Modified nonlocal heat-transport formula for steep temperature gradients

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A nonlocal heat-transport formula is derived by including the electrostatic potential and then solving the reduced Fokker-Planck equation for strongly inhomogeneous laser-produced plasmas. Our typical result shows a further reduction of the streaming heat-flow value which corresponds to a flux-limiting factor $f \sim 0.04$.

I. INTRODUCTION

In the past few years attempts have been made to understand the heat-transport phenomena in strongly inhomogeneous laser-produced plasmas. It is now well understood that the classical Spitzer-Harm theory[1] of heat conduction breaks down when the mean free path of high-energy electrons becomes comparable with the temperature scale length.[2] In fact, for steep gradients numerical simulations have shown that the problem of heat transport becomes nonlocal.[3,4] A number of nonlocal models have been proposed[5–12] to reproduce the simulation results. Using such models, the effect of the electrostatic potential on heat transport has been studied by Kishimoto *et al.*[10] with a simple Krook-type collisional model and by Luciani *et al.*[7] with a Fokker-Planck collisional term. The latter showed that the effect of an electric field on heat transport is simply to multiply the heat flux by a factor of 0.4, which is the usual Spitzer correction. Recently, Albritton *et al.*[11] included the electric field effect and proposed a new nonlocal heat-transport formula by solving the reduced Fokker-Planck equation, assuming the kernels are not modified by the electric potential since only the high-energy electrons take part in heat transport. On the other hand, Bendib *et al.*[12] challenged this assumption by suggesting that in the corona of a laser-produced plasma a strong ambipolar field may exist that would prevent the electrons from escaping toward the vacuum. They instead proposed a simple phenomenological nonlocal heat-transport model, showing good agreement with numerical simulation and indicating that the assumption of Albritton *et al.*[11] was not quite correct.

We, in this paper, attempt to solve the reduced Fokker-Planck equation in the presence of steep gradients as well as the electrostatic potential as Albritton *et al.* did, but free from their assumption that only high-energy electrons are responsible for heat transport. We therefore retain the electrostatic potential and systematically investigate its effect on heat transport. We solve the reduced Fokker-Planck equation without assuming $\varepsilon > e\phi$ and derive the expression for particle and heat fluxes by considering the local Maxwellian distribution function of the type considered by Cairns and Sanderson.[13] The limiting cases for steep and gentle gradients are also investigated.

II. DERIVATION OF THE HEAT-TRANSPORT FORMULA

We start with the Fokker-Planck equation for electrons

$$\frac{\partial f}{\partial t}+\mu v\frac{\partial f}{\partial x}-\frac{eE}{m}\left[\mu\frac{\partial f}{\partial v}+\frac{1-\mu^2}{v}\frac{\partial f}{\partial \mu}\right]=\left[\frac{\partial f}{\partial t}\right]_c . \quad (1)$$

Here, $f(x,v,\mu,t)$ is the electron distribution function, $\mu=\cos\theta$, where θ is the angle between the velocity and X direction and E represents the electric field. The term on the right-hand side of Eq. (1) is taken as the Fokker-Planck collision term,[14]

$$\left[\frac{\partial f}{\partial t}\right]_c=\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\left[\frac{D_\parallel}{2}\frac{\partial f}{\partial v}+Cf\right]\right]+\frac{D_\perp}{2v^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right], \quad (2)$$

where C, $D_\parallel$, and $D_\perp$ represent, respectively, the coefficients for slowing down, and diffusion in velocity space and angular space due to electron-electron and electron-ion collisions. We shall use the high-velocity asymptotic forms[15] of these coefficients because most of the energy transport takes place at velocities of three to four times the thermal speed.

Using the diffusion approximation in which $f=f_0+\mu f_1$ and taking the first two moments of Eq. (1) with the collision terms expressed in Eq. (2), we get

$$\frac{\partial f_1}{\partial t}+v\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\frac{\partial}{\partial v}\right]f_0=\frac{2v^2}{\lambda_\varepsilon}\frac{\partial}{\partial v}\left[\frac{T}{mv}\frac{\partial f_1}{\partial v}+f_1\right]-\frac{2v}{\lambda_{90}}f_1 , \quad (3)$$

$$\frac{\partial f_0}{\partial t}+\frac{v}{3}\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\left[\frac{\partial}{\partial v}+\frac{2}{v}\right]\right]f_1=\frac{2v^2}{\lambda_\varepsilon}\frac{\partial}{\partial v}\left[\frac{T}{mv}\frac{\partial f_0}{\partial v}+f_0\right] , \quad (4)$$

where λ_ε is the energy loss mean-free-path and λ_{90} is 90° scattering electron-ion mean free path.[11] We now assume high-Z plasma and approximate f_0 in the parallel

