PHYSICAL REVIEW A VOLUME 41, NUMBER 1 1 JANUARY 1990

Stimulated Raman scattering of colored chaotic light in dispersive media

A. T. Georges
Department of Physics, University of Crete and Research Center of Crete–FORTH, Heraklion, Crete, Greece
(Received 21 August 1989)

A theory is presented for forward- and backward-stimulated Raman scattering of colored chaotic light in dispersive media. Both amplification of a coherent Stokes seed and spontaneous generation are treated. It is shown that group-velocity dispersion reduces the gain for the average Stokes intensity, while the intensity fluctuations in the pump enhance it. For a given bandwidth, chaotic light is scattered more efficiently than coherent light when its average intensity exceeds the critical intensity for overcoming the decorrelating effects of group-velocity dispersion on the pump and Stokes waves. The figures presented demonstrate the asymmetry in the intensity of forward and backward Raman scattering of nonmonochromatic light.

I. INTRODUCTION

The problem of stimulated Raman and Brillouin scattering (SRS and SBS) of colored (finite-bandwidth) chaotic light in dispersive media arises in several real-life experimental situations. First, it arises whenever high-power broadband lasers are used in applications of Raman and Brillouin scattering, such as generation of frequency-shifted radiation,[1] wave-front correction, coherent beam combining,[2] and phase conjugation.[3] These types of lasers (excimer, nitrogen, Nd:YAG) that operate either multimode or in the amplified spontaneous-emission mode exhibit amplitude and phase fluctuations which are described adequately by Gaussian statistics, the synonym for chaotic behavior. Second, it arises in the use of the recently developed optical technique of induced spatial incoherence (ISI) for reducing Raman backscattering in laser-induced fusion.[4] The light produced by ISI with a broadband incident laser beam is chaotic, both spatially and temporally. Third, it arises internally in the spontaneous (unseeded) generation of second-order Stokes radiation from a coherent laser pump. Below saturation the first-order Stokes radiation, which generates the second-order in a cascade process, is inherently chaotic because it is amplified spontaneously scattered radiation;[5] The amplification increases the correlation time of the noise but leaves its statistics unchanged. The second Stokes radiation in turn is superchaotic, because it too starts spontaneously (generation via four-wave mixing is weak) and is amplified by the chaotic first Stokes.[6] There is in fact experimental evidence of the associated super-Gaussian statistics in recent experimental data on pulse energy distributions of the second Stokes,[7] although the analysis is complicated because the statistics change as the first and then the second Stokes saturate.

On the theoretical side, the problem has been receiving attention for many years. Akhmanov *et al.*[8] studied SRS of a zero-bandwidth (infinite correlation time) chaotic field and showed that the intensity fluctuations in the pump enhance the average Stokes intensity greatly over that in the case of a coherent pump. The enhancement is due to the large contribution from the tail of the negative exponential distribution of the random pump intensity. In a previous paper[6] we presented a mathematical technique for averaging exactly the integral equation of motion for the Stokes-field amplitude over the fluctuations of a Markovian chaotic pump of arbitrary bandwidth, in the presence of dispersion. We then used a Markovian expansion for the Stokes-field amplitude to calculate approximately the average Stokes intensity, spectrum, and two-time intensity correlation. Numerical results were reported for the special case of no dispersion. The Markovian modeling of the Stokes field turns out to be satisfactory in the case of a correlated Stokes seed for arbitrary pump bandwidths, and in the case of a coherent (uncorrelated) seed for pump bandwidths equal or less than the Raman linewidth of the medium. But in the case of a coherent seed and pump bandwidths greater than the Raman linewidth the calculation cannot account for the growth of a non-Markovian Stokes-field component that is quasicorrelated to the pump and grows with nearly the same gain as in the case of a coherent pump. Trippenbach *et al.*[9,10] studied forward SRS of colored chaotic light in nondispersive media. They were able to calculate exactly the average Stokes intensity and the pump-Stokes intensity cross correlation, taking advantage of the existence of analytical solutions for the Stokes-field operator in the case of an arbitrary stochastic pump and no dispersion. Unfortunately, in the theory of resonant, nonlinear, quantum-optical processes with propagation effects included, the existence of such analytical solutions is the exception to the rule. To treat the effect of incoherent laser fields in such processes in the absence of analytical solutions, one has to either try to average the relevant stochastic equations of motion or resort to Monte Carlo calculations. In this paper, we apply the mathematical technique introduced in Ref. 6 to average the coupled integral equations of motion for the Stokes intensity operator and the triple correlation of the amplitudes of the pump, Stokes field, and medium excitation. The theory presented here corrects the one deficiency of the Markovian modeling mentioned above. Calculations are presented for the first time for the average Stokes intensity in SRS of finite-bandwidth chaotic light in dispersive media. The calculations show that there is a competition between dispersion which reduces the Stokes gain by decorrelating the pump and the Stokes field, and the intensity fluctuations in the pump which enhance this highly nonlinear optical process. The case of backward SRS of incoherent light, where the counter-propagation of the fields introduces a group-velocity mismatch even in an ideal nondispersive medium, is treated by our general theory in the same manner as forward SRS in a dispersive medium, but with a different dispersion parameter.

