

Barkas effect and effective charge in the theory of stopping power

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Experimental and theoretical values of the Barkas function L_1 are compared for p , \bar{p} , α , and Li ions. For a gold absorber, existing theories of L_1 do not provide a good approximation for the experimental data. Therefore an empirical function is suggested. For Al, Si, Cu, and Ag, parameters b and χ for the Ashley, Ritchie, and Brandt theory [Phys. Rev. B 5, 2393 (1972)] have been determined that give fairly good agreement with experiment for particle energies above 1 MeV/u. The Bloch term L_2 is used without any modifications. For Li ions, the charge-state corrections z^* used so far appear to be too large. In order to derive reliable experimental values of L_1 from p - and α -particle data, measurements of stopping power should have an uncertainty of the order of 0.1%.

I. INTRODUCTION

The stopping power $S = -dT/dx$ is used to calculate the mean energy loss of fast charged particles traversing an absorber. For particles with charge ze , kinetic energy T , and speed $v = \beta c$, traversing an absorber of thickness x much smaller than the range of the particles, the mean energy loss is $\Delta = Sx$. Bethe¹ derived the following expression for S :

$$S = \frac{k}{\beta^2} B, \tag{1}$$

with

$$k = \frac{4\pi z^2 e^4 N_0 Z}{mc^2 A} = 0.30707z^2 \frac{Z}{A} \text{ MeV cm}^2,$$

N_0 represents Avogadro's number, e and m represent the charge and rest mass of the electron, Z represents the atomic number of the absorber, A represents its atomic weight in grams, and B represents the stopping number. At low energies,² the particles will carry a reduced charge z^* . For protons and α particles with the energies used here ($T > 0.5$ MeV/u) it is assumed that $z^* = z$. Values for Li ions will be considered in Sec. VII. It is convenient to introduce the experimental stopping number ${}_{\text{expt}}B$, given by

$${}_{\text{expt}}B = \frac{\beta^2}{k} {}_{\text{expt}}S, \tag{2}$$

where ${}_{\text{expt}}S$ is an experimental value of the stopping power.

The stopping number B is assumed to consist of three terms:

$$B = B_0 + zL_1 + L_2. \tag{3}$$

B_0 is the Bethe stopping number written as

$$B_0 = \ln 2mc^2 \beta^2 \gamma^2 / I - \beta^2 - \frac{C}{Z} - \frac{\delta}{2} \tag{4}$$

with $\gamma^2 = 1/(1 - \beta^2)$, I the mean excitation energy of the

absorber, C the total shell correction, and δ the density effect.

L_1 describes the Barkas effect, discovered when it was found that the ranges of positive mesons differed from those for negative ones.³ Later it was also shown that there were related differences in stopping power for heavy ions.⁴ The need for L_1 might also have been discerned if the Bloch term L_2 had been used consistently in the analysis of the data for α particles.⁵ L_2 is the Bloch⁶ term, given by

$$L_2 = -y^2 \sum_{j=1}^{\infty} j^{-1} (j^2 + y^2)^{-1}, \quad y = z/(137\beta), \tag{5}$$

for $y=0$, the sum is equal to 1.202. No parametric modification such as is given in Ref. 4 will be used for L_2 .

An extensive discussion of stopping power was given by Mikkelsen and Sigmund,⁷ and L_1 was discussed by Andersen.⁸ Basbas⁹ discussed and compared several of the theories of L_1 , some of which are not considered here. He also argued that it may not be plausible to separate B_0 from L_1 and L_2 . In particular, he stated that the shell corrections should not be independent of particle charge. This argument will not be considered here.

The Barkas effect has been discussed in many papers with various assumptions about the general theory of stopping power. In particular, in some papers the Bloch correction was not included,¹⁰ in others, the Bloch correction was modified,⁴ experimental data from many sources were considered,⁵ or experimental data for many elements at only one particle speed were studied.¹¹

Here, probative functions L_1 were obtained which can be used for the analysis of experimental stopping-power data for ions with $z \leq 3$. Because of the problems associated with experimental data from different laboratories (see the figures), only data from single sources were considered for the determination of L_1 ; also, data must be available for a range of particle speeds. Experimental functions L_1 were obtained, and parameters for best-fit theoretical ones were determined. Theoretical functions then were compared with experimental ones.