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diffusion term by the local Maxwell-Boltzmann distribution f_{MB} of the type considered by Cairns and Sanderson[13] $f_{MB}=n(m/2\pi T)^{3/2}\exp[-(mv^2/2+e\phi)/T]$, so that Eqs. (3) and (4) can be rewritten as

$$\frac{\partial f_1}{\partial t}+v\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\frac{\partial}{\partial v}\right]f_0=-\frac{2v}{\lambda_{90}}f_1\ , \tag{5}$$

$$\frac{\partial f_0}{\partial t}+\frac{v}{3}\left[\frac{\partial}{\partial x}-\frac{eE}{mv}\left[\frac{\partial}{\partial v}+\frac{2}{v}\right]\right]f_1=\frac{2v^2}{\lambda_\varepsilon}\frac{\partial}{\partial v}(f_0-f_{MB})\ . \tag{6}$$

Transforming the independent variables from (x,v,t) to (x,ε,t), where $\varepsilon=mv^2/2-e\phi(x,t)$, the above two equations become the usual reduced Fokker-Planck equations

$$\frac{\partial f_1}{\partial t}-\frac{e}{m}\frac{\partial\phi}{\partial t}\frac{\partial f_1}{\partial\varepsilon}+v\frac{\partial f_0}{\partial x}=-\frac{2v}{\lambda_{90}}f_1\ , \tag{7}$$

$$\frac{\partial f_0}{\partial t}-\frac{e}{m}\frac{\partial\phi}{\partial t}\frac{\partial f_0}{\partial\varepsilon}+\frac{v}{3}\frac{\partial f_1}{\partial x}+\frac{2}{3}\frac{\partial e\phi}{\partial x}\frac{f_1}{mv}=\frac{2v^2}{\lambda_\varepsilon}\frac{\partial}{\partial v}(f_0-f_{MB})\ . \tag{8}$$

If we assume slow temporal variation for the distribution function as well as for the potential[10,11] ϕ then the above two equations give

$$\frac{\lambda_\varepsilon}{6}\frac{\partial}{\partial x}\left[(\varepsilon+e\phi)\lambda_{90}\frac{\partial f_0}{\partial x}\right]=-4(\varepsilon+e\phi)^2\frac{\partial}{\partial(\varepsilon+e\phi)}(f_0-f_{MB}) \tag{9}$$

or

$$\frac{\partial}{\partial\xi}\left[(\varepsilon+e\phi)^3\frac{\partial f_0}{\partial\xi}\right]+\frac{\partial f_0}{\partial(\varepsilon+e\phi)}=-\frac{f_{MB}}{T}\ , \tag{10}$$

where $\xi=x/\tilde\lambda_s$, $\tilde\lambda=\lambda/4(\varepsilon+e\phi)^2$, and $\tilde\lambda_s=(\frac{2}{3}\tilde\lambda_{90}\tilde\lambda_\varepsilon)^{1/2}$. Since the potential ϕ is slowly varying, we can solve Eq. (10) by using a WKB physical optics approximation[12] and obtain

$$f_0=\int\int_Y d\xi' dY'\frac{\exp[-(\xi-\xi')^2/(Y'^4-Y^4)]}{[\pi(Y'^4-Y^4)]^{1/2}}\times\frac{f_{MB}(\xi',Y')}{T(\xi')},\quad Y=\varepsilon+e\phi\ . \tag{11}$$

Notice that the electrostatic potential ϕ is implicit in the distribution function f_{MB} as well as in Y. The particle and energy fluxes are defined as

$$[J,Q]=-4\pi\int_0^\infty dY[1,Y]Y\frac{\lambda_{90}}{3m^2}\frac{\partial f_0}{\partial x}\ . \tag{12}$$