II. THEORY

We start from the familiar equations for the Stokes field operator $\hat{E}_S^\dagger(z,\tau)$ and the collective atomic raising operator $\hat{Q}^\dagger(z,\tau)$ corresponding to the Raman transition in the medium[11–13]

$$\frac{\partial}{\partial z}\hat{E}_S^\dagger(z,\tau)=-i\kappa_2 E_p(\tau+\mu z)\hat{Q}^\dagger(z,\tau)\,, \tag{1}$$

$$\left[\frac{\partial}{\partial\tau}+\frac{\Gamma}{2}\right]\hat{Q}^\dagger(z,\tau)=i\kappa_1 E_p^*(\tau+\mu z)\hat{E}_S^\dagger(z,\tau)+\hat{F}^\dagger(z,\tau)\,, \tag{2}$$

where $\tau=t-z/v_S$ is a retarded time and $\mu=(1/v_S \pm 1/v_p)$ measures the group-velocity mismatch of the pump and Stokes waves for forward ($-$) and backward ($+$) Raman scattering. Note that for backward scattering it is the pump and not the Stokes field that is taken to propagate in the negative z direction. Γ is the full width at half maximum (FWHM) of the Raman line, and the detuning from resonance is assumed to be zero. The coupling parameters κ_1 and κ_2 are defined in Ref. 12 for electronic Raman scattering, and the relation between them in the MKSA system of units is $\kappa_2=N\hbar\omega_S\kappa_1^*/(2\epsilon_S v_S)$, where N is the number density of the medium and ϵ_S its permittivity at the Stokes frequency ω_S. $\hat{F}^\dagger(z,\tau)$ is a collective quantum Langevin force operator that accounts for the intrinsic noise of the medium which causes spontaneous Raman scattering. $E_p(\tau+\mu z)$ is the fluctuating complex amplitude of the chaotic pump which obeys Gaussian statistics. The pump is assumed to be Markovian and its correlation function is

$$\langle E_p(\tau_1+\mu z_1)E_p^*(\tau_2+\mu z_2)\rangle = E_{p0}^2\exp[-\tfrac{1}{2}\gamma_p|\tau_1-\tau_2+\mu(z_1-z_2)|]\,, \tag{3}$$

where γ_p is the FWHM of the Lorentzian spectrum and E_{p0} the root-mean-square amplitude.

From Eqs. (1) and (2) it can be shown that the average Stokes intensity is given by[11]

$$\langle I_S(z,\tau)\rangle=I_{S0}+\int_0^z[\langle E_p^*(\tau+\mu z_1)C(z_1,\tau)\rangle+\text{c.c.}]dz_1\,, \tag{4}$$

where I_{S0} is the intensity of the incident Stokes radiation which is assumed to be coherent. The angular brackets denote average over the classical fluctuations in the pump and $C(z,\tau)\equiv i\kappa_2'\langle\hat{E}_S^\dagger(z,\tau)\hat{Q}(z,\tau)\rangle_q$, where $\kappa_2'=2\epsilon_S v_S\kappa_2$ and $\langle\ \rangle_q$ denotes quantum average, is determined by the stochastic integral equation

$$\begin{aligned}C(z,\tau)={}&\frac{\kappa_2^2}{2\rho_L}\Gamma E_p(\tau+\mu z)\int_{\tau_0}^{\tau}e^{\Gamma(\tau_1-\tau)}d\tau_1+\kappa_1\kappa_2 I_{S0}\int_{\tau_0}^{\tau}e^{(1/2)\Gamma(\tau_1-\tau)}E_p(\tau_1+\mu z)d\tau_1\\&+\kappa_1\kappa_2\int_{\tau_0}^{\tau}e^{(1/2)\Gamma(\tau_1-\tau)}\int_0^z\Big\{[E_p(\tau_1+\mu z)E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)+E_p(\tau_1+\mu z)E_p(\tau_1+\mu z_1)C^*(z_1,\tau_1)]\\&\qquad+|E_p(\tau_1+\mu z_1)|^2 i\kappa_2'\langle\hat{E}_S^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\\&\qquad+\int_{\tau_0}^{\tau_1}e^{(1/2)\Gamma(\tau_2-\tau_1)}\left[\left[-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]\\&\qquad\times E_p^*(\tau_2+\mu z_1)i\kappa_2'\langle\hat{E}_S^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q d\tau_2\Big\}dz_1 d\tau_1\,,\end{aligned} \tag{5}$$

where $\rho_L=AN$ is the linear number density of the medium, with A being the cross-sectional area of the cylindrical interaction volume. Equation (5) is exact, except for the neglect of a transient term which vanishes in the stationary case ($\tau\gg\Gamma^{-1}$) considered here.