Many studies have been made to determine the functions z^* and B appearing in Eqs. (1), (3), and (4). If it is assumed that z^* , C , L_1 , and L_2 are independent functions, it is possible to study z^* , L_1 , and L_2 from data for particles with different charges ze , without a knowledge of C and I . This was done in Ref. 4 and is done here for their data, with different assumptions, and also using the data of Ref. 12.

II. THEORETICAL FUNCTIONS L_1

In their derivation of a function for the Barkas effect, based on the harmonic oscillator model, Ashley *et al.*¹³ argued that the effect could be neglected for close collisions (in which the electrons are considered to be free). Thus they introduced a lower limit a_ω of the impact parameter and assumed that the electrons were unbound for collisions at smaller distances. Using the statistical model, they assumed that a_ω was given approximately by the radius r of the shell of charge associated with the plasma frequency $\omega_0(r)$, i.e., $a_\omega = \eta r$, where η is of order 1. They derived a function

$$L_1 = \frac{\chi F(b/x^{1/2})}{Z^{1/2} x^{3/2}} \equiv \chi l_1 \quad (6)$$

where $x = v^2/(Zv_0^2) = (137\beta)^2/Z$, and $b = \eta\chi Z^{1/6}$. The factor $\chi \approx \sqrt{2}$ was needed to account for the binding forces acting on the electrons.¹⁴ b was expected to have values between 1 and 2. The function $\chi F(w)$ was given in Fig. 3 of Ref. 13. It must be noted that L_1 depends strongly on b : a change of 25% in b causes a change of L_1 by a factor of 2. Hill and Merzbacher¹⁵ obtained the same result with a quantum-mechanical harmonic oscillator approximation.

Jackson and McCarthy¹⁶ gave a function which can be approximated to better than $\pm 3\%$ by

$$L_1 = \frac{B_0}{V^2} [g - h \ln(V + \chi^2)] / Z^{1/2} \quad (7)$$

for $1 \leq V \leq 10$ where $V = v/(Z^{1/2}v_0)$ and, for $\chi^2 = 2$, $g = 0.477$, $h = 0.1385$, while for $\chi^2 = 3$, $g = 0.607$, $h = 0.175$.

Lindhard¹⁷ showed that there is a contribution from close collisions which is about the same as that from the distant collisions in Ref. 16. His model is too schematic to permit a realistic calculation of the effect.

A complete theory of L_1 for a harmonic oscillator was given by Mikkelsen and Sigmund.⁷ They showed that the energy loss term proportional to $(ze)^3$ has contributions from all impact parameters, close as well as distant, and they compared their calculations with the "classical calculation" of Ashley *et al.*¹³ in, e.g., their Fig. 3. There are significant differences between the two theories. In the figure it can be seen though that by choosing small values of η , it is possible to obtain large values of the classical L_1 , and thus effectively include the contribution from close collisions in the Ashley theory. This is a dubious approach, but it works in the present context because the experimental data do not extend over a very large range of speeds. Mikkelsen and Sigmund tried to adapt

their theory to some atoms by using a single harmonic oscillator with frequency $\omega = I/\hbar$ to represent all the electrons in the atoms. Results are shown in their Fig. 7; I consider this approximation somewhat too schematic.

Since for a gold absorber, and for other absorbers at $T < 1.5$ MeV, the above theoretical functions did not approximate the experimental functions well, an empirical function was also employed. It is defined by

$${}_b(L_1) = C\beta^{-2\alpha}, \quad (8)$$

where C and α are parameters determined by a search for a best fit. This function should only be used for the range of energies for which measurements were made.

