

**dc-field-coupled autoionizing states for laser action without population inversion**

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We consider how externally coupled (e.g., dc-field-coupled) autoionizing states can be useful in producing gain without population inversion. The dc fields produce new absorption minima that enable one to obtain laser action in different frequency regions depending on the strength of the coupling. We calculate absorption and emission profiles assuming that one of the autoionizing states is incoherently pumped as in most conventional laser models. We present numerical results that demonstrate gain regions without population inversion.

**I. INTRODUCTION**

The study of lasers without population inversion has attracted considerable attention.<sup>1-3</sup> In a recent paper Harris<sup>1</sup> has considered two upper levels of a three-level laser system that are purely lifetime broadened and decay to an identical continuum. These upper levels are initially populated or are pumped in some manner. He has shown that substantial gain at certain frequencies can be obtained without population inversion relative to the levels considered. This is because the interference effects reduce the absorption of lower-level atoms in a certain frequency range without, however, affecting significantly the stimulated emission from upper-level atoms. In a previous study Arkhipkin and Heller<sup>2</sup> showed that a single discrete level embedded in a continuum exhibits Fano-type absorptive interference,<sup>4</sup> whereas this interference is absent in emission. Clearly lasers without population inversion can be produced if one operates in the regions where the absorption profile exhibits a minimum,<sup>5</sup> and thus it is worthwhile to explore other mechanisms that can produce minima in the absorption profiles.

It is now well known that the presence of external fields leads to field-induced minima in the absorption spectrum.<sup>6-10</sup> Thus one could, for example, examine how the dc-field-induced minima in the absorption profiles could be useful in producing laser action without population inversion, and this is the problem this paper addresses. The organization is as follows: In Sec. II we consider two dc-field-coupled autoionizing (AI) states, with one of the AI states being pumped *incoherently* at some rate. In Sec. III we calculate both absorption and emission profiles. In Sec. IV we consider the possibility of gain on the laser transition by assuming that the incoherent pumping is such that there is no population inversion between the upper and lower states. We present numerical results to demonstrate different regions of gain.

**II. THE MODEL SYSTEM AND DYNAMICAL EQUATIONS**

The schematic diagram of the model system of interest is shown in Fig. 1. Here  $|a\rangle$  and  $|i\rangle$  are the two AI states coupled to each other by a dc field. The state  $|a\rangle$  is coupled by configuration mixing to the unperturbed continuum state  $|E\rangle$ . The states  $|E\rangle$  and  $|a\rangle$  are coupled to the discrete state  $|f\rangle$  by a weak laser field of frequency  $\omega$  and amplitude  $A$ . In addition, it is assumed that population is pumped into the state  $|a\rangle$  at the rate  $\Lambda$ . In the absence of the dc field and the laser field the population of the AI state is

$$\rho_{aa} = \frac{\Lambda}{\Gamma} \equiv p_a, \tag{2.1}$$

where  $\Gamma$  is the rate of AI. In addition, we assume that

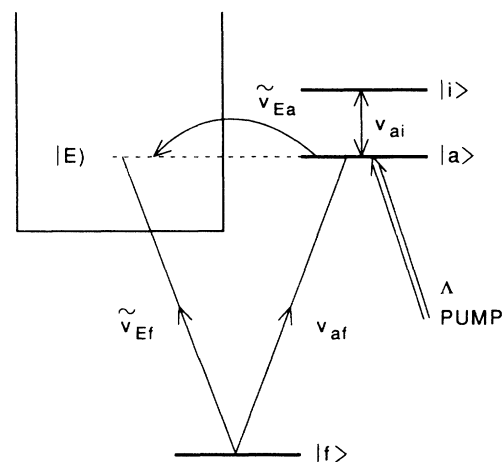


FIG. 1. Schematic diagram of the energy levels and various interactions for the model system of interest.

