Explicit N-soliton solution of the modified nonlinear Schrödinger equation

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By means of the technique of determinant calculation, an explicit N-soliton solution of the modified nonlinear Schrödinger equation is obtained for arbitrary N using the method of a meromorphic transformation matrix. The final expression is written in a form suitable for practical needs. Its expected asymptotic behavior is derived.

The modified nonlinear Schrödinger (MNLS) equation has been proposed[1,2] to describe the short-pulse propagation in a long single-mode optical fiber in consideration of the inherent property of asymmetric output pulse spectrum.[3,4] The MNLS equation has been shown to be completely integrable,[5] but it has never been solved except for its one-soliton solution obtained by simply integrating in a moving coordinate,[6] so that some works have to be based on the numerical analysis for suiting needs of practice.[7,8]

Recently, we proposed a method[9] based on an ansatz that the Jost solutions, and then the transformations among them, are meromorphic and have only simple poles. We also obtained a generalized Zakharov-Shabat system of linear algebraic equations for determining the N-soliton solution of the MNLS equation. From it we have obtained an expression in terms of a determinant of the known quantities.[10] However, even in the case of a two-soliton solution, it is tedious to obtain an explicit expression.[9,10]

In this work, we present an explicit expression of the N-soliton solution of the MNLS equation from the generalized Zakharov-Shabat equations with the aid of pure algebraic calculation. In addition, we introduce a new spectral parameter to express the final expression in a form suitable for practical needs. We also derive the expected asymptotic behavior of the N-soliton solution.

The MNLS equation can be written in the following normalized form:

$$iu_t+u_{xx}+i(|u|^2u)_x+2\rho|u|^2u=0\ , \tag{1}$$

where ρ is a real constant. In our previous paper,[9] a method based on the Darboux transformation in the form of pole expansion has been proposed for finding its soliton solutions. The N-soliton solution can be expressed as

$$u_N=i2A\bar{B}\ , \tag{2}$$

where the overbar denotes the complex conjugate, and

$$A=-2\sum_{j=1}^{N}a_j^{-1}\phi_jf_j^{-1}\zeta_j^{-2}\ , \tag{3}$$

$$B=1-2\sum_{j=1}^{N}a_j^{-1}\psi_jf_j^{-1}\zeta_j^{-1}\ , \tag{4}$$

$$a_j=\prod_{k(\neq j)}\frac{\zeta_j^2-\zeta_k^2}{\zeta_j^2-\bar{\zeta}_k^2}\frac{2\zeta_j}{\zeta_j^2-\bar{\zeta}_j^2}\ , \tag{5}$$

$$f_j^{-1}=b_j^{-1}\exp\{-i[(\zeta_j^{-2}-\rho)x+2(\zeta_j^{-2}-\rho)^2t]\}\ , \tag{6}$$

and ϕ_j and $\bar{\psi}_k$ satisfy the following linear algebraic equations:

$$-\bar{\psi}_k=\sum_{j=1}^{N}\bar{f}_k^{-1}\frac{2\bar{\zeta}_k}{\bar{\zeta}_k^2-\zeta_j^2}f_j^{-1}a_j^{-1}\phi_j\ , \tag{7}$$

$$\phi_k=f_k^{-1}+\sum_{j=1}^{N}f_k^{-1}\frac{2\bar{\zeta}_j}{\zeta_k^2-\bar{\zeta}_j^2}\bar{f}_j^{-1}\bar{a}_j^{-1}\bar{\psi}_j\ . \tag{8}$$

ζ_j and b_j are complex constants.

