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Mode coupling in a Penning trap: π pulses and a classical avoided crossing

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An inhomogeneous radiofrequency electric field can couple the cyclotron and axial modes of a single ion in a Penning trap. The classical equations of motion are the same as those of a driven quantum-mechanical two-level system. We discuss an analog of the π pulse, which can exchange mode actions, and an analog of the avoided crossing. Experimental illustrations are presented.

In experiments to perform precise resonance measurements on single charged particles in a Penning trap,[1-3] only one of the particle's normal-mode motions is typically detected directly. For example, in our recent single-ion cyclotron-resonance experiment,[3,4] only the axial mode couples to our detector. The cyclotron and the magnetron modes are undetected and essentially undamped. Cooling, driving, and measuring the frequency of the undetected modes require techniques for coupling them to the detected mode.

Wineland and Dehmelt[5] suggested that an inhomogeneous rf electric field at the sum or difference frequency of two modes will couple those modes, and that, in particular, the magnetron mode can be cooled by coupling it to the damped axial mode. The technique was demonstrated experimentally by Van Dyck, Schwinberg, and Dehmelt.[6] Cohen-Tannoudji[7] and Brown and Gabrielse[8] discuss rf coupling fields in some generality and rigor. In this paper we develop two particular effects of such coupling fields, using an analogy with a two-state quantum-mechanical system to motivate our purely classical results.

The data which we present here were taken on an apparatus designed to compare the cyclotron frequencies of single ions with the eventual goal of measuring ion mass ratios to parts in 10^{12}. The apparatus, an orthogonally compensated,[9] hyperbolic Penning trap in a cryogenic environment, is described in Refs. 3 and 4.

For work with a single particle of mass m and charge e, it is convenient to write the electric and magnetic fields in an ideal Penning trap as

$$\mathbf{E}=\frac{-\omega_z^2 m}{e} z\hat{\mathbf{z}}+\frac{1}{2}\frac{\omega_z^2 m}{e}(x\hat{\mathbf{x}}+y\hat{\mathbf{y}}) ,$$

$$\mathbf{B}=\frac{c\omega_c m}{e}\hat{\mathbf{z}} ,$$

where $\omega_z^2 \equiv eV_{\text{trap}}/(md^2)$, $\omega_c \equiv eB_0/mc$, d is the characteristic trap size, and e is the charge on the ion. The motion of the ion in these fields is a linear superposition of the three normal modes,

$$\mathbf{r}=\text{Re}[a_c e^{i\omega_c' t}(\hat{\mathbf{x}}+i\hat{\mathbf{y}})+a_z e^{i\omega_z t}\hat{\mathbf{z}}+a_m e^{i\omega_m t}(\hat{\mathbf{x}}+i\hat{\mathbf{y}})] ,$$

where

$$\omega_c'=\tfrac{1}{2}[\omega_c+(\omega_c^2-2\omega_z^2)^{1/2}] ,$$

$$\omega_m=\tfrac{1}{2}[\omega_c-(\omega_c^2-2\omega_z^2)^{1/2}] ,$$

and a_c, a_z, and a_m are the complex amplitudes of the cyclotron, axial, and magnetron motions, respectively.[8] We will work in the approximation $\omega_c \gg \omega_z \gg \omega_m$. For most of this paper we study the example of cyclotron-axial coupling, although, as explained below, this approach can be adapted to magnetron-axial coupling. For cyclotron-axial coupling the perturbation frequency ω_p must be near the difference frequency, with a small detuning δ: $\delta \equiv \omega_p-\omega_c'+\omega_z$. In our experiment the fields are produced by applying voltages to segments of the guard rings. Near the center of the trap, the coupling field to lowest order is an oscillating quadrupole field tilted with respect to the static electric field:

$$\mathbf{E}_P=\text{Re}(\mathcal{E}_p e^{i\omega_p t})(x\hat{\mathbf{z}}+z\hat{\mathbf{x}})$$

where $\mathcal{E}_p$ is the complex amplitude of the coupling-field gradients.