To calculate the correlation $\langle E_p^*(\tau+\mu z_1)C(z_1,\tau)\rangle$ we apply techniques that were developed in Refs. 6 and 14 for averaging stochastic integral equations over Markovian laser field fluctuations. First, we formally average Eq. (5) over non-Markovian fluctuations in $C(z,\tau)$. This is equivalent to defining *marginal averages* in the averaging of a system of linear stochastic ordinary differential equations with Markovian coefficients.[15] Then, we proceed to calculate the correlation between the pump amplitude $E_p^*(\tau+\mu z)$ and the marginal average of $C(z,\tau)$. To this end, we multiply both sides of Eq. (5) by the complex conjugate of an eigenfunction of the conditional average integral for a chaotic field[6,14]

$$\Phi_{Nm}(E_p(\tau+\mu z))=\sqrt{N!/(N+m)!}(E_p^m(\tau+\mu z)/E_{p0}^m)L_N^m(|E_p(\tau+\mu z)|^2/E_{p0}^2)\,, \tag{6}$$

$N=0,1,2,\ldots,$ $m=1$, where $L_N^m(x)$ are generalized Laguerre polynomials, and then average over the pump. The averaging involves multiplication by the joint probability density

$$f(E_p',E_p'',E_p''',E_p'''';z,\tau;z,\tau_1;z_1,\tau_1;z_1,\tau_2)$$
$$=f(E_p';z,\tau|E_p'';z,\tau_1)f(E_p'';z,\tau_1|E_p''';z_1,\tau_1)$$
$$\times f(E_p''';z_1,\tau_1|E_p'''';z_1,\tau_2)f(E_p'''';z_1,\tau_2)\ , \quad (7)$$

and integration over the random variables $E_p'=E_p(\tau+\mu z)$, E_p'',E_p''', and $E_p''''=E_p(\tau_2+\mu z_1)$, in the order that they are written. The conditional and marginal probabilities densities, $f(E_p';z,\tau|E_p'';z,\tau_1)$ and $f(E_p;z,\tau)$ are given in Ref. 14. The average is carried out easily using the orthonormality of the eigenfunctions and their conditional averages. For example, the conditional average of $\Phi_{Nm}(E_p(\tau+\mu z))$ given the value of E_p at (z,τ_1) is

$$\langle \Phi_{Nm}(E_p(\tau+\mu z))|E_p;z,\tau_1\rangle$$
$$=e^{-(1/2)(2N+m)\gamma_p(\tau-\tau_1)}\Phi_{Nm}(E_p(\tau_1+\mu z))\ . \quad (8)$$

The recursion relations for generalized Laguerre polynomials

$$xL_N^1(x)=(N+1)L_N^0(x)-(N+1)L_{N+1}^0(x) \quad (9)$$

and

$$L_N^0(x)=L_N^1(x)-L_{N-1}^1(x) \quad (10)$$

are used to express products of the type $E_p(\tau+\mu z)\Phi_{N1}^*(E_p(\tau+\mu z))$ in terms of eigenfunctions of the $\Phi_{N0}^*(E_p(\tau+\mu z))$ kind, and after conditional averaging to change them back to $\Phi_{N1}^*(E_p(\tau+\mu z))$ kind. Carrying out the average leads to the infinite system of deterministic integral equations

$$\langle C(z,\tau)\rangle_N=\frac{\kappa_2^2}{2\rho_L}\Gamma E_{p0}\int_{\tau_0}^{\tau}e^{\Gamma(\tau_1-\tau)}d\tau_1\delta_{N0}+\kappa_1\kappa_2E_{p0}I_{S0}\int_{\tau_0}^{\tau}e^{(1/2)(\Gamma+\gamma_p)(\tau_1-\tau)}d\tau_1\delta_{N0}$$
$$+\kappa_1\kappa_2E_{p0}^2\int_{\tau_0}^{\tau}e^{(1/2)[\Gamma+(2N+1)\gamma_p](\tau_1-\tau)}$$
$$\times\int_0^z\Bigg| e^{N\mu\gamma_p(z_1-z)}[(N+1)\langle C(z_1,\tau_1)\rangle_N-\sqrt{(N+1)N}\,\langle C(z_1,\tau_1)\rangle_{N-1}+\text{c.c.}]$$
$$-e^{(N+1)\mu\gamma_p(z_1-z)}[\sqrt{(N+2)(N+1)}\langle C(z_1,\tau_1)\rangle_{N+1}-(N+1)\langle C(z_1,\tau_1)\rangle_N+\text{c.c.}]$$
$$+e^{(1/2)(2N+1)\mu\gamma_p(z_1-z)}$$
$$\times\Bigg|\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_1)\rangle}[(N+1)\langle C(z_1,\tau_1)\rangle_N-\sqrt{(N+1)N}\,\langle C(z_1,\tau_1)\rangle_{N-1}$$
$$-\sqrt{(N+2)(N+1)}\langle C(z_1,\tau_1)\rangle_{N+1}+(N+1)\langle C(z_1,\tau_1)\rangle_N]$$
$$+\int_{\tau_0}^{\tau_1}e^{(1/2)\Gamma(\tau_2-\tau_1)}\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}$$
$$\times\{\tfrac{1}{2}(\gamma_p-\Gamma)e^{N\gamma_p(\tau_2-\tau_1)}[(N+1)\langle C(z_1,\tau_2)\rangle_N-\sqrt{(N+1)N}\,\langle C(z_1,\tau_2)\rangle_{N-1}]$$
$$+\tfrac{1}{2}(\gamma_p+\Gamma)e^{(N+1)\gamma_p(\tau_2-\tau_1)}[\sqrt{(N+2)(N+1)}\langle C(z_1,\tau_2)\rangle_{N+1}$$
$$-(N+1)\langle C(z_1,\tau_2)\rangle_N]\}d\tau_2\Bigg|\Bigg|dz_1d\tau_1\ , \quad (11)$$

