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Coupled wakes behind two circular cylinders

P. Le Gal and M. P. Chauve
Centre National de la Recherche Scientifique, Institut de Mécanique Statistique de la Turbulence, 12 Avenue Général Leclerc, 13003 Marseille, France

R. Lima
Centre de Physique Théorique, Centre National de la Recherche Scientifique, Luminy, 13288 Marseille, CEDEX 09, France

J. Rezende
Departamento de Matematica, Universidade de Lisboa, L. Luis de Vasconcelos, 1000 Lisboa, Portugal
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The wake of a pair of identical cylinders placed side by side in a uniform flow is visualized. Different flows appear when the distance between the cylinders is decreased. For large gaps, the study of the phase difference between the vortex shedding shows that locking occurs and can be associated with asymmetric flows. For small gaps, a new vortex pattern with a separated stagnant zone is visualized. Finally, a classical alternate vortex street is observed at very small gaps. An analogy with coupled oscillators is then presented in order to interpret the asymmetric regimes as beats between the wakes.

Since the early works of Bénard and von Kármán, the vortex street that forms the wake of a circular cylinder placed perpendicular to a uniform flow has been extensively studied through many experiments.[1] Less common are the published works concerning the double wake of a pair of cylinders placed side by side in a flow. And yet, a precise understanding of such flows would have multiple interests. On one hand, these studies are of practical importance: Heat exchangers or mechanical structures in air or water streams are typical examples where a good comprehension of the interactions between the flows and the structures can improve industrial design. On the other hand, fundamental hydrodynamics find in these flows a typical field for the study of coherent vortical structures that are of great importance in the general context of the transition to turbulence.[2]

In a review paper,[3] Zdravkovich gave a summary of some of the experimental works that have dealt with flows around pairs of identical circular cylinders and for Reynolds numbers between 10^3 and 10^5. At the time, the understanding of the coupled wakes seemed quite confused. The main feature, described by different authors, is the bistable nature of the flow. Indeed, when the distance separating the two cylinder surfaces is decreased under a critical value (around one or two cylinder diameters), the Strouhal number,[4,5] the drag coefficients,[6] and the base pressure[4] can take two different values. Some visualizations[7] allow an interpretation of these two states by the formation of an inclined flow between the cylinders that breaks the symmetry of the wake when deviating the flow towards one side: One cylinder will have a narrow wake and the other a wide wake. More recently, Williamson[8] performed some remarkable visualizations of these coupled wakes. Contrary to the previous studies, his experiments were carried out at rather low Reynolds numbers (less than 200). These observations confirm the existence of an asymmetric regime for a distance between the cylinders of less than one diameter. This regime is associated with vortex shedding at higher harmonics of the fundamental frequency. At large gaps, he observed both in-phase and antiphase shedding, as did Dumas, Domptail, and Daien[9] at higher Reynolds numbers. In a recent study, Kim and Durbin[10] realized a statistical study of the exchange of stability between the two asymmetric regimes. They observed that what they called the "flip-flopping" is a Poisson stochastic process and that each cylinder develops its own narrow or wide wake associated, respectively, with two different frequencies. They also show that the flip-flopping characteristic time is a highly decreasing function of the Reynolds number and therefore explains the lack of exchanges of stability in the low-Reynolds-number experiments.[8]

Our experimental work is based on two questions about the exchanges of stability between the different patterns that have already appeared in previous experiments. Why at large gaps can the shedding be observed in phase or in antiphase? Is there a link between this instability and the existence of the asymmetric regimes seen at smaller gaps? Indeed we think that the flip-flopping is directly related to the vortex shedding and that the difference of phase between the two oscillators constituted by the near wakes is directly related to one of the major parameters describing the flow. This paper describes the very first experimental runs that we performed in a water tunnel.

