Nonlinear interaction of intense laser pulses in plasmas

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A nonlinear one-dimensional theory is developed that describes some important aspects of intense laser-plasma interactions. The self-consistent laser-plasma analysis includes nonlinear plasma wake-field generation, relativistic optical guiding, coherent harmonic radiation production, as well as other related phenomena. Relativistic optical guiding is found to be most effective for long laser pulses having slow rise times. Short laser pulses are shown to be weakly guided. Coherent harmonic generation using a linearly polarized laser is found to be most efficient for short laser pulses and can be enhanced by the presence of large amplitude plasma wake fields. Aspects of particle acceleration by laser pulses as well as possible methods for upshifting the frequency of laser pulses are also discussed.

I. INTRODUCTION

The interaction of ultra-high-power laser beams[1] with plasmas is rich in a variety of wave-particle phenomena.[2] These phenomena become particularly interesting and involved when the laser power is high enough to cause the electron oscillation (quiver) velocity to become highly relativistic. Some of the interesting laser-plasma processes that are discussed include (a) relativistic optical guiding[3–12] of the laser beam, (b) the excitation of coherent radiation at harmonics of the fundamental laser frequency, (c) the generation of large amplitude plasma waves[13–16] (wake fields), (d) frequency shifts induced in the laser pulse by plasma waves,[17,18] (e) frequency amplification using an ionization front,[19] and (f) single-particle acceleration in a laser pulse.

In the following, a fully nonlinear one-dimensional (1D) model is developed that describes the self-consistent interaction of intense laser pulses with plasmas. By assuming a "quasistatic" cold fluid plasma response, a set of coupled nonlinear equations is derived for the vector potential of the radiation field and for the electrostatic potential of the plasma. The quasistatic approximation assumes that in a frame moving at the speed of light, the plasma fluid experiences a nearly steady-state radiation field (after the transients have decayed away). The resulting nonlinear equations are used to examine various laser-plasma interaction phenomena. The important issue of laser-plasma instabilities[2,3,20] is not addressed in this paper. Instabilities will certainly limit the laser propagation distance and, therefore, will have a profound effect on some of the potential applications discussed in this paper.

Relativistic optical guiding[3–12] is a result of the relativistic quiver motion of the electrons by the laser field. Analysis[6–9] has shown that as the laser power exceeds a critical threshold, diffraction can be overcome, resulting in optical guiding of the laser pulse. Previous analyses of relativistic guiding have included, for the most part, only the transverse electron motion in the plasma response current.[3–12] Relativistic guiding was believed to occur on a fast time scale (on order of the inverse laser frequency). However, the present analysis finds this not to be the case. In the following nonlinear analysis of relativistic optical guiding, the electron density response and longitudinal electron motion are included self-consistently. It is shown that for short laser pulses (pulse lengths less than a plasma wavelength), the combined effects of the plasma density response and the longitudinal motion significantly reduce the relativistic guiding effect. It is found that relativistic guiding occurs only for long pulses with slow rise times (greater than an inverse plasma frequency).

As the quiver motion of the electrons in a linearly polarized laser field becomes highly relativistic, the plasma response current will develop harmonic components. The harmonic content of the response current density can lead to the excitation of coherent radiation at harmonics of the fundamental laser frequency. It is shown that harmonic generation is more effective for short laser pulses (pulse lengths less than a plasma wavelength) than for long pulses with slow rise times.

An intense, short-pulse laser (pulse lengths near the plasma wavelength) interacting with a plasma can generate large amplitude plasma wave wake fields. The excited wake fields may be used to (i) accelerate a trailing electron bunch (laser wake-field acceleration),[13–16] (ii) optically guide a trailing laser pulse, and (iii) enhance the coherent harmonic radiation generated by a trailing laser pulse.

Three other laser-plasma interaction phenomena are briefly discussed. The first describes how frequency shifts are induced in a laser pulse by a plasma wave,[17,18] the second discusses how an ionization front may be used to amplify the frequency of a laser pulse,[19] and the third describes how a single particle may be accelerated by the passage of a laser pulse.

II. NONLINEAR FORMULATION

The 1D fields associated with the laser-plasma interaction can be described by the transverse vector and scalar potentials, $\mathbf{A}_\perp(z,t)$ and $\Phi(z,t)$, respectively (see Fig. 1). In what follows, we use the Coulomb gauge, i.e., $\nabla\cdot\mathbf{A}=0$ which implies $A_z=0$. The transverse polarization of the laser field is arbitrary. The normalized potentials satisfy the following equations:

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{a}=k_p^2\frac{n}{n_0}\boldsymbol{\beta}_\perp=k_p^2\frac{n}{n_0}\frac{\mathbf{a}}{\gamma}\ , \qquad (1a)$$

$$\frac{\partial^2\phi}{\partial z^2}=k_p^2\left[\frac{n}{n_0}-1\right]\ , \qquad (1b)$$

where $\mathbf{a}(z,t)=|e|\,\mathbf{A}_\perp/m_0c^2$, $\phi(z,t)=|e|\Phi/m_0c^2$, $k_p=\omega_p/c$, $\omega_p=(4\pi|e|^2n_0/m_0)^{1/2}$ is the ambient plasma frequency, $n(z,t)$ is the plasma density, n_0 is the ambient plasma density, $\boldsymbol{\beta}_\perp=\mathbf{v}_\perp/c=\mathbf{a}/\gamma$ is the normalized transverse plasma fluid velocity and $\gamma=(1-\beta_z^2-\beta_\perp^2)^{-1/2}=(1+a^2)^{1/2}/(1-\beta_z^2)^{1/2}$ is the relativistic mass factor. In obtaining the right-hand side of (1a) we used the fact that the transverse canonical momentum is invariant and prior to the laser pulse interaction the plasma is assumed stationary.