III. EXPERIMENTAL FUNCTIONS L_1 FROM DATA FOR PROTONS AND α

Experimental values of L_1 were determined from the relative difference Δ_α of the experimental stopping powers S_p for protons and S_α for α particles:⁴

$$\Delta_\alpha = 1 - \frac{4S_p}{S_\alpha} = 1 - \frac{\text{expt}(B_p)}{\text{expt}(B_\alpha)}. \quad (9)$$

It must be noted that an uncertainty of $\pm 1\%$ of one of the stopping powers causes an uncertainty of ± 0.01 of Δ_α (of the order of $\pm 25\%$ to $\pm 100\%$). By substituting for $\text{expt}B$ the expression given by Eq. (3), we can solve Eq. (9) to obtain

$$\begin{aligned} \text{expt}(L_1) &= \text{expt}(B_p) \left[\frac{1}{1-\Delta} - 1 \right] - {}_\alpha(L_2) + {}_p(L_2) \\ &\approx \text{expt}(B_p) \Delta (1+\Delta) - {}_\alpha(L_2) + {}_p(L_2), \end{aligned} \quad (10)$$

where Δ is the experimental value of Δ_α , $\text{expt}(B_p)$ is ob-

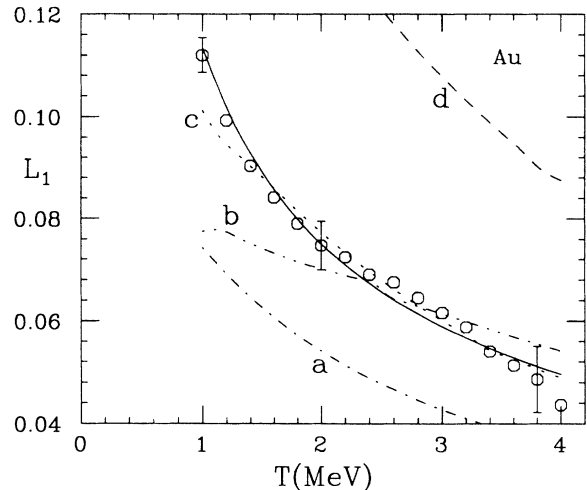


FIG. 1. Experimental and theoretical functions L_1 for the Barkas effect for gold as a function of proton energy T . Experimental values $\text{expt}(L_1)$, circles, were derived from Eq. (10) from the data given in Ref. 4. Five theoretical functions are shown. Solid line: the empirical approximation ${}_b(L_1)$ of Eq. (8) with the parameters $C = 0.002833$, $\alpha = 0.6$ (Table I). Line a: Eq. (7) for $\chi^2 = 3$; line b: Eq. (6) for $b = 1.72$, $\chi = 1.4$, line c: $b = 1.2$, $\chi = 0.69$; line d: $b = 1.26$, $\chi = 1.34$. The function with $b = 1.8$, $\chi = 1.56$ differs little from line b and is not shown.

tained with Eq. (2), and for L_2 the theoretical function given by Bloch is used. It is implicitly assumed that the shell corrections do not depend on the charge z of the particle.

The following example shows the order of magnitude of the quantities: for 1-MeV protons in gold, with the values $\text{expt}S = 61.5 \text{ MeV cm}^2/\text{g}$, $\text{expt}(B_p) = 1.0626$, $\text{expt}(\Delta_\alpha) = 0.028$, the theoretical Bloch functions $\rho(L_2) = -0.0295$, $\alpha(L_2) = -0.1109$ [note that $z^2\rho(L_2) = -0.118$ is 6% larger than $\alpha(L_2)$], we calculate

$$\begin{aligned} \text{expt}(L_1) &= 1.0626 \left[\frac{1}{1-0.028} - 1 \right] + 0.1109 \\ &= 0.0295 = 0.112. \end{aligned} \quad (10')$$

If the Bloch correction is neglected, as was done in some of the early papers, we get $\text{expt}(L_1) = 0.0306$. Clearly, the assumptions about L_2 influence the values of L_1 strongly.

The functions $\text{expt}(L_1)$ were calculated for all elements in Table I of Ref. 4. For gold, $\text{expt}(L_1)$ is given in Fig. 1.

IV. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF L_1

Theoretical L_1 were determined by searching for parameters that gave functions approaching $\text{expt}(L_1)$. For the empirical function $b(L_1)$, Eq. (8), the parameters C and α were determined by least-squares fits to $\text{expt}(L_1)$. Values are given in Table I, and the function for gold is shown as the solid line in Fig. 1. The function given by Eq. (7), with $\chi^2 = 3$, is also shown.¹⁶ It is about 30% below the experimental values and therefore will not be used further.