where $\langle C(z,\tau)\rangle_N$, $N=0,1,2,\ldots,$ stands for $\langle\Phi_{N1}^*(E_p(\tau+\mu z))C(z,\tau)\rangle$ and $Q(z,\tau)=\langle\hat{Q}(z,\tau)\rangle_q$. Note that $E_{p0}\langle C(z,\tau)\rangle_0=\langle E_p^*(\tau+\mu z)C(z,\tau)\rangle$ is the correlation that determines the average Stokes intensity in Eq. (4). In obtaining Eq. (11) from Eq. (5) we have used the approximations

$$i\kappa_2'\langle\hat{E}_S^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\simeq\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_1)\rangle}C(z_1,\tau_1)\ , \quad (12)$$

and

$$i\kappa_2'\langle\hat{E}_S^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\simeq\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}C(z_1,\tau_2)\ , \quad (13)$$

where we assume that $\hat{Q}(z,\tau_1)$ grows from $\hat{Q}(z_1,\tau_1)$ and $\hat{Q}(z_1,\tau_2)$ approximately the same way that $\langle Q(z,\tau_1)\rangle$ grows from $\langle Q(z_1,\tau_1)\rangle$ and $\langle Q(z_1,\tau_2)\rangle$, respectively. The same approximation was used in Ref. 11, where we investigated stimulated Raman scattering of a phase-diffusion laser field. The justification given there for this approximation is independent of the statistics of the field and, hence, is valid also in the present case of a chaotic pump.

To illustrate the averaging technique we carry out below the average of the last term in Eq. (5), which is the most complicated. Step by step the averaging proceeds as follows:

$$\left\langle \Phi^*_{N1}(E_p(\tau+\mu z))\left[\left[-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]E_p^*(\tau_2+\mu z_1)i\kappa_2'\langle \hat{E}^\dagger_S(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\right\rangle$$

$$\simeq \frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}\left\langle \Phi^*_{N1}(E_p(\tau+\mu z))\left[\left[-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\right\rangle$$

$$=\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}\left[-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]e^{-(1/2)(2N+1)\gamma_p[(\tau-\tau_1)+\mu(z-z_1)]}\langle \Phi^*_{N1}(E_p(\tau_1+\mu z_1))E_p(\tau_1+\mu z_1)E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\rangle$$

$$=\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}\left[-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_{p0}^2e^{-(1/2)(2N+1)\gamma_p[(\tau-\tau_1)+\mu(z-z_1)]}(N+1)$$
$$\times[\langle \Phi^*_{N0}(E_p(\tau_1+\mu z_1))E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\rangle-\langle \Phi^*_{N+1,0}(E_p(\tau_1+\mu z_1))E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\rangle]$$

$$=\frac{\langle Q(z,\tau_1)\rangle}{\langle Q(z_1,\tau_2)\rangle}E_{p0}^2e^{-(1/2)(2N+1)\gamma_p[(\tau-\tau_1)+\mu(z-z_1)]}$$
$$\times\{\tfrac{1}{2}(\gamma_p-\Gamma)e^{N\gamma_p(\tau_2-\tau_1)}[(N+1)\langle C(z_1,\tau_2)\rangle_N-\sqrt{(N+1)N}\,\langle C(z_1,\tau_2)\rangle_{N-1}]$$
$$+\tfrac{1}{2}(\gamma_p+\Gamma)e^{(N+1)\gamma_p(\tau_2-\tau_1)}[\sqrt{(N+2)(N+1)}\langle C(z_1,\tau_2)\rangle_{N+1}-(N+1)\langle C(z_1,\tau_2)\rangle_N]\}\ . \tag{14}$$

Note that the only approximation involved is the use of the approximate Eq. (13) in the first step of the calculation. In the second step, we have used the fact that the order of differentiation ($\partial/\partial\tau_1$) and averaging can be interchanged, and then carried out the conditional average from (z,τ) to (z_1,τ_1). In the third step we have expressed the product $\Phi^*_{N1}(E_p(\tau_1+\mu z_1))E_p(\tau_1+\mu z_1)$ in terms of $\Phi^*_{N0}(E_p(\tau_1+\mu z_1))$ and $\Phi^*_{N+1,0}(E_p(\tau_1+\mu z_1))$. Finally, in the last step we have carried out the conditional average from (z_1,τ_1) to (z_1,τ_2), and then expressed $\Phi^*_{N0}(E_p(\tau_2+\mu z_1))$ and $\Phi^*_{N+1,0}(E_p(\tau_2+\mu z_1))$ in terms of $\Phi^*_{N1}(E_p(\tau_2+\mu z_1))$-type eigenfunctions.

In the stationary case ($\tau \gg \Gamma^{-1}$), the averages $\langle C(z,\tau)\rangle_N$ are time independent and the infinite system of integral Eqs. (11) can be converted into the following infinite system of first-order differential equations

$$\frac{d}{d\zeta}X_N(\zeta)=a_NX_{N-1}(\zeta)+[b_N-\tfrac{1}{2}(2N+1)\gamma_p/\Gamma+G_Q(\zeta)]X_N(\zeta)+c_NX_{N+1}(\zeta)+\left[\frac{\gamma_p}{2\Gamma}-G_Q(\zeta)\right]Y_N(\zeta)$$
$$+\left[\frac{\gamma_p}{2\Gamma}+G_Q(\zeta)\right]Z_N(\zeta)+\left[\frac{\gamma_p}{2\Gamma}-G_Q(\zeta)\right]\delta_{N0}\ , \tag{15}$$