the level  $|f\rangle$  has some population  $p_f$ . We investigate the conditions under which the probe field acting on the transition  $|f\rangle \leftrightarrow |a\rangle$  can experience gain if

$$p_a < p_f, \quad (2.2)$$

i.e., if there is no population inversion between the levels

$|a\rangle$  and  $|f\rangle$ . In order to calculate gain, we need to find the induced polarization at the frequency  $\omega$ ,

$$\mathbf{p}(t) = \text{Tr}(\rho \mathbf{d}) = \int \tilde{\mathbf{d}}_{Ef}^* \rho_{Ef} dE + \tilde{\mathbf{d}}_{af}^* \rho_{af} + \text{c.c.}, \quad (2.3)$$

where  $\tilde{\mathbf{d}}_{\alpha\beta}$  represents the dipole matrix element.

The Hamiltonian for the model system of Fig. 1 is

$$\begin{aligned} H = & \int E|E\rangle\langle E|dE + E_a|a\rangle\langle a| + E_f|f\rangle\langle f| + \left[ \int \tilde{v}_{Ea}|E\rangle\langle a|dE + \text{H.c.} \right] \\ & + \left[ \int \tilde{v}_{Ef}|E\rangle\langle f|e^{-i\omega t}dE + v_{af}|a\rangle\langle f|e^{-i\omega t} + \text{H.c.} \right] + (v_{ai}|a\rangle\langle i| + \text{H.c.}), \quad E_i=0, \quad \hbar=1. \end{aligned} \quad (2.4)$$

Various terms in the above Hamiltonian have the following meaning. The first three terms represent the unperturbed Hamiltonian of the atomic system. The matrix element  $\tilde{v}_{Ea}$  represents the configuration interaction,<sup>11</sup> while  $\tilde{v}_{Ef}$  and  $v_{af}$  are due to the coupling of the continuum state and the AI state to the state  $|f\rangle$ .  $v_{ai}$  is responsible for the dc-field interaction. We ignore the effect of the dc-field coupling of the AI state to the continuum. The density matrix for the atomic system obeys the equation

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \Lambda|a\rangle\langle a| \equiv L\rho + \Lambda|a\rangle\langle a|, \quad (2.5)$$

where  $\Lambda$  represents the *incoherent* pumping of the AI level from some other levels of the system. Note that we have assumed that these levels form a large reservoir; otherwise, they would have to be included in the system of equations. As in the case of Raman scattering [considered in Ref. 5(d)], if the population of  $|f\rangle$  were to exceed that of the levels from which pumping occurs, laser action would no longer be possible.

To solve Eq. (2.5), we work with Laplace transforms whence we get

$$\hat{\rho}(z) = (z-L)^{-1}\rho(0) + \frac{\Lambda}{z}(z-L)^{-1}|a\rangle\langle a|, \quad (2.6)$$

which on using Eq. (2.1) becomes

$$\hat{\rho}(z) = p_f(z-L)^{-1}|f\rangle\langle f| + p_a \frac{\Gamma}{z}(z-L)^{-1}|a\rangle\langle a| \quad (2.7)$$

$$= p_f \hat{\rho}^{(f)}(z) + p_a \frac{\Gamma}{z} \hat{\rho}^{(a)}(z). \quad (2.8)$$

Here  $\hat{\rho}^{(f)}(z)$  and  $\hat{\rho}^{(a)}(z)$  are, respectively, solutions of Eq. (2.5) with  $\Lambda=0$  and for initial conditions  $\psi_f(0)=1$  and  $\psi_a(0)=1$ . Thus the solution of Eq. (2.5) in the presence of incoherent pumping of the autoionizing state has been expressed in terms of the solution in the absence of pumping. The long-time limit of Eq. (2.8) will be used for calculating the steady-state gain.<sup>12</sup>

In the absence of incoherent pumping  $\Lambda$ , we use the Fano-diagonalization procedure to diagonalize the part of the Hamiltonian responsible for configuration interaction and replace the unperturbed continuum state and the