Least-squares fits were made for the function given by Eq. (6): b and χ were used as free parameters. Using the ratio $\text{expt}(L_1)/l_1 = \text{expt}\chi$, calculations were made with several values of b , and a best fit was obtained if $\text{expt}\chi$ was approximately constant. The values b , χ , and $\eta = b/(\chi Z^{1/6})$ for the best fits to the experimental data are given in Table I. In addition, since the value $b = 1.8$ has been suggested as a suitable choice,^{5,18,19} the values χ_1 and η_1 obtained with $b_1 = 1.8$ are also given in Table I. For this case, it is interesting to consider the quantity $a_1 = I/(Z\chi_1)$, with values of I from Ref. 20. It is constant to within $\pm 2\%$ for $Z < 50$, suggesting some intriguing relations for the statistical model.¹⁴ It must be noted

though that a_1 is 20% smaller than the value for the Lenz-Jensen model ($K_{LJ} = 7.583 \text{ eV}$, Ref. 13). Finally, since the original suggested value of χ is $\sqrt{2}$, values b_2 calculated for $\chi_2 = 1.4$ are given. Functions L_1 calculated with these parameters for gold are shown in Fig. 1. Clearly the function with $b = 1.72$ differs substantially from $\text{expt}(L_1)$. The function with $b = 1.2$, $\chi = 0.69$ agrees reasonably well with the data, but χ is well outside of the range considered acceptable. The function with $b = 1.26$, $\chi = 1.34$ is also shown;¹¹ it greatly exceeds the experimental values.

Thus it appears that for gold only the empirical function of Eq. (8) approximates $\text{expt}(L_1)$ fairly closely. Graphical comparisons for the other elements are made in the next section.

V. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF Δ_α

All experimental values of Δ_α from Table I in Ref. 4 are shown in Figs. 2–5 as circles with error bars. A second experimental function is shown as the double-dashed-dotted line. It was calculated with Eq. (9), using the stopping-power functions S_p given by Andersen and Ziegler²¹ and S_α given by Ziegler.²² It differs greatly from those of Ref. 4 for all Z . Some additional experimental values measured at a single laboratory are also shown. They do not contribute to the determination of L_1 . Values of Δ_α derived from Table II in Ref. 11 are shown in Fig. 6. No simple relation between Δ_α and Z is discernible. Therefore I do not think that current experimental data from different sources can successfully be used for the analysis of the Barkas and Bloch corrections. To improve the data of Andersen *et al.*,⁴ the uncertainty of the stopping power must be much less than $\pm 0.3\%$.

Theoretical values of Δ_α were obtained from

$$\begin{aligned} \text{theor}(\Delta_\alpha) &= 1 - \frac{\text{expt}(B_0) + L_1 + \rho(L_2)}{\text{expt}(B_0) + 2L_1 + \alpha(L_2)} \\ &= 1 - \frac{\text{expt}(B_p)}{\text{expt}(B_0) + 2L_1 + \alpha(L_2)}, \end{aligned} \quad (11)$$

where $\text{expt}(B_0) = \text{expt}(B_p) - L_1 - \rho(L_2)$ is the experimental value of the Bethe stopping number defined by Eq. (4), L_1 is given by the theoretical functions, Eqs. (6) or (8), and L_2 by Eq. (5).

TABLE I. Parameters used for the theoretical functions L_1 which best approximate the experimental functions $\text{expt}(L_1)$ derived from Ref. 4. α , C : values for Eq. (8); b , χ : values used for Eq. (6). The parameter $\eta = b/(\chi Z^{1/6})$ is also given. In addition, the parameter χ_1 was obtained by setting $b = 1.8$ for all Z , the parameter b_2 was obtained if $\chi = 1.4$ was chosen for all Z . For gold, the functions L_1 are shown in Fig. 1. For $Z < 50$, the quantity $a_1(\text{eV}) = I/(Z\chi_1)$ is constant to within $\pm 2\%$.

