Emission and absorption by two atoms in a damped cavity

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Emission and absorption from two two-level atoms interacting with a single mode of a cavity field in the vacuum state are investigated. The role of cavity losses, detuning, and cooperativity on the Rabi oscillations is studied. Absorption from a weak probe field with the cavity at zero temperature computed to third order in the atom-probe coupling constant is analyzed to determine the effect of cavity damping and detuning on the Rabi splitting.

Recent advances in experimental techniques have made it possible to observe some interesting aspects of the atomic behavior in a cavity such as vacuum-field Rabi oscillations,[1] collapses and revivals,[2] and inhibition and enhancement of spontaneous emission,[3] etc. Much of the theoretical understanding of this behavior rests on the Jaynes-Cummings model,[4] which has been known to be exactly solvable in the absence of cavity damping. Recently, however, Agarwal and Puri[5] have presented exact quantum-electrodynamic results for the emission and absorption spectra of a single atom contained in a cavity with finite Q. In this paper we present a generalization of the results of Agrawal and Puri to the case of two atoms in a damped cavity.

Our system comprises two identical two-level atoms of resonance frequency ω_0, interacting with a single mode of the quantized radiation field of frequency ω in a cavity. The Hamiltonian of the system is

$$H=\hbar\omega_0 S^z+\hbar\omega a^\dagger a+\hbar g(aS^+ +a^\dagger S^-) , \tag{1}$$

where

$$S^i=\tfrac{1}{2}(\sigma_1^i+\sigma_2^i), \quad i=x,y,z$$

$$S^\pm=S^x\pm iS^y ,$$

σ_j^i $(j=1,2)$ are the Pauli-spin matrices for the jth atom, $a^\dagger$ and a are the creation and annihilation operators of the field, and g is the field-atom coupling constant.

We assume a cavity of finite Q. The leakage of photons from the cavity leads to decay of the field at a rate κ. The density matrix of the system satisfies the equation of motion[6]

$$\frac{\partial\rho}{\partial t}=\frac{-i}{\hbar}[H,\rho]-\kappa(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a)\equiv L\rho . \tag{2}$$

The states of the system may be labeled as $|S,M;n\rangle$, where $M=-S,-S+1,\ldots,S$ and $|n\rangle$ denotes the Fock state of the field. We consider transitions from the state $|1,1;0\rangle\equiv|0,e\rangle$ to the states $|1,0;1\rangle\equiv 1,i\rangle$ and $|1,-1;2\rangle\equiv|2,g\rangle$. Using Eq. (2) we obtain the following equations of motion for the density matrix elements:

$$\dot\eta_1=-i\sqrt{2}g\eta_5 , \tag{3}$$

$$\dot\eta_2=-2\kappa\eta_2+i\sqrt{2}g\eta_5-i2g\eta_9 , \tag{4}$$

$$\dot\eta_3=-4\kappa\eta_3+i2g\eta_9 , \tag{5}$$

$$\dot\eta_4=-4\kappa\eta_4+i\Delta\eta_5-i2g\eta_7 , \tag{6}$$

$$\dot\eta_5=-i2\sqrt{2}g\eta_1+i2\sqrt{2}g\eta_2+i\Delta\eta_4-\kappa\eta_5-i2g\eta_6 , \tag{7}$$

$$\dot\eta_6=-i2g\eta_5-2\kappa\eta_6+i2\Delta\eta_7+i\sqrt{2}g\eta_9 , \tag{8}$$

$$\dot\eta_7=-i2g\eta_4+i2\Delta\eta_6-2\kappa\eta_7+i\sqrt{2}g\eta_8 , \tag{9}$$

$$\dot\eta_8=i\sqrt{2}g\eta_7-3\kappa\eta_8+i\Delta\eta_9 , \tag{10}$$

$$\dot\eta_9=-i4g\eta_2+i4g\eta_3+i\sqrt{2}g\eta_6+i\Delta\eta_8-3\kappa\eta_9 , \tag{11}$$

where for the sake of economy of notation we have defined

$$\begin{aligned}
&\eta_1(t)=\langle 0,e|\rho|0,e\rangle, \quad \eta_2(t)=\langle 1,i|\rho|1,i\rangle ,\\
&\eta_3(t)=\langle 2,g|\rho|2,g\rangle ,\\
&\eta_{4,5}(t)=\langle 1,i|\rho|0,e\rangle\pm\langle 0,e|\rho|1,i\rangle ,\\
&\eta_{6,7}(t)=\langle 2,g|\rho|0,e\rangle\pm\langle 0,e|\rho|2,g\rangle ,\\
&\eta_{8,9}(t)=\langle 2,g|\rho|1,i\rangle\pm\langle 1,i|\rho|2,g\rangle .
\end{aligned} \tag{12}$$