AI state by a new continuum state  $|E\rangle$ . The state  $|E\rangle$  is related to  $|E\rangle$  and  $|a\rangle$  as

$$\begin{aligned} |E\rangle = & \frac{\sin\Delta}{\pi\tilde{v}_{Ea}}|a\rangle + \int \left[ \frac{\tilde{v}_{E'a}\sin\Delta}{\pi\tilde{v}_{Ea}(E-E')} \right. \\ & \left. - (\cos\Delta)\delta(E-E') \right] |E'\rangle dE'. \end{aligned} \quad (2.9)$$

The Hamiltonian (2.4) of the system in the new basis states after making a canonical transformation with  $\omega|f\rangle\langle f|$  has the form

$$\begin{aligned} H = & \int E|E\rangle\langle E|dE + \Delta_f|f\rangle\langle f| \\ & + \left[ \int v_{Ef}|E\rangle\langle f|dE + \text{H.c.} \right] \\ & + \left[ \int v_{Ei}|E\rangle\langle i|dE + \text{H.c.} \right], \end{aligned} \quad (2.10)$$

with  $\Delta_f = E_f + \omega$ .

In Eq. (2.10)  $v_{Ef}$  and  $v_{Ei}$  represent the interaction of states  $|f\rangle$  and  $|i\rangle$  with the Fano state  $|E\rangle$ . These can be calculated using Eq. (2.9) and have the form

$$v_{Ef} = \tilde{v}_{Ef} q_f \sin\Delta \left[ 1 + \frac{2}{\Gamma q_f} (E - E_a) \right], \quad (2.11)$$

$$v_{Ei} = (2/\Gamma\pi)^{1/2} v_{ai} \sin\Delta, \quad (2.12)$$

$$\tan\Delta = \frac{-\pi|\tilde{v}_{Ea}|^2}{(E-E_a)}, \quad \Gamma = 2\pi|v_{Ea}|^2, \quad (2.13)$$

$$q_f = \frac{\langle a|v|f\rangle}{\pi\tilde{v}_{Ea}\langle E|v|f\rangle}. \quad (2.14)$$

Note that  $v_{Ei}$  and  $v_{Ef}$  now represent the interaction of the structured continuum with the states  $|i\rangle$  and  $|f\rangle$ . Using the Schrödinger equation, the Hamiltonian (2.10) leads to the following equations for the wave functions:

$$\begin{aligned} \dot{\psi}_1 &= -i \int v_{Ei}^* \psi_E dE, \\ \dot{\psi}_f &= -i \Delta_f \psi_f - i \int v_{Ef}^* \psi_E dE, \\ \dot{\psi}_{E_1} &= -i E_1 \psi_{E_1} - i v_{E_1 f} \psi_f - i v_{E_1 i} \psi_i, \end{aligned} \quad (2.15)$$

where as usual  $\psi_\alpha = \langle \alpha | \psi \rangle$ . As pointed out before, we need to solve the above equations assuming the system to be initially either in the state  $|a\rangle$  or in the state  $|f\rangle$ , using Eqs. (2.9) and (2.13). We find the initial condition on the occupation of the Fano states,

$$\psi_E(0) = (2/\Gamma\pi)^{1/2} \sin\Delta \quad \text{if } \psi_a(0) = 1. \quad (2.16)$$

We need to solve Eqs. (2.15) twice, once with the initial condition  $\psi_i = \psi_f = 0$ ,  $\psi_E \neq 0$ , and then with  $\psi_i = \psi_E = 0$ ,  $\psi_f = 1$ .