	1000C	α	b	Best fit			$b_1 = 1.8$		$\chi_2 = 1.4$	
				χ	η	χ_1	η_1	a_1	b_2	
Au	2.833	0.6	1.2	0.69	0.84	1.56	0.56	6.41	1.72	
Ag	6.812	0.45	1.8	1.67	0.57	1.68	0.56	5.95	1.63	
Cu	2.415	0.65	1.88	1.88	0.57	1.77	0.58	6.27	1.55	
Al	1.054	0.8	1.78	2.04	0.57	2.08	0.56	6.14	1.19	

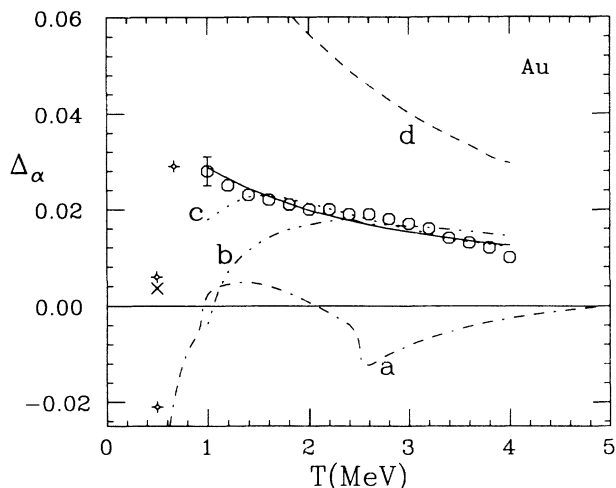


FIG. 2. The relative difference Δ_α , Eq. (9), between the stopping power of protons and α particles in gold, as a function of the energy T of a proton with the same speed as the α . The circles represent the experimental data given in Ref. 4. Line a represents experimental values of Δ_α calculated with the best-fit functions for proton stopping power S_p from Ref. 21 and for α particles, S_α from Ref. 22. The equivalent function is also shown in Figs. 3–5. Three values of Δ_α values derived from data for d , He^3 , and He^4 by Santry and Werner (Ref. 27) are shown as horizontal crosses. A value derived from Luomajärvi (Ref. 28) and Fontell and Luomajärvi (Ref. 29) is shown by the slanted cross. A value $\Delta_\alpha = 0.005 \pm 0.070$ from Alberts and Malherbe (Ref. 30) is not shown. The other lines give Δ_α calculated with Eq. (11) for different theoretical functions L_1 . For all functions, $\text{expt}(B_p)$ is the experimental value of the stopping number, calculated with Eq. (2), while $\text{expt}(B_0)$ is obtained from Eq. (3) with the appropriate L_1 and L_2 . For the solid line, L_1 defined by Eq. (8) was used ($C = 0.002833$, $\alpha = 0.6$). For the three functions (Ref. 13) defined by Eq. (6), the parameters are line b : $b = 1.72$, $\chi = 1.4$; line c : $b = 1.2$, $\chi = 0.69$; line d : $b = 1.26$, $\chi = 1.34$.

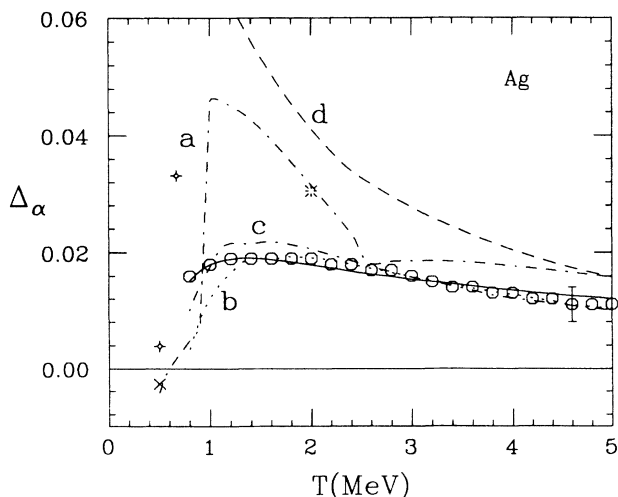


FIG. 3. Same as Fig. 2, for silver. An experimental value derived from Ref. 11 (see Fig. 6) at 2 MeV is shown by a star. Line b : $b = 1.8$, $\chi = 1.67$; line c : $b = 1.63$, $\chi = 1.4$; line d : $b = 1.26$, $\chi = 1.34$.