We used a vertical visualization water tunnel with a cross section of 20×20 cm^2. The length of the working area is about 1 m and the turbulence level is less than 1%. Two stainless-steel, 4-mm-diam cylinders are mounted on a special apparatus that permits us to change continuously the distance between the cylinders with an accuracy of 0.1 mm. The gap, defined as the distance between the cylinder surfaces divided by the cylinder diameter, can be set between 6.5 and 0. In order to minimize edge effects,[11] the cylinders cross the tunnel entirely. They are hollow, and 0.5-mm apertures in the middle of the rear face of each of them allow a white dye to be injected in the flow at the desired rate and with no initial speed. In fact, the alternate vortices draw up the dye when detaching from the cylinders, their centers are therefore colored, and the main features of the flow are easy to visualize when illuminating the tunnel through a slit perpendicular to the cylinders. The Reynolds number is fixed to 110, and we checked that three-dimensional instabilities are avoided.[12] The typical vortex shedding frequency is around 1.2 Hz (Strouhal number around 0.17). For each gap, the experimental runs are videotaped for a period of 10 min. In order to record the passage of the vortices at special locations on the monitor, two photoconductive cells, properly mounted in an optical setup, are placed in front of the screen at a distance of three diameters downstream of each cylinder and at one diameter on the side. As the near wake is magnified by a factor of 3, the spatial integration of the sensors is minimized. The passage of white-colored vortices in front of the sensors gives an electric signal which is recorded and analyzed by a computer. By this means, we can reconstitute the time history of the vortex shedding behind each cylinder with an accuracy of 0.02 s. In particular, we calculated the difference of phase (that we call phase in what follows) between the shed eddies and observed different flow configurations as a function of g.

Large gaps ($g \geq 2.5$). For very large gaps ($g > 4.5$), the wake takes the form of two separated alternate vortex streets. Apparently, the coupling between the two wakes is small and the phase curves are difficult to interpret. They consist of slow variations and long horizontal plateaus, meaning that the two wakes are only slightly coupled and that locking is not permanent. The position of these plateaus can vary and the double wake seems to be very sensitive to external upstream fluctuations. The video film proves also that the shedding of vortices can be synchronized at different phases: in-phase or antiphase shedding being particular ones.[8] No obvious asymmetric flows can be detected although weak dissymmetries cannot be excluded.

On the contrary, for gaps lower than 4.5, asymmetric flows are observed. From time to time, one of the streets has a very weak lateral expansion while the other one is quite wide. Although it is difficult to determine a critical gap for the appearance of these flows, $g = 4.5$ is an apparent threshold for their observations. The most impressive patterns are obtained for $g = 2.5$, where they consist of three rows of vortices: One of the cylinder wakes is so narrow that the counter-rotating eddies are aligned on the same axis (see Fig. 1). We note that the speed of this row is 10% greater than the speed of the wide street. It is also in these cases that the pairing of vortices is sometimes observed.[8]

Figure 2 presents the variation of the phase throughout the complete experiment for $g = 4.5$. The curve is associated with typical pictures of the vortex patterns. We observe that the phase is particularly attracted by odd multiples of $\pi/2$ where plateaus are observed. It is at these values that the more asymmetric patterns are observed even if these regimes can also be detected during the phase shifting. The symmetry of the system is restored for the two particular values 0 and π that can be taken by the

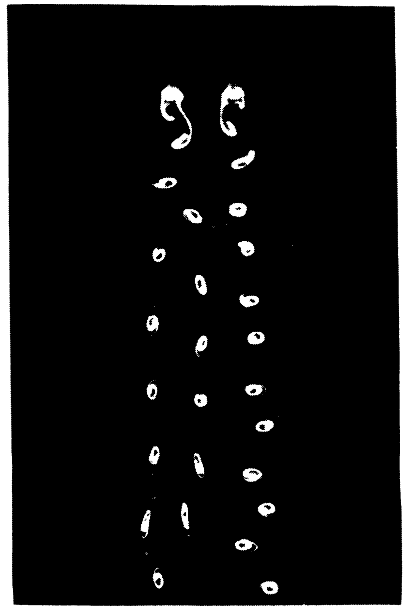

FIG. 1. Asymmetric regime for $g = 2.5$.

phase. Flip-flopping (we adopt this appellation although Kim and Durbin's experiment was realized at higher Reynolds numbers and with a smaller gap) occurs when the phase goes from one plateau to another by crossing the 0 or π value, the roles of the two cylinders being therefore exchanged. This phenomenon has been rarely seen in our visualizations. As shown by Kim and Durbin, this must be attributed to the low Reynolds number of our experiment.

For $g = 2.5$, the flow behind the pair of cylinders oscillates between two states with a typical time of 20 times the vortex shedding period. One of these states is similar to what has been previously described for larger gaps; the other state consists of a new arrangement between the vortices, and is more and more often observed as the gap is decreased. This new pattern will be described next.