$$\frac{d}{d\zeta}Y_N(\zeta)=-N\frac{\gamma_p}{\Gamma}Y_N(\zeta)-\sqrt{(N+1)N}\,d_NX_{N-1}(\zeta)+(N+1)d_NX_N(\zeta)\ , \tag{16}$$

$$\frac{d}{d\zeta}Z_N(\zeta)=-(N+1)\frac{\gamma_p}{\Gamma}Z_N(\zeta)-(N+1)d_NX_N(\zeta)+\sqrt{(N+2)(N+1)}\,d_NX_{N+1}(\zeta)\ , \tag{17}$$

where $\zeta=\mu\Gamma z$ is a dimensionless length, $X_N(\zeta)$ is the stationary value of the quantity $[E_{p0}\langle C(z,\tau)\rangle_N+\text{c.c.}]$, and $Y_N(\zeta)$ and $Z_N(\zeta)$ are auxiliary functions defined by Eqs. (16) and (17). The three functions are measured in units of

$$\left[\frac{\Gamma\hbar\omega_S}{4A}+I_{S0}\frac{\Gamma}{\Gamma+\gamma_p}\right]gI_p/\mu\Gamma\ ,$$

and their initial values are

$$X_0(0)=1,\quad X_N(0)=0,\quad N>0$$

and

$$Y_N(0)=Z_N(0)=0,\quad N=0,1,2,\ldots\ .$$

The parameters a_N, b_N, c_N, and d_N are given by

$$a_N=-\tfrac{1}{2}\sqrt{(N+1)N}\left[3-\frac{1-\gamma_p/\Gamma}{1+2N\gamma_p/\Gamma}\right]d_N\ , \tag{18}$$

$$b_N=\tfrac{1}{2}(N+1)\left[6-\frac{1-\gamma_p/\Gamma}{1+2N\gamma_p/\Gamma}-\frac{1+\gamma_p/\Gamma}{1+2(N+1)\gamma_p/\Gamma}\right]d_N\ , \tag{19}$$

$$c_N=-\tfrac{1}{2}\sqrt{(N+2)(N+1)}\left[3-\frac{1+\gamma_p/\Gamma}{1+2(N+1)\gamma_p/\Gamma}\right]d_N\ , \tag{20}$$

and

$$d_N=\frac{gI_p/\mu\Gamma}{1+(2N+1)\gamma_p/\Gamma}\ . \tag{21}$$

The function $G_Q(\zeta)\equiv\langle Q(\zeta,\tau)\rangle^{-1}d\langle Q(\zeta,\tau)\rangle/d\zeta$ is the stationary gain coefficient for $\langle Q(\zeta,\tau)\rangle$. Averaging both sides of Eq. (2) over quantum and pump fluctuations we find that in the stationary limit

$$\langle Q^*(z,\tau)\rangle=i\frac{2\kappa_1}{\Gamma}\langle E_p^*(\tau+\mu z)E_S(z,\tau)\rangle \ . \tag{22}$$

From this follows that the average $\langle Q^*(z,\tau)\rangle$ and the pump-Stokes field cross correlation $\langle E_p^*(\tau+\mu z)E_S(z,\tau)\rangle$ have the same coefficient.[11] For a chaotic pump, this cross correlation can be determined from equations given in Ref. 6. The stationary values of the averages $\langle \Phi_{N1}^*(E_p(\tau+\mu z))\langle \hat{E}_S^\dagger(z,\tau)\rangle_q\rangle$, $N=0,1,2,\ldots$, symbolized here by $\langle E_S(\zeta)\rangle_N$, satisfy the infinite tridiagonal system of linear differential equations

$$\frac{d}{d\zeta}\langle E_S(\zeta)\rangle_N=A_{N,N-1}\langle E_S(\zeta)\rangle_{N-1}+A_{NN}\langle E_S(\zeta)\rangle_N + A_{N,N+1}\langle E_S(\zeta)\rangle_{N+1} \ , \tag{23}$$

where

$$A_{NN}=-(2N+1)\frac{\gamma_p}{2\Gamma} + \frac{gI_p}{2\mu\Gamma}\left[\frac{N+1}{1+2N\gamma_p/\Gamma}+\frac{N+1}{1+2(N+1)\gamma_p/\Gamma}\right] , \tag{24}$$

$$A_{N,N+1}=A_{N+1,N}=-\frac{gI_p}{2\mu\Gamma}\frac{\sqrt{(N+2)(N+1)}}{1+2(N+1)\gamma_p/\Gamma} \ . \tag{25}$$

The common gain coefficient for $\langle E_p^*(\tau+\mu z)E_S(z,\tau)\rangle =E_{p0}\langle E_s(z)\rangle_0$ and $\langle Q(z,\tau)\rangle$ can be written in the form

$$G_Q(\zeta)=\frac{\sum_{k=0}^{N_{\max}} V_0(k)W_0(k)\beta(k)e^{\beta(k)\zeta}}{\sum_{k=0}^{N_{\max}} V_0(k)W_0(k)e^{\beta(k)\zeta}} , \tag{26}$$

where $\beta(k)$, $V_N(k)$, and $W_N(k)$ are the eigenvalues, eigenvectors, and reciprocal eigenvectors of the symmetric tridiagonal matrix A. $N_{\max}$ is the value of N at which the matrix A and the system of Eqs. (15)–(17) are truncated in a numerical solution so that $G_Q(\zeta)$ and $X_0(\zeta)$ converge with a desired accuracy. The stationary average Stokes intensity is given by