The induced polarization Eq. (2.3) can also be reexpressed in terms of the matrix elements with respect to Fano states as follows:

$$\mathbf{p}(t) = \int \mathbf{d}_{Ef}^* \rho_{Ef} e^{-i\omega t} dE + c.c. \quad (2.17)$$

Ignoring vectorial properties of the matrix elements, we can define the transient susceptibility at frequency  $\omega$  as

$$\chi(t, \omega) = - \int \frac{v_{Ef}}{A^2} \rho_{Ef}(t) dE, \quad (2.18)$$

where

$$v_{Ef} = -\mathbf{d}_{Ef} \cdot \mathbf{A}. \quad (2.19)$$

The long-time limit of the susceptibility can be obtained by Laplace transform of Eq. (2.18), which yields

$$\hat{\chi}(z, \omega) = - \int \frac{v_{Ef}^*}{A^2} \hat{\rho}_{Ef}(z) dE, \quad (2.20)$$

$$\lim_{z \rightarrow 0} z \hat{\chi}(z, \omega) = - \lim_{z \rightarrow 0} z \int \frac{v_{Ef}^*}{A^2} \hat{\rho}_{Ef}(z) dE.$$

On using Eq. (2.8), we can rewrite Eq. (2.20) as

$$\chi(\omega) = p_f \chi_{ab}(\omega) - p_a \chi_{em}(\omega), \quad (2.21)$$

where we have introduced the absorption and emission profiles defined by

$$\chi_{ab}(\omega) = - \lim_{z \rightarrow 0} z \int \frac{v_{Ef}^*}{A^2} \hat{\rho}_{Ef}^{(f)}(z) dE, \quad (2.22)$$

$$\chi_{em}(\omega) = \lim_{z \rightarrow 0} \Gamma \int \frac{v_{Ef}^*}{A^2} \hat{\rho}_{Ef}^{(a)}(z) dE. \quad (2.23)$$

The probe field will experience amplification if

$$\text{Im}\chi(\omega) < 0, \quad (2.24)$$

i.e., if

$$p_a \text{Im}\chi_{em}(\omega) > p_f \text{Im}\chi_{ab}(\omega). \quad (2.25)$$

For a laser without population inversion,  $p_a < p_f$  and hence one has to search for conditions under which Eq. (2.25) holds even if  $p_a < p_f$ .

### III. EMISSION AND ABSORPTION PROFILES

In this section we derive explicit results for the emission and absorption profiles. Eqs. (2.15) can be reduced to a single integral equation for the Laplace transform of  $\psi_E(t)$ . The integral equation turns out to have a separable kernel and hence is analytically soluble.

#### A. Emission profiles

For the emission profile the initial condition Eq. (2.16) and the above procedure lead to

$$\hat{\psi}_i(z) = -i\hat{\phi}_1(z)/z, \quad \hat{\phi}_1(z) = \int v_{Ei}^* \hat{\psi}_E(z) dE, \quad (3.1)$$

$$\hat{\psi}_f(z) = -i\hat{\phi}_2(z)/(z+i\Delta_f), \quad \hat{\phi}_2(z) = \int v_{Ef}^* \hat{\psi}_E(z) dE, \quad (3.2)$$

$$\hat{\psi}_E = \frac{a_E}{(z+iE)} - i(v_{Ef}\hat{\psi}_f + v_{Ei}\hat{\psi}_i)/(z+iE), \quad (3.3)$$

$$\psi_E(0) = a_E = (2/\Gamma\pi)^{1/2} \sin\Delta. \quad (3.4)$$

Equation (3.3) can be reduced by standard methods to two coupled equations for  $\hat{\phi}_1$  and  $\hat{\phi}_2$ :

$$\begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{bmatrix} = -m \begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{bmatrix} + \begin{bmatrix} \hat{f}_1 \\ \hat{f}_2 \end{bmatrix}, \quad (3.5)$$

where

$$m_{ij} = \int L_i(E) K_j(E) dE, \quad \hat{f}_1 = \int \frac{L_i(E) a_E}{(z+iE)} dE, \quad (3.6)$$

and where

$$L_1(E) = v_{Ei}^*, \quad K_1(E) = \frac{v_{Ei}}{z(z+iE)}, \quad (3.7)$$

$$L_2(E) = v_{Ef}^*, \quad K_2(E) = \frac{v_{Ef}}{(z+iE)(z+i\Delta_f)}.$$

The elements  $m_{ij}$  depend on the structure of the continuum. These can be evaluated in closed form if we assume that the unperturbed continuum is flat:

