# Kinetic equation of a plasma and the kinetic shielding potential

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The kinetic equation of a quantum plasma is given in the spatially quasihomogeneous case. The equation includes the quantum effect and the many-body effect. The kinetic shielding potential produced by the many-body effect is discussed.

## I. INTRODUCTION

The Boltzmann integro-differential equation and the Landau equation each have been applied in general to study the nonequilibrium properties for a plasma.[1] Although the form of the collision terms of these two equations are different, both use three fundamental hypotheses: two-body collision, elastic scattering, and the molecular chaos hypothesis. These equations can be used only in a diluted gas. Since there is a long-range Coulomb interaction in plasmas, the collision integral is divergent. In order to overcome this difficulty the Debye truncation or the Debye shielding potential is applied.

In order to consider the many-body effect the Liouville equation is used as a starting point. The correctional binary collision with many-body effects is considered. The Lenard-Balescu equation is obtained[2] from the Liouville equation, which uses several assumptions. Since one assumption is that the plasma is spatially homogeneous, the Lenard-Balescu equation can only be applied to the test-particle problem. On the other hand, the Lenard-Balescu equation is a classical equation that does not consider the quantum effect. In some cases the quantum effect is important. For example, a plasma may be composed by the electron and the hole in a semiconductor.

In this paper the kinetic equation of a quantum plasma is obtained in a spatially quasihomogeneous case. The equation includes the quantum effect and the many-body effect. In Sec. II the kinetic equation is given. The kinetic shielding potential will be discussed in Sec. III.

## II. KINETIC EQUATION

Recently a kinetic equation for nuclear gas was derived by means of the Wigner distribution function and a Bogoliubov approach from the Liouville-Von Neumann equation of quantum statistics.[3] Actually, this equation can be applied to all fermion gases. Of course, the equation can also be applied to a plasma composed of electrons and ions. Only the Coulomb interaction is employed.

Let us suppose there are $M$ components in the plasma. The number of particles is $N$. The number of the particles is $N_a$ for a component $a$. In this case the kinetic equation is[3]

$$\frac{\partial f_a}{\partial t}+\frac{\mathbf{p}_a}{m_a}\cdot\frac{\partial f_a}{\partial \mathbf{q}_a}+(\mathbf{F}_{\text{out}}+\mathbf{F}_{\text{in}})\cdot\frac{\partial f_a}{\partial \mathbf{p}_a}=\left[\frac{\partial f_a}{\partial t}\right]_{\text{coll}}, \tag{1}$$

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll}}=\frac{\pi v}{(2\pi)^3\hbar}\sum_{b=1}^{M} n_b'\int d\mathbf{k}\left(e^{(\hbar\mathbf{k}/2)\cdot(\partial/\partial\mathbf{p}_a)}-e^{-(\hbar\mathbf{k}/2)\cdot(\partial/\partial\mathbf{p}_a)}\right)\times\int d\mathbf{p}_b\,\delta\left[\mathbf{k}\cdot\left[\frac{\mathbf{p}_a}{m_a}-\frac{\mathbf{p}_b}{m_b}\right]\right]\frac{\tilde{V}_{ab}^2(k)}{\left|1+\frac{1}{\hbar}\sum_c n_c'\tilde{V}_{bc}(k)\Psi\right|^2}(f_a^+f_b^- - f_a^-f_b^+)\ . \tag{2}$$

Here $f_a=f_a(qpt)$ is the Wigner distribution function of component $a$, and $m_a$ is the mass. $\mathbf{F}_{\text{out}}$ is an external action which is the electromagnetic interaction. $\mathbf{F}_{\text{in}}$ is an internal self-consistent field action in the plasma. Its form is

$$\mathbf{F}_{\text{in}}\cdot\frac{\partial f_a}{\partial \mathbf{p}_a}=\frac{i}{\hbar}\left(e^{(i\hbar/2)\eta_a}-e^{-(i\hbar/2)\eta_a}\right)f_a-\frac{i}{\hbar}\int dx_b\left(e^{(i\hbar/2)\theta_{ba}'}-e^{-(i\hbar/2)\theta_{ba}'}\right)f_af_b\ , \tag{3}$$

where

$$\eta_a=\frac{\partial}{\partial \mathbf{q}_a}\left[\int V_{ab}(q_a-q_b)f_b\,dx_b\right]\cdot\frac{\partial}{\partial \mathbf{p}_a}\ ,$$

$$\theta_{ba}'=\frac{\partial V_{ab}(q_a-q_b)}{\partial \mathbf{q}_a}\cdot\frac{\partial}{\partial \mathbf{p}_a}\ .$$

In Eq. (2)

$$f_a^{\pm}=f\left[p_a\pm\frac{\hbar k}{2}\right]\left[1-f\left[p_a\mp\frac{\hbar k}{2}\right]\right],\quad n_c'=\frac{N_c}{N}$$

 

$$\tilde{V}_{ab}(k)=\int \frac{e_a e_b}{q} e^{i\mathbf{k}\cdot\mathbf{q}} d\mathbf{q}=\frac{4\pi e_a e_b}{k^2} .$$

Equations (1) and (2) are the new kinetic equation which is an improved Boltzmann-Uehling-Uhlenbeck equation. The three improvements in comparison to the general Boltzmann integro-differential equation are as follows.