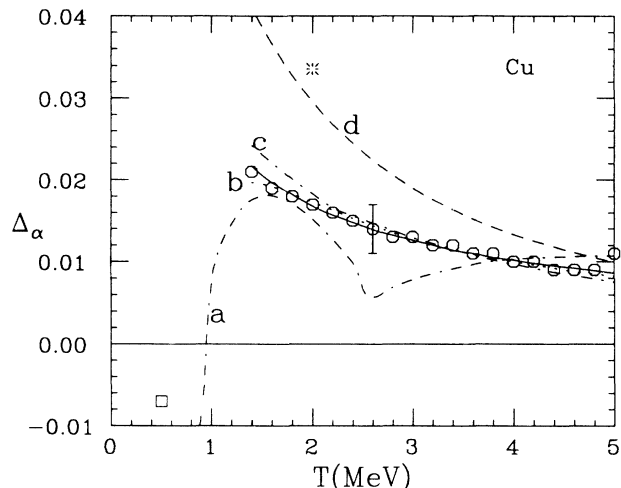


FIG. 4. Same as Figs. 2 and 3, for copper. An experimental value for nickel for 1-MeV d and 1.5-MeV He^3 obtained from Ref. 27 is shown by a square; values of -0.0215 for 1-MeV d and 2-MeV He^4 and 0.043 for 1.33-MeV d and 2-MeV He^3 are outside the scale of the figure. Line b : $b = \chi = 1.88$; line c : $b = 1.55$, $\chi = 1.4$; line d : $b = 1.26$, $\chi = 1.34$.

Values of $\text{theor}(\Delta_\alpha)$ are given in Figs. 2–5. All the functions calculated with Eq. (8) agree very well with experiment. The functions Δ_α calculated with the parameters¹¹ $b = 1.26$, $\chi = 1.34$ differ greatly from the experimental functions for Au, Ag, and Cu.

For gold, Fig. 2, none of the theoretical functions Δ_α calculated with L_1 from Eq. (6) agrees with the experimental values over the whole range of energies. The experimental values Δ_α for Ta ($Z = 73$) (Ref. 23) are not shown. They lie well above those for gold ($Z = 79$).⁴ For Al though, the old data agree with the newer ones.

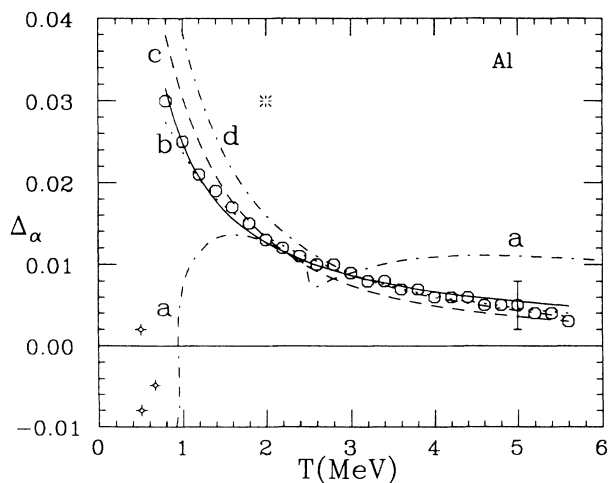


FIG. 5. Same as Figs. 2 and 3, for aluminum. Values for Si obtained from Ref. 27 are -0.012 for 1-MeV d and 1.5-MeV He^3 , and -0.024 for 1.33-MeV d and 2-MeV He^3 and are outside the scale of the figure. Line b : $b = 1.78$, $\chi = 2.04$; line c : $b = 1.26$, $\chi = 1.34$; line d : $b = 1.19$, $\chi = 1.4$.

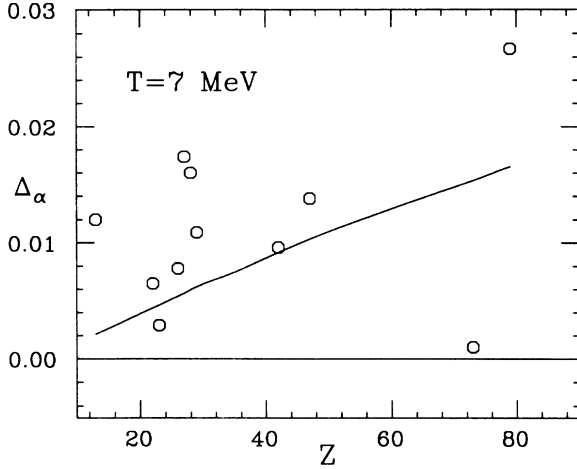


FIG. 6. Values of Δ_α calculated from the data for 7-MeV protons and 28-MeV α from Table 2 in Sakamoto *et al.* (Ref. 11) are plotted (circles) as a function of the atomic number of the absorber. The uncertainty of the numbers is at least ± 0.005 . The theoretical functions of Eq. (11) calculated with $b = 1.26$, $\gamma = 1.34$ is shown as a line.

