Longitudinal field components for laser beams in vacuum

Lorenzo Cicchitelli and H. Hora
Department of Theoretical Physics, University of New South Wales, Kensington 2033, Australia

R. Postle
Department of Theoretical Physics and the School of Fiber Science and Technology, University of New South Wales, Kensington 2033, Australia

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The discovery of Lax, Louisell, and Knight (LLK) [Phys. Rev. **9**, 378 (1974)] that electromagnetic beams in vacuum do have a longitudinal component can be proved experimentally from the polarization independence of the energy of electrons from the focus of a laser. For this purpose we had to develop the LLK paraxial approximation to a Maxwellian exact solution for a Gaussian beam. Inserting the exact solutions into the Maxwellian stress tensor expression of the nonlinear force for the electron acceleration demonstrates a polarization dependence if only the transversal optical components are used. Including the exact longitudinal fields results in the experimentally proven polarization independence.

I. INTRODUCTION

Electromagnetic waves in vacuum are transversal as, e.g., seen from the plane-wave solution of the Maxwellian equations. For beams of finite diameter, the pure transversality is no longer valid, even in vacuum. Apart from the evaluation of the field in the focus[1] showing a very complex structure of the field, it was discovered by Lax, Louisell, and Knight[2] that there is a longitudinal electric-field component in an electromagnetic beam in a vacuum if the paraxial approximation is evaluated up to first order. Longitudinal components of the electric and magnetic fields in microwave guides are well known, but it appears absolutely not trivial that a beam of electromagnetic waves in vacuum sufficiently far away from any walls or matter does have longitudinal components. This paper is devoted to the derivation of higher approximations than that of the paraxial approximation[2] by applying the exact Maxwellian theory.

This exercise is not of simple academic interest only. It helps in solving a problem of the different descriptions of the radial emission of electrons from intense laser beams by either using methods of energy relations based on the classical Hamiltonian function and time-averaged Lorentz forces[3] versus the description with forces given by the components of the Maxwellian stress tensor. The energy description can be summarized to the well-known result [Ref. 4, Eq. (12.21)] for the explanation of the experimental result that the electrons emitted from a laser beam with sufficiently high intensity receive a maximum energy [measured values of 100 or 1000 eV (Ref. 5)] which is equal to half the maximum oscillation energy in the focus. It is immediately evident that this energy is independent of the polarization direction as measured.

The problem arises if one would like to explain the radial electron emission from the laser beam by following up the detailed time-resolved quivering process where one can see how the oscillation energy is converted into translative kinetic energy of the electrons. For this case one has to use the expression of the electromagnetic force density as known from the interaction theory with plasmas[4,5] where all components of the Maxwellian stress tensor are necessary. From the signs of the electric and magnetic vectors **E** and **H** in the diagonal elements one can immediately see that there arises a discrepancy to the mentioned result of the energy-conservation model.

Such a discrepancy cannot be ignored though the mentioned alternative energy model[3,4] results in a satisfactory theory; moreover, what this discrepancy is teaching—as we shall see—is how the small longitudinal components of electromagnetic radiation in vacuum discovered by Lax, Louisell, and Knight[2] of Maxwellian exact solutions of the laser field change the theoretical prediction from "no" to "yes" with respect to agreement with the polarization independency of the experiment.

The motivation to this exercise was given before from the result (Ref. 4, p. 231ff) that the radial quiver drift of the electrons when being emitted from the laser beam arrived at the conversion of half of the oscillation energy into the translative motion energy if the electron followed the polarization direction of the electric vector. No action resulted from the quiver drift for the 90° different polarization; however, these calculations used as usual the transversal components of the laser field only. If for simplicity a triangular radial decay of the laser fields on the radius was assumed and if then the necessary exact solution of the longitudinal components of the laser field were calculated from the Maxwellian equations (showing also the diffraction limit of the beam), the quiver drift resulted exactly in the energy conservation known from the alternative model. What this also teaches is that in nonlinear physics even such a small addition as the longitudinal component for an exact solution against the transversal wave approximation can only change a theoretical prediction essentially.

The present paper solved the extensive task[6] of using a Gaussian radial intensity profile instead of the aforementioned triangular case. It involved the fact that the former known paraxial approximation of the longitudinal components discovered by Lax, Louisell, and Knight[2] was needed here to be generalized to higher approximations. The test of the correctness of these new solutions is then shown by evaluating the radial nonlinear force on the basis of the general Maxwellian stress tensor description[4,5] in order to show that a polarization-independent electron emission appears in agreement with the measurements and in agreement with the global energy transfer model [Refs. 3 and 4, Eq. (12.21)].

