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Dynamic phase transition in the kinetic Ising model under a time-dependent oscillating field

Tânia Tomé and Mário J. de Oliveira
Instituto de Física, Universidade de São Paulo, Caixa Postal 20 516, 01 498 São Paulo, São Paulo, Brazil
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We analyze within a mean-field approach the stationary states of the kinetic Ising model described by the Glauber stochastic dynamics and subject to a time-dependent oscillating external field. We have found that the magnetization of the system oscillates in time around a certain value that is zero at high temperatures or large field amplitudes and nonzero at low temperatures and small field amplitudes. The transition from one regime to the other, which corresponds to a spontaneous symmetry breaking, is found to be continuous for sufficiently small values of the field amplitudes. For higher values the transition becomes discontinuous and the system exhibits a dynamical tricritical point.

I. INTRODUCTION

The time evolution of nonequilibrium states of a thermodynamic system can be studied by establishing an appropriate stochastic dynamics that will drive the system. This approach is suitable for certain lattice systems, such as the Ising model, that have no deterministic dynamics. Stochastic dynamics are capable of describing not only the approach to equilibrium states[1–3] but also nonequilibrium states and dynamic phase transitions[4–7] of interacting lattice systems.

In this paper we analyze an interacting lattice system with a stochastic dynamics subject to a forced oscillating field. More precisely, we have studied, within a mean-field approach, the kinetic Ising model described by a Glauber dynamics[1] in the presence of a time-dependent oscillating external field. For high temperatures or large field strength, we have found that the system follows the oscillating field with a delay. At low temperatures and small field strengths, however, a spontaneous symmetry breaking sets in and the system is not able to accompany the oscillating field anymore.

The transition from one regime to the other is found to be continuous for sufficiently small values of the field amplitude. For higher values the transition becomes discontinuous and the system exhibits a dynamical tricritical point.

II. THE MODEL

We consider a kinetic Ising system with N spins described by the ferromagnetic mean-field Hamiltonian

$$\mathcal{H}=-\frac{J}{N}\sum_{\substack{i,j\\ i<j}}\sigma_i\sigma_j-H\sum_i\sigma_i \ , \tag{1}$$

where $\sigma_i=\pm 1$, $J>0$, and H is a time-dependent external field given by

$$H(t)=H_0\cos(\omega t) \ . \tag{2}$$

The system evolves according to the Glauber stochastic process[1] at a rate of $1/\tau$ transitions per unit time. Let $w_i(\{\sigma_j\})$ be the probability per unit time of flipping the ith spin at time t. The Glauber prescription gives[1,3]

$$w_i=\frac{1}{2\tau}[1-\sigma_i\tanh(\beta E_i)] \ , \tag{3}$$

where

$$E_i=\frac{J}{N}\sum_{j(\neq i)}\sigma_j+H(t) \ . \tag{4}$$

From the master equation associated to the stochastic process it follows that the average $\langle\sigma_i\rangle$ satisfies the following equation:[3]

$$\tau\frac{d}{dt}\langle\sigma_i\rangle=-\langle\sigma_i\rangle+\langle\tanh(\beta E_i)\rangle \ . \tag{5}$$

When the thermodynamic limit $N\rightarrow\infty$ is taken, the quantity $\sum_{j(\neq i)}\sigma_j/N$ approaches, by the law of large numbers, the average $\langle\sigma_i\rangle$ which is supposed to be independent of i. In this case $E_i=J\langle\sigma_i\rangle+H(t)$ and Eq. (5) is reduced to the simple equation

$$\tau\frac{d}{dt}\langle\sigma_i\rangle=-\langle\sigma_i\rangle+\tanh\{\beta[J\langle\sigma_i\rangle+H_0\cos(\omega t)]\} \ , \tag{6}$$

which involves only the average $\langle\sigma_i\rangle$. This equation is written in the form

$$\Omega\frac{dm}{d\xi}=-m+\tanh\left[\frac{1}{T}(m+h\cos\xi)\right] \ , \tag{7}$$

where $m=\langle\sigma_i\rangle$, $\xi=\omega t$, and T, h, and Ω are dimensionless parameters defined by $T=(\beta J)^{-1}$, $h=H_0/J$, and $\Omega=\omega\tau$.