The purpose of this paper is to give an explicit N-soliton solution. Introducing

$$p_j=-i\zeta_j\ , \tag{9}$$

$$\alpha_j=i\tfrac{1}{2}a_j=\prod_{k(\neq j)}\frac{p_j^2-p_k^2}{p_j^2-\bar{p}_k^2}\frac{p_j}{p_j^2-\bar{p}_j^2}\ , \tag{10}$$

$$C=(\alpha_1^{-1/2}f_1^{-1},\ldots,\alpha_N^{-1/2}f_N^{-1})\ , \tag{11}$$

$$\Phi=(\alpha_1^{-1/2}\phi_1,\ldots,\alpha_N^{-1/2}\phi_N)\ , \tag{12}$$

$$\Psi=(\alpha_1^{-1/2}\psi_1,\ldots,\alpha_N^{-1/2}\psi_N)\ , \tag{13}$$

$$Q_{jk}=C_j\frac{\bar{p}_k}{p_j^2-\bar{p}_k^2}\bar{C}_k\ , \tag{14}$$

(7) and (8) can be written as

$$\bar{\Psi}=-\Phi\underline{Q}\ , \tag{15}$$

$$\Phi=\underline{C}+\bar{\Psi}\,\underline{Q}^T\ . \tag{16}$$

In virtue of (15) and (16), (3) and (4) may be rewritten as

$$A=i\underline{C}(\underline{I}+\underline{Q}\,\underline{Q}^T)^{-1}\underline{p}^{-2}\underline{C}^T\ , \tag{17}$$

$$\bar{B}=1+\underline{C}(\underline{I}+\underline{Q}\,\underline{Q}^T)^{-1}\underline{Q}\,\underline{\bar{p}}^{-1}\underline{\bar{C}}^T\ , \tag{18}$$

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where $\underline{p}^{-2}$ is simply a diagonal matrix, i.e., $\mathrm{diag}(p_1^{-2},\ldots,p_N^{-2})$.

Since we have the known formula of linear algebra,

$$\det(x_i y_j + Z_{ij}) = \det(Z_{ij})\left[1+\sum_{i,j=1}^{M} x_i y_j (Z^{-1})_{ji}\right], \qquad (19)$$

which is valid for a nonsingular $M\times M$ matrix and arbitrary rows x and y, (17) and (18) can be expressed as

$$A = i\{[\det(\underline{I}+\underline{R})]^{-1}\det(\underline{I}+\underline{R}')-1\}, \qquad (20)$$

$$\bar{B} = [\det(\underline{I}+\underline{R})]^{-1}\det(\underline{I}+\underline{R}''), \qquad (21)$$

where

$$\underline{R} = \underline{Q}\,\underline{Q}^T, \qquad (22)$$

$$\underline{R}' = \underline{R} + \underline{p}^{-2}\underline{C}^T\underline{C}, \qquad (23)$$

$$\underline{R}'' = \underline{R} + \underline{Q}\,\bar{\underline{p}}^{-1}\bar{\underline{C}}^T\underline{C}. \qquad (24)$$

We have

$$\det(\underline{I}+\underline{R}) = 1+\sum_{r=1}^{N}\sum_{1\le j_1<j_2<\cdots<j_r\le N} R(j_1,j_2,\ldots,j_r), \qquad (25)$$

$$R(j_1,j_2,\ldots,j_r) = \sum_{1\le k_1<k_2<\cdots<k_r\le N} Q(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)^2, \qquad (26)$$

where we have taken account of the well-known Binet-Cauchy formula, and where $Q(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)$ denotes a minor that is a determinant of a submatrix of $\underline{Q}$ by remaining $(j_1,j_2,\ldots,j_r)$th rows and $(k_1,k_2,\ldots,k_r)$th columns. $Q(j_1,j_2,\ldots,j_r)$ means a principal minor, i.e., $Q(j_1,j_2,\ldots,j_r;j_1,j_2,\ldots,j_r)$.

Using the known formula

$$\det[(x_j+y_k)^{-1}] = \prod_{j<j'}(x_j-x_{j'})\prod_{k<k'}(y_k-y_{k'})\prod_{j,k}(x_j+y_k)^{-1}, \qquad (27)$$

we obtain

$$Q(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)^2 = \prod_j C_j^2 \prod_k \bar{C}_k^2\bar{p}_k^2 \prod_{j<j'}(p_j^2-p_{j'}^2)^2 \times \prod_{k<k'}(\bar{p}_k^2-\bar{p}_{k'}^2)^2\prod_{j,k}(p_j^2-\bar{p}_k^2)^{-2}, \qquad (28)$$

where

$$j,j'\in\{j_1,j_2,\ldots,j_r\}, \quad k,k'\in\{k_1,k_2,\ldots,k_r\}. \qquad (29)$$