(i) The effect of Pauli blocking is included in collision terms. The function $f \rightarrow f(1-f')$ is taken. This is important for collision processes of low-temperature plasma and solid plasma.

(ii) The modified mean-field interaction is introduced into interaction terms. That has a great influence on far nonequilibrium states.

(iii) The equation considers a correctional binary collision with a many-body effect wherein the many-body shielding effect can be obtained spontaneously. In the next section we shall discuss the third improvement further.

## III. KINETIC SHIELDING POTENTIAL

Define the function

$$\phi(k)=\frac{\tilde{V}_{ab}(k)}{1+\frac{1}{\hbar}\sum_c n_c' \tilde{V}_{bc}(k)\Psi} , \tag{4}$$

$$\xi=1+\frac{1}{\hbar}\sum_c n_c' \tilde{V}_{bc}(k)\Psi$$

$$=1+\frac{1}{\hbar}\sum_c \frac{N_c}{N}\tilde{V}_{bc}(k)\int d\mathbf{p}_c \frac{1}{\left[\frac{\mathbf{p}_b}{m_b}-\frac{\mathbf{p}_c}{m_c}\right]\cdot\mathbf{k}-i\epsilon} \times[f^{+}(x_c)-f^{-}(x_c)] . \tag{5}$$

When the system is nearly in the equilibrium state, the distribution function can be taken to be the equilibrium one in Eq. (5). The reason is that this term is a modified one. For convenience, the distribution function is taken from the Maxwell distribution

$$f(x_c)=\frac{1}{\pi^{3/2}\alpha_c^3}e^{-p_c^2/\alpha_c^2}, \quad \alpha_c^2=2m_c k_B T$$
$$f^{\pm}(x_c)=\frac{1}{\pi^{3/2}\alpha_c^3}e^{-[p_c\pm(\hbar k/2)]^2/\alpha_c^2} , \tag{6}$$

where $k_B$ is the Boltzmann constant.

In Eq. (5), when we orient the $k$ axis in the $z$ direction, the integration about $x$ and $y$ has been evaluated as

$$\xi=1+\frac{1}{\hbar}\sum_c \frac{N_c}{N}\frac{4\pi e_b e_c}{k^2}\left[ \mathrm{P}\int dp_{cz}\frac{1}{\mathbf{k}\cdot\left[\frac{\mathbf{p}_{bz}}{m_b}-\frac{\mathbf{p}_{iz}}{m_c}\right]}\frac{1}{\pi^{1/2}\alpha_c}(e^{-[p_{cz}+(\hbar k/z)]^2/\alpha_c^2}-e^{-[p_{cz}-(\hbar k/z)]^2/\alpha_c^2}) + i\pi\int\delta\left[\mathbf{k}\cdot\left[\frac{\mathbf{p}_{bz}}{m_b}-\frac{\mathbf{p}_{cz}}{m_c}\right]\right]\frac{dp_{cz}}{\pi^{1/2}\alpha_c}(e^{-[p_{cz}+(\hbar k/2)]^2}-e^{-[p_{cz}-(\hbar k/2)]^2/\alpha_c^2})\right] , \tag{7}$$

where P denotes the principal value of the integral. We calculate two integral in Eq. (7) as follows.

For the imaginary term, we define $p_{cz}=m_c v_{cz}$ and $\beta_c=\alpha_c/m_c$. The integral is completed by means of a $\delta$ function to be

$$\mathrm{Im}\xi'=\frac{\pi}{k}\frac{1}{\pi^{1/2}\beta_c}(e^{-[v_{bz}+(\hbar k/2m_c)]^2/\beta_c^2}-e^{-[v_{bz}-(\hbar k/2m_c)]^2/\beta_c^2}) . \tag{8}$$

Assuming $\hbar k/2m \ll v_{bz}$ we expand the exponents in Eq. (8), using a first-order approximation, to obtain

$$\mathrm{Im}\xi'=\frac{-\sqrt{\pi}\hbar}{m_c k_B T}\frac{v_{bz}}{\beta_c}e^{-v_{bz}^2/\beta_c^2} . \tag{9}$$

For the real term, because the integral is not divergent, the principal value operator P may be ignored. We find

$$\mathrm{Re}\xi'=\frac{1}{k}\left[e^{-[v_{bz}+(\hbar k/2m_c)]^2/\beta_c^2}\int_0^{2[v_{bz}+(\hbar k/2m_c)]/\beta_c}e^{(\beta_c^2 v^2)/4}dv-e^{-[v_{bz}-(\hbar k/2m_c)]^2/\beta_c^2}\int_0^{2[v_{bz}-(\hbar k/2m_c)]/\beta_c}e^{(\beta_c^2 v^2)/4}dv\right] . \tag{10}$$