The fluid quantities n, β_z, and γ are assumed to satisfy the cold relativistic fluid equations that can be written in the form

$$\frac{\partial n}{\partial t}+c\frac{\partial}{\partial z}(n\beta_z)=0\ , \qquad (2a)$$

$$\frac{d\beta_z}{dt}=-\frac{1}{\gamma^2}\left[c\frac{\partial}{\partial z}+\beta_z\frac{\partial}{\partial t}\right]\frac{a^2}{2}+\frac{c}{\gamma}(1-\beta_z^2)\frac{\partial\phi}{\partial z}\ , \qquad (2b)$$

$$\frac{d\gamma}{dt}=c\beta_z\frac{\partial\phi}{\partial z}+\frac{1}{2\gamma}\frac{\partial a^2}{\partial t}\ , \qquad (2c)$$

where $d/dt=\partial/\partial t+c\beta_z\partial/\partial z$. The term containing a^2 in Eq. (2b) contains the ponderomotive force. Thermal effects may be neglected provided (i) the electron quiver velocity is much greater than the electron thermal velocity, and (ii) the thermal energy spread is sufficiently small such that electron trapping in the plasma wave is avoided. Also, the ions are assumed to be stationary.

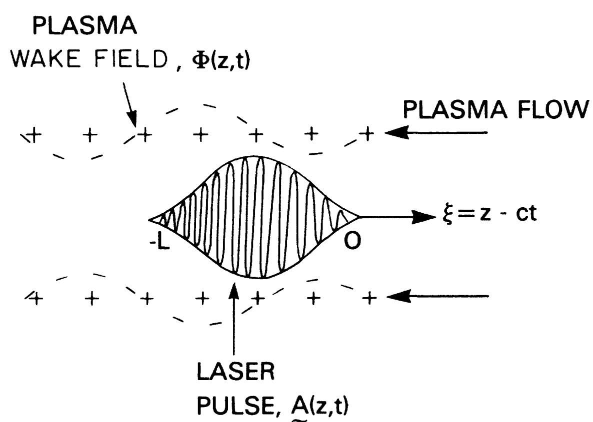

FIG. 1. Schematic showing the laser pulse in the speed of light frame (ξ,τ). The pulse extends from $\xi=-L$ to $\xi=0$, and the front of the pulse is at $\xi=0$. In this frame the plasma flows from right to left and a quasistatic state exists.

It proves convenient to perform an algebraic transformation from the laboratory frame independent space and time variables (z,t) to the independent variables (ξ,τ), where $\xi=z-c\beta_t t$ and $\tau=t$. Here $\beta_t=v_t/c\simeq 1$ is the normalized transformation velocity. To transform Eqs. (1) and (2) from z,t to ξ,τ variables we note that $\partial/\partial z=\partial/\partial\xi$ and $\partial/\partial t=\partial/\partial\tau-c\beta_t\partial/\partial\xi$. Using these transformations, Eqs. (1) and (2) become

$$\left[\frac{1}{\gamma_t^2}\frac{\partial^2}{\partial\xi^2}+\frac{2\beta_t}{c}\frac{\partial^2}{\partial\xi\partial t}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{a}=k_p^2\frac{n}{\gamma n_0}\mathbf{a}\ , \qquad (3a)$$

$$\frac{\partial^2\phi}{\partial\xi^2}=k_p^2\left[\frac{n}{n_0}-1\right]\ , \qquad (3b)$$

$$\frac{\partial}{\partial\xi}[n(\beta_t-\beta_z)]=\frac{1}{c}\frac{\partial n}{\partial\tau}\ , \qquad (3c)$$

$$\frac{\partial}{\partial\xi}[\gamma(1-\beta_t\beta_z)-\phi]=-\frac{1}{c}\frac{\partial}{\partial\tau}(\gamma\beta_z)\ , \qquad (3d)$$

where $\gamma_t^2=1/(1-\beta_t^2)$. Equations (3a)–(3d), together with $\gamma=(1+a^2)^{1/2}/(1-\beta_z^2)^{1/2}$, form a complete set of fully nonlinear, relativistic, cold fluid equations which describe the 1D laser-plasma interaction. The 1D model is valid as long as the radiation spot size is large compared to the plasma wavelength, i.e., $r_s\gg\lambda_p=2\pi/k_p$.