Since we can rewrite (23) as

$$\underline{R}' = \underline{Q}'\underline{Q}''^T, \qquad (30)$$

where $\underline{Q}'$ and $\underline{Q}''$ are $N\times(N+1)$ matrices whose rows are extended from 1 to N, columns from 0 to N,

$$Q'_{jk} = Q''_{jk} = Q_{jk}, \qquad (31)$$

$$Q'_{j0} = p_j^{-2}C_j, \quad Q''_{j0} = C_j, \qquad (32)$$

$j,k=1,2,\ldots,N$. We have also

$$R'(j_1,j_2,\ldots,j_r) = \sum_{0\le k_1<k_2<\cdots<k_r\le N} Q'(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)Q''(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r). \qquad (33)$$

The summation can be obviously decomposed into two parts: one is extended to $k_1=0$, the other to $k_1\ge 1$. The latter is just $R(j_1,j_2,\ldots,j_r)$ on account of (31). We thus obtain

$$\det(\underline{I}+\underline{R}')-\det(\underline{I}+\underline{R}) = \sum_{r=1}^{N}\sum_{1\le j_1<j_2<\cdots<j_r\le N}\;\sum_{1\le k_2<\cdots<k_r\le N} Q'(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r) \times Q''(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r). \qquad (34)$$

Using (27), we have

$$Q'(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r)Q''(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r) = \prod_j C_j^2\prod_k \bar{p}_k^2\bar{C}_k^2\prod_{j<j'}(p_j^2-p_{j'}^2)^2\prod_{k<k'}(\bar{p}_k^2-\bar{p}_{k'}^2)^2\prod_{j,k}(p_j^2-\bar{p}_k^2)^{-2}\prod_j p_j^{-2}\prod_k \bar{p}_k^2, \qquad (35)$$

where

$$j,j'\in\{j_1,j_2,\ldots,j_r\}, \quad k,k'\in\{k_2,\ldots,k_r\}. \qquad (36)$$

From (24), $\underline{R}''$ may be written as

$$\underline{R}'' = \underline{S}'\underline{S}''^T, \qquad (37)$$

where

$$S'_{jk} = C_j\frac{1}{p_j^2-\bar{p}_k^2}\bar{C}_k, \qquad (38)$$

$$S''_{jk} = C_j\frac{p_j^2}{p_j^2-\bar{p}_k^2}\bar{C}_k. \qquad (39)$$

We have

$$\det(\underline{I}+\underline{R}'')=1+\sum_{r=1}^{N}\sum_{1\le k_1<k_2<\cdots<k_r\le N}\sum_{1\le j_1<j_2<\cdots<j_r\le N} S'(k_1,k_2,\ldots,k_r;j_1,j_2,\ldots,j_r)\times S''(k_1,k_2,\ldots,k_r;j_1,j_2,\ldots,j_r)\ . \tag{40}$$

In analogy to (28), using (27), we have

$$S'(k_1,k_2,\ldots,k_r;j_1,j_2,\ldots,j_r)S''(k_1,k_2,\ldots,k_r;j_1,j_2,\ldots,j_r) =\prod_k C_k^2\prod_j \bar{C}_j^2\prod_{k<k'}(p_k^2-p_{k'}^2)^2\prod_{j<j'}(\bar{p}_j^2-\bar{p}_{j'}^2)^2\prod_{k,j}(p_k^2-\bar{p}_j^2)^{-2}\prod_k p_k^2\ , \tag{41}$$

where k,k' and j,j' satisfy (29). The complex conjugate of (41) is obviously just (28); we thus have an explicit expression of the N-soliton solution

$$u_N=-2\frac{\det(\underline{I}+\underline{R}')-\det(\underline{I}+\underline{R})}{\det(\underline{I}+\underline{R})}\frac{\overline{\det(\underline{I}+\underline{R})}}{\det(\underline{I}+\underline{R})} \tag{42}$$

by substituting (26), (28), (34), and (35).