For simplicity, we assume that the cyclotron mode may be treated as if it were a one-dimensional harmonic oscillator, with spring constant $k=(\omega_c')^2 m$. In the presence of a driving force in the $\hat{\mathbf{x}}$ direction, we ignore the $\hat{\mathbf{y}}$ motion and write the equation of motion:

$$\ddot{x}=(\omega_c')^2 x=\frac{F_x}{m} .$$

A Green-function treatment of the ion's motion in the x-y plane shows that this assumption is good for $\omega_c' \gg \omega_m$ when F_x is nearly resonant with the undriven cyclotron motion, at ω_c'.[8,10] Then the forces from the coupling field give two parametrically coupled simple harmonic oscillators:

$$\ddot{z}+\omega_z^2 z=\text{Re}\left[\frac{e\mathcal{E}_p}{m}e^{i\omega_p t}x\right] , \tag{1a}$$

$$\ddot{x}+(\omega_c')^2 x=\text{Re}\left[\frac{e\mathcal{E}_p}{m}e^{i\omega_p t}z\right] . \tag{1b}$$

We guess the following solutions:

TABLE I. Summary of mode properties and cooling limits. The action of the cyclotron or magnetron mode is just 2π times the magnitude of the canonical angular momentum [note that the magnetron canonical angular momentum is dominated by the field term, $\mathbf{r}\times e\mathbf{A}/c$ (Ref. 14)] The cooling limits given for the cyclotron and magnetron modes are reached after a single π pulse exchanges the action in the mode to be cooled with the action in the axial mode, which is assumed to have been cooled resistively to a rms radius $r_{z\text{th}}$, corresponding to a temperature T_z.

Mode	Angular momentum $\mathbf{r}\times(m\mathbf{v}+e\mathbf{A}/c)$	Action $\lvert\oint p\cdot dq\rvert$	Cooling limit for π pulses: rms radius	T_{eff}
Axial		$\pi m\omega_z a_z^2$	$a_{z\text{th}}$	T_z
Cyclotron	$-\frac{1}{2}m\omega_c' a_c^2$	$\pi m\omega_c' a_c^2$	$(\omega_z/\omega_c')^{1/2}a_{z\text{th}}$	$(\omega_c'/\omega_z)T_z$
Magnetron	$\frac{1}{2}m\omega_c' a_m^2$	$\pi m\omega_c' a_m^2$	$(\omega_z/\omega_c')^{1/2}a_{z\text{th}}$	$(\omega_m/\omega_z)T_z$

$$z=\text{Re}\left[\frac{Z(t)}{(\pi m\omega_z)^{1/2}}e^{i\omega_z t}\right],$$

$$x=\text{Re}\left[\frac{C(t)}{(\pi m\omega_c')^{1/2}}e^{i\omega_c' t}\right],$$

and define the coupling strength in units of frequency:

$$V\equiv\frac{ie\mathcal{E}_p}{2m(\omega_z\omega_c')^{1/2}}$$

Z and C are slowly varying functions of t, such that $|Z|^2$ and $|C|^2$ equal the classical action (i.e., $|\oint p_{\text{canon}}\cdot dq|$; see Table I) in each mode. Making the adiabatic approximation, and keeping only secular terms, Eq. (1) becomes

$$\dot{Z}=\frac{+V^*}{2}e^{-i\delta t}C\ , \tag{2a}$$

$$\dot{C}=\frac{-V}{2}e^{+i\delta t}Z\ . \tag{2b}$$

We recognize the standard equations for a driven two-level system.[11] Two particular properties of these equations are of experimental importance to us.

ACTION-EXCHANGING PULSES

The first property concerns the special case $\delta=0$. Imagine that the coupling drive is on between $t=0$ and $t=\tau$. Before the pulse, the initial conditions are:

$$Z(t)=Z_0,\quad t\le 0 \tag{3a}$$

$$C(t)=C_0\ , \tag{3b}$$

where C_0 (Z_0) is a complex number proportional to the initial phase and action of the cyclotron (axial) motion.

