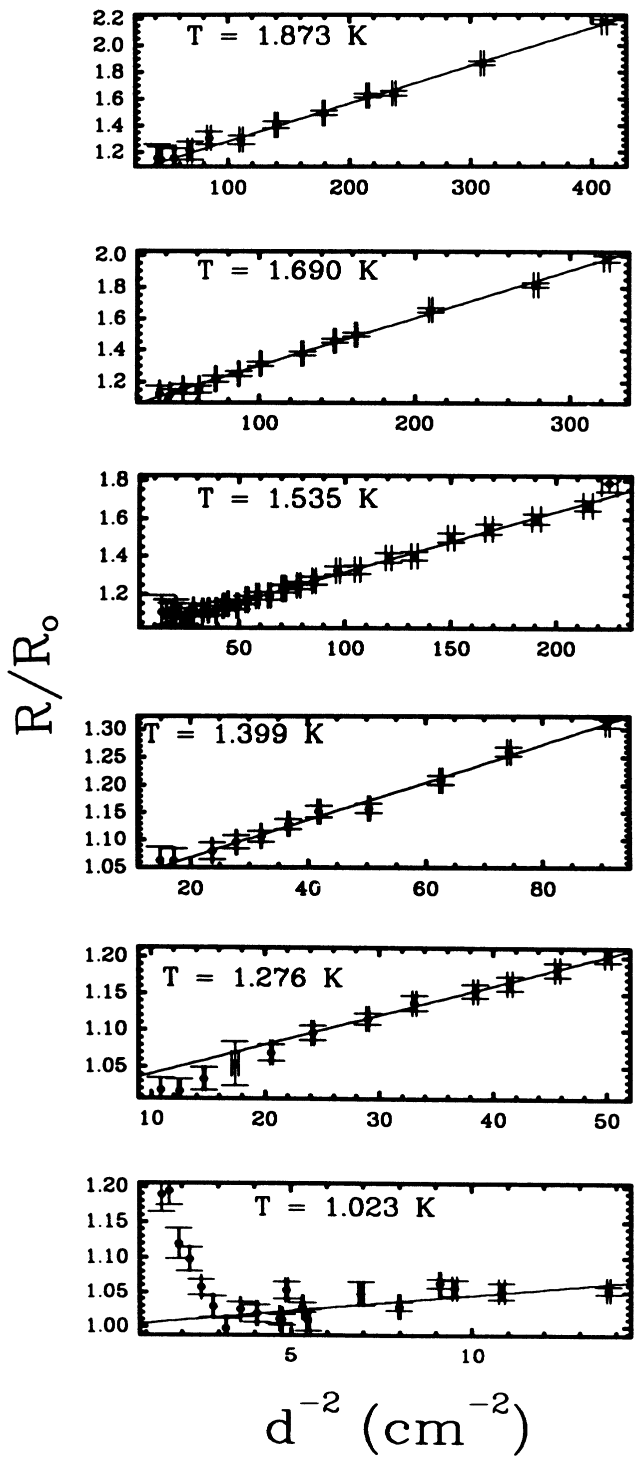

FIG. 2. Data for R_c/R_{c0} vs d^{-2}, the inverse square of the layer height. The mean temperature and Prandtl number of each data set is indicated.

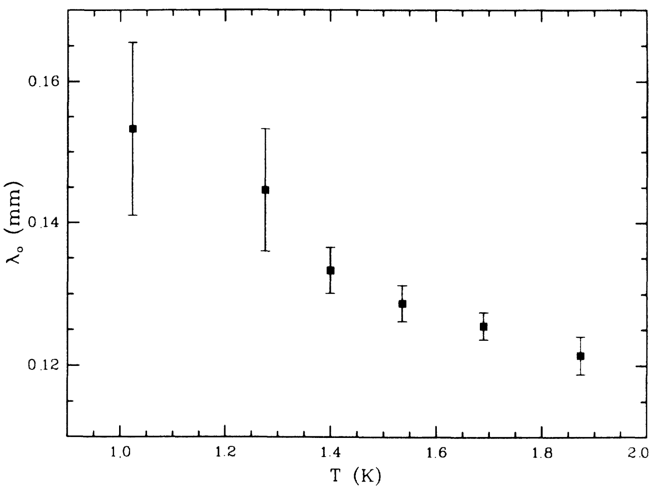

FIG. 3. Data for λ_0 vs temperature, as obtained from the slope of the data in Fig. 2