Introducing a scale transformation,

$$x'=\gamma^{-1}x,\quad t'=\gamma^{-2}t\ , \tag{43}$$

where γ is a real constant, from (1), we have

$$iu_{t'}+u_{x'x'}+i\gamma(|u|^2u)_{x'}+2\beta|u|^2u=0\ , \tag{44}$$

where $u(x't')=u(xt)$, and $\beta=\pm1$,

$$\rho=\beta\gamma^{-2}\ . \tag{45}$$

Equation (44) is the MNLS equation in its practical form. Soliton solutions of (44) can be obviously obtained from (42) with the aid of the scale transformation (43).

We now introduce new spectral parameters λ_j,

$$\lambda_j=\gamma(\zeta_j^{-2}-\beta\gamma^{-2}) \tag{46}$$

or

$$q_j=i\gamma(p_j^{-2}+\beta\gamma^{-2}),\quad q_j=-i\lambda_j\ . \tag{47}$$

We notice that (10), (28), and (35) can be expressed in terms of p_j^{-1},

$$\alpha_j=\prod_{k(\neq j)}\frac{p_j^{-2}-p_k^{-2}}{p_j^{-2}-\bar{p}_k^{-2}}\frac{-1}{p_j^{-2}-\bar{p}_j^{-2}}p_j^{-3}\prod_{l=1}^{N}\left[\frac{\bar{p}_l^{-2}}{p_l^{-2}}\right]\ , \tag{48}$$

$$Q(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)^2 =\prod_j C_j^2p_j^{-2}\prod_k \bar{C}_k^2\bar{p}_k^{-2}\prod_{\substack{j,j'\\ j<j'}}(p_j^{-2}-p_{j'}^{-2})^2\prod_{\substack{k,k'\\ k<k'}}(\bar{p}_k^{-2}-\bar{p}_{k'}^{-2})^2\prod_{j,k}(p_j^{-2}-\bar{p}_k^{-2})^{-2}\prod_j p_j^{-2}\ , \tag{49}$$

and

$$Q'(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r)Q''(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r) =\prod_j C_j^2p_j^{-2}\prod_k \bar{C}_k^2\bar{p}_k^{-2}\prod_{\substack{j,j'\\ j<j'}}(p_j^{-2}-p_{j'}^{-2})^2\prod_{\substack{k,k'\\ k<k'}}(\bar{p}_k^{-2}-\bar{p}_{k'}^{-2})^2\prod_{j,k}(p_j^{-2}-\bar{p}_k^{-2})^{-2}\prod_k \bar{p}_k^{-2}\ . \tag{50}$$

In virtue of (46) and (47), we have

$$Q(j_1,j_2,\ldots,j_r;k_1,k_2,\ldots,k_r)^2=\prod_j(C_j')^2\prod_k(\bar{C}_k')^2\prod_{\substack{j,j'\\ j<j'}}(q_j-q_{j'})^2\prod_{\substack{k,k'\\ k<k'}}(\bar{q}_k-\bar{q}_{k'})^2\prod_{j,k}(q_j+\bar{q}_k)^{-2}\prod_j(\beta+i\gamma q_j)\ , \tag{51}$$

$$Q'(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r)Q''(j_1,j_2,\ldots,j_r;0,k_2,\ldots,k_r) =\prod_j(C_j')^2\prod_k(\overline{C}_k')^2\prod_{\substack{j,j'\\ j<j'}}(q_j-q_{j'})^2\prod_{\substack{k,k'\\ k<k'}}(\bar{q}_k-\bar{q}_{k'})^2\prod_{j,k}(q_j+\bar{q}_k)^{-2}\prod_k(\beta-i\gamma\bar{q}_k)\ , \tag{52}$$

where

$$(C_j')^2=(\alpha_j')^{-1}(f_j')^{-2}\ , \tag{53}$$

$$(f_j')^{-2}=(b_j')^{-2}\exp[-2i(\lambda_jx'+2\lambda_j^2t')] =(b_j')^{-2}\exp[2(q_jx'+i2q_j^2t')]\ . \tag{54}$$