In the following analysis, we choose $v_t=c$, since $v_t=c$ lies between the group velocity and phase velocity of the laser pulse. Furthermore, we take the laser pulse to propagate in the positive z direction and to cross the $z=0$ plane at $t=0$. The $\xi=0$ plane defines the front of the pulse which extends into the $\xi\le 0$ region. In the ξ,τ frame we are, therefore, interested only in the region where $\xi\le 0$ since for $\xi\ge 0$ we have $\mathbf{a}=0$, $n=n_0$, $\beta_z=0$, and $\gamma=1$.

Quasistatic approximation

Equations (3) can be greatly simplified by noting that in the speed of light frame ($\beta_t=1$) we expect that under certain conditions a quasistatic state will exist in the macroscopic plasma quantities, n, β_z, and γ. That is, if the laser pulse is sufficiently short, the fields $\mathbf{a}$ and ϕ which drive the plasma are expected to change little during a transit time of the plasma through the laser pulse. From (3a) we find that the envelope of $\mathbf{a}$ changes on a characteristic time $\tau_e\sim 2\gamma|n_0/n|(\omega/\omega_p)/\omega_p$, where ω is the laser frequency. Since we will henceforth assume that $\omega\gg\omega_p$, the radiation envelope changes on a time scale which is long compared to a plasma period. If the laser pulse duration τ_L is small compared to τ_e, i.e., $\tau_L\ll\tau_e$, then the quasistatic approximation is valid. In addition, the validity of the 1D model requires that the laser beam diffraction time (transverse spreading time), $\tau_d=\pi r_s^2/(\lambda c)$, be long compared to τ_e. This condition is satisfied as long as $r_s\gg\lambda_p$. More formally, the quasistatic approximation [neglecting the right-hand side of (3c) and (3d)] implies

$$\left|\frac{1}{c}\frac{\partial}{\partial\tau}\int_\xi^0 n\,d\xi'\right|\ll n_0\ , \qquad (4a)$$

$$\left| \frac{1}{c} \frac{\partial}{\partial \tau} \int_{\xi}^{0} \gamma \beta_z d\xi' \right| \ll 1 \ . \tag{4b}$$

In this case, Eqs. (3c) and (3d) can be integrated to give

$$n(1-\beta_z)=n_0 \ , \tag{5a}$$

$$\gamma(1-\beta_z)-\phi=1 \ . \tag{5b}$$

Using (5a) and (5b) together with the expression for γ, the coupled field equations become

$$\left[\frac{2}{c} \frac{\partial}{\partial \xi} - \frac{1}{c^2} \frac{\partial}{\partial \tau} \right] \frac{\partial \mathbf{a}}{\partial \tau} = k_p^2 \frac{\mathbf{a}}{1+\phi} \ , \tag{6a}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = \frac{k_p^2}{2} \left[\frac{(1+a^2)}{(1+\phi)^2} - 1 \right] , \tag{6b}$$

where the plasma quantities in terms of the fields are

$$n/n_0 = 1 + \tfrac{1}{2}[(1+a^2)/(1+\phi)^2 - 1] \ , \tag{7a}$$

$$\gamma = [1+a^2+(1+\phi)^2]/[2(1+\phi)] \ , \tag{7b}$$

$$\beta_z = [1+a^2-(1+\phi)^2]/[1+a^2+(1+\phi)^2] \ . \tag{7c}$$

The above expressions for the fields and plasma quantities are fully nonlinear and hold for arbitrary polarization of the laser pulse. [Equations similar to (6a) and (6b) have been used by Akhiezer and Polovin[21] to study nonlinear plasma oscillations. Their equations were later used by Noble[22] to study beat wave excitation of plasma waves.] Consistent with the quasistatic assumption, a number of important points can be made concerning intense laser pulse propagation in plasmas.

III. LASER-PLASMA INTERACTION PHENOMENA

A. Relativistic optical guiding

The nonlinear index of refraction of the laser beam within the plasma determines, among other things, the optical guiding properties of the plasma. For the purpose of the present discussion, we assume that the laser field $\mathbf{a}$ is given by

$$\mathbf{a} = \mathbf{a}_L(\xi,\tau) e^{ik\xi}/2 + \text{c.c.} \ , \tag{8}$$

where $\mathbf{a}_L$ represents the complex amplitude and k is the wave number. The characteristic spatial variation in the laser envelope $|a_L|$ is assumed to be of order L and is long compared to the laser wavelength λ, i.e.,

$$\partial |a_L|/\partial \xi \simeq |a_L|/L \ll k|a_L| = 2\pi |a_L|/\lambda \ .$$

Using the representation in Eq. (8) we find from Eq. (6a) that the refractive index $\eta = ck/\omega$ is

$$\eta = 1 - \tfrac{1}{2}(k_p/k)^2/(1+\phi_s) \ , \tag{9}$$

where ϕ_s is the slow part of the scalar potential and $k_p \ll k$. In obtaining Eq. (9), we replaced $[1/(1+\phi)]_s$ with $1/(1+\phi_s)$, which is valid as long as $k_p \ll k$. This implies that $|\phi_f| \ll |\phi_s|$, where ϕ_f is the rapidly varying part of ϕ. (The work of Ref. 22 also implies that $\phi \simeq \phi_s$.)