$$m_{11} = \frac{\Gamma^2 \Omega_0}{2z(2z + \Gamma + 2iE_a)},$$

$$m_{22} = \frac{\Gamma \Omega_f}{2(z+i\Delta_f)} \left[ \frac{(q_f - i)^2 \Gamma}{(2z + \Gamma + 2iE_a)} + 1 \right],$$

$$zm_{21} = (z+i\Delta_f) m_{12} = (\Omega_0 \Omega_f)^{1/2} \frac{\Gamma^2 (q_f - i)}{2(2z + \Gamma + 2iE_a)},$$

with

$$\Omega_0 = 4|v_{ai}|^2/\Gamma^2, \quad \Omega_f = 2\pi|\bar{v}_{Ef}|^2/\Gamma. \quad (3.8)$$

Similarly, the quantities  $f$  are found to be

$$\hat{f}_1(z) = \frac{\sqrt{\Omega_0} \Gamma}{(2z + \Gamma + 2iE_a)}, \quad \hat{f}_2(z) = \frac{\sqrt{\Omega_f} \Gamma (q_f - i)}{(2z + \Gamma + 2iE_a)}. \quad (3.9)$$

Equations (3.5), (3.8), and (3.9) yield completely the wave functions of the system. We next use the solution to derive the explicit expression for  $\chi_{em}(\omega)$ . On writing Eq. (2.23) in terms of wave functions, we get

$$\begin{aligned} \chi_{em}(\omega) &= \lim_{z \rightarrow 0} \frac{\Gamma}{A^2} \int \left[ \int v_{Ef}^* \psi_E(t) dE \right] \psi_f^*(t) e^{-zt} dt \\ &= \lim_{z \rightarrow 0} \frac{\Gamma}{A^2} \int dt \phi_2(t) \psi_f^*(t) e^{-zt}, \end{aligned} \quad (3.10)$$

where the definition (3.2) is used. The weak-field emission profile is obtained by evaluating  $\phi_2(t)\psi_f^*(t)$  to second order in  $A$ , i.e., to order  $\Omega_f$ . Note that  $\hat{\psi}_f$  is proportional to  $\hat{\phi}_2$  and hence it is sufficient to calculate  $\hat{\phi}_2$  to order  $\sqrt{\Omega_f}$ . From Eqs. (3.5), (3.8), and (3.9), we find  $\hat{\phi}_2$  to order  $\sqrt{\Omega_f}$ :

$$\chi_{em}(\omega) = \frac{i\Gamma^3}{4A^2}(1+q_f^2) \left[ \left( \frac{z_+^*z_+}{(z_++z_+^*)(z_+^*-z_-^*)(z_+-z_-)(z_+^*-i\Delta_f)} + \frac{z_+z_+^*}{(z_++z_-^*)(z_-^*-i\Delta_f)(z_-^*-z_+^*)(z_+-z_-)} \right) + \frac{i\Delta_f z_+}{(z_+-z_-)(z_++i\Delta_f)(z_+^*-i\Delta_f)(z_-^*-i\Delta_f)} \right] + \{z_+ \rightarrow z_-\}, \quad (3.13)$$

where  $\{z_+ \rightarrow z_-\}$  represents terms switching  $z_+$  and  $z_-$ . This is the final expression for the emission profile. It depends on (i) the Fano asymmetry parameter  $q_f$ , (ii) the dc-field strength contained in  $\Omega_0$ , (iii) the energy separation between the two AI states  $E_a - E_i$ , (iv) the frequency of the probe field, and (v) the width  $\Gamma$  of the autoionizing state.