$$\alpha_j'=\prod_{k(\neq j)}\frac{q_j-q_k}{q_j+\bar{q}_k}\frac{1}{q_j+\bar{q}_j}\ , \tag{55}$$

and the new constant b_j' is defined by

$$(b_j')^{-2}=b_j^{-2}\frac{1}{-i\gamma p_j^{-1}}\prod_{l=1}^{N}\left[\frac{p_l^{-2}}{\bar{p}_l^{-2}}\right]\ . \tag{56}$$

By substituting (51) and (52), we obtain an explicit expression of the N-soliton solution of the MNLS equation (44).

We now turn to derive the expected asymptotic behavior of the N-soliton solution of the MNLS equation. From (54), we have

$$(f_j')^{-2}=\exp(2i\{(\mathrm{Im}q_j)x'+2[(\mathrm{Re}q_j)^2-(\mathrm{Im}q_j)^2]t'+\phi_j'\})\times\exp\{2(\mathrm{Re}q_j)[x'-x_j'-4(\mathrm{Im}q_j)t']\}\ , \tag{57}$$

where

$$b_j'=\exp[i\phi_j'+(\mathrm{Re}q_j)x_j'] \ . \tag{58}$$

Suppose all $(\mathrm{Re}q_j)$ are positive, and

$$(\mathrm{Im}q_1)>(\mathrm{Im}q_2)>\cdots>(\mathrm{Im}q_N) \ . \tag{59}$$

The vicinity of $x'=x_j'+4(\mathrm{Im}q_j)t'$ is denoted by Ω_j. In the limit as $t\to+\infty$, these vicinities must be separated from left to right as

$$\Omega_N,\Omega_{N-1},\ldots,\Omega_1 \ . \tag{60}$$

In the vicinity Ω_m, we have

$$x'-x_l'-4(\mathrm{Im}q_l)t'\to-\infty,\quad |f_l'|^{-1}\to 0,\quad l<m \tag{61}$$

$$x'-x_n'-4(\mathrm{Im}q_n)t'\to+\infty,\quad |f_n'|^{-1}\to\infty,\quad n>m \ . \tag{62}$$

From (51) and (52), taking account of (61) and (62), we have

$$\begin{aligned}\frac{\det(\underline{I}+\underline{R}')-\det(\underline{I}+\underline{R})}{\det(\underline{I}+\underline{R})}&\approx\frac{Q'(m,m+1,\ldots,N;0,m+1,\ldots,N)Q''(m,m+1,\ldots,N;0,m+1,\ldots,N)}{Q(m+1,m+2,\ldots,N;m+1,m+2,\ldots,N)^2+Q(m,m+1,\ldots,N;m,m+1,\ldots,N)^2}\\&=\frac{(C_m')^2\prod_{n=m+1}^{N}\left[\frac{q_m-q_n}{q_m+\bar q_n}\right]^2\prod_{n=m+1}^{N}\frac{\beta-i\gamma\bar q_n}{\beta+i\gamma q_n}}{1+(C_m')^2(\bar C_m')^2\frac{\beta+i\gamma q_m}{(q_m+\bar q_m)^2}\prod_{n=m+1}^{N}\left|\frac{q_m-q_n}{q_m+\bar q_n}\right|^4} \ .\end{aligned} \tag{63}$$

We can write it as

$$\frac{\det(\underline{I}+\underline{R}')-\det(\underline{I}+\underline{R})}{\det(\underline{I}+\underline{R})}\approx\frac{(q_m+\bar q_m)(f_m')^{(+)-2}}{1+|(f_m')^{(+)}|^{-4}(\beta+i\gamma q_m)} \ , \tag{64}$$

where

$$(f_m')^{(+)-2}=(f_m')^{-2}(\alpha_m')^{-1}\beta(q_m)^2(q_m+\bar q_m)^{-1} \ , \tag{65}$$

$$\beta(q_m)^2=\prod_{n=m+1}^{N}\left[\frac{q_m-q_n}{q_m+\bar q_n}\right]^2\prod_{n=m+1}^{N}\frac{\beta-i\gamma\bar q_n}{\beta+i\gamma q_n} \ . \tag{66}$$