$$\langle I_S(\zeta)=I_{S0}+\left[\frac{\Gamma\hbar\omega_s}{4A}+I_{S0}\frac{\Gamma}{\Gamma+\gamma_p}\right]\frac{gI_p}{\mu\Gamma} \times\int_0^\zeta X_0(\zeta')d\zeta' , \tag{27}$$

where the quantity $\Gamma\hbar\omega_S/4A$ plays the role of an effective Stokes input intensity for spontaneously initiated Raman scattering in the plane-wave approximation. The fraction $\Gamma/(\Gamma+\gamma_p)$ multiplying the intensity of the coherent Stokes seed represents the fraction of the pump intensity that couples resonantly with the monochromatic Stokes seed in the initial stage of amplification, before the spectrum of the seed begins to broaden.

Before closing this section we should point out that in the case of $\gamma_p \gg\Gamma$ the system of Eqs. (15)–(17) and Eq. (23) can be truncated after $N=0$. Retaining only terms of the lowest order in Γ/γ_p we obtain

$$G_Q(z)=\tfrac{1}{2}(gI_p-\mu\gamma_p) , \tag{28}$$

and

$$\frac{d}{dz}X_0(z)=(gI_p-\mu\gamma_p)X_0(z) -(\tfrac{1}{2}gI_p-\mu\gamma_p)\frac{gI_p}{1+\gamma_p/\Gamma}\int_0^z X_0(z)dz -(\tfrac{1}{2}gI_p-\mu\gamma_p) \ . \tag{29}$$

The solutions for $X_0(z)$ and $\langle I_s(z)\rangle$ consist of two exponential terms whose coefficients are the roots of the quadratic polynomial

$$G_{1,2}^2-(gI_p-\mu\gamma_p)G_{1,2}+(\tfrac{1}{2}gI_p-\mu\gamma_p)\frac{gI_p}{1+\gamma_p/\Gamma}=0 \ . \tag{30}$$

For $I_p \ll I_{cr}\equiv\mu\gamma_p/g$ the two gain coefficients are given by

$$G_1=gI_p\frac{\Gamma}{\Gamma+\gamma_p} , \tag{31}$$

$$G_2=gI_p-\mu\gamma_p<0 , \tag{32}$$

while for $I_p \gg I_{cr}$ they are given by

$$G_1=gI_p-\mu\gamma_p , \tag{33}$$

$$G_2=\tfrac{1}{2}gI_p\frac{\Gamma}{\Gamma+\gamma_p} \ . \tag{34}$$

Note that in both cases G_1 is much larger than G_2 and determines the Stokes gain. These results are in agreement with those obtained in the case of SRS of a phase-diffusion pump field.[11] The agreement reflects the fact that for $\gamma_p \gg\Gamma$ Raman scattering is sensitive only to the first-order correlation of the pump field [Eq. (3)], which is the same in the two cases. The higher-order correlation functions play an increasing role as γ_p decreases and becomes much less than Γ. In this case SRS responds to all the details in the fluctuating pump field, and can distinguish between pumps having the same spectrum but different statistics. For $I_p \gg I_{cr}$, Eq. (33) gives $G_1\simeq gI_p$. Therefore the gain for spontaneous Stokes generation or amplification of a coherent seed in the case of a broadband chaotic pump is the same as in the case of a coherent pump. However, because of the approximation in Eqs. (12) and (13), our theory is missing a factor of $(\pi gI_pz)^{-1/2}$ which in the exact theory[12,13] for SRS of coherent light multiplies e^{gI_pz} and accounts for the lack of perfect correlation between the pump and the Stokes field.[11] This factor, which is missing also from all the less detailed theories on SRS of incoherent light in dispersive and nondispersive media,[2,3,8] affects the accuracy of our calculations only for $I_p > I_{cr}$, when the terms in Eq. (11) involving the approximate Eqs. (12) and (13) become important. Moreover, as seen in Fig. 4 of Ref. 12, for high gain this factor becomes unimportant, and in the case of a chaotic pump should become even more unimportant. This is because the enhancement of the average Stokes intensity comes from the contribution of the very high intensities in the tail of the negative exponential distribution for the random pump intensity.

III. NUMERICAL RESULTS

In this section we present results of computer calculations for Stokes amplification of a coherent seed, $\langle I_0(z)\rangle/I_{S0}$, and spontaneous generation, $\langle I_0(z)\rangle/(\Gamma\hbar\omega_S/4A)$, with a chaotic pump. Three characteristic lengths of the dispersive medium and three characteristic pump bandwidths, $\gamma_p=0.1\Gamma$, Γ, and 10Γ, are considered. Figure 1 is for $z=100L_{\rm coh}$, where $L_{\rm coh}\equiv 1/\mu\Gamma$ is the coherence length associated with the decorrelating effects of dispersion for a pump with $\gamma_p=\Gamma$. In this case the effect of dispersion is strong ($z\gg L_{\rm coh}$) and since $I_p\ll I_{cr}$ the Stokes field is uncorrelated to the pump. For $\gamma_p=\Gamma$ and 10Γ the uncorrelated gain $\sim gI_p\Gamma/(\Gamma+\gamma_p)$ is less than the gain in the case of a coherent pump. For $\gamma_p=0.1\Gamma$, however, the effect of dispersion is weak and the intensity fluctuations in the pump enhance the average Stokes output intensity. Compared to the case of SRS of a phase-diffusion pump (see Fig. 2 of Ref. 11) the main difference is this enhancement, which is absent there since the intensity of the pump is constant, and the consequent larger spread of the threshold pump intensities in the case of the chaotic pump. The calculations for this figure were carried out with $N_{\rm max}=25$, 15, and 5 for $\gamma_p=0.1\Gamma$, Γ, and 10Γ, respectively.