There are two cases that can be considered, depending on the envelope scale length L compared to the plasma wavelength λ_p.

1. Long rise-time pulse

In this limit, $L \gg \lambda_p = 2\pi c/\omega_p$, the first term on the left-hand side of (6b) can be neglected and ϕ_s can be approximated by $\phi_s \simeq (1+|a_L|^2/2)^{1/2} - 1$. The refractive index becomes

$$\eta = 1 - \tfrac{1}{2}(\lambda/\lambda_p)^2/(1+|a_L|^2/2)^{1/2} \ . \tag{10}$$

Although the present analysis is 1D, we expect that for a slowly varying transverse laser profile the index of refraction will depend on the transverse coordinates through the laser amplitude $|a_L|$. Since the actual bounded laser beam amplitude falls off transversely, i.e., $\partial |a_L|/\partial r < 0$, so will the refractive index, i.e., $\partial \eta/\partial r < 0$. The negative transverse gradient of the refractive index can lead to optical guiding. Since the refractive index is a function of the laser amplitude, the condition for optical guiding places a lower limit on $|a_L|$. It is well known that if the refractive index is of the form given by Eq. (10), a critical laser power necessary for relativistic optical guiding exists and is given by[6–9] $P_{\text{crit}} \simeq 17.4(\lambda_p/\lambda)^2$ GW.

2. Short pulse

Consider the short laser pulse limit $L \lesssim \lambda_p$. When $|\phi| \ll 1$, Eq. (6b) can be solved for an arbitrary laser field $\mathbf{a}(\xi)$,

$$\phi \simeq \frac{k_p}{2} \int_{\xi}^{0} a^2(\xi') \sin[k_p(\xi'-\xi)] d\xi' \ , \tag{11}$$

where the boundary conditions $\phi = \partial\phi/\partial\xi = 0$ at $\xi = 0$ have been used.

If the pulse envelope is given by $a_L = a_{L_0} \sin(\pi\xi/L)$ for $-L \le \xi \le 0$ and $a_L = 0$ otherwise, we find that the scalar potential within the laser pulse is given by

$$\phi \simeq (a_{L_0}^2/8)\{1 - (k_p^2 - 4\pi^2/L^2)^{-1} \times [k_p^2 \cos(2\pi\xi/L) - (4\pi^2/L^2)\cos(k_p\xi)]\} \ , \tag{12}$$

where terms of order $(\lambda/\lambda_p)^2 \ll 1$ have been neglected. For $L \ll \lambda_p$, this gives $\phi_s \simeq (a_{L_0} k_p/4)^2 g(\xi)$, where $g(\xi) = \xi^2 - 2(L/2\pi)^2[1 - \cos(2\pi\xi/L)]$. Notice that for $L \ll \lambda_p$, ϕ_s is maximum at $\xi = L$ where $\phi_s \simeq (a_{L_0} k_p L/4)^2$. Also notice that even for $|a_{L_0}| > 1$, the assumption that $\phi_s \ll 1$ is still valid as long as $L \ll \lambda_p$. The index of refraction in the short-pulse limit is, therefore,

$$\eta = 1 - \tfrac{1}{2}(\lambda/\lambda_p)^2/[1 + (\pi/2)^2 (a_{L_0}/\lambda_p)^2 g(\xi)] \ , \tag{13}$$

where $-L \le \xi \le 0$.

In the short-pulse limit, the fact that $\phi_s \ll 1$ implies that the optical guiding effect is reduced significantly, by more than the factor $(\pi^2/2)(L/\lambda_p)^2 \ll 1$. The critical power, therefore, is increased by the inverse of this factor and, in addition, the degree of guiding varies along the pulse. Hence, it is unlikely that relativistic optical guiding can be effectively utilized in short, $L \lesssim \lambda_p$, laser pulses.

Although it may appear that a long laser pulse may undergo guiding, assuming the various laser-plasma instabilities can be controlled, the front of the pulse will diffract. Initially, that portion of the head of a long rise-time pulse in which the local power is less than $P_{\rm crit}$ will diffract. Once this portion has diffracted away, the pulse will exhibit "short-pulse" diffractive behavior, i.e., the front region ($\sim\lambda_p$) will continue to diffract. The erosion of the front of the pulse due to diffraction will propagate back through the body of the pulse. The erosion velocity back through the body of the pulse (in the $\xi = z - ct$ frame) may be estimated by $v_E \simeq (\lambda_p / z_R)c$, where $z_R = \pi r_s^2/\lambda$ is the vacuum Rayleigh length.

B. Harmonic excitation

The nonlinearities associated with the plasma waves can provide a source for the generation of coherent radiation at harmonics of the laser frequency. To examine this process we generalize Eq. (8) and represent the full radiation field by

$$\mathbf{a} = \sum_l \mathbf{a}_l(\xi,\tau) e^{ilk\xi}/2 + \text{c.c.} \ , \tag{14}$$

where $l = 1,2,3,\ldots$ and $\mathbf{a}_1 = \mathbf{a}_L(\xi)$ is the envelope of the dominant fundamental laser pulse, $|a_L| \gg |a_l|$, for $l \geq 2$.