We have also

$$\frac{\overline{\det(\underline{I}+\underline{R})}}{\det(\underline{I}+\underline{R})}\approx\frac{1+|(f_m')^{(+)}|^{-4}(\beta-i\gamma\bar q_m)}{1+|(f_m')^{(+)}|^{-4}(\beta+i\gamma q_m)} \ . \tag{67}$$

Similarly, when $t\to-\infty$, in the vicinity Ω_m, we have

$$\frac{\det(\underline{I}+\underline{R}')-\det(\underline{I}+\underline{R})}{\det(\underline{I}+\underline{R})}\approx\frac{(q_m+\bar q_m)(f_m')^{(-)-2}}{1+|(f_m')^{(-)}|^{-4}(\beta+i\gamma q_m)} \ , \tag{68}$$

$$\frac{\overline{\det(\underline{I}+\underline{R})}}{\det(\underline{I}+\underline{R})}\approx\frac{1+|(f_m')^{(-)}|^{-4}(\beta-i\gamma\bar q_m)}{1+|(f_m')^{(-)}|^{-4}(\beta+i\gamma q_m)} \ , \tag{69}$$

where

$$(f_m')^{(-)-2}=(f_m')^{-2}(\alpha_m')^{-1}\alpha(q_m)^2(q_m+\bar q_m)^{-1} \ , \tag{70}$$

$$\alpha(q_m)^2=\prod_{l=1}^{m-1}\left[\frac{q_m-q_l}{q_m+\bar q_l}\right]^2\prod_{l=1}^{m-1}\frac{\beta-i\gamma\bar q_l}{\beta+i\gamma q_l} \ . \tag{71}$$

We thus derive the expected asymptotic behavior of the N-soliton solution of the MNLS equation. The total phase shift and the total displacement of center of the mth peak δ_m and Δ_m can be obtained from (64)–(71)

$$\delta_m=\arg[\alpha(q_m)]-\arg[\beta(q_m)] \ , \tag{72}$$

$$\Delta_m=(\mathrm{Re}q_m)^{-1}[\ln|\beta(q_m)|-\ln|\alpha(q_m)|] \ . \tag{73}$$

This result shows that the MNLS equation has regular soliton solutions even in the case of normal dispersion, i.e., $\beta=-1$, provided $\gamma\neq 0$. In the case of anomalous dispersion, i.e., $\beta=1$, the N-soliton solution of the MNLS equation clearly reduces to that of the NLS equation as γ approaches zero.

We have given an explicit expression of the N-soliton solution of the MNLS equation in a form suitable for practical needs. It will provide a proper basis for analyzing soliton formation in the case of a single initial short pulse and wave-packet decay and soliton interactions in the multisoliton case.

[1] N. Tzoar and M. Jain, Phys.Rev. A **23**, 1266 (1981).
[2] D. Anderson and M. Lisak, Opt. Lett. 7, 394 (1982).
[3] L. F. Mollenauer, R. M. Stolen, and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[4] H. Nakatsuka, D. Grischkowsky, and A. C. Balant, Phys. Rev. Lett. **47**, 910 (1981).
[5] M. Wadati, K. Konno, and Y. H. Ichikawa, J. Phys. Soc. Jpn. **46**, 1965 (1979).
[6] D. Anderson and M. Lisak, Phys. Rev. A **27**, 1393 (1983).
[7] E. A. Golovchenko, E. M. Dianov, A. M. Prokhonov, and V. N. Serkin, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 74 (1985) [JETP Lett. **42**, 88 (1985)].
[8] K. Ohkuma, Y. H. Ichikawa, and Y. Abe, Opt. Lett. **12**, 516 (1987).
[9] Z. Y. Chen and N. N. Huang, Phys. Lett. A **142**, 31 (1989).
[10] Z. Y. Chen and N. N. Huang, Opt. Lett. (to be published).