We are reporting here on the derivation of the exact Maxwellian formulation with intent on applying this to the action of radial emission of electrons.[6] Though the solutions to Maxwell's equations are only to an approximation[7] beyond the paraxial approximation, the formulation is exact with respect to the number of field components necessary to describe beamed electromagnetic radiation. This implies that not only does one need the usual transverse components, but requires longitudinal electric- and magnetic-field components to appropriately describe a beam, with a radially varying profile, in a vacuum. Using the angular spectrum method[8-10] each field component, including the longitudinal electric and magnetic fields, is described by a superposition integral of plane waves.[10] Following the work of Agrawal and Pattanayak,[11] using their expansion for the exponential term in the previously mentioned integrals for the component fields, we evaluate the longitudinal electric and magnetic fields implicitly. It should be noted that more than two decades earlier a rigorous calculation of electromagnetic fields of wave beams including longitudinal components was evaluated by van Nie,[12] however the coefficients of these solutions are left in the integral form and were shown by Agrawal and Pattanayak[11] not to be in agreement with the paraxial solutions of Lax, Louisell, and Knight.[2]

The solutions of all the components (a correct formulation) of a Gaussian beam are applied to the Maxwellian stress tensor formulation of the nonlinear force. The results clearly demonstrate the polarization independence of the energy emitted of the electrons from the Gaussian beam if all components including the longitudinal fields are used in the Maxwellian stress tensor in agreement with the experiments. If the longitudinal components are neglected a distinct polarization dependence appears contrary to the experiments. Though these longitudinal components are small compared with their transversal counterparts, it is clear that they must be included for an exact formulation if one is to expect correct results at least when dealing with nonlinear phenomena. This situation shows quite clearly the sensitivity and caution one must take when dealing with the nonlinear ponderomotive force. Corroborating this theoretical result is the experimental evidence by Boreham and co-workers,[13] in which they observed the polarization independence of the energy of the emitted electrons. The energy conservation model of Kibble[3] was implicitly using the Maxwellian exact field, without one having to be aware what an important role the (small) longitudinal optical field components of the laser beam are playing.

II. EVALUATION OF FIELD COMPONENTS

Based on the angular spectrum method one assumes that the electromagnetic action or disturbance at some point in positive half space can be related to an earlier electromagnetic disturbance, through the Fourier transform.[14] Considering monochromatic radiation, the profile of this initial electromagnetic disturbance is assumed to be Gaussian with a radial decay on a circular cross section at $z=0$:

$$E_x(x,y,0)=\exp[-(x^2+y^2)/2\omega_0^2] \ . \tag{1}$$

This initial disturbance, without loss of generality, is assumed to be polarized in the x direction while the beam propagates in the z direction and is normalized relative to the centerline amplitude. The spot size parameter ω_0 is by definition where the amplitude is $1/e$ smaller than it's centerline amplitude profile.[15] The components expressed by Carter[10] may also be expressed in the integro-differential form as follows:

$$E_x(x,y,z)=\int\int_{-\infty}^{\infty} A_x(p,q)e^{ik(px+qy+mz)}dp\,dq \ , \tag{2}$$

$$E_y(x,y,z)=\frac{i}{k}\frac{\partial}{\partial x}\int\int_{-\infty}^{\infty} A_x(p,q)\frac{e^{ik(px+qy+mz)}}{m}dp\,dq \ , \tag{3}$$

$$H_x(x,y,z)=\left[\frac{1}{k\omega\mu_0}\right]\frac{\partial}{\partial y}\frac{\partial}{\partial x} \times\int\int_{-\infty}^{\infty} A_x(p,q)\frac{e^{ik(px+qy+mz)}}{m}dp\,dq \ , \tag{4}$$

$$H_y(x,y,z)=\left[\frac{k}{\omega\mu_0}\right]\frac{\partial^2}{\partial x^2} \times\int\int_{-\infty}^{\infty} A_x(p,q)\frac{e^{ik(px+qy+mz)}}{m}dp\,dq - \left[\frac{k}{\omega\mu_0}\right]\int\int_{-\infty}^{\infty} A_x(p,q)me^{ik(pz+qy+mz)} \ , \tag{5}$$