It is clear from the right-hand side of (6a) that harmonic excitation is solely due to the excitation of the fast part of ϕ. In particular, since the fundamental component of the radiation field dominates, the fast part of the scalar potential is

$$\phi_f = -(k_p/4k)^2 a_L^2 (1+\phi_s)^{-2} \cos(2k\xi) \ , \tag{15}$$

where we have taken the fundamental laser pulse to be $a_L(\xi)\cos k\xi$. The source term in Eq. (6a) becomes

$$\begin{aligned} \mathbf{S} &= k_p^2 \mathbf{a}_L (1+\phi_s+\phi_f)^{-1} \cos(k\xi) \\ &= k_p^2 \mathbf{a}_L (1+\phi_s)^{-1} \cos(k\xi) \sum_m [\alpha \cos(2k\xi)]^m \ , \end{aligned} \tag{16}$$

where ϕ_f is given by (15), $\alpha = [(k_p/4k)a_L]^2 (1+\phi_s)^{-3}$, and $m = 0,1,2,\ldots$.

As an illustration we consider the excitation of third harmonic radiation (3ω). Since $|\phi_f| \ll |\phi_s|$, the third harmonic component of the source is

$$S_3 = (k_p^2/2)(k_p/4k)^2 a_L^3 (1+\phi_s)^{-4} \cos(3k\xi) \ , \tag{17}$$

where we have used the $m = 1$ term in Eq. (16). Substituting (17) into the right-hand side of Eq. (6a) and solving for the third harmonic field we find that

$$|a_3| = \tfrac{1}{3}(\lambda/4\lambda_p)^3 (1+\phi_s)^{-4} a_L^3 \omega_p \tau \ , \tag{18}$$

where τ is the laser-plasma interaction time. The ratio of the third harmonic power to the fundamental laser power is

$$P_3/P_1 = [(\lambda/4\lambda_p)^3 (1+\phi_s)^{-4} a_L^2 \omega_p \tau]^2 \ , \tag{19}$$

where $P_1 = 2.15 \times 10^{10} (r_s a_L/\lambda)^2$ W (for a Gaussian transverse laser profile).

Equation (16) shows that the generation of harmonics is a strong function of the plasma wake field, described by ϕ_s, in the region of the fundamental laser pulse. For a single long-pulse, large amplitude laser, $|a_L| \gg 1$, the slow part of the scalar potential is $\phi_s \sim |a_L|/\sqrt{2}$. In this case, the harmonic content of the source term in Eq. (16) is exceedingly small, and for the third harmonic, we have

$$P_3/P_1 = [4(\lambda/4\lambda_p)^3 \omega_p \tau / |a_L|^2]^2 \ .$$

Taking τ to be a diffraction time, $\tau_d = \pi r_s^2/\lambda c$, and $r_s \gtrsim \lambda_p$, the third harmonic power becomes

$$P_3/P_1 \simeq (\pi/2)^4 (\lambda/\lambda_p)^4 / (4|a_L|^4) \ .$$

In the case of a short, $L \ll \lambda_p$, large amplitude laser pulse, $\phi_s \lesssim |a_L|^2 (\pi L/2\lambda_p)^2$, the third harmonic power is

$$\begin{aligned} P_3/P_1 &= [|a_L|^2 (\lambda/4\lambda_p)^3 \omega_p \tau]^2 \\ &\simeq |a_L|^4 (\pi/4)^4 (\lambda/\lambda_p)^4/4 \end{aligned}$$

for $\tau = \tau_d$ and $r_s \gtrsim \lambda_p$. For $|a_L|^2 \gg 1$, a short pulse is more efficient than a long pulse for harmonic generation.

Harmonic generation can be enhanced in regions of a plasma wave where $\phi_s < 0$. This can be achieved by positioning a short laser pulse at the appropriate location in a large amplitude plasma wave having a phase velocity close to the speed of light. The large amplitude plasma wave can be the wake field generated by a laser pulse[13–16] or by an electron beam pulse.[23]

C. Wake-field generation

The self-consistent evolution of the nonlinear plasma wave can be studied by numerically solving Eqs. (6a) and (6b). Figures 2 and 3 show the plasma density variation $\delta n/n_0 = n/n_0 - 1$ and the corresponding axial electric field E_z for a laser pulse envelope given by $a_L = a_{L_0} \sin(\pi\xi/L)$ for $-L \leq \xi \leq 0$. In these figures, $L = \lambda_p = 0.03$ cm, $\lambda = 10$ μm, and $a_{L_0} = 0.5$ in Fig. 2 and $a_{L_0} = 2$ in Fig. 3. The steepening of the electric field and the increase in the period of the wake field[21,22,24,25] are apparent for the highly nonlinear situation shown in Fig. 3 ($|a_{L_0}| = 2$) as compared to the slightly nonlinear case shown in Fig. 2 ($|a_{L_0}| = 0.5$). Figure 4 shows that the electrostatic potential ϕ is predominantly slowly varying within the laser pulse even though $\delta n/n_0$ has rapidly varying components. In addition, the figure shows that ϕ can be negative behind the laser pulse.