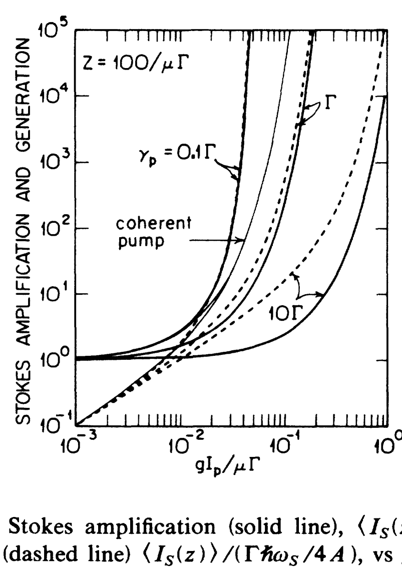

FIG. 1. Stokes amplification (solid line), $\langle I_S(z)\rangle/I_{S0}$, and generation (dashed line) $\langle I_S(z)\rangle/(\Gamma\hbar\omega_S/4A)$, vs $gI_p/|\mu\Gamma$, for three different bandwidths of the chaotic pump, $\gamma_p=\Gamma/10$, Γ, 10Γ, where Γ is the Raman linewidth. The two fine solid lines correspond to Stokes amplification (e^{gI_pz}) and generation ($e^{gI_pz}-1$) in the case of a coherent pump. The length of the medium is $z=100/|\mu|\Gamma$.

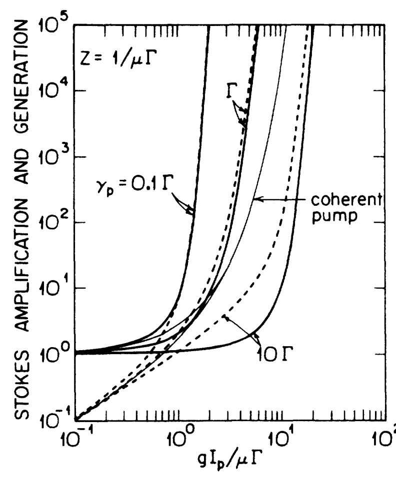

FIG. 2. Stokes amplification (solid line) and generation (dashed line) for $z=1/|\mu|\Gamma$.

In Fig. 2 the length of the medium is $z=L_{\rm coh}$ and the effect of dispersion is moderate. As the average pump intensity goes from $I_p\ll I_{cr}$ to $I_p\gg I_{cr}$, the instantaneous intensity of the Stokes field tends to become more correlated with that of the pump, following its spikes. The enhancement of the Stokes gain becomes greater. For $\gamma_p=0.1\Gamma$ the average Stokes intensity grows more steeply than in Fig. 1. Now there is enhancement even for $\gamma_p=\Gamma$. But for $\gamma_p=10\Gamma$ the effect of dispersion keeps the gain lower than in the case of a coherent pump. Compared to the case of a phase-diffusion pump (see Fig. 3 of Ref. 11) there is again a larger spread of the threshold pump intensities due to the gain enhancement for $\gamma_p\le\Gamma$. The calculations for this figure were carried out with $N_{\rm max}=450$, 40, and 5 for $\gamma_p=0.1\Gamma$, Γ, and 10Γ, respectively.

In Fig. 3 the length of the medium is $z=L_{\rm coh}/100$ and the effect of dispersion is very weak. Since $I_p\gg I_{cr}$ for all

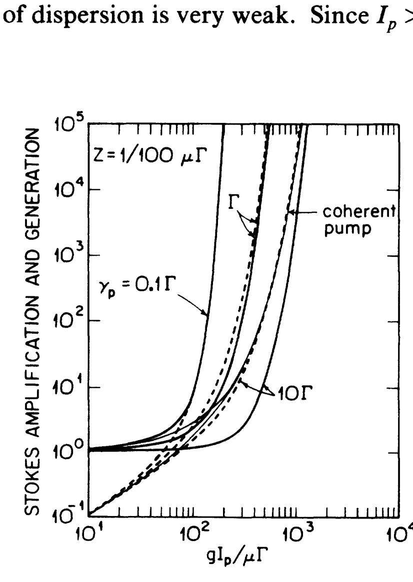

FIG. 3. Stokes amplification (solid line) and generation (dashed line) for $z=1/100|\mu|\Gamma$.