$$H_z(x,y,z)=\left[\frac{i}{\omega\mu_0}\right]\frac{\partial}{\partial y} \times\int\int_{-\infty}^{\infty} A_x(p,q)e^{ik(px+qy+mz)}dp\,dq \ , \tag{6}$$

and are time time-independent parts of the field components. The time dependence is assumed to be of the form $e^{-i\omega t}$, which is not shown explicitly here. The radiation condition,[10] a consequence of the wave equation, provides the usual definition for the direction cosines m, p, and q to be

$$m=(1-p^2-q^2)^{1/2} \quad \text{if } p^2+q^2\le 1 \ , \tag{7}$$

$$m=i(p^2+q^2-1)^{1/2} \quad \text{if } p^2+q^2>1 \ , \tag{8}$$

for the radiant (inhomogeneous) and reactive (evanescent) field, respectively.

The complex amplitude $A_x(p,q)$ is related to the boundary condition of Eq. (1) by the Fourier transform

$$A_x(p,q)=(k/2\pi)^2\int\int_{-\infty}^{\infty}E_x(x,y,0)e^{-ik(px+qy)}dx\,dy\ , \tag{9}$$

$$A_x(p,q)=(k/2\pi)^2\int\int_{-\infty}^{\infty}e^{[(x^2+y^2)/2\omega_0^2]}\times e^{-ik(px+qy)}dx\,dy\ , \tag{10}$$

$$A_x(p,q)=\begin{cases}\dfrac{1}{2\pi f^2}\exp[-(p^2+q^2)/2f^2],\quad p^2+q^2\le 1 & (11)\\[2ex] 0,\quad p^2+q^2>1\ , & (12)\end{cases}$$

where $f=1/k\omega_0$. The condition $A_x(p,q)=0$ corresponds to the reactive fields which decay exponentially with increasing z. Effectively, the condition $A_x(p,q)=0$ is not the evaluation of the integral (10) for $p^2+q^2>1$ but is an assignment in which we are stating to neglect the evanescent wave fields. This neglection of the reactive energy fields of the beam is only justified in that the wave fields decay exponentially with a decay constant comparable with the magnitude of the wave vector of the electromagnetic radiation. The implications of this is that at the order of a few wavelengths from the initial boundary condition these reactive fields are exceedingly small.

Substituting Eq. (11) into expressions (2)–(6) provides integral expressions for the field components in terms of explicitly complexed amplitude. However, as usual in Fourier optics,[16,17] where circular symmetry is evident, the transformation to the cylindrical coordinate allows the reduction of the double integral to a single integral in the cylindrical domain. The field expressions for the radiant fields are then as follows:

$$E_x(r,z)=\int_0^1\frac{\exp(-b^2/2f^2)}{f^2}e^{ikmz}J_0(krb)b\,db\ , \tag{13}$$

$$E_z(r,z)=\frac{i}{k}\frac{\partial}{\partial x}\int_0^1 f^{-2}e^{-b^2/2f^2}\frac{e^{ikmz}}{m}J_0(krb)d\,db\ , \tag{14}$$

$$H_x(r,z)=\left[\frac{1}{k\omega\mu_0}\right]\frac{\partial}{\partial y}\frac{\partial}{\partial x}\int_0^1 f^{-2}e^{-b^2/2f^2}\times\frac{e^{ikmz}}{m}J_0(krb)b\,db\ , \tag{15}$$

$$H_y(r,z)=\left[\frac{k}{\omega\mu_0}\right]\int_0^1 f^{-2}e^{-b^2/2f^2}me^{ikmz}J_0(krb)b\,db+\left[\frac{1}{k\omega\mu_0}\right]\frac{\partial^2}{\partial x^2}\int_0^1 f^{-2}e^{-b^2/2f^2}\times\frac{e^{ikmz}}{m}J_0(krb)b\,db\ , \tag{16}$$

$$H_z(r,z)=\frac{i}{\omega\mu_0}\frac{\partial}{\partial y}\int_0^1 f^{-2}e^{-b^2/2f^2}e^{ikmz}J_0(krb)b\,db\ , \tag{17}$$

where $J_0(krb)$ is the zeroth-order Bessel function, $b=(p^2+q^2)^{1/2}$, and $r=(x^2+y^2)^{1/2}$ (recalling that the reactive fields were neglected, hence $0\le b\le 1$).

