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Soliton switching and energy coupling in two-mode fibers: Analytical results

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Analytic solutions describing soliton interaction in bimodal optical fibers are obtained in the case of equal cross- and self-phase modulation effects. Conditions are established for integral pulse-shape switching and a periodic linearlike behavior of the energy coupling is emphasized.

Nonlinear pulse propagation in an optical fiber is usually described by the nonlinear Schrödinger equation (NLSE).[1] This equation is completely integrable and has been solved with the inverse scattering method.[2] In the anomalous dispersion regime, soliton pulses can propagate without distortion as a result of a perfect balance between dispersive and nonlinear (self-phase modulation) effects.[3]

In many circumstances, a more complete description of the propagation would rather involve an interaction between two (or more) coupled modes. For example, birefringence will give rise to two nondegenerate polarization modes interacting through a linear coupling.[4] The birefringence may result from a nonuniform core or may be twist induced.[5] The coupling could also be between the modes of two optical guides as in a dual-core fiber.[6]

A nonlinear coupling may also occur through the third-order polarization density[7]

$$P_{\mathrm{NL}_i}=A(\mathbf{E}^*\cdot\mathbf{E})E_i+B(\mathbf{E}\cdot\mathbf{E})E_i^*\,, \tag{1}$$

where $\mathbf{E}$ is the total electric field and the constants A and B represent the material-dependent susceptibilities. They can be normalized so that $A+B=1$. It has been shown elsewhere[4,8] that the interplay between linear and nonlinear coupling may lead to interesting and useful applications in optical communication systems. In particular, switching between two coupled modes in an optical waveguide has been a subject of intensive research recently.[9]

In many cases, it is a good approximation to consider that the group velocity and dispersion parameters can be taken as equal for both modes.[10] The case of small birefringence represents a typical example.[11] Then, in a reference frame moving at the common group velocity, the dynamics of soliton interaction can be reduced to a pair of coupled NLSE's:[1,4,5,10–12]

$$\begin{aligned} i\frac{\partial U}{\partial z}&=\frac{\partial^2 U}{\partial \tau^2}+\kappa V+[\,|U|^2+\sigma|V|^2]U\,,\\ i\frac{\partial V}{\partial z}&=\frac{\partial^2 V}{\partial \tau^2}+\kappa U+[\,|V|^2+\sigma|U|^2]V\,, \end{aligned} \tag{2}$$

where z is the normalized propagation distance and τ is a reduced local time. $U(z,\tau)$ and $V(z,\tau)$ are slowly varying envelopes, κ is the linear coupling constant, and σ is the ratio between the cross- and self-phase modulation contributions to the nonlinear effects.[10]

Besides the numerical simulations[6,10–13] related to this system of coupled NLSE's, only a few particular exact solutions have been published. First, in the cw regime, Winful[4] has already shown that the system (2) (with $\partial^2/\partial t^2=0$) can be solved exactly and useful switching effects can be anticipated. Considering the dispersive term, but not the linear coupling ($\kappa=0$), Inoue[14] has presented a series of exact solutions. More generally, for the same situation ($\kappa=0$), and when $\sigma=1$, the system (2) reduces to a Manakov system which is completely integrable by the inverse scattering method.[15] But, to our knowledge, all the solutions published so far[16] (including some particular ones with $\kappa\neq0$) represent solitary waves for which no energy exchange between the modes occurs during the propagation.

In this Rapid Communication, we show that the system (2) has a wide class of exact solutions exhibiting energy exchange between the modes. When $\sigma=1$, the cross- and self-phase modulation effects make an equal contribution to the nonlinearity. This situation corresponds to at least two possible cases: (i) For a purely electrostrictive nonlinearity, the constant $B=0$ [Eq. (1)]. This implies $\sigma=1$ in (2);[4,12] (ii) in the case of elliptical birefringence when $\theta\simeq35°$, θ being the angle between the major and minor axes of the birefringence ellipse.[5]

From now on, we will then consider that $\sigma=1$. Assuming U and V to be of the form

$$\begin{aligned} U(z,\tau)&=u_0(z,\tau)\cos(\kappa z)-iv_0(z,\tau)\sin(\kappa z)\,,\\ V(z,\tau)&=v_0(z,\tau)\cos(\kappa z)-iu_0(z,\tau)\sin(\kappa,z)\,, \end{aligned} \tag{3}$$

the original system (2) reduces to the following set of equations for u_0 and v_0

$$\begin{aligned} i\frac{\partial v_0}{\partial z}&=\frac{\partial^2 v_0}{\partial \tau^2}+[\,|v_0|^2+|u_0|^2]v_0\,,\\ i\frac{\partial u_0}{\partial z}&=\frac{\partial^2 u_0}{\partial \tau^2}+[\,|u_0|^2+|v_0|^2]u_0\,. \end{aligned} \tag{4}$$

This is immediately recognized as the same system of nonlinear equations solved by Manakov.[15] It is well known that this system can be integrated by the inverse scattering method and we therefore conclude that for $\sigma=1$, a wide class of solutions of the system (2) has now been obtained.