Incidentally, in the simplest case of $\Delta=\kappa=0$, the solution for $\eta_3(t)$ is given by

$$\eta_9(t)=\tfrac{1}{9}(3-4\cos\sqrt{6}gt+\cos 2\sqrt{6}gt) . \tag{13}$$

We are in a position now to study the physics of radiation-matter interaction in terms of the influence of cavity relaxation and detuning on the vacuum-field Rabi oscillations and the role of cooperativity. Numerical solutions for $\eta_3(t)$ for a general case (Δ and κ both nonzero) are shown in Fig. 1. The cascade structure between the adjacent peaks is to be seen only in the presence of detuning: it disappears when we set $\Delta=0$ and smooth oscillations result, albeit of decreasing amplitude due to damping. The cascade periodicity is $5.2gt$. The oscillation frequency, however, depends on Δ and it decreases with increase in Δ. The fact that the single-atom case does not show any structure between the adjacent peaks leads us to infer that we are witnessing here the effects of cooperativity. Another manifestation of cooperativity appears in the different oscillation frequencies of the one- and two-atom on-resonance cases with κ

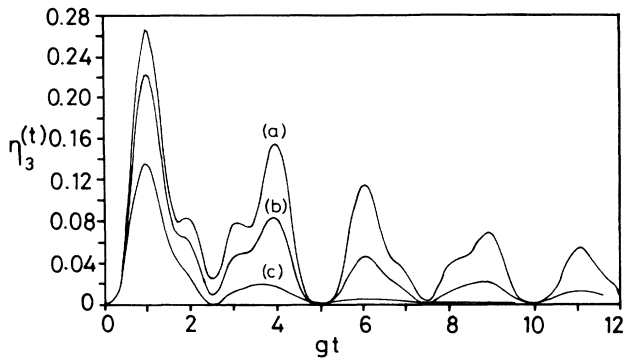

FIG. 1. The probability η_3 vs time with $\Delta=2g$ and different values of κ. Curve (*a*) $\kappa=0.1$; curve (*b*) $\kappa=0.2$; curve (*c*) $\kappa=0.5$.

nonzero which are found to be $3.1gt$ and $2.6gt$, respectively. The problem of two atoms in a damped cavity has also been discussed by others[7] though with a different emphasis and for the on-resonance cavity only. We have seen that detuning is not merely an inessential complication; it reveals some important structure between adjacent peaks in the transition probability which upon comparison with the single-atom case can clearly be attributed to cooperativity.

In order to see how the cavity damping affects the absorption spectrum, we consider absorption of energy by the cavity bound atoms from a weak probe field of frequency ν. The time rate of absorption is proportional to $\langle S^+\rangle$. To compute this expectation value we expand the density matrix in powers of the coupling constant G of the probe field with the atoms. The first nonzero contribution to $\langle S^+\rangle$ comes from $\rho^{(1)}$, the expression for which is given by

$$\rho^{(1)}(t)=-iGe^{-i\nu t}\int_0^t dt_1 e^{i\nu t_1}e^{iLt_1}[S^+,\rho(0)]+\text{H.c.} \quad (14)$$

We start with a cavity with zero temperature so that the initial density matrix can be written as

$$\rho^{(0)}=|0,g\rangle\langle 0,g| \ .$$

To determine the corresponding contribution to $\langle S^+\rangle$, we proceed as follows. Define

$$\phi(t)=(1/\sqrt{2})e^{Lt}[S^+,\rho(0)] \ , \quad (15)$$

so that we may write

$$\phi(t)=e^{Lt}|0,i\rangle\langle 0,g|$$
$$=\alpha(t)|0,i\rangle\langle 0,g|+\beta(t)|1,g\rangle\langle 0,g| \ . \quad (16)$$