The multiplication theorem of Bessel and modified Bessel functions[14] allow the reexpression of the exponential factor in the above integral equations. The exponential term in the integral equations (13) and (17) can be rewritten as

$$e^{ikmz}=e^{iv(1-b^2)^{1/2}}=\sum_{n=0}^{\infty}\frac{1}{n!}(b^2/2)^n v^{n+1}h_{n-1}^{(1)}(v)\quad |b^2|<1\ . \tag{18}$$

For the first integral in Eq. (16) we substitute the following:

$$me^{ikmz}=(1-b^2)e^{iv(1-b^2)1/2}=i\sum_{n=0}^{\infty}\frac{1}{n!}(vb^2/2)^n\times[vh_n^{(1)}(v)-2nh_{n-1}^{(1)}(v)],\quad |b^2|<1\ , \tag{19}$$

while for remaining integral expressions the following applies:

$$\frac{e^{ikmz}}{m}=\frac{e^{iv(1-b^2)^{1/2}}}{(1-b^2)^{1/2}}=i\sum_{n=0}^{\infty}\frac{1}{n!}(b^2/2)^n v^{n+1}h^{(1)}(v),\quad |b^2|<1\ , \tag{20}$$

where $v=kz$ and $h_n^{(1)}(v)$ represents the nth-order spherical Bessel function of the third kind. The substitution of expressions (18), (19), and (20) for the corresponding term in integral equations (13)–(17) yields the following expressions for the field components:

$$E_x(r,v)=\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_{n-1}^{(1)}(v)I_n(r)\ , \tag{21}$$

$$E_z(r,v)=\frac{-1}{k}\frac{\partial}{\partial x}\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_n^{(1)}(v)I_n(r)\ , \tag{22}$$

$$H_x(r,v)=\left[\frac{i}{k\omega\mu_0}\right]\frac{\partial^2}{\partial y\partial x}\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_n^{(1)}(v)I_n(r)\ , \tag{23}$$

$$H_y(r,v)=\left[\frac{i}{k\omega\mu_0}\right]\frac{\partial^2}{\partial x^2}\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_n^{(1)}(v)I_n(r)-\left[\frac{ik}{\omega\mu_0}\right]\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}\times[vh_n^{(1)}(v)-2nh_{n-1}^{(1)}(v)]I_n(r) \tag{24}$$

$$H_z(r,v)=\frac{i}{\omega\mu_0}\frac{\partial}{\partial y}\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}H_{n-1}^{(1)}(v)I_n(r)\ . \tag{25}$$

The integral $I_n(r)=\int_0^1 f^{-2}e^{-b^2/2f^2}b^{2n+1}J_0(krb)db$ can be written as

$$I_n(r)=\int_0^\infty f^{-2}e^{-b^2/2f^2}b^{2n+1}J_0(krb)db - \int_1^\infty f^{-2}e^{-b^2/2f^2}b^{2n+1}J_0(krb)db \ , \tag{26}$$

since the integrand is continuous and well behaved. The second integral term may be neglected if $f<0.4$ since the Gaussian exponential factor dominates and rapidly decreases as b becomes large. On exclusion of the second integral term in expression (26), the remaining integral term is readily solved:[18]

$$I_n(u)=2^n n! f^{2n}e^{-u}L_n(u) \ , \tag{27}$$

where $u=r^2/2\omega_0^2$ and $L_n(u)$ are the nth-order Laguerre polynomials. In view of Eq. (27) the electric-field component in the polarization direction is

$$E_x(u,v)=e^{-u}\sum_{n=0}^{\infty} f^{2n}v^{n+1}h_{n-1}^{(1)}(v)L_n(u) \ . \tag{28}$$

Expression (28) was originally derived by Agrawal and Pattanayak[7] for unpolarized electromagnetic waves. Here we have derived the remaining components which entirely constitute the exact Maxwellian formulation. The derived electric field is polarized in the x direction, hence the longitudinal electric field in vacuum becomes, from (22),

$$E_z(u,v)=\frac{x}{\omega_0}e^{-u}\sum_{n=0}^{\infty} f^{2n+1}v^{n+1}h_n^{(1)}(v)L_n^{(1)}(u) \ , \tag{29}$$

where $L_n^{(1)}(u)$ is the nth-order first generalized Laguerre polynomials, while the longitudinal magnetic field and the remaining transverse fields are