TABLE I. Shielding effects for the incident electron.

| $v_{ez}/\beta_e$ | $v_{ez}/\beta_i$ | $a_{e\text{-}e}$ | $b_{e\text{-}e}$ | $a_{e\text{-}i}$ | $b_{e\text{-}i}$ |
|---|---|---|---|---|---|
| 0.1 | 4.291 | 0.9939 | $8.829\times10^{-2}$ | $2.229\times10^{-7}$ | $1.649\times10^{-1}$ |
| 1 | 42.91 | 0.5389 | 0.6053 | $6.287\times10^{-797}$ | $1.648\times10^{-2}$ |

TABLE II. Shielding effects for the incident ion.

| $v_{iz}/\beta_i$ | $v_{iz}/\beta_e$ | $a_{i\text{-}e}$ | $b_{i\text{-}e}$ | $a_{i\text{-}i}$ | $b_{i\text{-}i}$ |
|---|---|---|---|---|---|
| 0.1 | $2.33\times10^{-3}$ | 1 | $2.065\times10^{-3}$ | 0.9939 | $8.839\times10^{-2}$ |
| 1 | $2.33\times10^{-2}$ | 0.9996 | $2.065\times10^{-2}$ | 0.5387 | 0.6055 |
| 10 | 0.233 | 0.9676 | $2.065\times10^{-1}$ | $4.624\times10^{-42}$ | $0.7071\times10^{-1}$ |

Using $\hbar k/2m_c \ll v_{bz}$ we expand the exponent in Eq. (10) to obtain

$$\mathrm{Re}\xi'=\frac{\hbar}{k_B T}\left[1-\frac{2v_{bz}}{\beta_c}e^{-v_{bz}^2/\beta_c^2}\int_0^{v_{bz}/\beta_c}e^{x^2}dx\right] . \tag{11}$$

Inserting Eqs. (9) and (11) into Eq. (7) we find

$$\xi=1+\frac{1}{D^2k^2}(A-iB) , \tag{12}$$

where

$$D=\frac{k_BT}{4\pi n_e e^2} .$$

$D$ is the Debye length,

$$A=\sum_c\frac{N_c}{N}\left[1-\frac{2v_{bz}}{\beta_c}e^{-v_{bz}^2/\beta_c^2}\int_0^{v_{bz}/\beta_c}e^{x^2}dx\right] ,$$

$$B=\sum_c\frac{N_c}{N}\sqrt{\pi}\frac{v_{bz}}{\beta_c}e^{-v_{bz}^2/\beta_c^2} .$$

Next we insert Eq. (12) into Eq. (4) and then take an inverse Fourier transformation for $\phi(k)$. The kinetic shielding potential is obtained as follows:

$$\begin{aligned}\phi(q)&=\frac{1}{8\pi^3}\int\phi(k)e^{i\mathbf{k}\cdot\mathbf{q}}d\mathbf{k}\\&=\frac{e_ae_b}{\pi}\int_0^\infty k^2dk\int_0^\pi\frac{e^{ikq\cos\theta}}{k^2+\frac{1}{D^2}(A-iB)}\sin\theta\, d\theta\\&=\frac{e_ae_b}{|q|}e^{-(|q|/D)m} ,\end{aligned} \tag{13}$$

where

$$m=\left[\frac{A+(A^2+B^2)^{1/2}}{2}\right]^{1/2}-i\frac{B}{2}\left[\frac{A+(A^2+B^2)^{1/2}}{2}\right]^{-1/2}=a-ib .$$

It is shown from Eq. (13) that the kinetic shielding potential is similar to the Debye potential. What is different is that the kinetic shielding potential is relative to the velocity of a test particle and the state of field particles. For example, let us calculate for a hydrogen plasma. Suppose the ion and the electron have the same density and temperature, i.e., $n_i=n_e$ and $T_i=T_e$. The test particle selects an ion or electron, respectively. We calculate some $a$ and $b$ values in some special velocities. The results are given in Tables I and II.

In the tables, the following effects are shown: (1) the many-body shielding effect of a binary interaction is relative to the velocity of a test particle. When the velocity is increased the effect is reduced.

(ii) In the plasma, the shielding effects around a electron depend on electrons, and the influence of ions can be neglected. The shielding effects around a ion depend on electrons and ions. Though the shielding effects by ions are incomplete, the total shielding effects are stronger than the Debye shielding. Thus when the kinetic shielding potential is applied to the electrical conductivity and thermal conductivity, the modification may be very small. However, in the diffused process, since ions take part in this processes, the modification may be larger. (iii) Since there is a complex number in the shielding exponent, the interaction potential can be negative some distance between both particles of the same charge. The attraction interaction of particles of the same charge appears in some regions.

[1]R. Balescu, *Transport Processes in Plasma* (North-Holland, Amsterdam, 1988).

[2]D. R. Nichelson, *Introduction to Plasma Theory* (Wiley, New York, 1983).

[3]Bo-jun Yang and Shu-guang Yao, Phys. Rev. C **36**, 667 (1987).