During the pulse, the solution to Eq. (2) satisfying Eq. (3) is

$$C(t)=C_0\cos\left[\frac{|V|t}{2}\right]-\frac{V}{|V|}Z_0\sin\left[\frac{|V|t}{2}\right],\quad 0<t\le\tau$$

$$Z(t)=\frac{V^*}{|V|}C_0\sin\left[\frac{|V|t}{2}\right]+Z_0\cos\left[\frac{|V|t}{2}\right].$$

If the strength and duration of the pulse is such that $|V|\tau=\pi$, then after the pulse

$$C(t)=\frac{-V}{|V|}Z_0,\quad t>\tau$$

$$Z(t)=\frac{V^*}{|V|}C_0\ .$$

Note that the action and phase information of the cyclotron motion is preserved now in the axial motion (but shifted by the phase of the perturbing field). Similarly, the π pulse has put the initial phase and action of the axial mode into the cyclotron mode. The total action, $|Z|^2+|C|^2$, is a constant of the motion. Figure 1 illustrates the effect of mode-coupling pulses of varying strengths.

This π pulse is used in a novel technique for measuring the cyclotron frequency ω_c'. We begin the measurement by driving the (initially cold) ion into a cyclotron orbit of known phase with a pulse of rf electric field directly at the cyclotron frequency. The cyclotron motion evolves

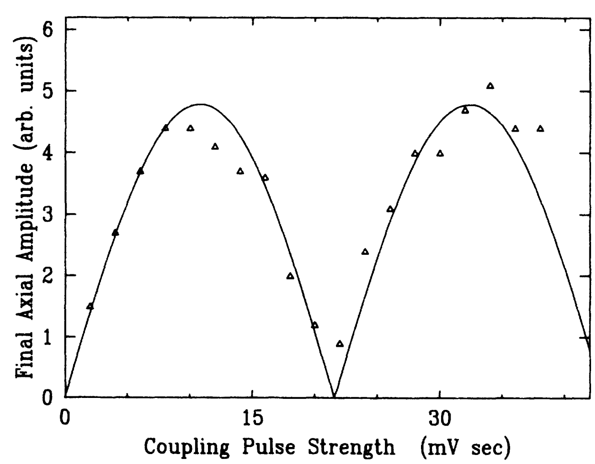

FIG. 1. For each plotted point, the following experiment is performed: An N_2^+ ion is excited into a 0.2-mm-radius cyclotron orbit, a 40-msec coupling pulse (of indicated strength) is applied, and the resulting axial amplitude measured. The solid curve, the absolute value of a sine wave, is fit to the points. The peak at pulse strength 11 mV sec corresponds to a π pulse, the zero at 22 mV sec to a 2π pulse, and so on.

in the dark, unperturbed by coupling fields, for a precise length of time T, and then, with a π pulse, the cyclotron motion is swapped into the axial mode. We then detect the current induced on the endcaps and determine the phase. The procedure is repeated with a variety of lengths of time between pulses T, to determine the cyclotron phase as a function of T. The cyclotron frequency is simply the time derivative of the cyclotron phase. Reference 3 describes a precision mass comparison made using this technique. The procedure is essentially a variant of Ramsey's method of separated oscillatory fields,[12] except that it is the final *phase,* rather than the transition probability, that is measured after the two pulses.

The π pulse may be used to cool rapidly the cyclotron mode by exchanging its action with that of the resistively cooled axial mode. The cooling limit for this scheme (Table I) is the same as the limit for cw sideband cooling,[8] but the π-pulse cooling *rate* is higher.

AVOIDED CROSSING

The second interesting property, which we call a "classical avoided crossing," is again easily understood in analogy with a near-resonantly driven two-level system. In this case, the analogy is to the dressed-atom formalism.[13] Instead of thinking of the motion of the perturbed ion as swinging back and forth between the axial and cyclotron modes, we can find time-independent linear superpositions of cyclotron and axial motions, the normal modes of an ion "dressed" by the oscillatory perturbative field.

By analogy with driven systems generally, we expect that the two components will oscillate with frequencies which differ by the driving frequency ω_p. We guess that the dressed modes consist of the ion moving in the axial direction with a frequency ω near ω_z with $\epsilon \equiv \omega - \omega_z$, and at the same time moving in the cyclotron direction with frequency $\omega + \omega_p$, so that $(\omega + \omega_p) - \omega_c' = +\delta + \epsilon$.