Qualitative aspects of nonlinear wake-field generation may be examined by considering a circularly polarized laser pulse with a square pulse profile, i.e., $a_L = a_{L_0}$ for $-L \leq \xi \leq 0$ and $a_L = 0$ otherwise (also, see Ref. 26). For this case, Eq. (6b) may be solved analytically in terms of elliptic integrals. In particular, one can show that the optimal pulse length $L = L_{\rm op}$ (which corresponds

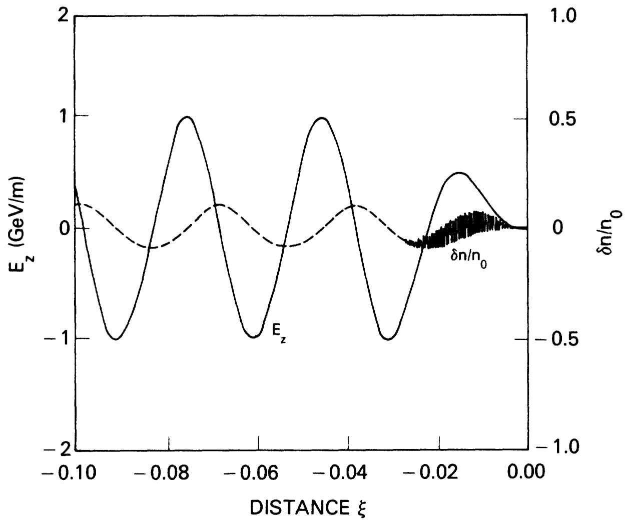

FIG. 2. Density variation $\delta n/n_0 = n/n_0 - 1$ and axial electric field E_z in GeV/m for a laser pulse located within the region $-L \le \xi \le 0$, where $L = \lambda_p = 0.03$ cm and $a_{L_0} = 0.5$.

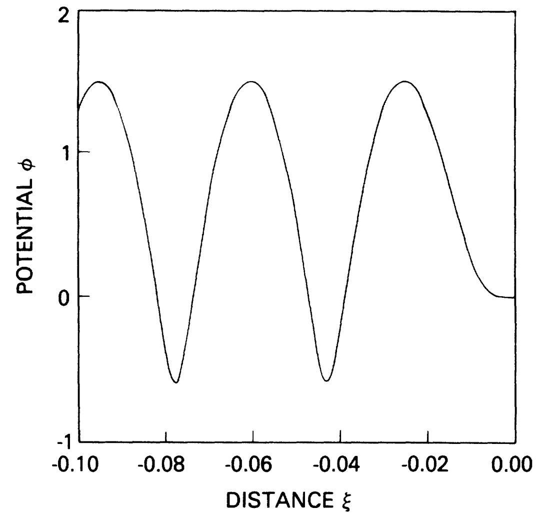

FIG. 4. Electrostatic potential ϕ for a laser pulse located within the region $-L \le \xi \le 0$, where $L = \lambda_p = 0.03$ cm and $a_{L_0} = 2.0$.

to the maximum wake-field amplitude) is given by $L_{op} = 2\gamma_{\perp 0} E(\rho)/k_p \to 2\gamma_{\perp 0}/k_p$ for $\gamma_{\perp 0}^2 \gg 1$, where $E(\rho)$ is the complete elliptic integral of the second kind, $\rho^2 = (\gamma_{\perp 0}^2 - 1)/\gamma_{\perp 0}^2$, and $\gamma_{\perp 0}^2 = 1 + a_{L_0}^2$. This gives a wake field behind the pulse for which $\gamma_{\perp 0}^2 \ge 1 + \phi_s \ge 1/\gamma_{\perp 0}^2$ and the maximum axial electric field of this wake field is given by $\hat{E}_{\max} \simeq (\gamma_{\perp 0}^2 - 1)/\gamma_{\perp 0}$, where $\hat{E} = |e|E_z/(m_0 c^2 k_p)$. The nonlinear wavelength of the wake field is given by $\lambda_p^{NL} = 4\gamma_{\perp 0} E(\rho_0)/k_p \to 4\gamma_{\perp 0}/k_p$ for $\gamma_{\perp 0}^2 \gg 1$, where $\rho_0^2 = (\gamma_{\perp 0}^4 - 1)/\gamma_{\perp 0}^4$.