In the absence of dc field  $\Omega_0=0$ , the roots are

$$z_+ = 0 \quad \text{and} \quad 2z_-/\Gamma = -1 - 2iE_a/\Gamma \quad (3.14)$$

and Eq. (3.13) reduces to

$$\chi_{em}(\omega) = \frac{i\Gamma\Omega_f(1+q_f^2)(1+i\delta)}{2A^2(1+\delta^2)} \quad (3.15)$$

and hence

$$\text{Im}\chi_{em}(\omega) = \frac{\Gamma\Omega_f(1+q_f^2)}{2A^2(1+\delta^2)}. \quad (3.16)$$

Here the detuning parameter  $\delta$  is defined by

$$\delta = \frac{2}{\Gamma}(\omega + E_f - E_a). \quad (3.17)$$

### B. Absorption profiles

The absorption profiles can be calculated by following the same procedure as above and these in fact are given in Ref. 6. Writing Eq. (2.22) in terms of the wave functions, we get

$$\chi_{ab}(\omega) = -\lim_{z \rightarrow 0} \frac{z}{A^2} \int_0^\infty \phi_2(t)\psi_f^*(t)e^{-zt} dt, \quad (3.18)$$

where  $\phi_2$  and  $\psi_f$  are to be evaluated under the initial condition  $\psi_i = \psi_E = 0$ ,  $\psi_f = 1$ . The detailed calculations show that

$$\chi_{ab}(\omega) = \frac{i\Gamma\Omega_f}{2A^2} \left[ \frac{(\delta + 2q_f + iq_f^2)(\delta - \alpha) - \Omega_0}{(\delta - \delta_+)(\delta - \delta_-)} \right], \quad (3.19)$$

where  $\delta$  is defined by (3.17) and  $\delta_\pm$  are the roots of

$$(\delta - \alpha)(\delta + i) - \Omega_0 = 0, \quad \alpha = \frac{2}{\Gamma}(E_i - E_a). \quad (3.20)$$

$$\hat{\phi}_2(z) = \sqrt{\Omega_f}(q_f - i)z\Gamma/[2(z - z_+)(z - z_-)], \quad (3.11)$$

where  $2z_\pm/\Gamma$  are the roots of the equation

$$z(2z + \Gamma + 2iE_a) + \Gamma^2\Omega_0/2 = 0. \quad (3.12)$$

On using (3.2) and (3.11) and on carrying out the algebra, Eq. (3.10) reduces to

The roots  $\delta_\pm$  are related to the roots  $z_\pm$  by

$$\delta_\pm = \alpha + \frac{2i}{\Gamma}z_\pm. \quad (3.21)$$

In the limit of zero dc field, Eq. (3.19) goes over to standard result:

$$\chi_{ab}(\omega) = \frac{i\Gamma\Omega_f}{2A^2} \left[ \frac{\delta + 2q_f + iq_f^2}{\delta + i} \right], \quad (3.22)$$

which leads to the Fano formula

$$\text{Im}\chi_{ab}(\omega) = \frac{\Gamma\Omega_f(q_f + \delta)^2}{2A^2(1 + \delta^2)}. \quad (3.23)$$

### IV. GAIN ON LASER TRANSITION WITHOUT POPULATION INVERSION

Having obtained the absorption and emission profiles, we can now examine the regions of gain for a range of parameters like  $q$  values, dc-field strengths, etc. A rather

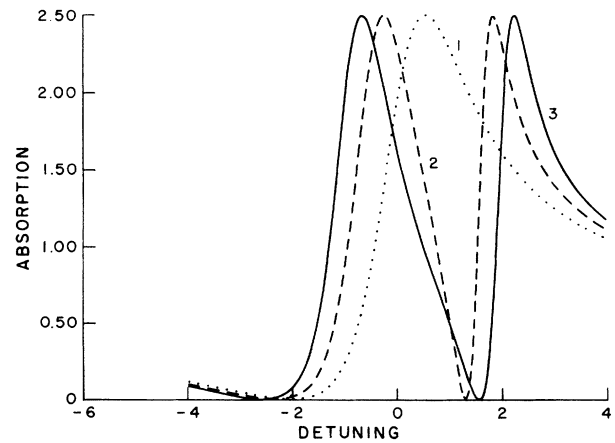


FIG. 2. Absorption profiles  $[\text{Im}\chi_{ab}(\omega)A^2/\Gamma\Omega_f]$  as a function of detuning  $\delta = (2/\Gamma)(E_f + \omega - E_a)$  for  $q_f = 2$ ,  $\alpha = (2/\Gamma)(E_i - E_a) = 1$ , and for the dc-field values given by  $\Omega_0 = 4|v_{ai}|^2/\Gamma^2 = 0(1), 1(2), 2(3)$ .