The dynamics associated with solutions (3) and (4) can be either simple (solitary waves) or complex, depending on the choice of the input distributions $u_0(0,\tau)$ and $v_0(0,\tau)$. As shown below, the solution U,V can also be periodic in z. Here, we limit ourselves to two particular cases of special interest. First, we consider the case $u_0=\pm v_0$. Then system (4) reduces to the single regular NLSE which has soliton solutions. In this case, Eq. (3) becomes

$$U(z,\tau)=u_0(z,\tau)\exp(\mp i\kappa z)\,,$$
$$V(z,\tau)=\pm u_0(z,\tau)\exp(\mp i\kappa z)\,,$$

and no energy is transferred between the modes. The stability of that solution has been investigated recently by Wright, Stegeman, and Wabnitz[10] for the particular case when $u_0(z,\tau)$ is the $N=1$ sech soliton.

The other special case pertains to the experimental situation when only one of the modes is excited at the input. To analyze this case, we put $v_0(z,\tau)=0$ and then, from (3),

$$U(z,\tau)=u_0(z,\tau)\cos(\kappa z)\,,\quad V(z,\tau)=-iu_0(z,\tau)\sin(\kappa z)\,, \tag{5}$$

$u_0(z,\tau)$ being the solution of the single NLSE. At $z=\pi/2\kappa$, all the energy will be switched from one mode (U) to the other mode (V), for any input pulse $U(0,\tau)$. However, for practical purposes related to communication systems, it may be desirable to recover *exactly the same pulse shape* as injected at $z=0$. Inspection of (3) and (4) then reveals that this requirement is automatically satisfied *when the input pulse is a fundamental soliton.* However, if we inject a higher-order soliton ($N\geq 2$), then the sinusoidal period must also be adapted to the soliton period $\pi/2$. This means $\kappa=2m$ ($m=1,2,\ldots$). When the input pulse does not correspond to a soliton, the output pulse will generally be distorted. Numerical simulations would be needed for an estimate of the importance of the distortions.

In general, solutions (3) and (4) will correspond to a more complex behavior more so than the special cases described above. But a striking simplicity underlies this apparent complexity. Let $E_U(z)$ and $E_V(z)$ represent the energy in each mode, this is $E_U(z)=\int_{-\infty}^{\infty}|U(z,\tau)|^2d\tau$, etc. Then, from (3) and (4), we find for the general situation $u_0\neq v_0\neq 0$,

$$E_U(z)=E_U(0)+[E_V(0)-E_U(0)]\times\sin^2(\kappa z)-C\sin(2\kappa z)\,, \tag{6}$$
$$E_V(z)=E-E_U(z)\,,$$

where

$$C=\mathrm{Im}\left[\int u_0(0,\tau)v_0^*(0,\tau)d\tau\right]$$

and

$$E=E_U(0)+E_V(0)$$

is the conserved total energy. In other words, in terms of energy, the system exhibits a cw-linear-like behavior, i.e., the energy is periodically exchanged and the period (π/κ) is independent of the input energy, as in the cw linear regime. This is in contrast with the general case $\sigma\neq 1$, where the energy transmission depends on the nonlinearity.[6,11-13,17]

Equation (6) also implies that if the input distributions do not have the same energy $[E_V(0)\neq E_U(0)]$, then the propagation will give rise to a periodic energy exchange and no stabilization to a stationary energy repartition may occur. Moreover, the parameter C in (6) confirms the importance of the relative phase. For example, when $u_0(0,\tau)=v_0(0,\tau)e^{i\phi}$ the pulses have the same energy, but from (6), we then have

$$E_U(z)=E_U(0)[1-\sin(\phi)\sin(2\kappa z)] \tag{7}$$

showing that unless $\phi=0$ or π (i.e., the first particular case considered above), the energy will be periodically exchanged along the guide.

In conclusion, we have presented a wide class of exact solutions to the coupled NLSE's. The solution encompasses simple solitary waves as well as intricate behaviors. At the same time, energy considerations reveal a particularly simple periodicity. The analysis also shows that only solitons can be entirely switched (in shape and energy). Finally, we believe that the new solution given here can also be of interest for the general case $\sigma\neq 1$. It could be used as a zero-order solution in a perturbative treatment[18] or a variational method.[17]

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