For silver, Fig. 3, the function calculated with $b = 1.8$ agrees within experimental errors with $\text{expt}(\Delta_\alpha)$ for $T > 1.2$ MeV, but an extrapolation to energies below that is inadvisable. The function calculated with $b = 1.63$ should not be extrapolated to $T < 1$ MeV. For copper, Fig. 4, the best-fit function, $b = \chi = 1.88$, agrees well with experiment, but appears to drop below experiment at 1.4 MeV, that with $b = 1.55$, $\chi = 1.4$ tends to exceed experiment below 2 MeV. Further experimental results below 1.4 MeV are desirable. Similar trends are seen for aluminum, Fig. 5. Here, the function given in Ref. 11 agrees quite well with $\text{expt}(\Delta_\alpha)$ above 1.5 MeV.

VI. COMPARISON WITH DATA FROM ANTIPARTICLES

If the experimental stopping numbers of particles, $\text{expt}(B_p)$ and of antiparticles, $\text{expt}(B_a)$ (with charge $\pm ze$,

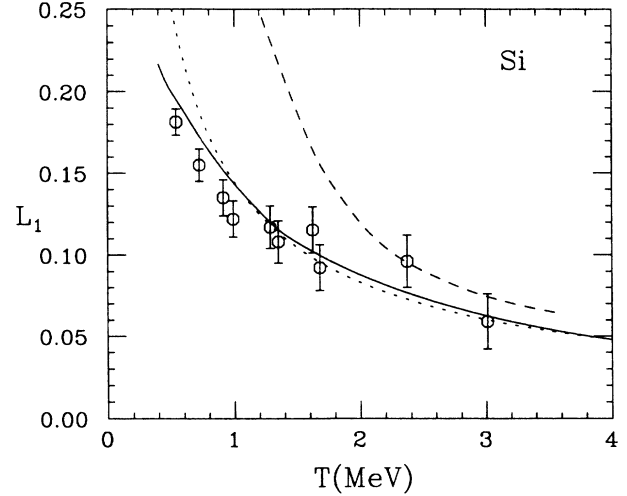


FIG. 7. Experimental values of the Barkas function L_1 for protons and antiprotons in silicon (circles with error bars: an uncertainty of much less than $\pm 1\%$ for the difference in w for p and \bar{p} was assumed) are plotted vs proton energy T . The Ashley function derived for aluminum, with $b = 1.78$, $\gamma = 2.04$ (solid line), the empirical function of Eq. (8), with $C = 0.001054$, $\alpha = 0.8$ (dotted line), and the function L_1 from Fig. 7 of Andersen *et al.* (Ref. 4) (dashed line) are also shown.

speed v) are known, we can solve Eq. (3) for $\text{expt}(L_1)$:

$$\text{expt}(L_1) = \frac{\text{expt}(B_p) - \text{expt}(B_a)}{2z}. \quad (12)$$

Measurements of the ionization in thin silicon detectors have been made for protons and antiprotons.¹² If it is assumed that the energy needed to produce an electron-hole pair w is the same for p and \bar{p} at all speeds^{24,25} the experimental values shown in Fig. 7 are obtained. The function L_1 of Eq. (6) calculated with the parameters for aluminum, $b = 1.78$, $\chi = 2.04$, from Table I is also shown and is in good agreement with $\text{expt}(L_1)$, but the empirical function, Eq. (8), begins to exceed $\text{expt}(L_1)$ below about 0.8 MeV. This shows that the use of the unmodified

TABLE II. Values of $\text{expt}(\Delta_{Li})$ from Ref. 4 and the difference $\delta = \text{theor}\Delta - \text{expt}\Delta$ between experimental and theoretical values calculated with Eqs. (8) and (11) for Li ions, as a function of ion energy per mass unit T/u (MeV). The uncertainty of $\text{expt}(\Delta_{Li})$, and thus also of δ , is ± 0.003 .