III. RESULTS

We are concerned here with the analysis of the stationary solutions of Eq. (7) when the parameters T, h, and Ω are varied. It is clear that for $h=0$ the usual mean-field equation for a ferromagnet is recovered. In this case it is well known that a symmetry breaking takes place at $T=1$. We shall see that for a nonzero value of h the system also exhibits a symmetry breaking as long as $h<1$.

The stationary solutions of Eq. (7) will be a periodic function of ξ with period 2π; that is, $m(\xi+2\pi)=m(\xi)$. Moreover, they can be one of two types according to whether they have or do not have the property

$$m(\xi+\pi)=-m(\xi)\ . \tag{8}$$

A solution with this property is called a symmetric or paramagnetic solution. It occurs at high temperatures or at large values of h. The magnetization $m(\xi)$ oscillates around the zero value and is delayed with respect to the external field. Examples of this type of solution are shown in Figs. 1(a) and 2(a).

By decreasing the temperature at constant h the symmetric solution becomes unstable and the symmetry given by Eq. (8) is broken as long as $h<1$. A new type of solution is then obtained which we call a nonsymmetric or

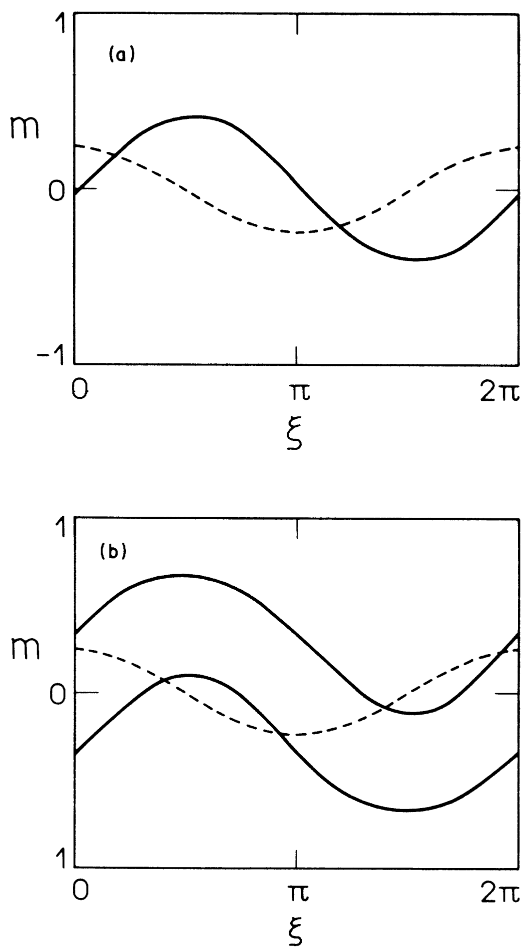

FIG. 1. The solid lines represent the steady-state solutions of Eq. (7) for $\Omega/2\pi=0.1$, $h=0.25$, and $T=0.9$ (a) and $T=0.85$ (b). In (a) only the symmetric solution exists and is stable, and in (b) only the nonsymmetric solutions are stable. The dashed lines correspond to the input field $h\cos\xi$.