Evidently α and β satisfy the equations of motion

$$\dot{\alpha}(t)=-i\omega_0\alpha(t)-i\sqrt{2}g\beta(t) \ , \quad (17)$$

$$\dot{\beta}(t)=-i\sqrt{2}g\alpha(t)-(i\omega+\kappa)\beta(t) \ . \quad (18)$$

Since $\alpha(0)=1$ and $\beta(0)=0$, their Laplace transforms are

$$\hat{\alpha}(s)=(s+i\omega+\kappa)/(s-s_1)(s-s_2) \ ,$$
$$\hat{\beta}(s)=-i\sqrt{2}g/(s-s_1)(s-s_2) \ , \quad (19)$$

where

$$s_{1,2}=-\tfrac{1}{2}[i(\omega+\omega_0)+\kappa]\pm(i/2)(\Delta^2+8g^2-\kappa^2+2i\Delta\kappa)^{1/2} \ .$$

If $\Delta=0$,

$$s_{1,2}=-i\omega-\tfrac{1}{2}\kappa\pm(i/2)(8g^2-\kappa^2)^{1/2} \ .$$

Incidentally, the good and bad cavity values ($\kappa\ll 4g^2$ and $\kappa\gg 4g^2$, respectively) are

$$s_{1,2}=-i\omega-(1/2)\kappa\pm i\sqrt{2}g, \quad -i\omega-(1/2)\kappa\mp(1/2)\kappa \ .$$

The first-order contribution to W is thus

$$W=-4\hbar\nu|G|^2\text{Re}\hat{\alpha}(-i\nu) \ , \quad (20)$$

where $\hat{\alpha}(-i\nu)$ may be obtained by setting $s=-i\nu$ in the first of Eqs. (19).

In this order the final state reached is $|1,i\rangle$. To reach the ground state we need a higher-order calculation. It turns out that the second-order contribution to W vanishes. We have also evaluated the third-order contribution. The analytic expressions being lengthy are not reproduced here. However, the curves shown in Fig. 2 include the third-order contribution.

Figure 2 shows the absorption spectrum of the two atoms contained inside the cavity for $\Delta=-1.0g$. Splitting of the spectrum is observed for the good-cavity case. It is caused by the vacuum-field Rabi oscillations. As we increase the value of κ, splitting begins to disappear. For example, for $\kappa=5g$ (bad cavity), the spectrum has only a single peak. A comparison with the one-atom case reveals some similarities but quantitative differences arise in the heights and widths of the peaks and their locations. These differences are to be attributed to the cooperative effects.

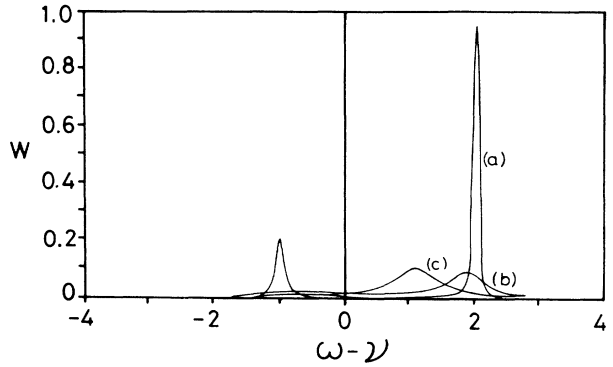

FIG. 2. Absorption spectrum as a function of $\omega-\nu$ for $\Delta=-1$ and different values of κ. Curve (*a*) $\kappa=0.1$; curve (*b*) $\kappa=1.$; curve (*c*) $\kappa=5$.

It is instructive to compare the present work with Agarwal's in Ref. 5(b), where he discusses the N-atom case. In his work the cavity decay term is not calculated explicitly and the use of the limit $g\sqrt{N} \gg \kappa$ and the secular approximation eliminates explicit reference to κ, so that the dependence of the absorption spectrum on κ cannot be studied. The present work does not suffer from such constraints and the change in the absorption spectrum with cavity damping is computed and manifested in Fig. 2.

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