The large amplitude axial electric fields associated with the plasma waves can be utilized to accelerate an injected beam of electrons to high energies (LWFA).[13-16] In the region where $\delta n/n_0 < 0$, the transverse profile of the plasma wake field can lead to a negative transverse gradient of the refractive index,[27] $\partial\eta/\partial r < 0$. Equation (9) indicates that a properly phased trailing laser pulse (located at a maximum in ϕ_s) may, therefore, be optically guided. From Eq. (16), it is seen that harmonic generation can be substantially enhanced in a properly phased (located at a minimum of ϕ_s) short trailing laser pulse propagating in the wake field generated by a leading laser pulse when $-1 \lesssim \phi < 0$. In addition, the sharp axial gradient in $\delta n/n_0$ for a highly nonlinear plasma wake field could induce large frequency shifts in a short trailing laser pulse.[17,18]

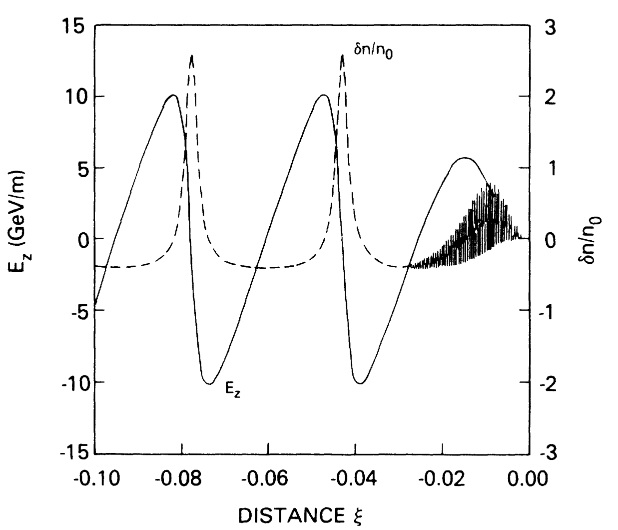

FIG. 3. Density variation $\delta n/n_0 = n/n_0 - 1$ and axial electric field E_z in GeV/m for a laser pulse located within the region $-L \le \xi \le 0$, where $L = \lambda_p = 0.03$ cm and $a_{L_0} = 2.0$.

D. Laser pulse frequency variations

As the laser pulse propagates in the plasma, the excited plasma wave can modify the laser frequency.[17,18] To analyze this effect we use the representation in Eq. (8) and set $a_L(\xi,\tau) = |a_L(\xi)|\exp[i\psi(\xi,\tau)]$, where $|a_L(\xi)|$ denotes the laser pulse envelope, and $\psi(\xi,\tau)$ is real and denotes the slowly varying phase shift. Substituting (8) into (6a) we find that

$$\psi(\xi,\tau) = -\frac{ck_p^2}{2k}\int_0^\tau [1 - \phi/(1+\phi)]d\tau' . \tag{20}$$

The laser frequency is

$$\begin{aligned} \omega_L(\xi,\tau) &= -\partial(k\xi + \psi)/\partial t \\ &= ck + (c\partial/\partial\xi - \partial/\partial\tau)\psi \\ &= ck[1 + (k_p/k)^2/2] + \Delta\omega , \end{aligned}$$

where $\Delta\omega$ is the frequency shift due to the plasma wave,

$$\Delta\omega = -\frac{ck_p^2}{2k}\left[\frac{\phi}{1+\phi} - c\frac{\partial}{\partial\xi}\int_0^\tau \frac{\phi}{1+\phi}d\tau'\right] . \tag{21}$$

For completeness, we note that the wave number is given by

$$k_L(\xi,\tau)=\partial(k\xi+\psi)/\partial\xi=k+\Delta k \ ,$$

where

$$\Delta k=\frac{ck_p^2}{2k}\frac{\partial}{\partial\xi}\int_0^\tau\frac{\phi}{1+\phi}d\tau' \ , \tag{22}$$

and the refractive index $\eta=ck_L/\omega_L$ is given by Eq. (9). The validity of (21) is based on the assumption that ψ is slowly varying; this breaks down when the frequency shift approaches the laser frequency, i.e., $|\Delta\omega|\sim ck$.

For $|\phi|\ll 1$, ϕ is given by Eq. (11) and the laser frequency shift for $c\tau/L\gg 1$ is given by

$$\Delta\omega=-ck(k_p/k)^2(\omega_p\tau/8)k_p \times\int_\xi^0|a_L(\xi')|^2\cos k_p(\xi'-\xi)d\xi' \ . \tag{23}$$

For a short-pulse laser ($L\ll\lambda_p$), assuming an interaction time $\tau=\tau_d$, and spot size $r_s\gtrsim\lambda_p$, the frequency variation within the pulse is given by $\Delta\omega/\omega=(\pi/2)^3(\lambda/\lambda_p^2)a_{L_0}^2\partial g/\partial\xi\ll 1$. Frequency variations induced on a short-pulse laser propagating in a plasma wave generated by a leading laser pulse can also be analyzed.[17,18]