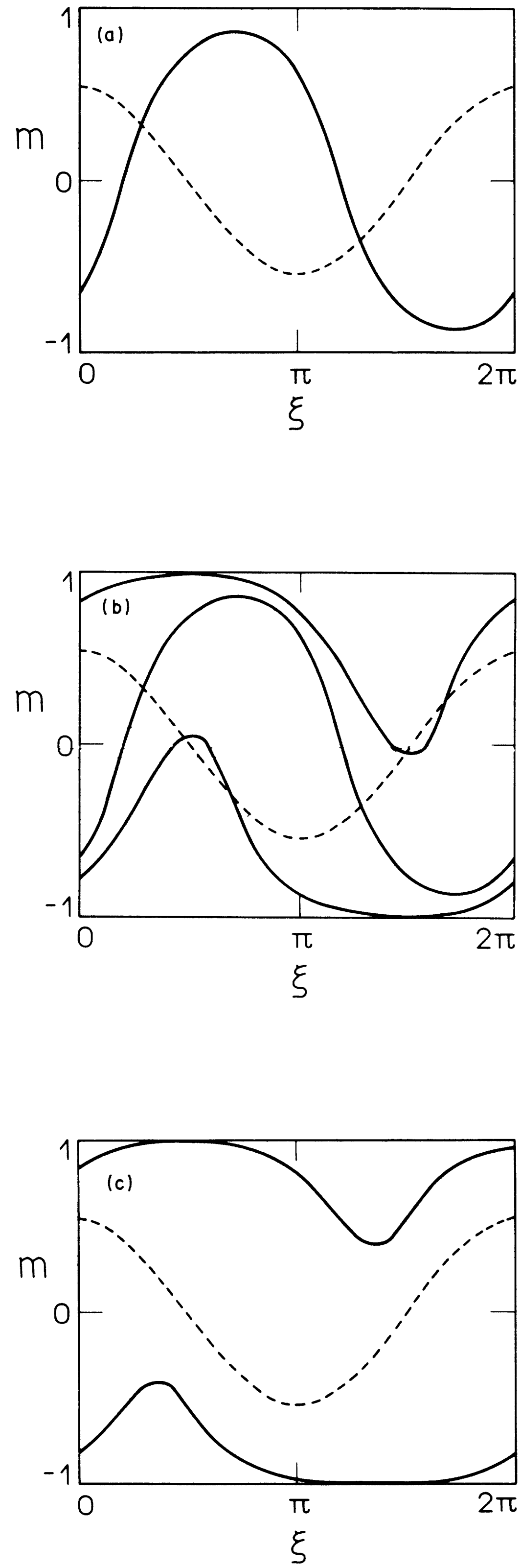

FIG. 2. The solid lines represent the steady-state solutions of Eq. (7) for $\Omega/2\pi=0.1$, $h=0.55$, and $T=0.395$ (a), $T=0.39$ (b), and $T=0.385$ (c). In (a) only the symmetric solution exists and is stable, in (c) only the nonsymmetric solutions are stable, and in (b) the symmetric as well as the nonsymmetric solutions are stable. The dashed lines correspond to the input field $h\cos\xi$.

ferromagnetic solution. In this case the magnetization does not follow the external field any more, but instead oscillates around a nonzero value, as can be seen in Figs. 1(b) and 2(c). There are always two independent solutions of this type. If we denote them by $m_+(\xi)$ and $m_-(\xi)$ we see that they are related to each other by the property

$$m_+(\xi+\pi)=-m_-(\xi)\ . \tag{9}$$

The spontaneous breaking of symmetry (8) is described by the order parameter $m_0(\xi)$ defined by

$$m_0(\xi)=\tfrac{1}{2}[m(\xi)+m(\xi+\pi)]\ . \tag{10}$$

For a paramagnetic solution $m_0(\xi)$ vanishes identically. Instead of dealing with $m_0(\xi)$, which is actually a multicomponent order parameter, we will make use of one of its Fourier components, namely the component M given by

$$M=\frac{1}{2\pi}\int_0^{2\pi} m(\xi)d\xi\ , \tag{11}$$

which is just the mean magnetization. Figures 3 and 4 show the behavior of M as a function of temperature for $h=0.25$ and 0.55, respectively. In the first case M vanishes continuously as the temperature is increased. In the second case, however, M behaves discontinuously and jumps from a nonzero to a zero value. In this case there is a range of values of h where the symmetric as well as the two nonsymmetric solutions are stable. An example of such a case is shown in Fig. 2(b).

The stability of a solution may be verified by calculating the Liapunov exponent λ. If we write Eq. (7) as

$$\Omega\frac{dm}{d\xi}=F(m,\xi)\ , \tag{12}$$

then the exponent λ is given by

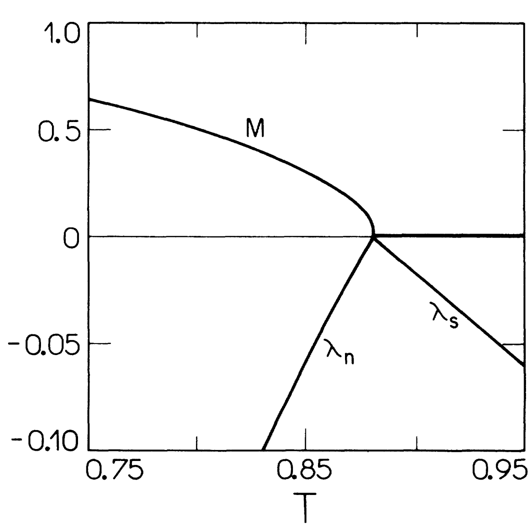

FIG. 3. The component M of the order parameter and the Liapunov exponents λ_s and λ_n are shown as a function of temperature for $h=0.25$ and $\Omega/2\pi=0.1$. They all vanish at the same temperature.