Small gaps ($g < 2.5$). For values of g between 2.5 and 1, the inner vortices collapse into vortical sheets which are bent and stretched by the flow. This inner flow has a very low speed and can be considered as a separated stagnant zone. The extremities of these sheets are eventually wound up around the outer vortices. These outer vortices are shed alternatively and the global wake looks like a unique Bénard-Karman street. We suppose that the very low speed of the inner region is due to an induction effect produced by the velocity field of the outer eddies. This state is nearly always observed and the time history of the phase presents plateaus at each even multiple of π. Between these synchronizations, the phase changes rapidly (a few vortex shedding periods) and the flow passes through very unstable states corresponding to strongly asymmetric regimes. The vortex trajectories are, in these cases, very complex and lead very often to three-dimensional breakdown. Figure 3 illustrates these descriptions for $g = 1.5$ and 2, where vortex pairing can be observed in both cases. To our knowledge, it is the first time

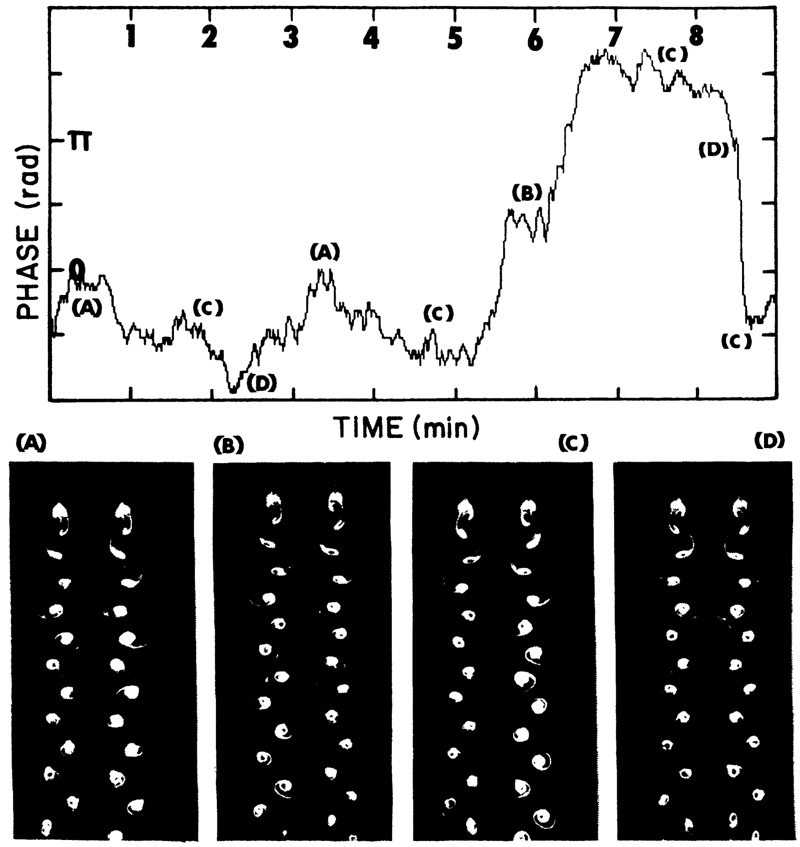

FIG. 2. Time history of the phase between the sheddings for $g=4.5$. Typical values of the phase are illustrated by pictures of the corresponding vortex patterns: A, in-phase symmetric shedding; B, asymmetric shedding, narrow right street; C, asymmetric shedding, narrow left street; and D, antiphase symmetric shedding.