$$H_x(u,v)=\frac{i}{\omega\mu_0}\frac{xy}{\omega_0^3}e^{-u}\sum_{n=0}^{\infty} f^{2n+1}v^{n+1}h_{n-1}^{(1)}(v)L_n^{(2)}(u) \ , \tag{30}$$

$$H_y(u,v)=\left[\frac{ik}{\omega\mu_o}\right]e^{-u}\sum_{n=0}^{\infty} f^{2n}v^{n+1}\left[2nh_{n-1}^{(1)}(v)-vh_n^{(1)}(v)L_n^{(u)}+\frac{x^2}{\omega_0^2}f^2h_n^{(1)}(v)L_n^{(2)}(u)\right] \ , \tag{31}$$

$$H_x(u,v)=-\left[\frac{i}{\omega\mu_0}\right]\frac{y}{c\mu_0}e^{-u}\sum_{n=0}^{\infty} f^{2n+1}v^{n+1}h_n^{(1)}(v)L_n^{(1)}(u) \ , \tag{32}$$

respectively.

III. APPLICATION TO LASER ACCELERATION OF ELECTRONS

A question which arises at this point is the following: why is it important or necessary to have an exact formulation to describe a beam of electromagnetic radiation? Its answer is best understood by what follows. Documented experimental evidence[13] shows a polarization independence of the energy of the emitted electrons in laser irradiated tenuous plasma. However, simply assuming this is the case theoretically is not quite as trivial. If the assumption is, as in the classical case, that beamed electromagnetic radiation is purely transversal similar in many respects to plane waves and only differ in the nonuniform distribution of the intensity,[19] then—theoretically based on this assumption—the nonlinear force is polarization dependent.

Given the general expression for the ponderomotive nonlinear (NL) force density,[5] for stationary (nontransient) conditions in vacuum,

$$f_{\rm NL}=\nabla\cdot T \ , \tag{33}$$

where T is the Maxwellian stress tensor whose components in the mixed covariant and contravariant form are $T=T_k^1$:

$$T_1^1=\frac{1}{8\pi}(E_x^2-E_y^2-E_x^2+H_x^2-H_y^2-H_z^2) \ ,$$

$$T_2^2=\frac{1}{8\pi}(-E_x^2+E_y^2-E_x^2-H_x^2+H_y^2-H_z^2) \ ,$$

$$T_3^3=\frac{1}{8\pi}(-E_x^2-E_y^2+E_2^2-H_x^2-H_y^2-H_z^2) \ ,$$

$$T_3^1=T_1^3=\frac{1}{4\pi}(E_xE_z+H_xH_z) \ ,$$

$$T_3^2=T_2^3=\frac{1}{4\pi}(E_yE_z+H_yH_z) \ ,$$

$$T_2^1=T_1^2=\frac{1}{4\pi}(E_xE_y+H_xH_y) \ ,$$

it is possible to evaluate the force density in the transverse directions. It is noted that, in this particular case, the electric field component E_y is identically zero and the remaining field components are to be taken from Eqs. (29)–(32).

To numerically evaluate the time average force density according to Eq. (33), it is necessary to expand the spheri-

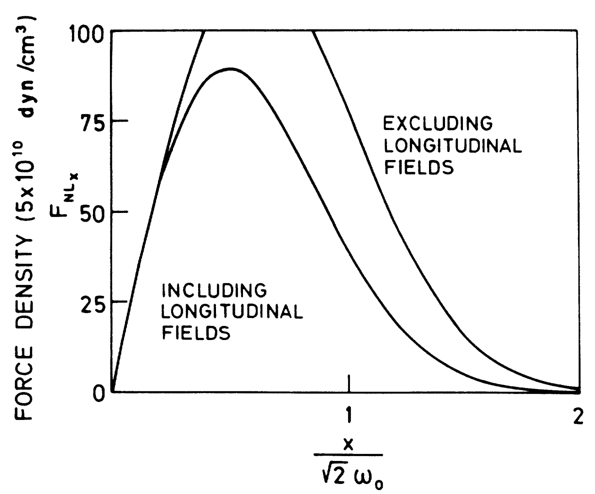

FIG. 1. Radial force density (dyn/cm^3) in a linearly polarized Gaussian neodymium glass laser beam of 13 μm diameter (half maximum intensity width) of 1×10^{15} W/cm^2 intensity in the direction of the **E** vector for the two cases including and excluding longitudinal laser field components, plotted against the distance, relative to the spot-size parameter from the center of the beam. [Note that $(0,0,z)$ is the axis of symmetry of the beam].