E. Frequency amplification using an ionization front

A laser-induced ionization front may also be used to upshift the frequency of a laser pulse.[19] An intense pump laser with a short rise time (< 1.0 psec) may be used to ionize a gas as it propagates. This creates a moving ionization front in which the plasma density goes from zero to large values ($\sim 10^{21}$ cm^{-3}) within a very short distance ($\lesssim 0.01$ cm). This sharp plasma density gradient propagates at the group velocity of the pump laser, which may be approximated by $v_{g0}/c=(1-\omega_p^2/\omega_0^2)^{1/2}$, where ω_0 is the frequency of the pump (ionizing) laser and ω_p is the plasma frequency of the ionized gas. One possible method for upshifting the frequency of a laser pulse is the following. A secondary short (< 1.0 psec) laser pulse propagates nearly parallel with the pump laser. By phasing the secondary pulse such that it "rides" the large density gradient of the ionization front, the frequency of the secondary pulse may be upshifted in much the same manner in which a plasma wave may be used[17,18] to upshift the frequency of a laser pulse. For a linear gradient in the ionization front moving at velocity c, it can be shown that the frequency of the secondary laser pulse is given by

$$\omega(z)\simeq\omega(0)\left[1-z\left[\frac{\omega_p}{\omega(0)}\right]^2\frac{\partial}{\partial\xi}\frac{n(\xi)}{n_0}\right]^{1/2} \ , \tag{24}$$

where $\omega(0)$ is the initial frequency of the secondary laser pulse, $\partial(n/n_0)/\partial\xi=-1/d$, d is the spatial gradient length of the ionization front, and z is the interaction length. As an example, consider a KrF laser [$\lambda(0)=0.26$ μm], an ionization density of $n_0=10^{21}$ cm^{-3} ($\lambda_p=1.0$ mm) with $d=0.01$ cm and $z=1.0$ m. This gives $\omega(z=1\text{ m})\simeq 26\omega(0)$ or $\lambda=0.01$ μm. Notice that it may be possible to tune the upshifted frequency by varying the interaction length. However, due to the difference between the laser pulse velocity and the ionization front velocity, the laser pulse will eventually slip out of the region of the ionization front, thus limiting the interaction distance.[28]

Alternatively, more dramatic "amplifications" in the frequency[19] may be achieved by considering a secondary laser pulse propagating nearly collinear to the pump laser, but in the opposite direction. In this case the ionization front of the pump laser appears as a relativistic plasma mirror moving towards the secondary pulse. Hence, part of the incident secondary pulse will reflect off this ionization front. The frequency of the reflected radiation, ω_s, will be relativistically Doppler shifted, $\omega_s=4(\omega_0/\omega_p)^2\omega_i$, where ω_i is the frequency of the incident radiation. This frequency amplification may be quite large, and one can envision ω_s to be in the x-ray regime. The power of the reflected radiation, however, will most likely be small for two reasons: (i) The power of the incident radiation needs to be sufficiently low so as not to appreciably ionize the gas, and (ii) the ionization front may be a "poor" mirror, the reflectivity of which is dependent on the profile of the plasma density as well as on the ratio ω_i/ω_p. It should be noted that an electron beam may also be used in place of the pump laser to create an ionization front.

F. Single-particle acceleration

As a final point we note that electrons, initially at rest, can be accelerated axially by experiencing only the transverse laser fields. The axial acceleration is due to the $\mathbf{v}\times\mathbf{B}$ force on the electrons. In the absence of a plasma, the scalar potential vanishes, $\phi=0$. Equation (7b) shows that after experiencing the fields of a laser pulse, an electron, initially at rest, will acquire a final energy given by $\varepsilon_f=(a_f^2/2)m_0c^2$, where a_f is the final value of the laser's vector potential. This is reasonable since a_f is proportional to the area under the electric field. From Eqs. (7b) and (7c) we find that the final ratio of the magnitude of the axial to transverse velocity is $|\beta_z/\beta_\perp|_f=|a_f|/2$.

IV. DISCUSSION AND CONCLUSIONS

Based on a 1D nonlinear quasistatic model, a number of different laser-plasma interaction phenomena have been discussed. Relativistic optical guiding is shown to depend strongly on the laser pulse duration. In the long-pulse regime, optical guiding requires a minimum level of total laser power, i.e., the critical power. However, the leading portion of the pulse will experience diffraction. In the short-pulse regime, relativistic guiding effects are greatly diminished by the density response and the longitudinal motion of the electrons. As the electron quiver motion becomes highly relativistic, the plasma response current develops harmonics. This process is analyzed and it is shown that coherent harmonic generation is more effective for short pulses than it is for long pulses with slow rise times. The generation of nonlinear plasma wake fields by intense, short pulses was examined, and wave steepening is observed for very intense laser pulses. Various applications of the plasma wake fields may be possible, including (i) the acceleration of a trailing electron bunch (laser wake-field acceleration), (ii) optical guiding of a trailing laser pulse, and (iii) enhancing the coherent harmonic radiation generated by a trailing laser pulse. In addition, the 1D nonlinear model has been used to calculate the frequency shifts induced in a laser pulse by a plasma wave, as well as single-particle acceleration by a laser pulse. The possibility of frequency amplification by using a relativistic ionization front may lead to a source of coherent x rays. The viability of this process requires further investigation. The above results are based on the 1D model, which requires that the laser pulse be wide ($r_s > \lambda_p$). For the case of a narrow laser pulse ($r_s < \lambda_p$), the plasma response may be quite different than predicted by the 1D model. Generalization of the above results to include 3D effects will be the subject of future research.

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