Then solutions to Eqs. (2) will have the form

$$Z(t) = D_z e^{i\epsilon t}, C(t) = D_c e^{i(\epsilon+\delta)t} ,$$

where (D_z, D_c) describes the eigenvector of the dressed mode. Inserting these solutions into Eq. (2), and solving the characteristic equation for ϵ, we get two solutions:

$$\epsilon = \frac{-\delta}{2} \pm \tfrac{1}{2}(\delta^2 + |V|^2)^{1/2} . \tag{4}$$

We can observe the dressed modes directly by exciting the axial motion of an ion with a short pulse and then detecting the axial component of its ring-down signal in the presence of a coupling drive. As the coupling drive approaches resonance, the observed axial frequency shifts from its unperturbed frequency. For small detunings both modes have significant axial components and it is possible to detect the axial component of both modes simultaneously (Fig. 2). By fitting the observed frequency shifts to the avoided crossing line shape [Eq. (4)], one obtains a value for the cyclotron frequency and a calibration for the strength of the coupling drive, $|V|$, a quantity which is difficult to calculate from electrode geometry *a priori.*

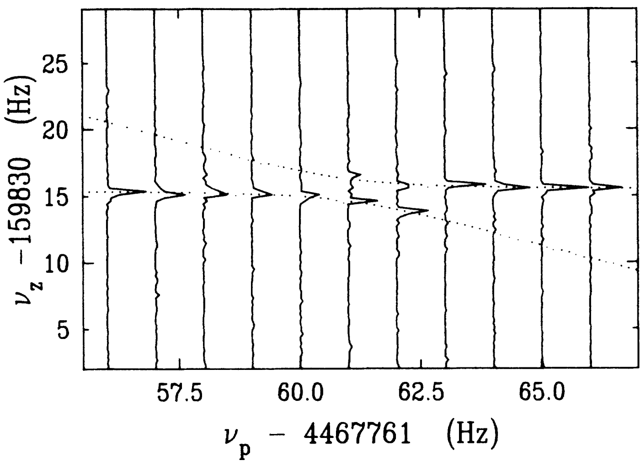

FIG. 2. Experimental illustration of the avoided crossing effect, using a single N_2^+ ion. We adjust the coupling frequency in 1-Hz increments and then excite the axial motion by pulsing. Each trace is the fast Fourier transform of the detected signal from the axial motion after the excitation. The dotted lines are a fit of the peak centers to the avoided-crossing line shape [Eq. (4)]. The fit yields $|V| = 1.5(1)$ Hz and $\nu_c' - \nu_z = 4\,467\,761.36(15)$ Hz.

MAGNETRON MOTION

Extending the preceding results to magnetron-axial couplings involves a few subtleties. To begin with, the magnetron motion, driven near resonance in the $\hat{\mathbf{x}}$ direction, does *not* act like a simple harmonic oscillation with spring constant $k = \omega_m^2 m$. But, again using a Green-function approach, we find that by rescaling the applied force

$$F_x' = -F_x \frac{\omega_m}{\omega_c'} ,$$

we can write the equation for the near-resonantly driven magnetron motion in the familiar form

$$\ddot{x} + \omega_m^2 x = \frac{F_x'}{m} .$$

In order to get coupled equations of motion in the form of Eq. (2), it is necessary for the coupling frequency to be near the *sum,* rather than the difference frequency, so we define the detuning $\eta \equiv \omega_p - \omega_z - \omega_m$.

Guessing the solutions,

$$z = \mathrm{Re} \left[\frac{Z(t)}{(\pi m \omega_z)^{1/2}} e^{i\omega_z t} \right] ,$$

$$x = \mathrm{Re} \left[\frac{M(t)}{(\pi m \omega_c')^{1/2}} e^{i\omega_m t} \right] ,$$

and defining V exactly as before, we get the equations

$$\dot{Z} = \frac{-V}{2} e^{+i\eta t} M^* ,$$

$$\dot{M}^* = \frac{+V^*}{2} e^{-i\eta t} Z .$$

The π-pulse and avoided-crossing results follow from here.

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