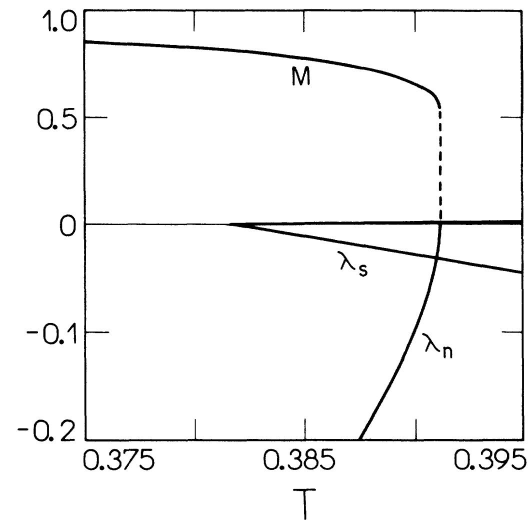

FIG. 4. The component M of the order parameter and the Liapunov exponents λ_s and λ_n are shown as a function of temperature for $h=0.55$ and $\Omega/2\pi=0.1$. Notice that M jumps from a nonzero value to a zero value at a temperature where $\lambda_n=0$.

$$\Omega\lambda=\frac{1}{2\pi}\int_0^{2\pi}\frac{\partial F}{\partial m}d\xi\ . \tag{13}$$

If $\lambda<0$ then the solution is stable.

Let us call the paramagnetic (ferromagnetic) region of the (T,h) plane the region where the symmetric (nonsymmetric) solution is stable. If we denote by $\lambda_s(\lambda_n)$ the Liapunov exponent associated to the symmetric (nonsymmetric) solution then the boundary of the paramagnetic (ferromagnetic) region is given by $\lambda_s=0$ $(\lambda_n=0)$. In the paramagnetic region we have also $M=0$ whereas in the

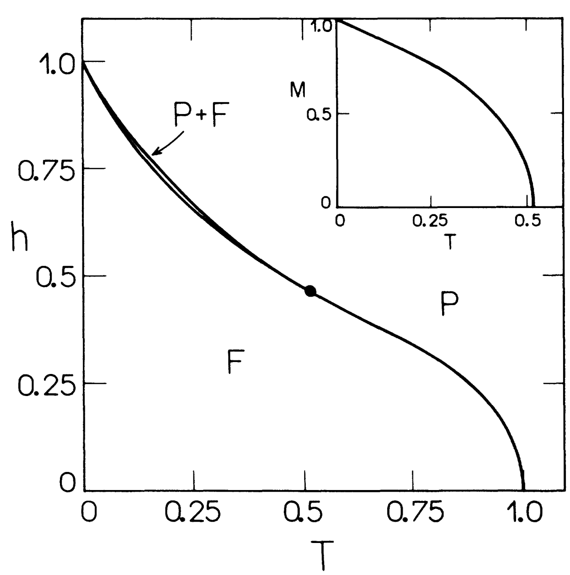

FIG. 5. Phase diagram in the (T,h) plane for $\Omega/2\pi=0.1$. The paramagnetic (P) and ferromagnetic (F) regions overlap in the region indicated by P+F. The dot represents the tricritical point. The inset shows the jump in the order parameter along the boundary of the ferromagnetic region.