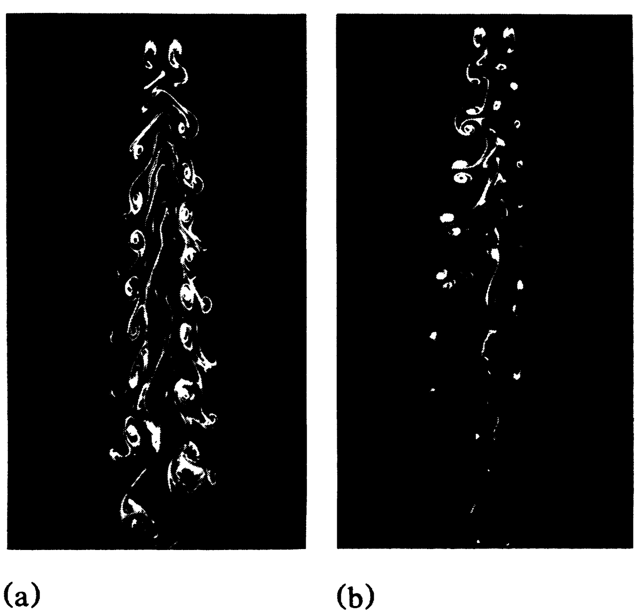

FIG. 3. (a) The nearly stable pattern with the inner stagnant zone for $g=1.5$; (b) asymmetric state for $g=2$ with aligned vortices on the left street. Vortex pairing can be observed on both patterns.

that such visualizations are obtained. Finally, for g less than 1, a unique vortex street is observed. The dimensions of the vortices show that they are associated with a body with a width equivalent to two diameters. We did not study this regime in great detail because the vortex street is quickly broken.

So, contrary to previous authors, we observed an interaction between the wakes even at gaps greater than 1. Even if desynchronization can occur for long periods of time, phase lockings are observed even for g as large as 6.5. Asymmetric flows appear clearly in our experiment for g between 4.5 and 2.5. The magnitude of the dissymmetry is directly linked to the phase difference between the two wakes. In particular, it is very pronounced during a synchronization with a phase difference around odd multiples of $\pi/2$. We do not know if the variations of the phase between the plateaus are created by a natural two-dimensional dynamical effect, or by a three-dimensional effect like oblic shedding, or simply by small upstream fluctuations in the flow. Flip-flopping has been difficult to observe but it is apparently linked to a phase variation crossing the values 0 or π. A critical gap appears around $g=2.5$, where a new state which looks like the classical vortex street is particularly stable. Contrary to some observations at high Reynolds numbers,[10] the speed of the inner flow is very weak. This effect is due to the induction of the outer vortices, and we verified that it loses its stability as soon as the velocity is increased. We attribute this stability to a balance between the flow between the cylinders and the counter flow produced by the outer vortices. Finally, as it is described in the literature, a classical vortex street is recovered for very small gaps. This street is created by the whole pair that acts as a unique body. The Reynolds number is therefore multiplied by a factor of $2+g$ and three-dimensional patterns are observed. Our acquisition process is then impossible to use and does not permit a comparison with previous studies, where unfortunately the essential part of the results was obtained.

To achieve a better understanding of the asymmetric regimes, we try to represent each wake by a temporal oscillator. We know that this dramatic reduction can be justified if we consider only the near wakes at low Reynolds numbers.[13] The simple dynamical system, constituted by two coupled oscillators, can give some clues. In the case of linear coupled oscillators with a natural frequency f_0, two stationary eigenmodes are known: inphase oscillations at a frequency $f_1=f_0(1+\beta^2)^{1/2}$, where β is a corrective term due to the coupling, and antiphase oscillations at a frequency $f_2=f_0$. The global solution consists of combinations of these two regimes and gives the well-known linear beats. We recognize in these beats the origin of the asymmetric flows with their different lateral spreads and phase variations. Their frequency is therefore f_1-f_2 and is small in the case of weak coupling. Dimensional analysis shows that β is proportional to g^{-1} and that the beat frequency is a hyperbolic function of g^{-1}. Of course, the nonlinearity of the hydrodynamic system complicates this simple modeling: Flip-flopping looks more like a heteroclinic orbit than a sinusoidal oscillation. But these beats, which are at the origin of so many phenomena in physics, seem very attractive to explain the hydrodynamic behavior of the coupled wakes. Also, this simple model allows us to predict the existence of a critical value of g separating the region of symmetric to asymmetric flow regimes, as well as the appearance of the periodic regime described in Ref. 5. Another consequence of the nonlinear coupling will be phase locking, which is by now a well-known consequence of Peixoto's theorem;[14] frequency entrainment is a generic and stable situation. Moreover, since the coupling of the two oscillators is related to the width of the gap, different regimes are expected, going from regular to chaotic behavior as g is varied.