T/u	Aluminum		Copper		Silver		Gold	
	$\text{expt}(\Delta_{Li})$	δ	$\text{expt}(\Delta_{Li})$	δ	$\text{expt}(\Delta_{Li})$	δ	$\text{expt}(\Delta_{Li})$	δ
1.2	-0.002	0.031			0.010	0.008		
1.4	0.006	0.019			0.015	0.006		
1.6	0.010	0.012	0.013	0.014	0.018	0.005	0.024	0.003
1.8	0.012	0.008	0.017	0.009	0.021	0.003	0.027	0
2.0	0.013	0.005	0.016	0.008	0.023	0.001	0.029	-0.003
2.2	0.014	0.003	0.015	0.008	0.024	0	0.029	-0.004
2.4	0.014	0.001	0.014	0.008	0.025	-0.001	0.028	-0.004
2.6	0.013	0.001	0.013	0.008	0.025	-0.001	0.026	-0.002
2.8	0.013	0	0.012	0.008	0.024	0		
3.0	0.012	0.001			0.023	0.001		
3.2	0.012	0						
3.4	0.011	0						

Bloch correction in Eq. (11) is justified, provided the assumption about w made in Ref. 12 is correct. The function L_1 taken from Fig. 7 of Ref. 4 is shown as the dashed line, and lies well above $\text{expt}(L_1)$. Thus the function L_2 shown in that figure appears to be too large; see Eq. (10').

VII. DATA FOR LI IONS AND EFFECTIVE CHARGE

Heavy ions at small speeds will have electrons attached, and thus will have a reduced charge $z^* < z$. There is no complete theory for z^* for light ions. Values given in the literature² were based on the analysis of experimental measurements in which L_1 and L_2 were neglected. If we calculate Δ for Li ions, with the equivalent of Eq. (11), we should expect that the theoretical values will be too large at small speeds: if the experimental stopping power is reduced, the experimental value of Δ will also be reduced, Eq. (9). The difference $\delta = \text{theor.}\Delta - \text{expt.}\Delta$ for Li ions is shown in Table II; the uncertainty of δ is ± 0.003 .

For gold, the theoretical values $\text{theor.}\Delta$ agree with the experimental ones $\text{expt.}\Delta$ within the experimental uncertainty for all energies. For silver and aluminum, theory and experiment agree for $T/u > 2$ MeV (14-MeV Li ions), below this energy, $\text{theor.}(\Delta_{\text{Li}})$ exceeds the experimental values increasingly. Thus we see the behavior expected for a reduced charge z^* at small speeds. For copper, the theoretical values exceed the experimental ones for $T/u > 2$ MeV by about 0.008. I suspect that experimental problems caused this difference. The variations in the δ for the different elements preclude a simple statement about z^* . It must be noted that the functions for z^* used so far^{2,19,26} would give values of δ much larger than those seen in Table II.

VIII. CONCLUSIONS

Experimental data^{4,12} permit the calculation of the Barkas corrections with some confidence. Evaluated data sets such as the functions given by Andersen and Ziegler²¹ for protons and by Ziegler²² for α particles are not suitable for this purpose (see Figs. 2–5). For improvements of our knowledge of L_1 from experimental data for p and α , further measurements with an uncertainty of the order of $\pm 0.1\%$ are needed.

Within the present frame, Eqs. (3) and (4), the function L_1 given by Ashley *et al.*¹³ is in agreement with the experimental data only for $Z \leq 50$ and for energies above about 1.5 MeV. In addition it must be kept in mind that the use of relatively small values of η (Table I) is of dubious validity. For heavier elements, the empirical function given by Eq. (8) may be used for protons and α particles until a better theory is developed. These functions can be used for Li ions with energies above 2 MeV/u, except possibly for copper. From an examination of Table II, I conclude that our understanding of effective charge z^* must be completely revised. In view of the sensitivity of the determination of L_1 to the assumptions about the Bloch correction L_2 for particles with positive charge [Eq. (10')], it would be very desirable to have further accurate measurements of stopping power with particles of the same mass but opposite charge. The fact that the values of a_1 (Table I) are constant may be useful for speculations with the statistical model.

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