Substituting the value of f_0 in Eq. (12) we obtain

$$[J,Q]=-\frac{1}{4}\left[\frac{\lambda_{90}}{3m\lambda_\varepsilon}\right]^{1/2}\int dx' nT^{-1/2}[1,T]\times\left\{[I(\Theta),K(\Theta)]\frac{\partial T}{\partial x'}-[J(\Theta),L(\Theta)]\times\left[\frac{5}{2}\frac{\partial T}{\partial x'}-\frac{T}{n}\frac{\partial n}{\partial x'}+2T\frac{\partial}{\partial x'}\left[\frac{e\phi}{T}\right]\right]\right\}\exp(-e\phi/T)\ , \tag{13}$$

where $I(\Theta)$, $J(\Theta)$, $K(\Theta)$, and $L(\Theta)$ are the same propagators as described by Albritton *et al.*[11] with $\Theta=\int_x^{x'}dx''/\tilde\lambda_s(x'')T^2(x')$. Notice that our J and Q expressions are more general than those of the previous model, since $\varepsilon>e\phi$ or $e\phi$ has not been made zero everywhere except under the differentiation. The extra exponential term $\exp(-e\phi/T)$ plays a critical role in the limiting cases of steep and gentle temperature gradients. First, it is convenient to split the electrostatic potential into local and nonlocal parts, $e\phi=e\phi_L-e\phi_{NL}$, where $e\phi_L=5T/2$ as obtained by Spitzer and Harm[1] on ignoring density gradient.

In the limit of steep gradients (which corresponds to small Θ), the charge neutrality condition gives the following result:

$$e\phi_{NL}/T=\ln(T^{1/4})+C_1\ ,$$

where the constant of integration C_1 may be obtained by the condition that ϕ_{NL} becomes zero when the problem becomes local so that

$$e\phi_{NL}/T=\ln(T/T_C)^{1/4}\ . \tag{14}$$

The heat-flow expression becomes

$$Q_{max}=0.25\left[\frac{\lambda_{90}}{\lambda_\varepsilon}\right]^{1/2}\frac{n}{\sqrt{m}}T_C^{3/2}[(T_H/T_C)^{5/4}-1] \tag{15}$$

where T_C and T_H are the temperatures of hot and cold species of electrons, respectively.

On the other hand, for large Θ, the problem becomes local and $J=0$ condition gives

$$e\phi_{NL}/T=\tfrac{5}{4}\ln(T)+C_2$$

where C_2 is determined through the condition that ϕ_{NL} becomes ϕ_L when T becomes T_C, i.e.,

$$e\phi_{NL}/T=\tfrac{5}{2}-\tfrac{5}{4}\ln(T/T_C)\ . \tag{16}$$

With this value of ϕ_{NL}, the classical value of heat flux is recovered for $T=T_C$:

$$Q=-25.532n(T/M)^{1/2}\lambda_{MFP}\frac{\partial T}{\partial x}\text{with }\lambda_{MFP}=T^2\tilde\lambda_{90}\ . \tag{17}$$

TABLE I. Inhibition factor f as proposed by different nonlocal models (for $Z=5$).

f	Model
0.12	Luciani, Mora, and Virmont (Ref. 6)
0.1	Luciani, Mora, and Pellat (Ref. 7), for $T_H=2T_C$
0.3	Albritton *et al.* (Ref. 11), for $T_H=2T_C$ and $\ln\Lambda_{e\text{-}e}=2\ln\Lambda_{e\text{-}i}$
0.04	Our result for $T_H=2T_C$ and $\ln\Lambda_{e\text{-}e}=2\ln\Lambda_{e\text{-}i}$

III. RESULTS AND DISCUSSION

Our main result is the maximum heat-flow expression in Eq. (15). The temperature dependence differs from the result of Albritton *et al.* ($T_H^{3/2}-T_C^{3/2}$). Table I compares the values of $Q_{\max}/Q_{FS}$ for various models, choosing $Z=5$ and $T_H=2T_C$ where $Q_{FS}=n(T_H^3/m)^{1/2}$ is the free streaming heat flux. The values of f in previous nonlocal models lie between 0.1 and 0.3, which is in agreement with the result of Wyndham *et al.*[16] On the other hand, our model calculations suggest $f\sim 0.04$, resulting in substantial inhibition. This is in agreement with the experiments of Mead *et al.*,[17] of Fabbro *et al.*,[18] and of Yaakobi *et al.*[19,20] for which $f\sim 0.03$ was observed. We therefore claim that our modified expression of heat flow, as given in Eq. (14) with $e\phi=e\phi-e\phi_{\rm NL}$, describes fairly well the heat-transport phenomenon in steep temperature gradient. It not only describes the inhibited heat flow but also reproduces the classical expression of heat flow in the local limits.

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