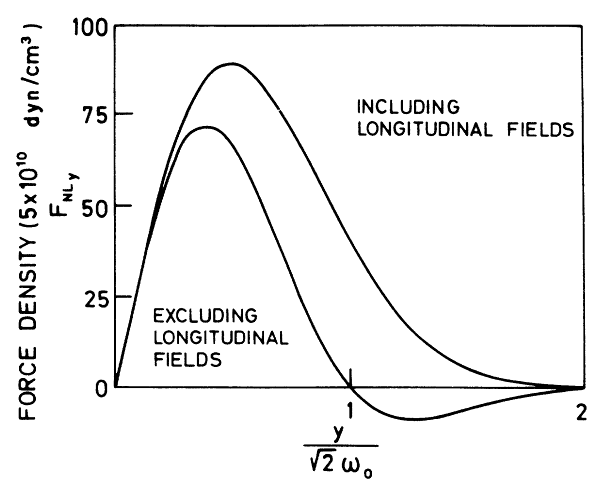

FIG. 2. Same as Fig. 1, for the direction of the **H** vector.

cal Bessel functions in the field components to a summation series[20] of the form

$$h_n^{(1)}(v)=\sum_{j=0}^{n}(n+\tfrac{1}{2};j)2^{-j+1}\exp i\left[(m-n-1)\frac{\pi}{2}+v\right], \tag{34}$$

where

$$(n+\tfrac{1}{2};j)=\frac{(n+j)!}{j!\Gamma(n-j+1)}$$

and $\Gamma(n)$ is the gamma factorial function. The field components [(28)–(31)] are now expressible in terms of a double summation from $1=0$ to n and $n=0$ to infinity. The time harmonic part of the fields are introduced and the real part representing the physical wave is taken $\mathrm{Re}(F)=\frac{1}{2}(F+F^*)$. The variable $u=r^2/[2\omega_0^2(1+iz/1)]$ is rewritten as $u=\beta e^{i\phi}$, $u^*=\beta e^{i\phi}$ where $1=k\omega_0^2$, $\beta=r^2/\{2\omega_0^2[1+(z/1)^2]\}$, and $\phi=-\tan^{-1}(z/1)$, and is independent of time, thus unaffected by time averaging. The field components, or rather the constituents of the Maxwellian stress tensor, are averaged over time for one cycle $\langle E_x^2\rangle, \langle E_xE_z\rangle$, etc., and are substituted for the appropriate component of stress tensor formulation of the nonlinear force density. A computer numeric code[6] is then used to evaluate the average force density as a function of the relative radial distance along the coordinate axis (see Figs. 1 and 2). Figure 1 is the force density at $y=0$ which implies along the direction of the electric-field polarization and Fig. 2 at $x=0$. The parametric condition were taken for a hypothetical neodymium-doped yttrium aluminum garnet (Nd:YAG) laser with a cross-sectional focal diameter of 1.3×10^{-5}m. The theoretical results clearly show the distinction of including and excluding the longitudinal fields. Figures 1 and 2 show a polarization preference in the direction of the electric field if the longitudinal laser field components are excluded or neglected, along the direction of the H vector even a confining force would result erroneously if the longitudinal laser components were neglected. Alternatively, if these small longitudinal fields are included we see a distinct symmetry or polarization independence. The electrons, upon inclusion of the longitudinal fields, are expelled with similar energies in both the transverse directions. The situation shows quite clearly the sensitivity and caution one must take when dealing with nonlinear phenomena. The exclusion of the longitudinal components clearly revert the answer of polarization independence very strongly, even though these longitudinal components are of an order of magnitude smaller than the transverse counterpart. Corroborating this theoretical result is the experimental evidence by Boreham and coworkers,[13] in which they observed the polarization independence of the energy of the emitted electrons.