three bandwidths, we are approaching the dispersionless case. For $\gamma_p = 0.1\Gamma$ the average Stokes intensity grows with approximately the same rate as in Fig. 2, indicating that it has reached its dispersionless limit. For $\gamma_p = \Gamma$ the enhancement is somewhat greater than in the previous figure. Finally, for $\gamma_p = 10\Gamma$ the Stokes generation becomes more efficient than in the case of a coherent pump when $I_p \geq 70 I_{cr}$. The enhancement is small in this case and the dispersion has a residual effect. Figure 3 can be compared with Fig. 1 of Ref. 6 and Fig. 2 of Ref. 9 which show the average Stokes intensity for SRS of a chaotic pump in nondispersive media. In the first case there is reasonable agreement, except in the Stokes amplification of a coherent seed with $\gamma_p = 10\Gamma$, where the Markovian modeling of the Stokes field in Ref. 6 accounts only for the uncorrelated Stokes field component. In the second case the agreement is better. For $\gamma_p = 0.1\Gamma$, our figure shows that $\langle I_S \rangle = 10^5$ for $gI_p z = 2$, while the figure in Ref. 9 gives $gI_p z \simeq 2.13$, a difference of 6.5% which is due to the approximation in Eqs. (12) and (13). For $\gamma_p = 10\Gamma$, the enhancement from the statistics of the pump is less in our case because of the residual effect of dispersion. Compared to the case of a phase-diffusion pump (see Fig. 4 of Ref. 11), where under the same conditions the intensity for Stokes generation is independent of the pump bandwidth, in this case the Stokes generation is bandwidth dependent because the gain enhancement depends on the correlation time of the fluctuations in the pump intensity.

As in the case of a phase-diffusion pump,[11] our calculations for SRS of chaotic light demonstrate very clearly the asymmetry in forward and backward scattering for nonmonochromatic light. For a medium with a 2% difference in the group velocities of the pump and Stokes field we have $\mu_+ = 100|\mu_-|$. Therefore, if Fig. 2 corresponds to forward SRS, then Fig. 1 corresponds to backward SRS for the same medium length. The range of absolute pump intensity in the two figures is the same. Likewise, if Fig. 3 corresponds to forward SRS, then Fig. 2 corresponds to backward SRS. At this point a comment seems appropriate on the reduction of Raman backscatter using ISI which was mentioned in the introduction. The experimental conditions reported in Ref. 4 ($I_p \gg I_{cr}$, $\gamma_p \ll \Gamma$, and $z \gg L_{\text{coh}}$) correspond to a situation where in the plane-wave approximation Raman backscattering should be enhanced rather than reduced. The observed reduction has been attributed to phenomena in the plasma that occur on a relatively long time scale, such as density profile fluctuations or beam filamentation.[4] From the laser physics point of view, however, one thing that seems to happen as the coherence time of the laser is varied from 2 nsec to 2 psec, becoming equal to the 2-psec differential delay between the 350 overlapping beamlets, is that the temporal and transverse spatial fluctuations become unfactorable. It is known from experiments and simplified theories of phase conjugation that in the case of an unfactorable pump, the backward Stokes wave cannot become correlated to the pump and its gain is less than that in the case of a pump whose temporal and spatial fluctuations factorize.[2,3] This possible explanation of the reduction warrants further investigation, but to generalize the present theory to the case of an unfactorable chaotic pump is a nontrivial problem.

ACKNOWLEDGMENTS

The author wishes to acknowledge the support and the hospitality of the Department of Physics, especially to P. Lambropoulos, of the University of Southern California, where part of this work was carried out. He also acknowledges many useful discussions with S. N. Dixit, and thanks A. Lyras and X. Tang for teaching him new computer skills. This work was supported in part by NATO Grant No. 0375/88.

[1]J. Rifkin, M. L. Bernt, D. C. MacPherson, and J. L. Carlsten, J. Opt. Soc. Am. B **5**, 1607 (1988).

[2]For a review of SRS with broadband lasers of large beam divergence see Yu. E. D'yakov and S. Yu. Nikitin, Kvanto. Electron. **14**, 1925 (1987) [Sov. J. Quantum Electron. **17**, 1227 (1987)].

[3]For a review of SBS with broadband lasers see G. C. Valley, IEEE J. Quant. Electron. **22**, 704 (1986).

[4]S. P. Obenschain, C. J. Pawley, A. N. Mostovych, J. A. Stamper, J. H. Gardner, A. J. Schmitt, and S. E. Bodner, Phys. Rev. Lett. **62**, 768 (1989).

[5]L. A. Westling and M. G. Raymer, Phys. Rev. A **36**, 4835 (1987).

[6]A. T. Georges, Opt. Commun. **41**, 61 (1982).

[7]M. D. Duncan, R. Mahon, L. L. Tankersley, and J. Reintjes, in Proc. SPIE, **1060** (to be published).

[8]S. A. Akhmanov, Yu. E. D'yakov, and L. I. Pavlov, Zh. Eksp. Teor. Fiz. **66**, 520 (1974) [Sov. Phys.—JETP **39**, 259 (1974)].

[9]M. Trippenbach, K. Rzazewski, and M. G. Raymer, J. Opt. Soc. Am. B **1**, 671 (1984).

[10]M. Trippenbach and Cao Long Van, J. Opt. Soc. Am. B **3**, 879 (1986).

[11]A. T. Georges, Phys. Rev. A **39**, 1876 (1989).

[12]M. G. Raymer and J. Mostowski, Phys. Rev. A **24**, 1980 (1981).

[13]S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **59**, 485 (1970) [Sov. Phys.—JETP **32**, 266 (1971)].

[14]A. T. Georges, Phys. Rev. A **21**, 2034 (1980).

[15]N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981), p. 407.