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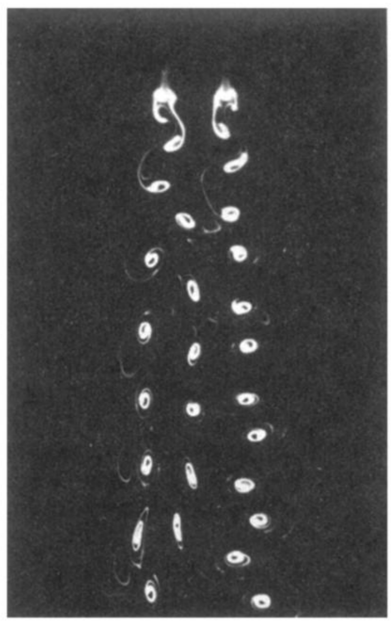
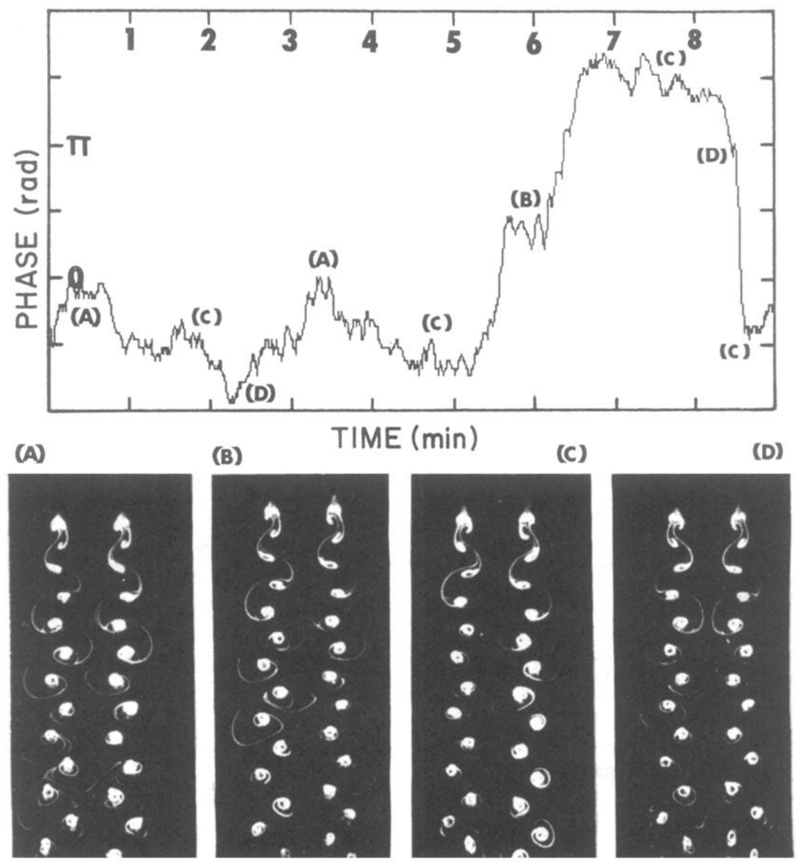


FIG. 1. Asymmetric regime for $g=2.5$.



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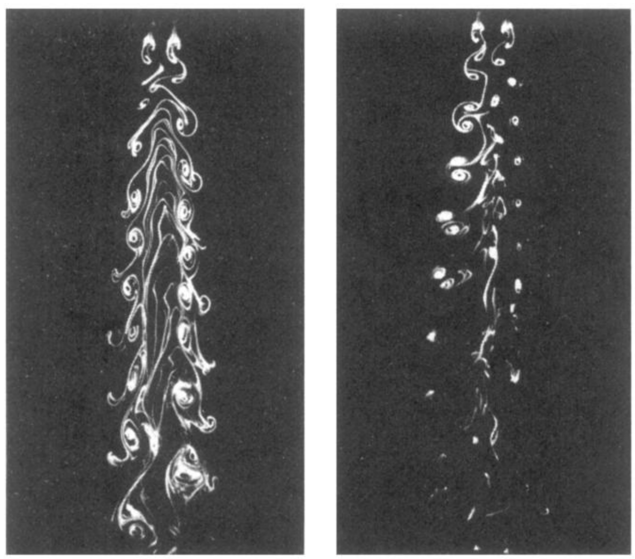

(a) (b)

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