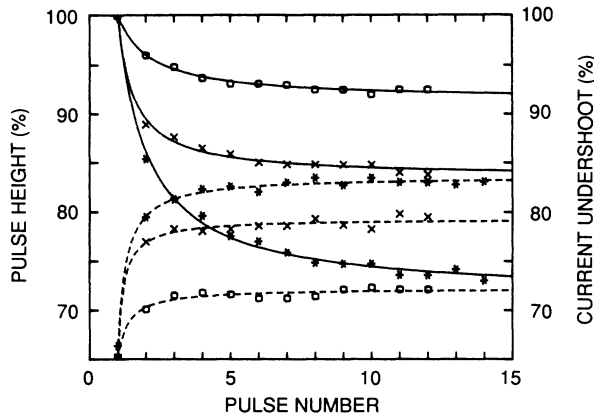


FIG. 3. Emission profiles ( $\text{Im}\chi_{\text{em}} A^2/\Gamma\Omega_f$ ) as a function of detuning for the same parameters as in Fig. 2.

simple result is obtained in the absence of dc field. Using (3.16) and (3.23) in (2.25), we get the condition

$$p_a(1+q_f^2) > p_f(q_f+\delta)^2. \quad (4.1)$$

Clearly the region corresponding to Fano minimum  $q_f+\delta \sim 0$  is the region in which one can obtain gain without population inversion. We next show the usefulness of dc fields in creating new frequency regions where the laser action can occur without population inversion. The absorption profiles in the presence of dc fields are shown in Fig. 2. The dc field creates a new absorption minimum, which is in addition to the Fano minimum in the absorption profiles. The position of the Fano minimum depends weakly on the strength of the dc field. The dc fields create a new absorption channel. The different absorption channels interfere to produce a new (dc-field) minimum in the absorption curve. The dc field mixes the two AI states to produce new states (dressed states), say,  $|a\rangle'$  and  $|i\rangle'$ , and the photons can now be absorbed in the transition  $|f\rangle \rightarrow |a\rangle'$ ,  $|f\rangle \rightarrow |i\rangle'$ ,  $|f\rangle \rightarrow |E\rangle$ . The emission profiles are plotted in Fig. 3. The emission profiles also exhibit a dc-field-induced minimum that, however, occurs at a different position from the dc-field minimum in the absorption profiles. For the purpose of laser action, the minimum in emission is uninteresting. A comparison of Figs. 2 and 3 shows the frequency regions in which one can have gain without population inversion. In particular we show gain regions in Fig. 4 for the case when the pumping is such that the population inversion is zero, i.e.,  $p_a = p_f$ . Figure 4 shows different regions of gain. It also demonstrates how the regions of gain can be controlled by dc fields. The above