It should be mentioned that the problem of radial emission of electrons from a laser beam included a very interesting test of the multiphoton ionization if the laser energies were so low that the emission of electrons were in the range of eV only. At these low laser intensities the quantum effects are evident; in particular the multiphoton process[21] in laser-induced free-free transition in electron scattering off argon atoms[22,23] exhibited a gain or loss of up to 11 discrete quanta of photon energies. In the cases of large laser intensities, the emitted electrons were of too high energies to be attributed to the multiphoton process alone. The observation of these keV electrons[24,25] could be concluded only on the basis of the classical electrodynamic ponderomotive force. In this case of the emission of keV electrons, it was possible to test the Keldysh tunnelling theory of the ionization contrary to the multiphoton ionization,[26] a question which led to a distinguishing between the quantum process of multiphoton ionization and more classical tunnelling process by means of the correspondence principle of electromagnetic interaction.[27]

The presented exact description of the laser beam is important also for filamentation and self-focusing of laser beams in plasmas[28] and applications for laser fusion.[29]

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[1]E. Wolf, Proc. R. Soc. London, Ser. A **204**, 533 (1951).

[2]M. Lax, W. H. Louisell, and W. B. Knight, Phys. Rev. A **11**, 1365 (1975).

[3]T. W. B. Kibble, Phys. Rev. **150**, 1060 (1966); Phys. Rev. Lett. **16**, 1054 (1966).

[4]H. Hora, *Physics of Laser Driven Plasmas* (Wiley, New York, 1981).

[5]H. Hora, Phys. Fluids **12**, 182 (1969).

[6]L. Cicchitelli, Ph.D. thesis, University of New South Wales, Australia 1988.

[7]G. P. Agrawal and D. N. Pattanayak, J. Opt. Soc. Am. **69**, 575 (1979).

[8]C. Goubau and F. Schwering, IRE Trans. Antennas Propag. **AP-9**, 248 (1961).

[9]J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941); G. Borgiotti, Alta Freq. **32**, 808 (1963); H. G. Booker and P. C. Clemmow, Proc. IEEE **97**, 11 (1950).

[10]W. H. Carter, J. Opt. Soc. Am. **62**, 1195 (1972).

[11]G. P. Agrawal and D. N. Pattanayak, J. Opt. Soc. Am. **69**, 575 (1979).

[12]A. G. van Nie, Phillips Res. Rep. **19**, 378 (1964).

[13]B. W. Boreham and H. Hora, Phys. Rev. Lett. **42**, 776 (1979); B. W. Boreham and B. Luther-Davies, J. Appl. Phys. **50**, 2533 (1979).

[14]J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968), pp. 33–54.

[15]R. Pratesi and L. Ronchi, J. Opt. Soc. Am. **67**, 1274 (1977).

[16]E. Hecht and A. Zajac, *Optics* (Addison-Wesley, Reading, MA, 1974).

[17]W. Magnus and F. Oberhettinger, *Formeln und Satze fur die Speziellen Funktionen der Mathematischen* (Springer-Verlag, Berlin, 1943), pp. 27–32.

[18]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965), p. 718.

[19]H. Kogelnik and T. Li, Proc. IEEE **54**, 1312 (1966).

[20]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).

[21]H. Kurger and Ch. Jung, Phys. Rev. A **17**, 1706 (1977).

[22]A. Weingartshofer, J. K. Holmes, G. Ganelle, E. M. Clarke, and H. Kruger, Phys. Rev. Lett. **39**, 269 (1977); A. Weingartshofer, J. K. Holmes, J. Sabbagh, and S. L. Chin, J. Phys. B **16**, 1805 (1983).

[23]D. Andrik and L. Langhans, J. Phys. B **9**, L459 (1976).

[24]E. A. Martin, Jr. and L. Mandel, Appl. Opt. **15**, 2378 (1976); P. K. Kruit, J. K. Kimman, H. G. Müller, and M. J. Van der Weil, Phys. Rev. A **28**, 248 (1983); G. Mainfray, Laser and Particle Beams **8**, No. 4 (1990).

[25]M. J. Hollis, Opt. Commun. **25**, 395 (1978).

[26]K. G. H. Baldwin and B. W. Boreham, J. Appl. Phys. **52**, 267 (1981).

[27]H. Hora and P. H. Handel, in *Advances in Electronics and Electron Physics,* edited by P. Hawke (Academic, New York, 1987), Vol. 69, p. 55.

[28]D. J. Nicholas and S. G. Sajjadi, J. Plasma Phys. **41**, 209 (1989); H. Hora, J. Opt. Soc. Am. **65**, 882 (1975); T. Häuser, W. Scheid and H. Hora, J. Opt. Soc. Am. B **5**, 2029 (1988).

[29]S. Nakai, Laser and Particle Beams **7**, 475 (1989); K. Mima, H. Tahabe and S. Nakai, *ibid.* **7**, 249 (1989).