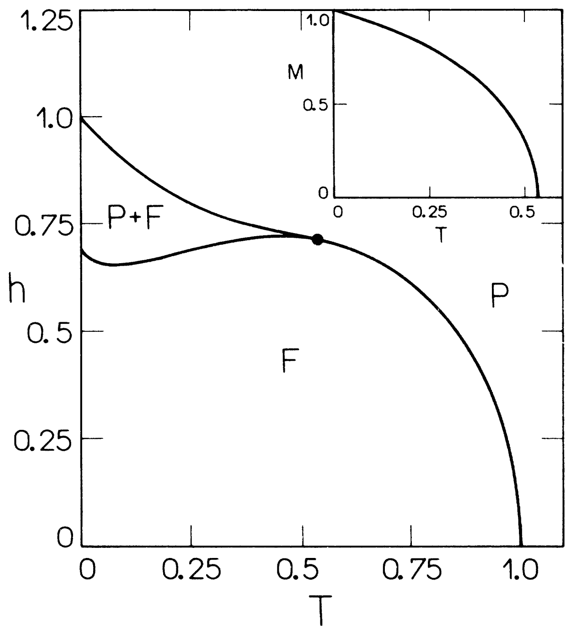

FIG. 6. The same as Fig. 5 for $\Omega/2\pi=1$.

ferromagnetic region $M\neq 0$. The behavior of λ_s and λ_n as a function of temperature at constant h is shown in Figs. 3 and 4 together with M.

For small values of h we have found that the boundary lines of the paramagnetic and ferromagnetic regions coincide. For large values of h, however, they are distinct and the two regions overlap as can be seen in Figs. 5 and 6. The phase diagram exhibits then a tricritical point, the point where the two boundary lines merge.

The tricritical point is present for any value of Ω. For small values, however, the overlapping region becomes very narrow. Our results indicate that, in the limit $\Omega\to 0$, the width of this region vanishes and the tricritical temperature approaches the static critical temperature.

IV. BEHAVIOR AROUND $T=1$, $h=0$

The behavior of the transition line around the static critical point $T=1$ and $h=0$ is given by $h\sim(1-T)^{1/2}$. This result is obtained as follows. Let us define the variable x by $x=m+h\cos\xi$. Equation (7) can then be written as

$$\Omega\frac{dx}{d\xi}=-x+\tanh\frac{x}{T}+h(1+\Omega^2)^{1/2}\cos(\xi+\phi)\ , \quad (14)$$

where ϕ is such that $\tan\phi=\Omega$. Around the static critical point we expect x to be small so that the right-hand-side of Eq. (14) can be expanded in a power series of x. By retaining up to the third-order term in x we obtain

$$\Omega\frac{dx}{d\xi}=(1-T)x-\frac{x^3}{3}+h(1+\Omega^2)^{1/2}\cos(\xi+\phi)\ . \quad (15)$$

The Liapunov exponent λ is then given by

$$\Omega\lambda=\frac{1}{2\pi}\int_0^{2\pi}[(1-T)-x^2]d\xi\ . \quad (16)$$

For a fixed value of Ω and for T sufficiently close to 1, that is, for $|1-T|\ll\Omega$, the symmetric solution of Eq. (15) in first order of approximation in h will be

$$x=\frac{h}{\Omega}(1+\Omega^2)^{1/2}\sin(\xi+\phi) \quad (17)$$

from which we get

$$\Omega\lambda_s=(1-T)-\frac{h^2(1+\Omega^2)}{2\Omega^2}\ . \quad (18)$$

The transition line $\lambda_s=0$ will then be given by

$$h=\frac{\Omega}{(1+\Omega^2)^{1/2}}\sqrt{2(1-T)}\ . \quad (19)$$

V. CONCLUSION

We have analyzed within a mean-field approach the kinetic Ising model under a time-dependent oscillating field. The time evolution of the system was described by a stochastic dynamics of the Glauber type. We have found that the system exhibits a continuous-phase transition for sufficiently small values of h. For higher values the transition becomes discontinuous and the system shows a dynamical tricritical point. We point out that the mean-field approach is usually valid for high dimensions. It would then be important to see whether the results found here persist for low dimensions. In this case a Monte Carlo simulation on a Bravais lattice would be useful. To our knowledge, however, such a simulation has not been performed yet.

Finally, we remark that the stochastic approach makes it necessary to assume that the number of spin flips occurring in a period of the oscillating external field is large. Since the number of spin flips per unit time equals $1/\tau$, the results reported here or any result coming from Monte Carlo simulation should be compared to real systems whose relaxation time is small when compared to the period $2\pi/\omega$ of the oscillating field.

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