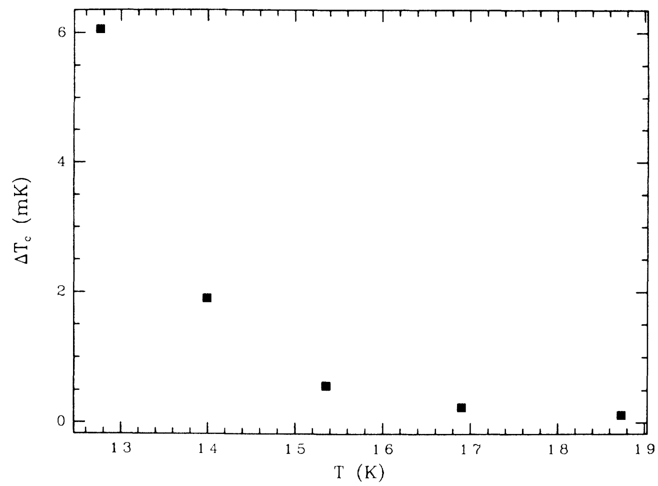

FIG. 4. Values of the critical temperature differences for each of the temperatures represented in Fig. 1.

dently by measuring the thermal conductance of the layer.

The experimental data for a given temperature and height are conveniently summarized by plots of the Nusselt number N vs r ($r \equiv R/R_c - 1$), and several examples are shown in Fig. 1. N is defined as the total heat flux across the layer normalized by the effective conductive heat flux; thus, $N=1$ in the preconvective state ($R < R_c$) and rises rapidly above 1 when convection begins.[13] (Note that the preconvecting state of a superfluid layer has nonzero vertical velocity due to the one-dimensional counterflow.)

Several corrections must be made to the directly measured quantities in order to obtain a good measure of d and the combination $d^3\Delta T_c$. (Here, ΔT_c is the value of ΔT at R_c.) These corrections arise from the self-heating of the top plate resistance thermometer by the bridge driving voltage and from the fact that at onset the average temperature across the layer, $T = T_{\text{bottom}} + \frac{1}{2}\Delta T_c$, increases somewhat as ΔT_c increases. All corrections have been determined by direct precise measurement, and in no case is the resulting correction to $d^3\Delta T_c$ more than 10%.

In Fig. 2 we present data for R_c vs d^{-2} for six values of the average temperature. We show the data in a normalized form, $R_c(d)/R_{c0}$. The normalization is made by fitting the data for $d^3\Delta T_c$ at each temperature to a polynomial in d^{-2}:

$$d^3\Delta T_c = A^*(T) + B^*(T)d^{-2} + C^*(T)d^{-4} + \cdots . \qquad (4)$$

The zeroth-order term of the fit A^* is then used as a divisor for the original data for $d^3\Delta T_c$, in order to experimentally obtain R_c/R_{c0}. This procedure was chosen because, of the elements comprising R_c, only d and ΔT_c are well known; the fluid properties, such as ν, are not so well known. But with this normalization, the effect of uncertainties in the thermohydrodynamic parameters is removed. The solid lines in Fig. 2 are fits using only up to the $O(d^{-2})$ term of Eq. (4). The quality of the fits is generally good. Fits including higher-order terms of Eq. (2) were not significantly better. For the lowest temperature, there is a significant increase in R_c/R_{c0} as d^{-2} becomes small, which we attribute to a small aspect ratio effect.[14]

From the above fits we obtain λ_0 as $(B^*/AA^*)^{1/2}$, which is shown versus T in Fig. 3. It is this quantity in particular which has not been well characterized in the past. Note that λ_0 actually *decreases* slightly with increasing temperature. This result is surprising, since it was anticipated that the superfluid corrections would become more important as T was increased towards T_λ. However, it is noteworthy that although λ_0 decreases with increasing T, the values of ΔT_c (at fixed d) decrease very rapidly as T increases. This is borne out by Fig. 4, which shows, for the five temperatures of Fig. 1, the corresponding values of ΔT_c. From the experimental point of view, we see that it becomes rapidly more difficult to resolve the temperature differences near the onset as T_λ is approached without compensating by making d very small. Nevertheless, the data imply that with the highest-resolution thermometry now available,[15] it would be possible to work in a regime much closer to T_λ without incurring significant superfluid effects. Prandtl numbers in the range $0.04 < \mathcal{N}_{\text{Pr}} < 1.5$ are accessible with minimal corrections, although at present, this conclusion applies only to the regime close to the onset of convection. Additional experimental and theoretical work is required to understand the effect of the superfluid correction terms on the convective flows well above R_c.

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