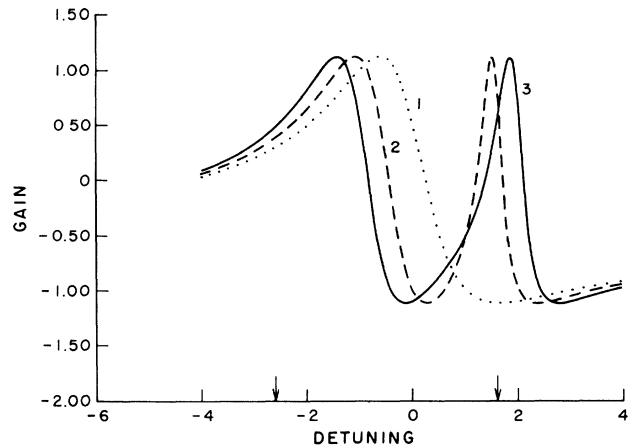


FIG. 4. The gain on the laser transition ( $(A^2/\Gamma\Omega_f)\text{Im}[\chi_{\text{em}}(\omega)-\chi_{\text{ab}}(\omega)]$ ) as a function of detuning for the same parameters as in Fig. 2. Observe the regions of gain and loss. Equal populations in  $|a\rangle$  and  $|f\rangle$  have been assumed. The left and right arrows give, respectively, the position of Fano minimum and dc-field-induced minimum in absorption for  $\Omega_0=2$ .

analysis also shows that if one had two AI states coupled by internal interaction,<sup>7</sup> then the laser action can also be produced without population inversion.

Finally, we emphasize, as noted above, that the pumping of the system plays a crucial role. In this work pumping occurred from a reservoir sufficiently large that the states associated with the reservoir did not have to be explicitly included in the calculations. In this way we found gain without an inversion existing between levels  $|a\rangle$  and  $|f\rangle$ . Similar results were found in Ref. 5(d) where state  $|a\rangle$  was coherently pumped and Raman-like gain was observed. In that case it was clear that, while an inversion did not exist relative to the populations of  $|a\rangle$  and  $|f\rangle$ , there was an inversion relative to the states *dressed* by the coherent pumping field. Thus we would expect in general that the pumping leads to modified (dressed) states and that an inversion in this global system of modified states is required for laser action.

#### ACKNOWLEDGMENTS

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<sup>1</sup>S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989).

<sup>2</sup>V. G. Arkhipkin and Yu. I. Heller, Phys. Lett. **98A**, 12 (1983).

<sup>3</sup>M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).

<sup>4</sup>U. Fano, Phys. Rev. **124**, 1866 (1961).

<sup>5</sup>(a) A. Imamoglu and S. E. Harris (unpublished); (b) S. E. Harris

and J. J. Macklin, Phys. Rev. A **40**, 4135 (1989); (c) A. Lyras, X. Tang, P. Lambropoulos, and J. Zhang, *ibid.* **40**, 4131 (1989); (d) G. S. Agarwal, S. Ravi, and J. Cooper, *ibid.* **41**, 4727 (1990).

<sup>6</sup>E. B. Saloman, J. W. Cooper, and D. E. Kelleher, Phys. Rev. Lett. **55**, 193 (1985).

<sup>7</sup>For an example of such autoionizing states, see J. Neukammer, H. Rinneberg, G. Jonsson, W. E. Cook, H. Hieronymus, A. Konig, and H. Springer-Bolk, *Phys. Rev. Lett.* **55**, 1979 (1985).

<sup>8</sup>G. S. Agarwal, J. Cooper, S. L. Haan, and P. L. Knight, *Phys. Rev. Lett.* **56**, 2586 (1986).

<sup>9</sup>G. S. Agarwal, S. L. Haan, and J. Cooper, *Phys. Rev. A* **29**, 2552 (1984); **29**, 2565 (1984), and references therein.

<sup>10</sup>S. Ravi and G. S. Agarwal, *Phys. Rev. A* **35**, 3354 (1987).

<sup>11</sup>In this work we assume that the autoionization of the state  $|i\rangle$

is negligible. This assumption can, however, be relaxed. See S. Ravi and G. S. Agarwal, *Phys. Rev. A* **35**, 3291 (1987).

<sup>12</sup>The present work has some similarities to the work of Ref. 5(a). We consider the states  $|i\rangle$  and  $|a\rangle$  which are close together to be coupled by a dc field. In Ref. 5(a), the states  $|i\rangle$  and  $|a\rangle$  could be coupled by an optical field. Reference 5(a) considers pumping to the state  $|i\rangle$ , whereas we consider the pumping to the state  $|a\rangle$ . We leave the pumping quite arbitrary, whereas in Ref. 5(a) the pumping strength seems to be determined by an internal mechanism.